ANALYSIS OF EXPLICIT FINITE DIFFERENCE METHODS ON A SURGE TANK TO OBTAIN WATER LEVEL OSCILLATIONS

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ABSTRACT

This method aims at conducting four different methods, viz, Explicit Euler method, Runge-Kutta fourth, third and second order methods on surge tank governing equations with were a set of non-linear Ordinary Differential Equations (ODE). Summaries of two case studies are presented where the first case study compares the four methods while simulation the experimental results. The second case studies the change in oscillating water depth by varying head loss coefficient and surge tank area, accordingly a suggestion for design consideration is made. The simulations are done using PYTHON in GOOGLE COLAB.

These two case studies are an extension of Chapter 3 of the thesis “Modeling of Hydraulic Transients in Closed Conduits” by Ali EL-Turki (2013) and an article on “Surge Tank Design Considerations for Controlling Water Hammer Effects at Hydro-electric Power Plants” by Dr. Karan Abdulghani Ramadan and Dr. Hatem Mustafa, 2013.

According to this study RK4 and RK3 become suitable for Case 1. Also, cross-sectional area of 400 m^2 of surge tank can be considered or parallel installation of surge tank of 200-300 m^2 can also be considered.

Chapter 1

Introduction

There are several methods to solve ordinary and partial differential equations, some of them are characteristic method, finite element method and finite difference method. Explicit and implicit methods are used in numerical analysis to obtain numerical approximations to the solutions of a time-dependent ordinary and partial differential equations, as is required in computer simulations of physical processes [5].

For time-dependent (i.e., time marching or initial value problems) partial differential equations, the finite-difference techniques fall into broad categories of explicit and implicit schemes [3]. Explicit methods calculate the state of a system at a later time from the state of the system at the current time, while implicit methods find a solution, solving an equation involving both the current and later state of the system [6]. On comparison explicit schemes are easier to program and solve, but it displays numerical stability issue, thereby, focusses on the use of smaller time step. Implicit scheme allows the use of large time step and ensures numerical stability, but it integrates higher computation cost for any simulation.

When choosing a numerical method for solving a surge tank problem, there are several factors to consider, including the accuracy and stability required, the complexity of the model, and the computational resources available. The surge tank problem involves modelling the behaviour of a water tank that is used to absorb pressure variations in a water distribution system. The governing equations for this problem are typically a set of nonlinear ordinary differential equations [1].

The continuity and momentum equations for the surge tank, expressed in the form of ordinary differential equations, will be replaced with finite difference approximations where the unknown quantities at the end of the time step are expressed as a function of known variables at the beginning of the time step [3]. Therefore, four explicit methods: Euler method, Second-order Runge-Kutta method, third-order Runge-Kutta method and fourth-order Runge-Kutta method, will be used to solve the two equations.

Chapter 2

NUMERICAL METHOD

3.1 Euler Method:

The Explicit Euler method is a numerical technique which solves ordinary differential equations (ODEs) numerically. It is a first-order method which approximates the solution at the next time step using information from the current time step.

The method begins by discretizing the continuous time domain into a sequence of discrete time points. Let t_n be the current time and t_{n+1} be the next time point. The Explicit Euler method then computes an approximate solution y_{n+1} at the next time point using the formula shown below:

$y_{n+1} = y_n + h \times f(t_n, y_n)$

where h is the step size and $f(t_n, y_n)$ is the derivative of the function y(t) at time t_n and y_n. This formula is derived by approximating the derivative using a forward difference approximation:

$f(t_n, y_n) = (y_{n+1} - y_n)/h$

and solving for y_{n+1}.

Euler method is a simple method, but it can give unstable results for larger time steps. This is because the method is of first-order, which implies that its accuracy increases linearly with the size of the time step. Therefore, as the time step increases, the errors in the approximation also increase, leading to less accurate results. Furthermore, this method assumes that the rate of change of the system is constant over each time step. But this may not be the case for all system, and the rate of change may vary significantly over larger time steps, leading to inaccurate results. Additionally, some ODEs may have unstable or oscillatory solutions, which means that small changes in the initial conditions or the numerical method can lead to large changes in the solution. The Euler method is particularly susceptible to these instabilities, which can result in wildly divergent or oscillatory solutions.

Overall, the Euler method is a widely used numerical method, but its accuracy can be limited for larger time steps and certain types of ODEs. Other numerical methods, such as the Runge-Kutta method or the Adams-Bashforth method, may be more accurate and stable for these cases.

3.2 Second order Runge-Kutta Method:

It is a higher-order method than the Euler method. The principle behind the Runge-Kutta second order (RK2) method is to use two stages to estimate the solution of an ordinary differential equation (ODE) at the next time step.

At the first stage, the method calculates the slope of the solution curve at the current time step, using the derivative of the ODE at that point. Then, it uses this slope to estimate the value of the solution at a point halfway between the current time step and the next time step.

At the second stage, the method calculates the slope of the solution curve at this intermediate point, again using the derivative of the ODE at that point. Then, it uses this slope to estimate the value of the solution at the next time step.

The final estimate of the solution at the next time step is obtained by taking a weighted average of the estimates from the two stages, with higher weight given to the estimate obtained at the second stage.

The RK2 method is based on the principle of Taylor expansion, which approximates the solution of an ODE using a polynomial of increasing degree. The RK2 method uses a second-order Taylor polynomial to estimate the solution at the next time step, which provides a better approximation than the first-order Taylor polynomial used in the Euler method.

The general formula for the RK2 method is:

$k_1 = f(t_n, y_n)$

$k_2 = f(t_{n+h}, y_n + h \times k_1)$

$y_{n+1} = y_n + (h/2) \times (k_1 + k_2)$

Where t_n is the current time, y_n is the solution at t_n, h is the time step size, k_1 and k_2 are intermediate slopes, and f(t, y) is the ODE being solved.

The RK2 method is less accurate than higher-order methods like RK3 or RK4, but it is simpler to implement and can still provide reasonably accurate solutions for many ODEs. It is often used when a higher-order method is not necessary or when computational efficiency is important.

3.3 Third order Runge-Kutta Method:

This method is also noted as RK3. Instead of two stages in case of RK2 method, RK3 method uses three stages to calculate the solution at the next time step. Similar to RK2 at each stage, the method uses a weighted average of the slopes at various points within the time step to estimate the solution at that stage. The final estimate is then obtained by taking a weighted average of the estimates from the three stages.

The general formula for the RK3 method is:

$k_1 = f(t_n, y_n)$

$k_2 = f(t_n + h/2, y_n + (h/2)\times k_1)$

$k_3 = f(t_{n+h}, y_n - h\times k_1 + 2\times h\times k_2)$

$y_{n+1} = y_n + (h/6)\times(k_1 + 4k_2 + k_3)$

Where t_n is the current time, y_n is the solution at t_n, h is the time step size, k_1, k_2, and k_3 are intermediate slopes, and f(t, y) is the ODE being solved.

The RK3 method is known for its simplicity and stability, making it a popular choice for many scientific and engineering applications.

3.4 Fourth order Runge-Kutta Method:

The Runge-Kutta 4th order method, commonly known as RK4, is a numerical method used to solve ordinary differential equations (ODEs). It is one of the most widely used numerical methods due to its high accuracy and stability. The principle behind the RK4 method is to approximate the solution of an ODE by taking four weighted steps. At each step, the method calculates the slope of the solution at the current point, and uses that information to determine the value of the solution at the next point.

The general formula for RK4 method is as follows:

$k_1 = hf(t_n, y_n)$

$k_2 = hf(t_n + h/2, y_n + k_1/2)$

$k_3 = hf(t_n + h/2, y_n + k_2/2)$

$k_4 = hf(t_n + h, y_n + k3)$

$y_{n+1} = y_n + (1/6)(k_1 + 2k_2 + 2k_3 + k_4)$

Here, h is the step size, t_n is the current time, y_n is the current value of the solution, f is the function defining the ODE, and k_1, k_2, k_3, and k_4 are intermediate values calculated at each step.

The RK4 method combines the slopes obtained from these intermediate values to calculate the final slope at each step, which is used to determine the value of the solution at the next point. The weighted average of these four slopes is used to estimate the solution at the next time step, with the weights chosen to minimize the error in the approximation.

By iteratively applying this method, we can obtain an approximation of the solution of an ODE over a given interval with high accuracy.

