Asteroseismology

Seismic analysis applying MCMC

Data Analysis Report M2 IRT
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1 Models

The reported analysis is made over the Kepler Mission dataset of ID = 1723700. The oscillation modes of a sphere are described by the spherical harmonics. These modes are parametrized by a radial order n, a degree l and rotational splittings m for $m \in [-l, l]$ where n, l, m are integers. Acoustic waves traveling

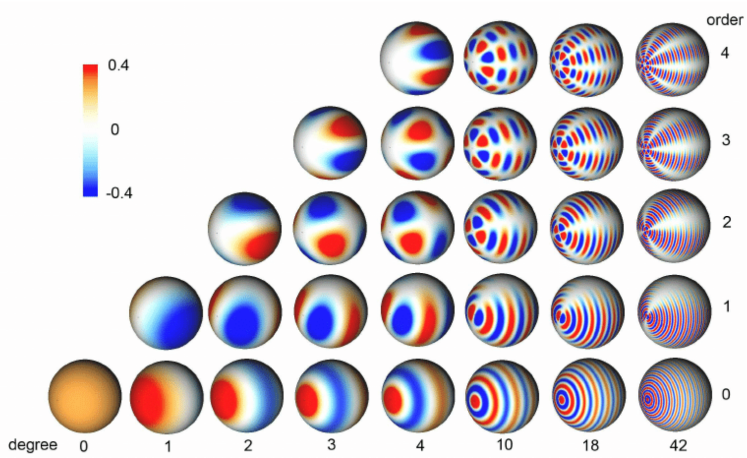

Figure 1: Representation of spherical harmonics on a sphere. In a star, the pressure field can be modeled as a combination of pressure modes corresponding to spherical harmonics. Taken From Chung et al. 2008[1]

through star can be modeled as a sphere subjected to deformations in which the pressure plays the role of the restoring force. These perturbations can be decomposed in **pressure modes** (p modes) using the model of a combination of spherical harmonics. The seismic activity of a star, in this case **Solar like oscillations**, can be estimated using the power spectrum of photometric measurements, since the star observed activity resonate at specific frequencies (modes)[2] given by the numbers n and l[1].

The quantities of Δ_ν (asymptotic large separation between peaks) and ν_{max} (frequency of maximum oscillation signal) of any observed compared to the values of our sun can give information on the mass, radius and stellar density [3]. Observing the spectrum 8 its evident a signal excess around ν_{max} and a trend corresponding to singal noise. The purpose of further sections is to isolate the signal excess, correct trending, fit a multiplet corresponding to peaks in the frequencies of oscillating pressure modes and extract the optimal parameters that can explain the observations.

1.1 Modes

In the near-asymptotic conditions , observations of oscillation frequencies of radial order n and degree l are fitted assuming low degree solar-like oscillations where $l << n$

$$\nu_{n,l} \approx \left(n + \frac{l}{2} + \epsilon + d_{0l} + \frac{\alpha}{2}(n - n_{max})^2 \right) \Delta\nu_{obs} \tag{1}$$

Every mode is splitted in rotational multiplets that depend on the azimuthal order m which is in the range $[-l, l]$ where n, l, m are integers. Each peak corresponding to a $\nu_{n,l,m}$ is modeled with a Lorenzian profile such that

$$M_{n,l,m}(\nu) = \frac{H_{n,l,m}}{1 + \frac{4}{\Gamma^2_{n,l,m}}(\nu - \nu_{n;l;m})^2} \tag{2}$$

Then, the mutiplet for n, l is modeled as a collection of lorenzian peaks assigned to every possible $\nu_{n,l,m}$ and for each one, the corresponding height $H_{n,l,m}$ is given by a Gaussian envelope centered in the Central frequency $\nu_{n,l}$:

$$H_{n,l,m}(\nu) = A \exp \frac{(\nu - \nu_{n,l})^2}{\sigma^2} \tag{3}$$

Where A is the maximum height corresponding to the central frequency $\nu_{n,l}$.

