Linear equations in two variables

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10th Maths - Chapter 3

This is Problem-3.5 from Exercise 3.2

1. On comparing the ratios $\frac{a1}{a2}, \frac{b1}{b2}$ and $\frac{c1}{c2}$,find out whether the following pairs of linear equations are consistent,or inconsistent

$$\frac{4}{3}x + 2y = 8 \tag{1}$$

$$2x + 3y = 12 \tag{2}$$

Solution:

This can also be written as:

$$\begin{pmatrix} \frac{4}{3} & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} \tag{3}$$

$$x = \frac{\begin{vmatrix} \mathbf{b} & \mathbf{a_2} \end{vmatrix}}{\begin{vmatrix} \mathbf{a_1} & \mathbf{a_2} \end{vmatrix}} = \frac{\begin{vmatrix} 8 & 12 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} \frac{4}{3} & 2 \\ 2 & 3 \end{vmatrix}} = \frac{(8)(3) - (2)(12)}{(\frac{4}{3})(3) - (2)(2)} = \frac{24 - 24}{4 - 4} = 0 \tag{4}$$

$$y = \frac{\begin{vmatrix} \mathbf{a_1} & \mathbf{b} \end{vmatrix}}{\begin{vmatrix} \mathbf{a_1} & \mathbf{a_2} \end{vmatrix}} = \frac{\begin{vmatrix} \frac{4}{3} & 2 \\ 8 & 12 \end{vmatrix}}{\begin{vmatrix} \frac{4}{3} & 2 \\ 2 & 3 \end{vmatrix}} = \frac{(\frac{4}{3})(12) - (8)(2)}{(\frac{4}{3})(3) - (2)(2)} = \frac{16 - 16}{4 - 4} = 0 \tag{5}$$

$$\tag{6}$$

Therefore the equation is consistent as $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$