Chapter 3

Surge Tank Schematic and Governing Equation

3.1 Schematic of the Surge Tank Set Up:

Under normal operating conditions, the water flows from the reservoir to the pipe and is discharged through the control valve into a collection tank. The collection tank was used to measure the flow rate under steady state conditions. A schematic representation of the experimental set up is shown in Fig. 3.1.

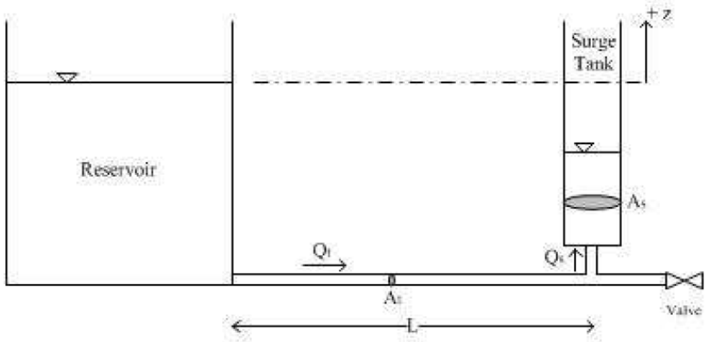


Fig. 3.1 Schematic of surge tank experiment (adopted from "Modelling of Hydraulic Transients in Closed Conduits", 2013)

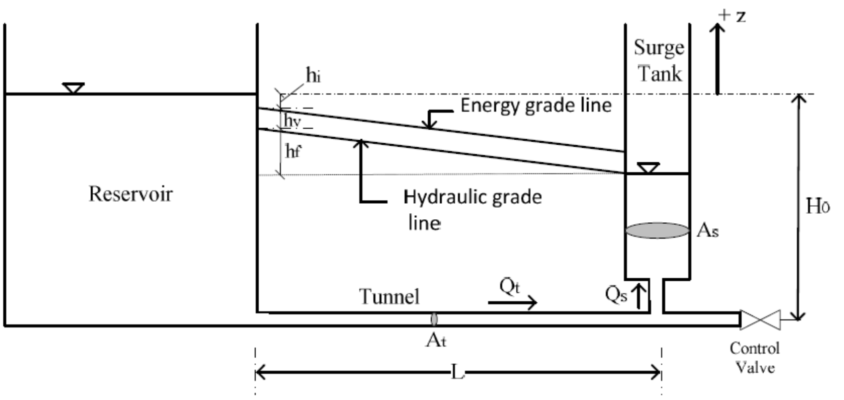
3.2 Governing Equations for Surge Tank:

The assumptions followed to derive the continuity and momentum equations which in turn describes the oscillating water depth in surge tank are [2]:

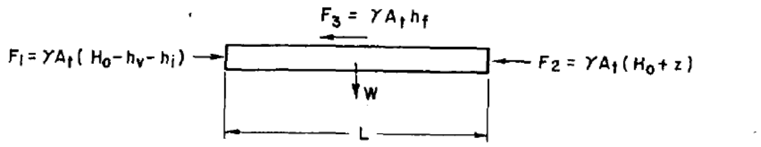
1. The liquid is incompressible.
2. The conduit walls are rigid.
3. Inertia of fluid in surge tank is small in comparison to fluid in tunnel and hence it is neglected.
4. Head losses in the system during the transient state can be computed using steady state formulae for corresponding flow velocities.
5. The conduit is horizontal with same cross-sectional area.

6. Flow variables do not vary with distance and are functions of time only. Therefore, there are no spatial derivatives and the resulting equations are ordinary differential equation instead of partial differential equations

A schematic diagram of surge tank for deriving the governing equations along with its free body diagram are shown below.



(a)



(b)

Fig. 3.2 (a)Schematic of a simple surge tank system, (b) Free body diagram (adopted from M Hanif Chaudry, Applied Hydraulic Transients, 1987)

3.2.1 Momentum Equation:

Considering the forces shown in Fig. 3.2(b), the forces acting on the fluid are:

$F_1 = \gamma A_t(H_0 - h_i - h_v)$ (1)

$F_2 = \gamma A_t(H_0 + z + h_{orf})$ (2)

$F_3 = \gamma A_t h_f$ (3)

Where, A_t = cross-sectional area of the conduit

H_0 = the static head

γ = specific weight of liquid;

h = velocity head;

h_i = intake head losses;

h_f = frictional head and form losses in the conduit between the reservoir and the surge tank;

z = water level in the surge tank measured above the reservoir level (considered positive upward).

Considering the downstream flow direction as positive, the resultant forces acting on the fluid element are

$$\Sigma F = F_1 - F_2 - F_3 \quad (4)$$

Substituting equation (1), (2) and (3) on (4), we get

$$\Sigma F = \gamma A_t(- h_v - h_i - z - h_{orf} - h_f) \quad (5)$$

Let length of the conduit be L and acceleration due to gravity be g, then mass of fluid element in the conduit can be expressed as $\gamma A_t L/g$. Therefore, the rate of change of momentum will be

$$\frac{dM}{dt} = \frac{\gamma AtL}{g}\frac{d}{dt}\frac{Qt}{At} \quad (6)$$

$$\frac{dM}{dt} = \frac{\gamma L}{g}\frac{d}{dt}Qt \quad (7)$$

Where M is momentum, Q_t is the flow rate in the conduit and t is the time.

Equating equation (7) to (5) following Newton's second law of motion, we get

$$\frac{\gamma L}{g}\frac{d}{dt}Qt = \gamma A_t(- h_v - h_i - z - h_{orf} - h_f) \quad (8)$$

$$\text{Let } h = h_v + h_i + h_{orf} + h_f \quad (9)$$

h is total head loss. This total head loss can be expressed as a function of discharge Q_t. Therefore,

$$h = c\, Q_t\, |Q_t| \quad (10)$$

c is a coefficient.

Substitution equation (10) to equation (8)

$$\frac{d}{dt}Qt = \frac{gAt}{L}(-z - c\, Q_t\, |Q_t|) \quad (11)$$

Here the main losses are the loss due to pipe friction and due to sudden enlargement at surge tank from pipe. Frist taking into account the Darcy-Weisbach equation for losses due to friction in pipe,

$$h_f = f\,\frac{LV^2}{2gD} = f\,\frac{LQt^2}{2gAt^2} = c_f Q_t^2 \tag{12}$$

$$c_f = f\,\frac{L}{2gAt^2} \tag{13}$$

Where f is friction factor and D is diameter of closed conduit.

Now taking into account the losses due to enlargement

$$h_i = k_l\,\frac{V^2}{2g} = k_l\,\frac{Qt^2}{2gAt^2} = c_s Q_t^2 \tag{14}$$

k_l is empirically determined loss coefficient and

$$c_s = k_l\,\frac{1}{2gAt^2} \tag{15}$$

Therefore, equation (11) becomes

$$\frac{d}{dt}Qt = \frac{gAt}{L}\{-z-(c_s+c_f)\,Q_t\,|Q_t|\} \tag{16}$$

This is the momentum equation.

3.2.2 Continuity Equation:

Considering the junction of the conduit and surge tank from Fig 3.2 (a), the flow rates can be expressed as

$$Q_t = Q_s + Q_v \tag{17}$$

Where Q_s is inflow of surge tank which is considered as positive; Q_v is the flow through the valve. Let A_s be the cross-sectional area of the surge.

equation (17) is valid for cases where the valve can be replaced by a turbine or pump and the flow through the turbine or the pump is designated as Q_v.

As $Q_s = A_s(dz/dt)$, equation (17) can be written as

$$\frac{dz}{dt} = \frac{1}{As}(Q_t - Q_v) \tag{18}$$

This is the continuity equation.

Both equations (16) and (18) are ordinary differential equations (ODE) corelating the water level oscillations of the surge tank system with the governing equations.

These equations exhibit nonlinearity (noting that the discharge through the turbine Q_v could also be nonlinear) it does not readily permit closed form solutions. Thus, numerical methods are often used to solve these equations [3].

Chapter 4

Case Study 1

With the derivation of the governing equation, the objective of the first case study is to verify the choices of the author of the thesis named “Modeling of Hydraulic Transients in Closed Conduits” regarding the explicit numerical methods (Euler method and RK4) for the experimental surge tank set-up. The author examined an experimental data performed by Dr. Venayagamoorthy in South Africa and compared the data with the results of numerical simulations conducted. This paper will extend the comparison study with RK2 and RK3 methods and will see if same level of stability and accuracy can be obtained will lesser computation.