Up to this point, a multiplet can be generated knowing the following values:

[1]assuming a spherically symmetric star

- n Radial order of the oscillation. Integer.
- l Degree of radial oscillation. Integer.
- A scale corresponding to the Maximum high of the central peak $\nu_{n,0}$. This value is fitted using $\log A$ since can vary in orders of magnitude.
- Γ mode width (FWMH)
- σ Deviation of the Gaussian envelope centered in $\nu_{n,0}$
- S_o Gauge constant to move vertically the modeled curve. This value is fitted using $\log S_0$ since can vary in orders of magnitude.

In the Figure 2 some multiplets are plotted to show the inferred shape of the modes we're looking for.

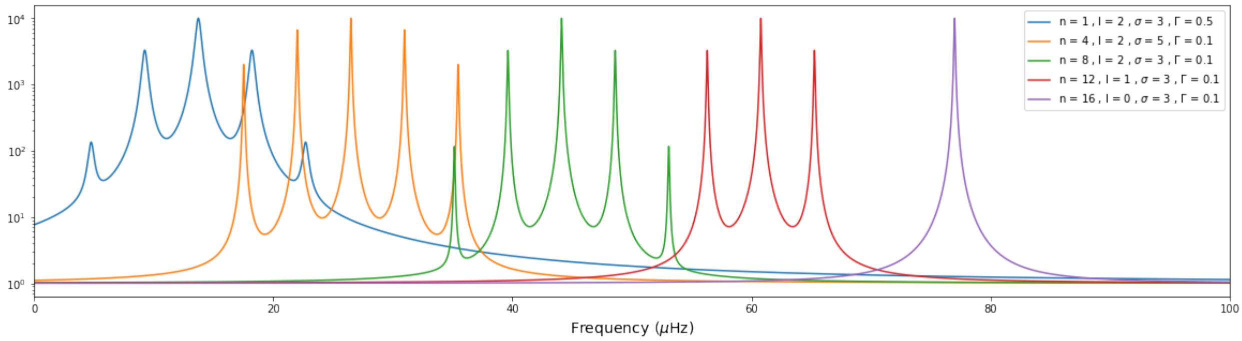

Figure 2: Synthetic multiplets (in arbitrary PSD intensity units) for different parameters. The max heights for the peaks and the constant gauge are defined with the same value for all multiplets but these scales can change as well.

1.2 Noise

the spectrum is clouded by a random noise signal. The measured signal (and spectrum) can be understood as the sum of a True signal associated with a PSD intensity $S_s(\nu)$ and a Noise signal associated with $S_n(\nu)$

$$S(\nu) = S_s(\nu) + S_n(\nu)$$

The noise is giving the spectral data a trend that can be modeled as:

$$S_n(\nu) = \frac{10^{a_n}}{\nu^{b_n}} + 10^{c_n} \tag{4}$$

For dealing with this bias, the spectrum is binned in $B = 400$ binning interval where the value of the bin is estimated as the average of the n_b frequencies included in the bin b:

$$S_b(\nu_b) = \frac{1}{n_b} \sum_{i}^{n_b} S(\nu_i) \tag{5}$$

The result of this binning is displayed in Figure 3. This binning is used to fit the noise and extract the optimal parameters a_n,b_n,c_n. The exponent of the denominator is set to 1 (Flicker Noise) to approximate easily the other vaalues. In Figure 8 the effect of different exponents can be compared.

Once the spectrum is corrected to suppress the influence of the noise in the signal,the corrected binned spectrum is used to find the optimal parameters to generate the observed multiplet.

1.3 Prior, Likelihood and hypothesis testing

The **prior** function is defined based on what we already can say about the data. For example, assuming that the values are indeed within a limited range and reflecting this in the prior function is a way of defining the boundaries of the parameters space. The value of the Prior function for a set of parameters Θ is then set to 1 for a set whose parameters are all within the desired space, 0 otherwise. This corresponds to $\log(Prior)$ values of 0 and $-\infty$ respectively.

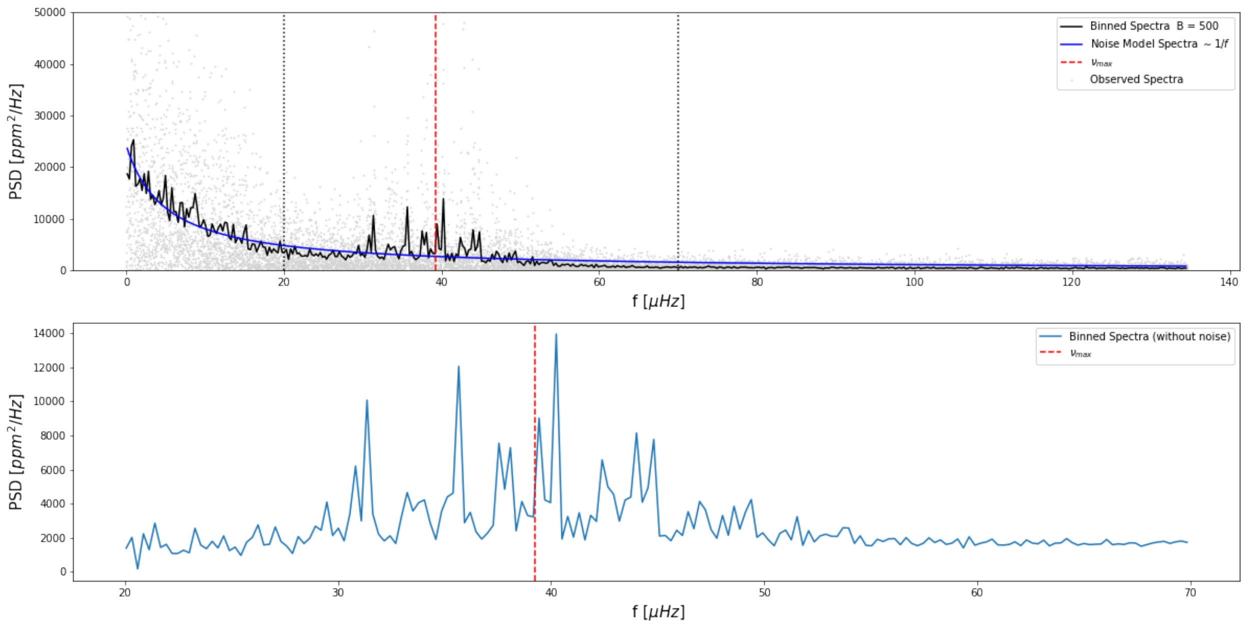

Figure 3: Binning of the provided spectral data for $B = 400$ where each bin has a width of 0.71 μHz. The Noise is also displayed where the optimal parameters are $a_n = 5.08$, $b_n = 1$, $c_n = -8.1$. The b_n is set to 1 for convenience since the exponent was not converging. In the inferior panel is the binned spectra when subtracted the trending caused by noise. The main frequency $\nu_{max} = 39.23\mu Hz$ is signaled in red.

1.3.1 Likelihood estimation

The likelihood of the parameters Θ is estimated by comparing the observed binned spectra $S_b(\nu_b)$ and a multiplet $S_m(\nu_b)$ generated with the parameters Θ. The spectra are compared using the following relation for the logarithm of the likelihood $\log L$:

$$\log L(\Theta) = -\frac{1}{B}\sum_{b}^{B}\left(\frac{S_b(\nu_b) - S_m(\Theta, \nu_b)}{\sigma}\right)^2 \tag{6}$$

Where ν_b are the binned frequencies. The binned spectra is used for fitting the MCMC model, but the obtained parameters can be used to create the estimated multiplets with any set of input frequencies. The model likelihood for the MCMC is based on the binned spectra, and not in the whole spectra, for faster convergence since the binned spectra is more *soft*.