Adopting the same surge tank set up from “Modeling of Hydraulic Transients in Closed Conduits” as taken by the author, i.e., an upstream reservoir providing energy head and enabling pipe flow through a 45 mm supply pipe. A cylindrical surge tank of 122 mm diameter is connected to the reservoir through the supply pipe of 10.4 m.

Under this case study we are considering an experimental data performed in a laboratory. The experiment was performed with the following steps:

i. At the beginning the water level of the surge tank was recorded by closing all the valves except for the surge tank isolator valve. This water level is later designated are reference water level or initial water level of the surge tank.
ii. Allowing a steady flow rate of Q_o by adjusting the valves.
iii. Then all the valves are kept in open position and its corresponding initial water level in the surge tank was recorded.
iv. Then, discharge (outlet) valve downstream of the surge tank was rapidly closed and the time history at which the water level in the surge tank crossed the still water level and reached the extreme (maximum and minimum) levels, together with water surface elevations, were recorded for three full cycles.
v. Same steps were followed for different Q_o.

Considering the observation table presented on “Modeling of Hydraulic Transients in Closed Conduits” for 0.001723m^3/s of steady flow rate is shown in Table no. 1.

Table No. 1

First set Time (sec)	Level (mm)
3	4923
8	5075
12	4923
16	4832
20	4923
24	4989
29	4923
34	4877
39	4923
44	4961
49	4923
52	4897
57	4923

Graphical presentation of the water level oscillation for the experimental data is shown in Fig. 4.1.

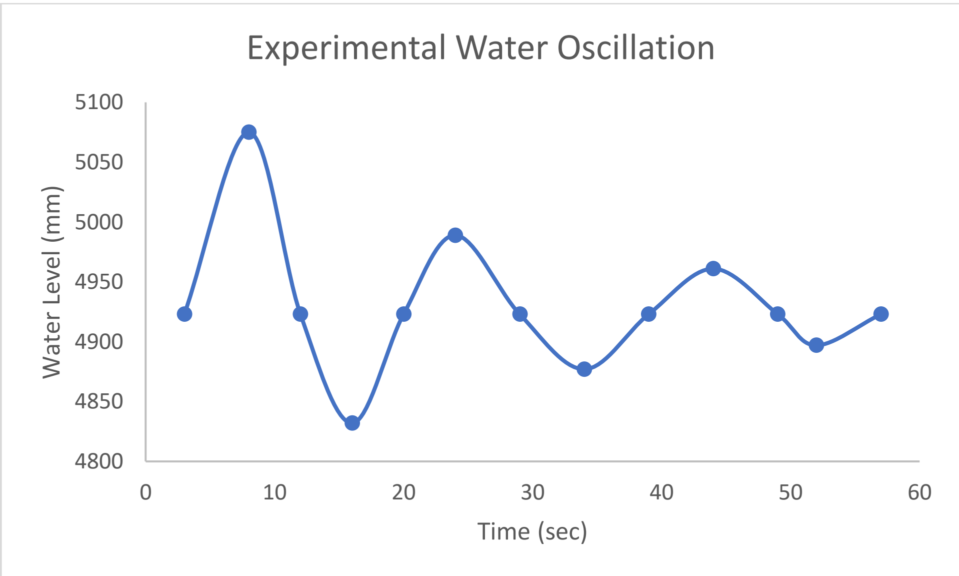


Fig. 4.1 Water Level Oscillations for experimental results

4.1 Computational Analysis:

To achieve a similar level oscillation for the same setup of surge tank as observed in the experiment, computation is done on Google Colab using Python. The computation is done by solving the dynamic equation and the continuity equation shown in equation 16 and 18 respectively. These equations are solved using four methods separately, viz. Euler method, RK4 method, RK3 method and RK2 method. The objective is to search a comparatively better method to duplicate the experimental values, hence, the computation is done for different time steps for each method.

The initial conditions adopted are:

For equation (16), Q(0) = assumed (taken $Q_o = 0.001723\ m^3/s$)

For equation (18), z(0) = 0, $Q_v(0) = 0$

4.1.1 Results of Euler Method (EM):

Figure 4.2 represents the numerical solutions to equations 16 and 18 using Euler Method. Adopting different time step sizes reflects both stability and accuracy of the method. It is observed that the solutions for time step sizes smaller than 1 second are in proximity with the experimental data and shows similar behaviour in wave oscillation. The solution is also stable only for a time step smaller than 3 second and fails for time step of 5 second. The first upsurge value for time step 0.1 second is near to the first upsurge value of experimental data.

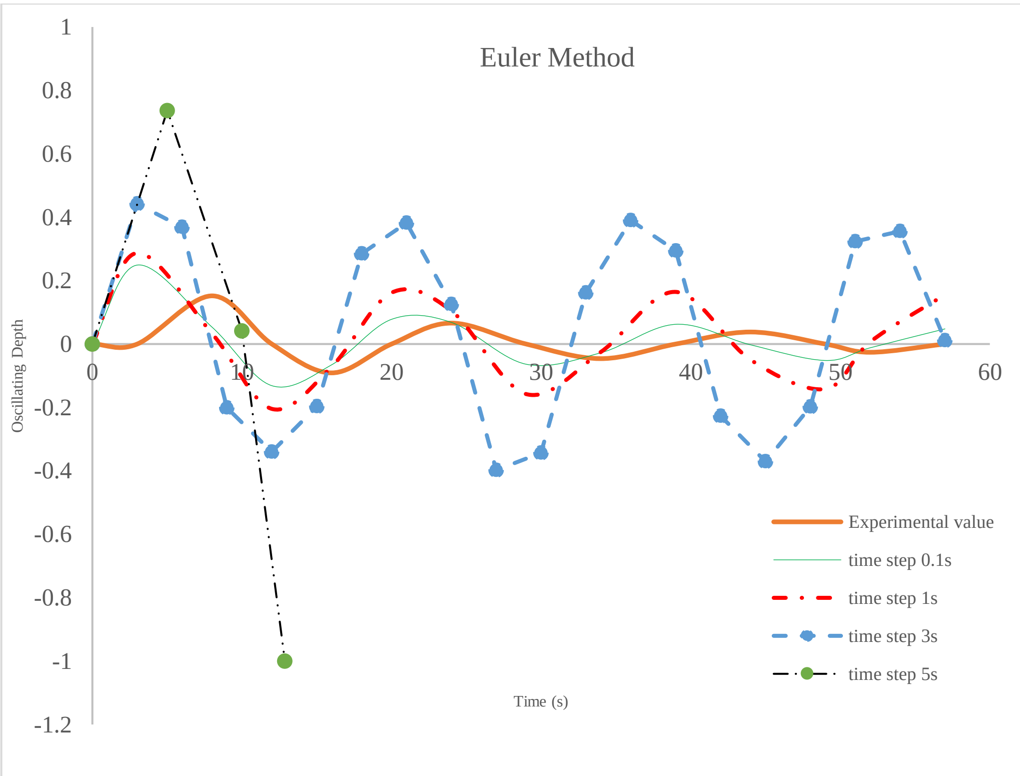


Fig. 4.2 Water level oscillation using Euler method.

4.1.2 Results of Fourth-Order Runge-Kutta Method (RK4):

Figure 4.3 represents the numerical solutions to equations 16 and 18 using RK4 method. Unlike Euler Method RK4 becomes unstable for a time step of 8 second, clearly indicating the superior stability properties of the RK4 method compared to the explicit Euler method. Furthermore, the solution for time step of 3 second are in close proximity to the experimental values. Also, avoiding the first upsurge, after 15 seconds the oscillating values for $\Delta t = 5$ seconds is same to the respective experimental values.

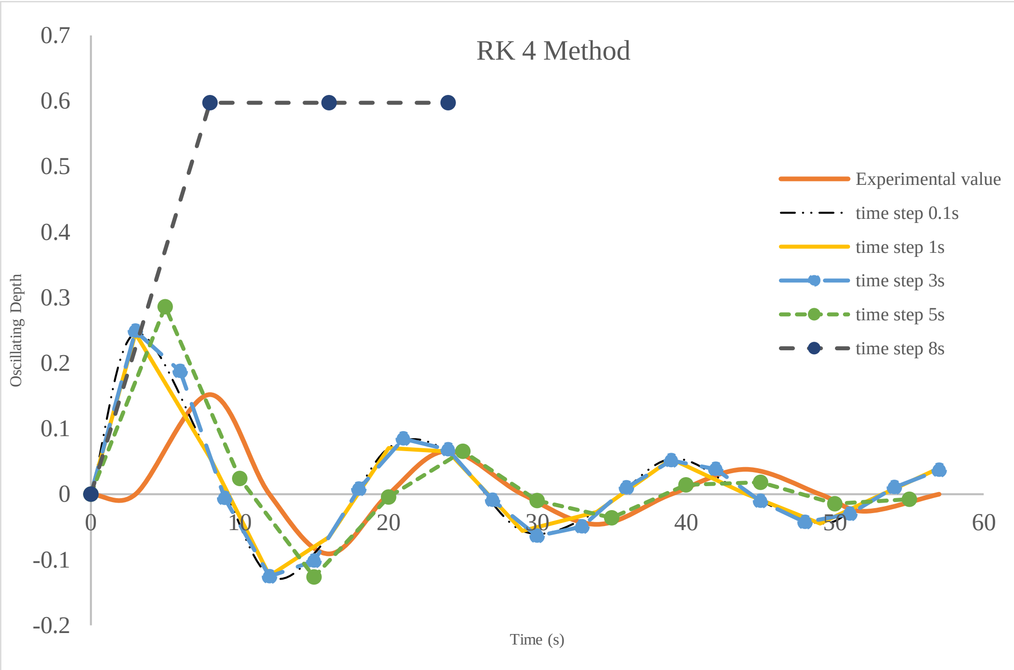


Fig. 4.3 Water level oscillation using RK4 method

4.1.3 Results of Third-Order Runge-Kutta Method (RK3):

Figure 4.4 represents the numerical solutions to equations 16 and 18 using RK3 method. Similar to RK4, RK3 too becomes unstable for a time step of 8 second, indicating the similar stability properties of the RK3 method to RK4 method.

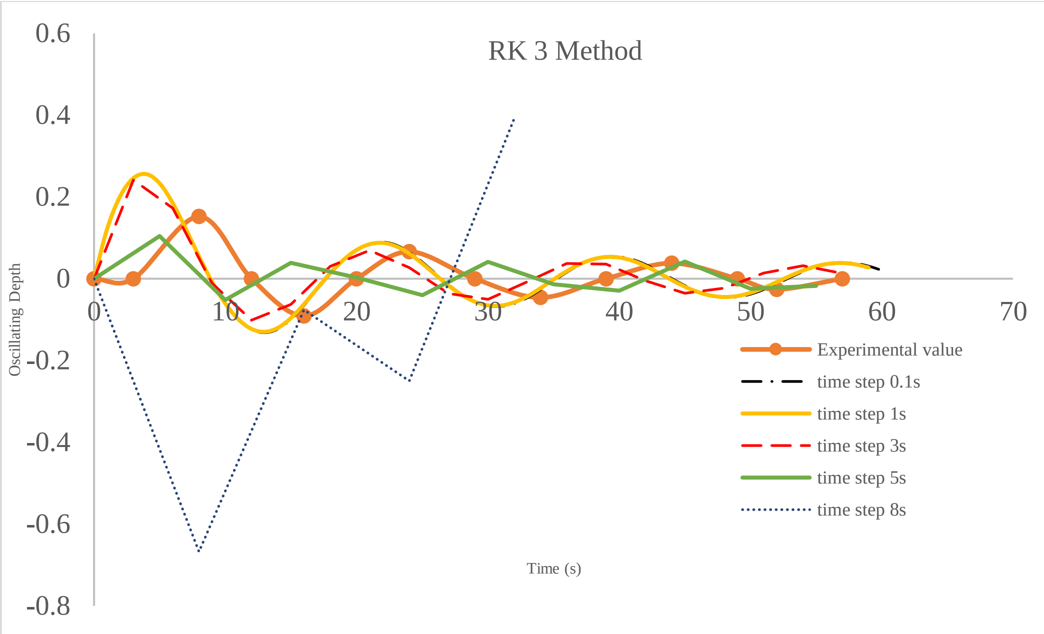


Fig. 4.4 Water level oscillation using RK3 method

But the behaviour of the water oscillations for time step of 5 second is not similar to the oscillations for time step less than or equal to 3 second, which was not in the case of RK4. For time step less than or equal to 3 second the surge values are similar.

4.1.4 Results of Second-Order Runge-Kutta Method (RK2):

Figure 4.5 represents the numerical solutions to equations 16 and 18 using RK2 method. The solution unstable for time step greater than 5 second. The stability of RK2 is similar to EM. For time step of 3 seconds the surge values are closer to experimental observations.

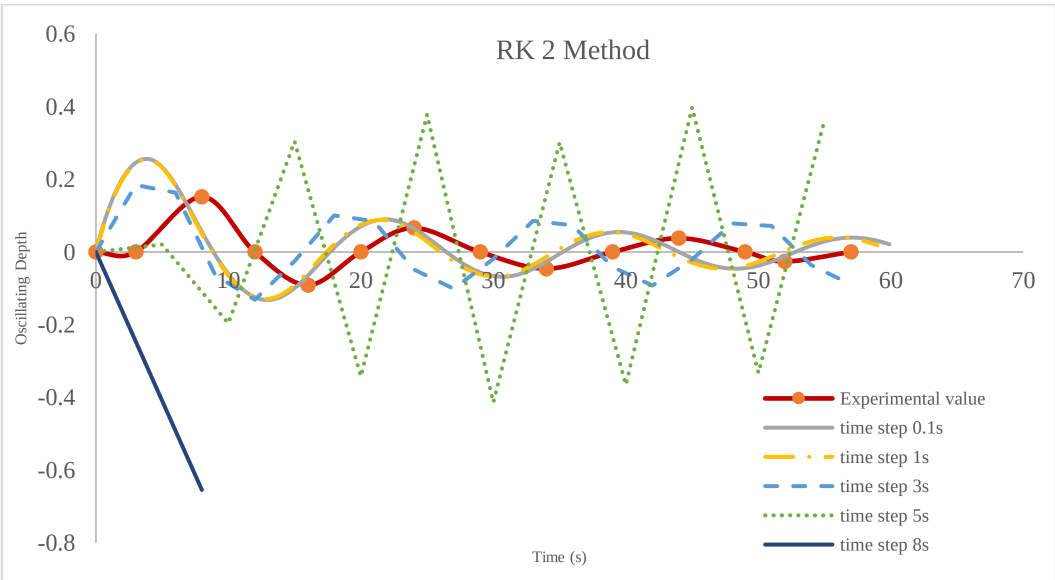


Fig. 4.5 Water level oscillation using RK2 method

4.1.5 Comparison Between EM, RK4, RK3 and RK2:

Since the maximum time step for which the solution was stable for all the four methods was 3 seconds, the variation of water level from all the four methods along with experimental values with Δt = 0.3 second is shown in Fig. 4.6. RK4 results are closer to the experimental observations, RK3 is also considerable.

For much smaller time step (Δt = 0.1 s) all the methods show identical variations but the first upsurge value is comparatively more than experimental first upsurge. This is similar for Δt = 1 second. Within the stability limits, EM exhibits comparatively higher values of surges than the experimental, whereas RK4 and RK3 shows nearby results. The only difference between RK4 and RK3 is that the later shows instability is lesser time scale than the former. Further RK2 outcomes are somewhat similar to RK3, except RK2 shows closer first-upsurge value than RK3 for Δt = 3 second.