The log(Posterior) distribution is estimated as:

$$\log(Posterior)(\Theta) = \log(Prior)(\Theta) + \log L(\Theta) \tag{7}$$

Note that for a very good fit $\log L(\Theta) \approx 0$ and, assuming its within the provided bounds, $\log(Posterior)(\Theta) \approx 0$. In the same way, when the difference between the observed spectra and the model is huge, $\log(Posterior)(\Theta)$ goes to $-\infty$.

A set of parameters Θ_a is said to be a better approximation than Θ_b if $\log(Posterior)(\Theta_a) > \log(Posterior)(\Theta_b)$

2 Fitting with MCMC

In order to approximate the optimal multiplet parameters Θ_{opt} a random walk in the parameter space is done with $N_{steps} = 10^7$ steps. The steps are not completely random, but follow a path where regions of the parameter space with high $\log(Posterior)$ are favored and more probable [4]. To avoid the random walk of remaining trapped around local maxima of $\log(Posterior)$, the walk sometimes shall choose a path that do not optimize $\log(Posterior)$. To achieve this the Metropolis Hastings method is used to explore the parameters space.

2.1 Metropolis Hastings algorithm

Once the bounds for each parameter p are chosen, an initial guess Θp_0 and a step length δ_p are chosen for each one. The implemented algorithm is the following:

The walk is started in the point of parameters space corresponding to the initial guesses. The $\log(Posterior)$ is calculated and saved.

now for each of the following $N_{steps} - 1$ steps:

- propose a new set of parameter Θ_{prop}.
- Calculate the $\log(Posterior)(\Theta_{prop})$
- if the $\log(Posterior)(\Theta_{prop})$ is higher than the log posterior value of the parameters in the previous step $\log(Posterior)(\Theta_{old})$, then take the step towards $\log(Posterior)(\Theta_{prop})$
- if $\log(Posterior)(\Theta_{prop}) < \log(Posterior)(\Theta_{old})$

  calculate the likelihood ratio R = min(1,exp($\log(Posterior)(\Theta_{prop}) - \log(Posterior)(\Theta_{old})$)) and choose a random number U between 0 and 1. if U < R then take the step towards $\log(Posterior)(\Theta_{prop})$, if not , stay in Θ_{old} (Appourchaux 2011)[5]

At the end , I took the Top 10 $\log(Posterior)(\Theta_{prop})$ and assuming there's no degeneracy in the parameter space, I averaged the top Θ obtaining the optimal parameters. The $\log(Posterior)(\Theta)$ evolution through the walk can be seen in the Figure 4

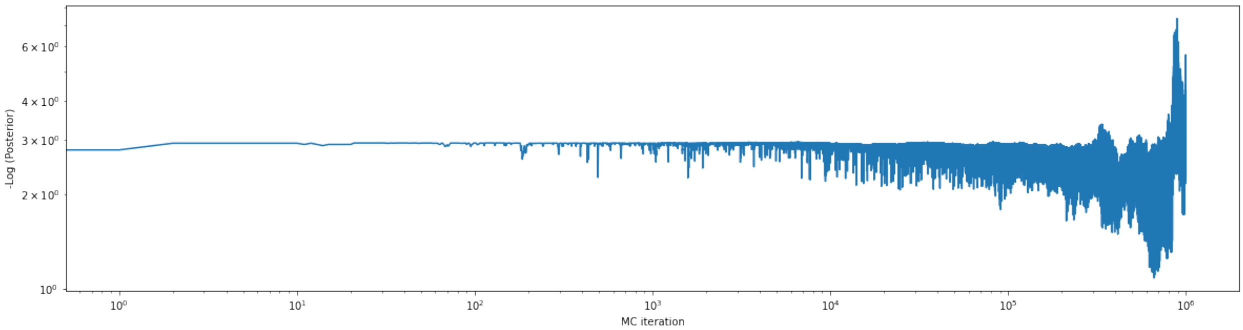

Figure 4: Evolution of the value $-\log(Posterior)(\Theta)$ over the Random walk.