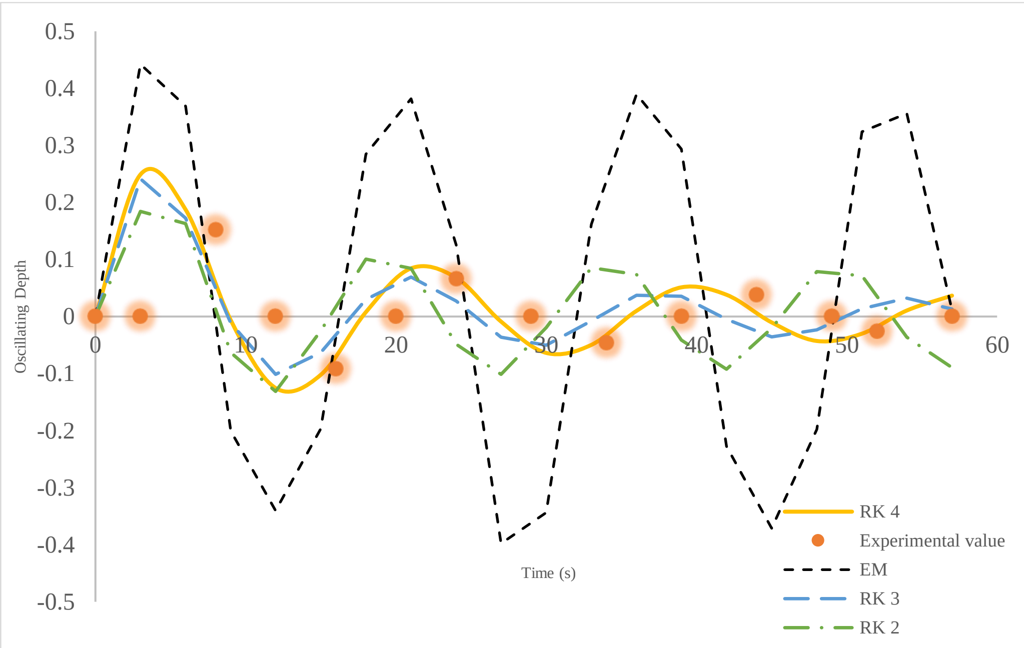


Fig. 4.6 Water level oscillations using explicit Euler, RK4, RK3 and RK2

4.2 Summary:

RK4 (short for Runge-Kutta 4th order) is generally considered to be a more accurate numerical integration method than the Euler method. This is because RK4 uses a higher order polynomial approximation to estimate the next value of the function being integrated, while the Euler method uses a first-order approximation [6].

In the context of the surge tank problem, the Runge-Kutta 4th order method is used to approximate the solution to the differential equations that describe the system's behaviour. This method works by breaking down the time step into smaller sub-steps, and computing the solution at each sub-step using a series of weighted averages. This approach reduces the error associated with assuming a constant slope over the time step and leads to more accurate results.

Therefore, the Runge-Kutta 4th order method is a popular choice for solving the surge tank problem because it provides accurate and efficient solutions to the system's behaviour over time.

On the other hand, Euler method estimates the next value of the function based on the slope of the function at the current point, assuming that the slope remains constant over the entire time step. This leads to errors that accumulate over time, especially for functions with non-linear behaviour or steep slopes. However, RK4 is more computationally expensive than the Euler method because it requires more evaluations of the function being integrated [7].

Other Runge-Kutta (RK) methods such as RK3 and RK2 can be used to solve the surge tank problem, but the choice of method depends on the desired level of accuracy and computational efficiency. RK3 is more accurate than RK2 because it uses a higher number of intermediate steps to compute the solution at each time step. Specifically, RK3 uses three intermediate stages, while RK2 uses only two intermediate stages. In general, the error of the RK method is proportional to the step size raised to the order of the method. Since RK3 is a third-order method, its error is proportional to the step size raised to the power of 4 (h^4). On the other hand, RK2 is a second-order method, so its error is proportional to the step size raised to the power of 3 (h^3).

This means that for the same step size, the error of RK3 is lower than that of RK2, making RK3 more accurate. However, it is important to note that RK3 requires more computational resources than RK2 since it requires the computation of three intermediate stages instead of two. Therefore, the choice of method should be made based on the balance between accuracy and computational efficiency.

Chapter 5

Case Study 2

Our second case study is based on design considerations for surge tank and its effects on the surge tank level. The parameters under considerations are area of the surge tank, A_S and frictional losses coefficients, c. The dimensions for the surge tank are adopted from "Surge Tank Design Considerations for Controlling Water Hammer Effects at Hydro-electric Power Plants" by Dr. Abdulghani Ramadan and Dr. Hatem Mustafa, 2013. The data used for this design analysis are: Steady state discharge, $Q_o = 300\ m^3/s$, supply pipe length, L=500 m, supply pipe diameter, $A_T = 80\ m^2$.

Unlike in Case Study 1 the setup for this case is more likely to be a field setup. The analysis is also done for different time step for stability check. For conducting the analysis, we will have to solve equations 16 and 18. The solution procedure is based on the numerical analysis scheme. In order to solve these non-linear ordinary differential equations, the initial conditions can be taken as follows:

For equation (16), Q(0) = assumed (taken $Q_o = 300\ m^3/s$)

For equation (18), z(0) = 0, $Q_v(0) = 0$

Also, c_s+c_f is assumed to be c.

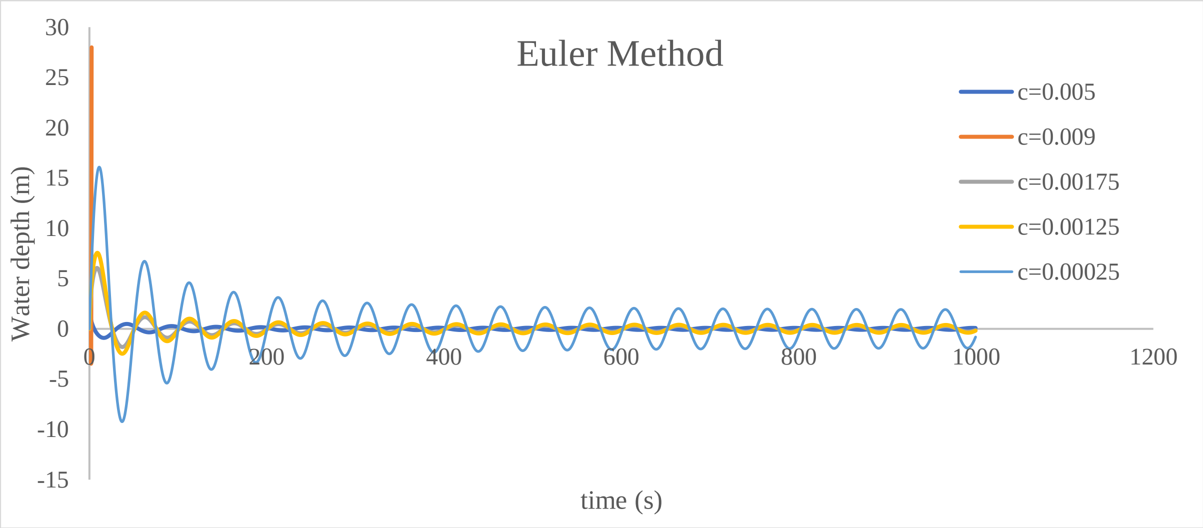
To find the solution of the system of equations, the Euler method and the Runge-Kutta (RK-2,3 and 4) methods are used. These equations are solved simultaneously. Google Colab was used to test and run the program written in Python. Various values are assumed for different design consideration and are the output for varying depth of surge tank is represented on the graph. The program calculates the required parameters at each time step.

First, we check for varying friction losses coefficient, c, on the surge tank level, Z.

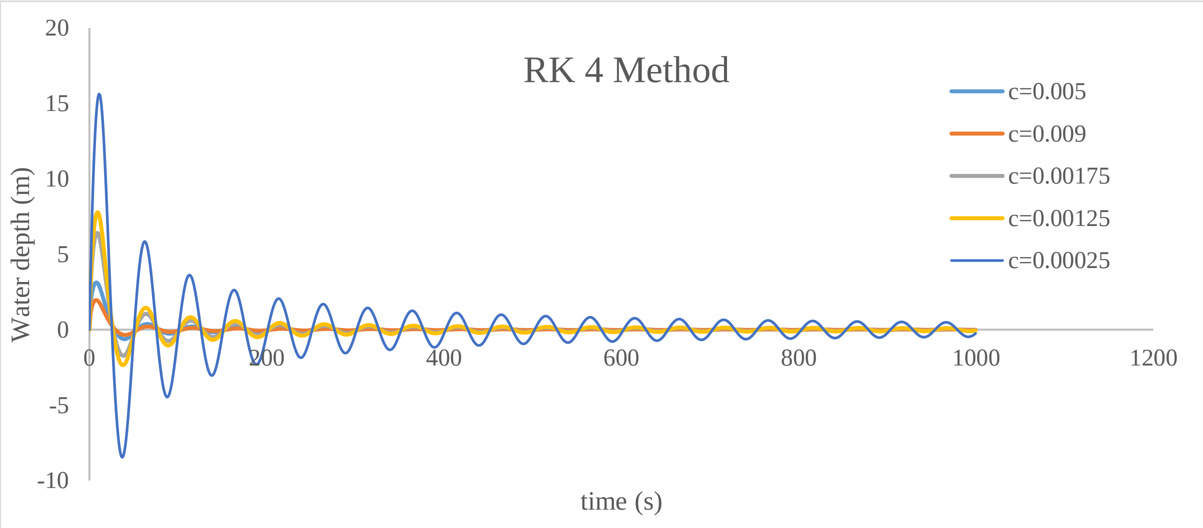
5.1 Computational Results for Varying c Value:

We have considered a range of frictional loss coefficient from c = 0.00025 to c = 0.009. A time step of 0.5 s is considered along with 100 m^2 of cross-sectional area for surge tank (i.e., A_S =100 m^2).

The variations of water level with respect to time are shown in Fig 5.1. We have observed that both Euler and RK2 methods have failed for higher frictional loss coefficient value, i.e., for c = 0.009, therefore suggesting inefficiency of these methods for higher c value, and for higher time step, i.e., Δt = 0.5 s. On other hand RK 4 and RK 3 have given similar results, also RK 3 was better in terms of time complexities.



(a)



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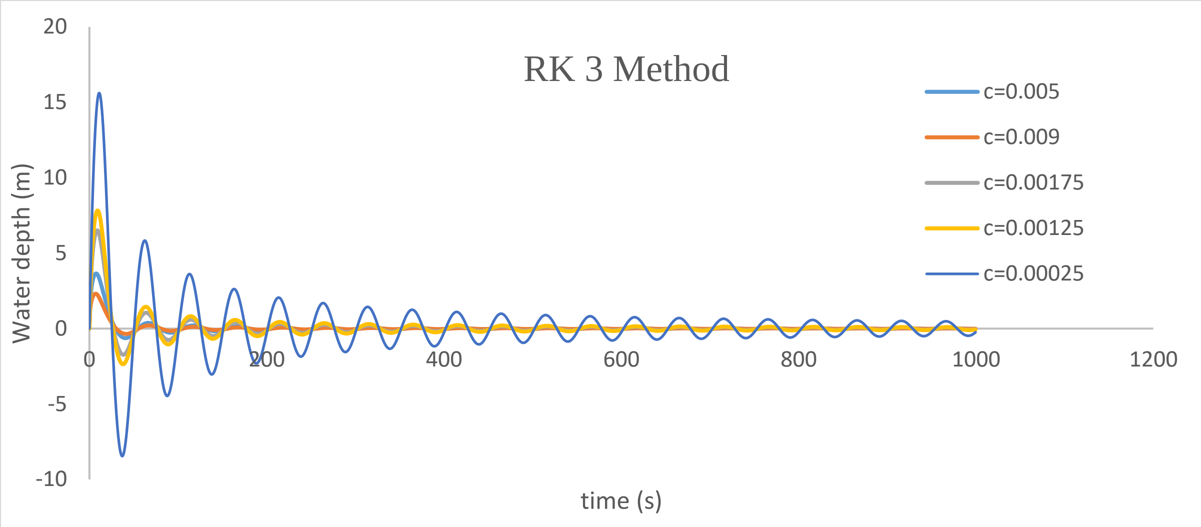
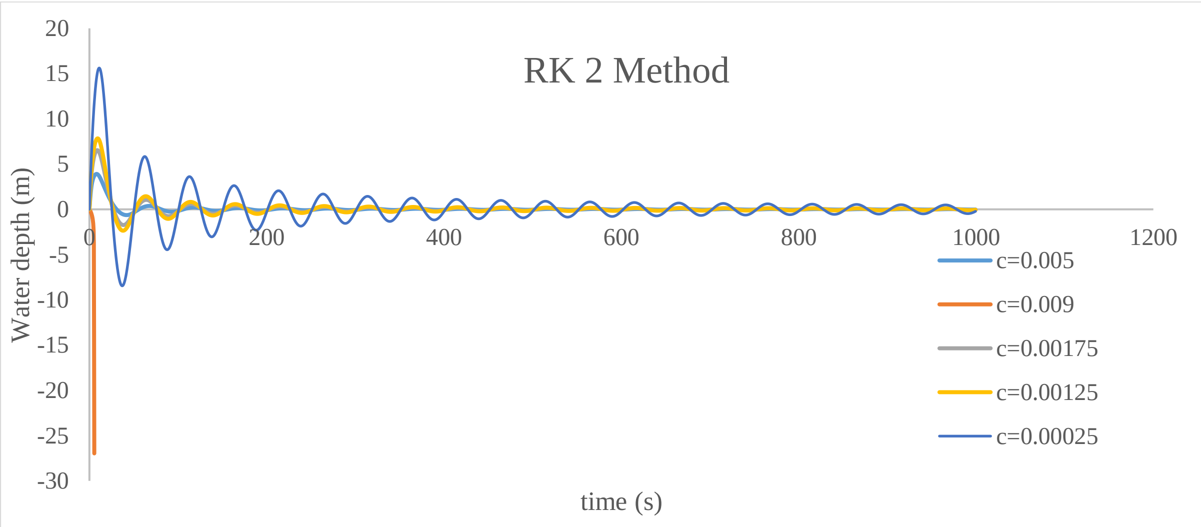


(c)



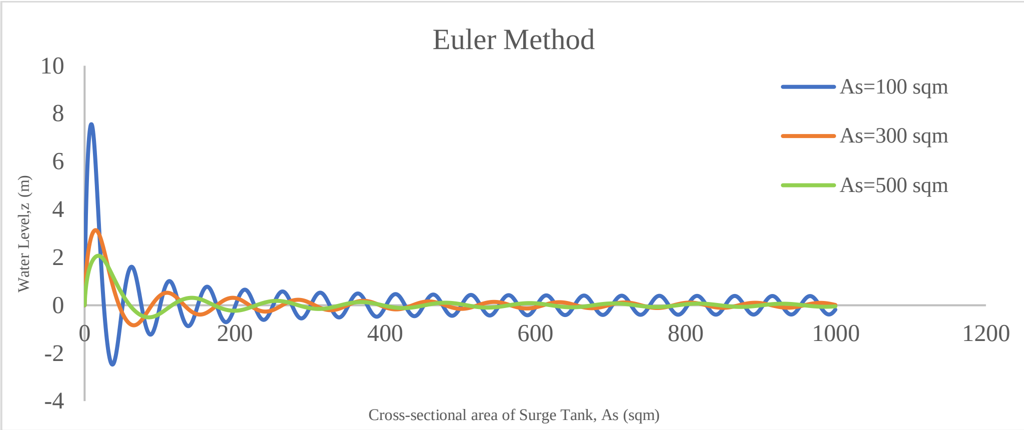
(d)

Fig. 5.1 Variation of surge tank level with time for c=0.00025, c=0.00125, c=0.00175, c=0.005, c=0.009 (a)Euler Method, (b) RK 4 Method, (c) RK 3 Method, (d) RK 2 Method

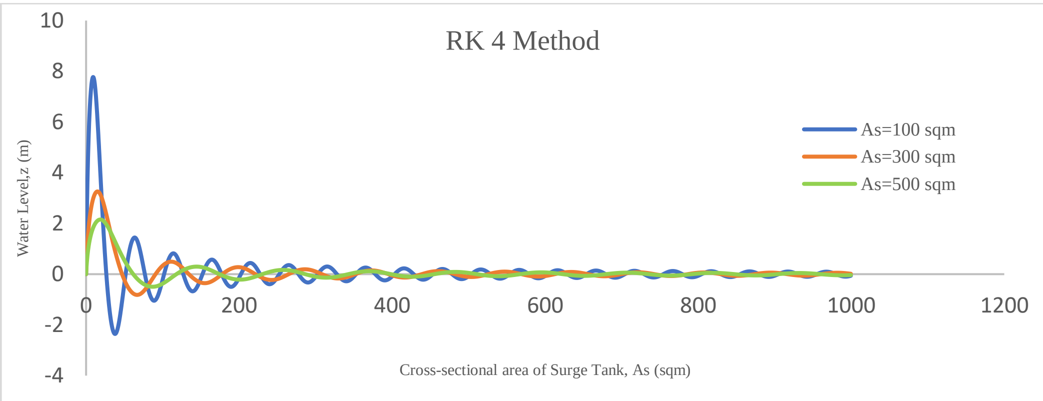
From Fig 5.1 we can infer the with increase in frictional loss coefficient the water level in surge tank decreases. The surge tank level decreases by 12m according to RK 4, RK 3 and RK 2 methods; and about 15 m according to results obtained from Euler method.