2.2 Extracting relevant values

Once the random walk has finished, the optimal parameters Θ_{opt} are estimated by averaging the top 10 performing Θ sets. I expected the MCMC to converge at the end towards the optimal parameters but since this was not the case I used the best parameters found during the walk, not the latest. The optimal set of parameters is shown in table 2

The parameters found fit the binned and corrected spectra as shown in figure 5. The model is then added to the noise model to be compared with the observed spectra. The obtained ν_{max} value is close to the reported value and the curve fits relatively well the binned spectra.

Values for the mass and radius of the star are proposed based on the new estimated ν_{max}. The value of $\Delta\ \nu$ was not fitted since I could not manage to make it converge easily, so I used the provided value all along.

Parameter	code name	guess	lower bound	upper bound	optimal	deviation
n	n	3	1	8	7	0
l	l	1	0	3	2	0
$logA$	log_A	3.5	1	9	3.89	0.003
$logS_o$	log_S0	2	0.01	8	2.51	0.01
σ	sigma	6	0.01	10	6.05	7.67
Γ	Gamma	1	0.01	5	1.47	0.01

Table 1: Optimal set of parameters found with Metropolis Hastings. Since n and l were defined as integer and the polled parameter set had the same values, the deviation of these parameters is 0. With this parameters the value of the central frequency is estimated to be $\nu_{max} = 39.648 \mu Hz$.