5.2 Computational Results for Varying A_S Value:

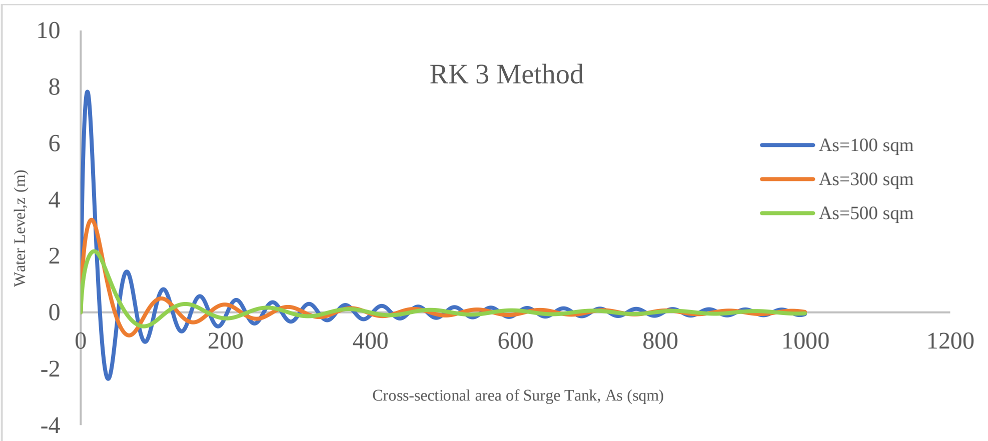
Our next objective is to observe the variation of surge tank level with respect to varying cross sectional area of the tank with time. Since, an upsurge of about 8 m is achieved for c = 0.00125 we will assume the same value of loss coefficient to get higher upsurge which in turn will give us an idea for maximum upsurge possible for various cross-sections. A range of cross-sections from A_S =100 m^2 to A_S = 1500 m^2 is analysed for the same time step as earlier and the results are shown in Fig 5.2 and Fig 5.3.



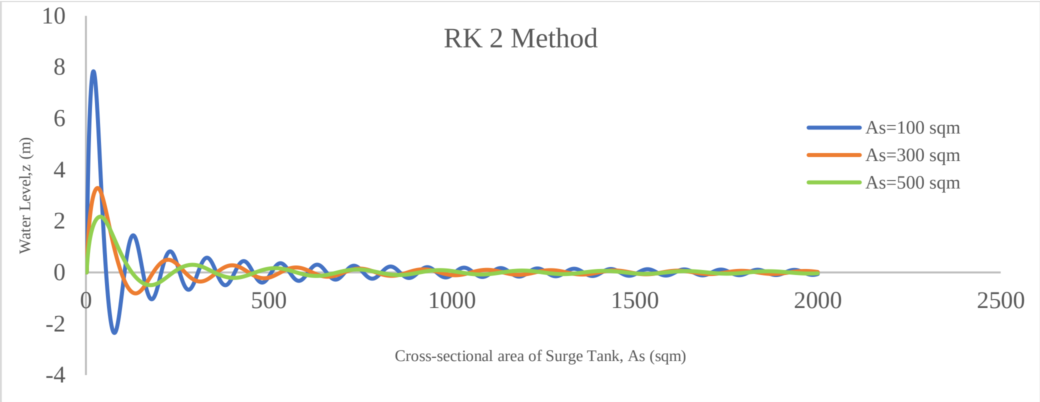
(a)



(b)

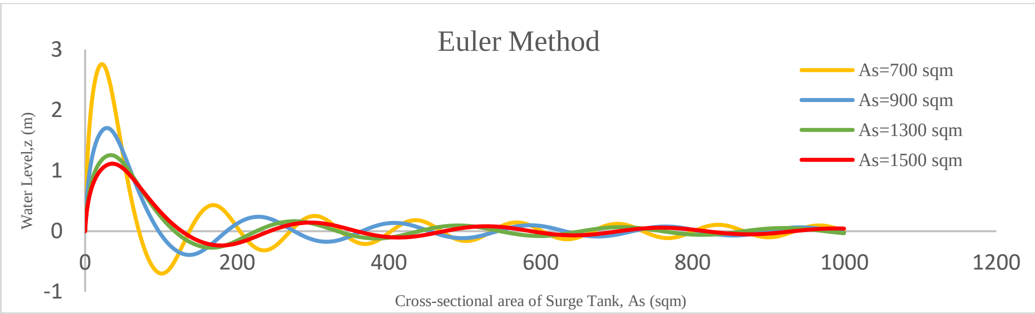


(c)

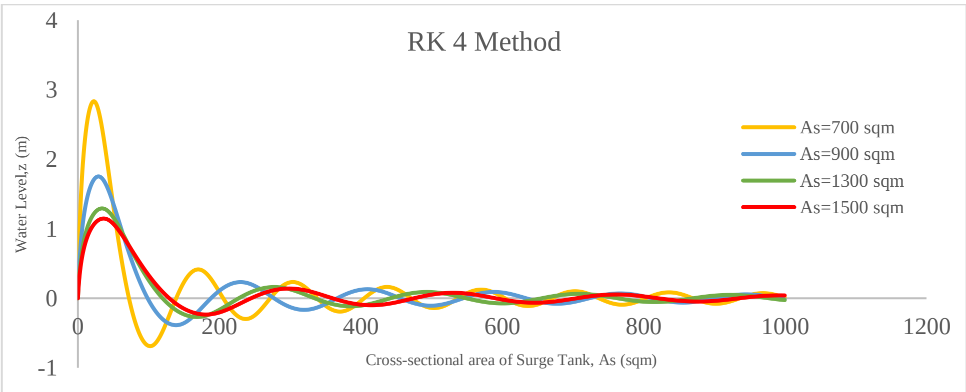


(d)

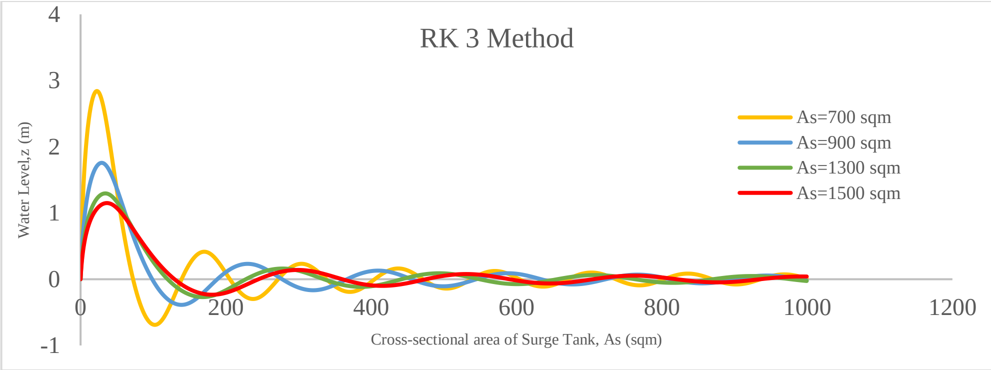
Fig. 5.2 Variation of surge tank level with time for A_S =100 m^2, A_S =300 m^2, A_S =500 m^2 (a)Euler Method, (b) RK 4 Method, (c) RK 3 Method, (d) RK 2 Method



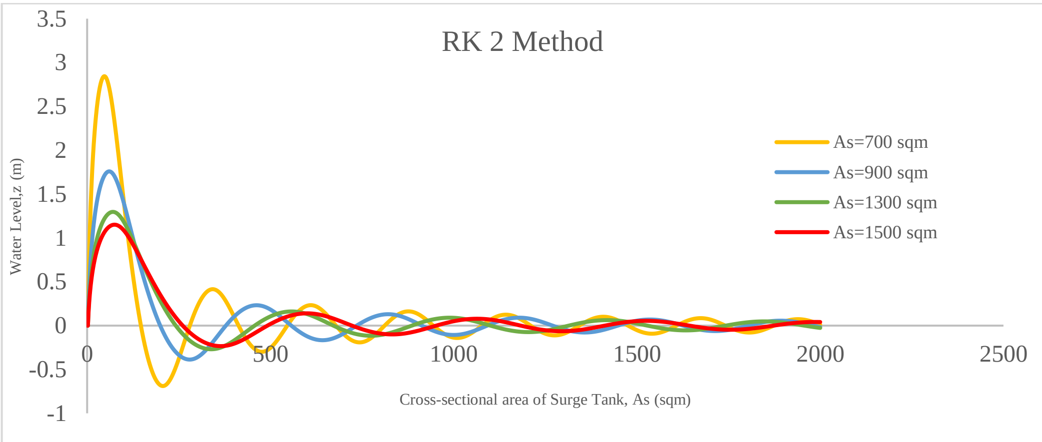
(e)



(f)



(g)



(h)

Fig. 5.3 Variation of surge tank level with time A_S =500 m^2, A_S =700 m^2, A_S =1300 m^2, A_S =1500 m^2 (a)Euler Method, (b) RK 4 Method, (c) RK 3 Method, (d) RK 2 Method

From Fig 5.2 and 5.3 with increase in cross-sectional area of the surge tank the upsurge decreases. The first upsurge decreases from 7.8 m to 1.5 m for varying A_S from 100m^2 to 1500m^2. All the four methods have shown similar variations of water depth with time.