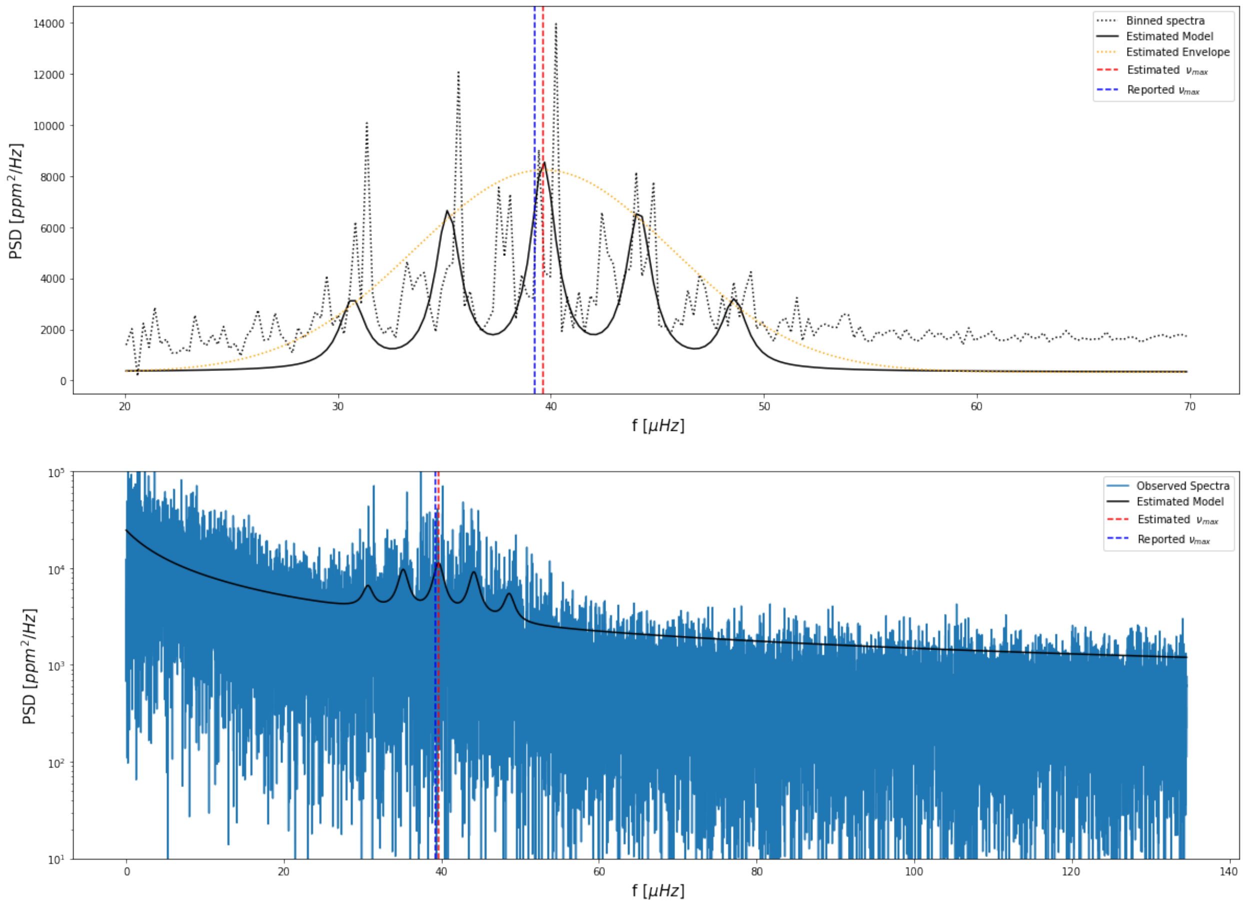

Figure 5: Overlapping the binned corrected spectra, the estimated Gaussian envelope and the estimated Multiplet modes (Top panel). Once obtained the multiplet model it is added with the model of noise spectra in order to be compared with the observed spectra. The observed mutiplet corresponds to $n = 7$ and $l = 2$. In both panels the estimated (from multiplet model) and provided (file) ν_{max} values are displayed, with values of 39.648 μHz and 39.23 μHz respectively.

Stellar parameter	provided	estimated
ν_{max}	39.23	39.65
$\Delta\nu$	4.483	-
R	10.48	10.59
M	1.27	1.31

Table 2: The mass and stellar radius were calculated with the values of $\delta\nu =$ and ν_{max} provided by the data file. New values were calculated with the estimated value for ν_{max}

References

[1] M. Chung, K. Dalton, and R. Davidson, "Tensor-based cortical surface morphometry via weighted spherical harmonic representation," *IEEE transactions on medical imaging*, vol. 27, pp. 1143–51, 09 2008.

[2] B. Mosser, "Stellar oscillations - i - the adiabatic case," *EAS Publications Series*, vol. 73-74, p. 3–110, 2015.

[3] D. Stello, W. J. Chaplin, S. Basu, Y. Elsworth, and T. R. Bedding, "The relation between ν and $\nu_{max}$for solar-like oscillations," *Monthly Notices of the Royal Astronomical Society: Letters*, vol. 400, p. L80–L84, Nov 2009.

[4] R. Handberg and T. L. Campante, "Bayesian peak-bagging of solar-like oscillators using mcmc: a comprehensive guide," *Astronomy Astrophysics*, vol. 527, p. A56, Jan 2011.

[5] T. Appourchaux, "A crash course on data analysis in asteroseismology," 2011.

Appendices

A Signal

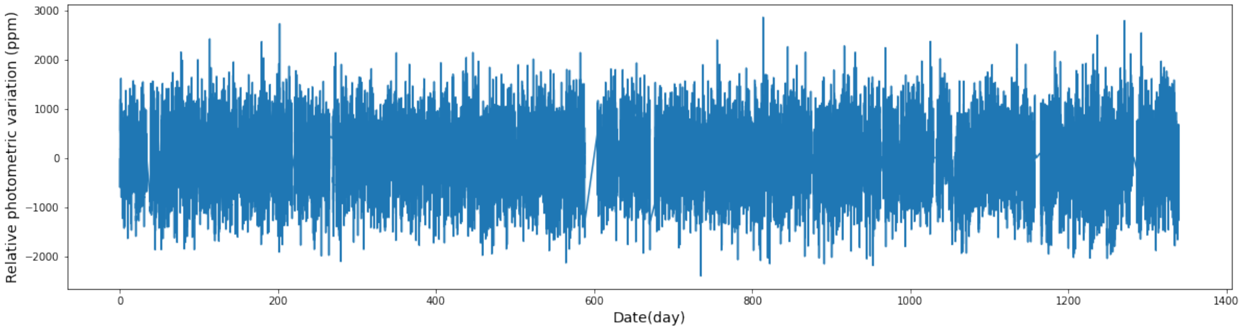

Figure 6: Measured signal with a sampling time of 1857.602 seconds.

B Noise Spectra Approximation

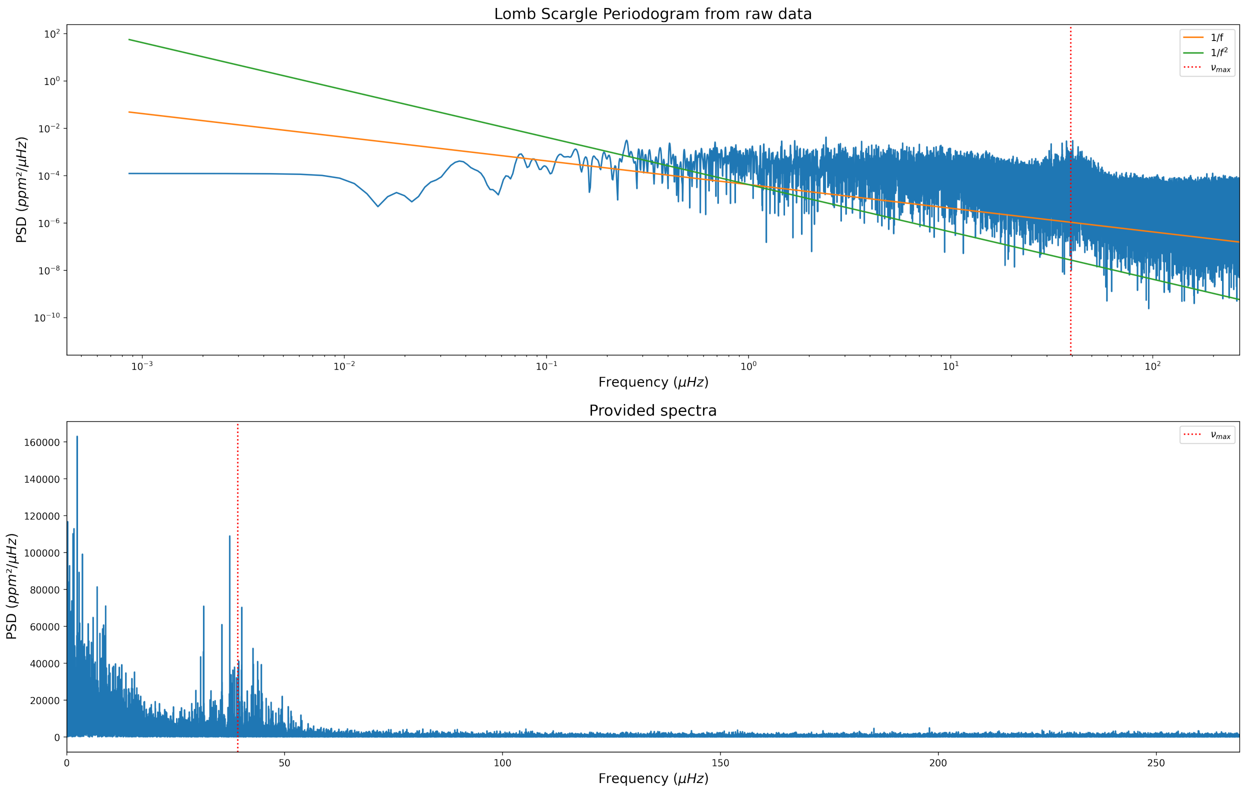

Figure 7: Lomb Scargle periodogram and provided spectra for the reference star. Trending lines proportional to $1/f$ and $1/f^2$ are drawn for comparison

C MCMC random walk

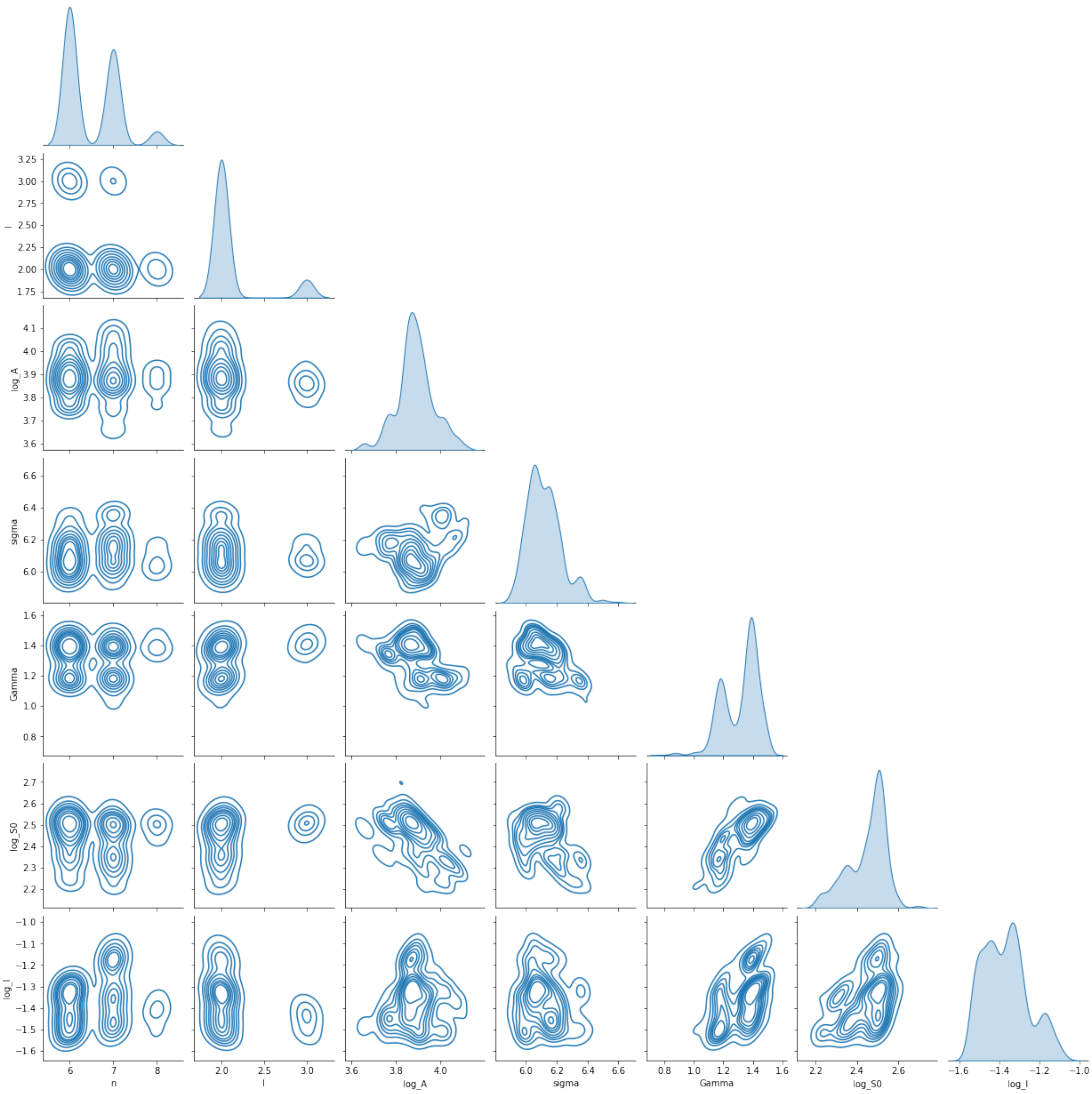

Figure 8: KDE profiles for the 700 iterations with the best log Posterior ($logL$) value. Remember that the found optimal parameters are $n = 7$, $l = 2$, $\log A = 3.89$, $\sigma = 6.05$, $\Gamma = 1.47$, $logS_o = 2.5$. For all the crossed plots, the iterations that reached higher log posterior values in each subspace were for parameter values around the proposed optimal parameters.