5.3 Comparison of Results for Different Methods:

The first upsurge is the most important parameter to consider. Fig. 5.1 shows Euler are RK 2 methods failed for c = 0.009, therefore, we will only consider the upsurge value for c=0.00025, c=0.00125, c=0.00175, c=0.005 for all the four methods. The first upsurge for various loss coefficients is shown in Fig. 5.4.

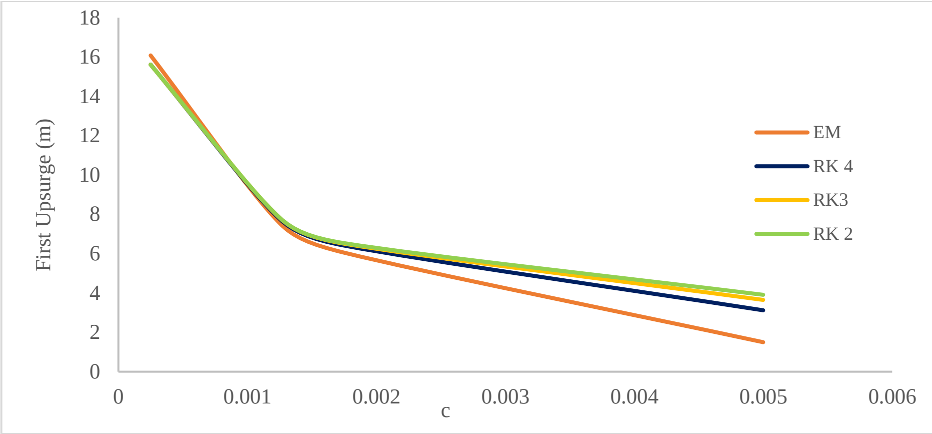


Fig. 5.4 The variation of first upsurge with friction losses coefficient, c

Here RK3 and RK2 have shown identical variation of first upsurge. RK4 value shown a similar trend with slightly lesser value of upsurge for c>0.003. But with Euler method the difference for first upsurge value is for as we increase c value. This can be because surge tank problems can be stiff, i.e., the solution changes rapidly in some regions of the solution space and slowly in others. The Euler method may not be well-suited for stiff problems, as it may require very small-time step sizes to maintain stability.

Similarly, the variation of first upsurge for different surge tank area is shown in Fig 5.5.

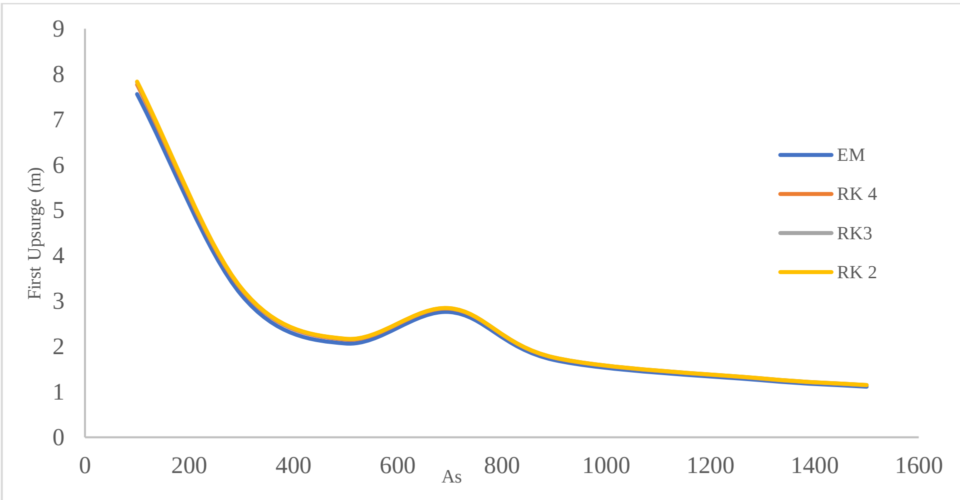


Fig. 5.5 The variation of first upsurge with A_S

All four methods have shown identical variation in Fig. 5.5, hence, suggesting preference of any method for the above-mentioned variation analysis. We can observe that the first upsurge value beyond A_S = 500 m^2 is very small and there is an increase in the trend of the upsurge near same A_S value for the otherwise decreasing trend, thus making it infeasible to increase the cross-sectional area of the tank to values more than that limit.

5.4 Summary:

i. As the head losses coefficient increases, the level of surge tank decreases. Up to c=0.005 all the four methods show the same trend, for c = 0.009 Euler and RK2 fails. Therefore, we can limit our consideration for head losses coefficient from 0.00025 to 0.005. And to avoid an upsurge of 15-16 m, c=0.002-0.003 can be adopted.

ii. As the surge tank cross-sectional area increases, the level of surge tank decreases. Beyond 500 m^2 the upsurge level is very small, along with a sudden increase in upsurge level as shown in Fig. 5.5. Hence, for preliminary design we can consider surge tank area less than 500 m^2.

iii. According to our result, we can theoretically suggest a surge tank of cross-sectional area of 400-500 m^2 corresponding to first upsurge of 2 ~ 3m respectively. Since we observed a change in the decreasing trend of Fig. 5.5 near A_S = 500 m^2, we can adopt parallel installation of smaller cross-sectional area surge tank (of about A_S = 200-300 m^2). Taking into account the total losses between the main reservoir and the surge tank, the height of the surge tank could be roughly estimated as the sum of (the height of water column in the surge tank, the total losses between the main reservoir and the surge tank, the first upsurge result from surge tank area variation and the upsurge results from the effect of friction losses coefficient) in addition to a factor of safety.

iv. There are other parameters for final design stage such as: cost, operation conditions, topography, location, type of equipment, load requirements, labour, etc.

Chapter 6

Conclusion

In a nutshell, the choice of numerical method for solving a surge tank problem will depend on the specific requirements and conditions of the problem. The accuracy and stability requirements, model complexity, stiffness of the problem, and available computational resources are all important factors to consider when selecting a numerical method.

From Case 1 we can conclude that the author of "Modeling of Hydraulic Transients in Closed Conduits" was right regarding the preference of RK4 compared to Euler Method. But RK3 also showed suitable results with a good range of time step. It is possible to use RK3 and RK2 to solve the surge tank problem, but the choice of method depends on the desired level of accuracy and computational efficiency. RK3 is more accurate than RK2 but requires more computational resources. RK2 is less accurate than RK3 but is computationally less expensive. Therefore, the choice of method should be made based on the trade-off between accuracy and efficiency.

While the Euler method can be a useful and straightforward method for solving surge tank problems, it may not always be the best choice for accurately reproducing experimental data. Surge tank problems can be stiff, suggesting that the solution changes rapidly in some regions of the solution space and slowly in others. The Euler method may not be well-suited for stiff problems, as it may require very small time-step sizes to maintain stability. Higher-order methods, such as the Runge-Kutta methods, can provide higher accuracy and stability, and may be better suited for stiff problems and nonlinear boundary conditions.

Case 2 provides an elaborate view on the effects of head loss coefficients and surge tank area on the first upsurge magnitude. The four methods have shown the variation in oscillating surge for the two parameters using a singe time-step of 0.5 second. For its larger time-step Euler and RK2 fails for large c value, i.e., c=0.009.

Case 1 have also presented the accuracy of all the four methods for smaller time step. Therefore, with smaller time-step RK2 can show accurate results for both the cases without failing for large c value.

For checking the stability of the four methods in different time-step sizes, we take the conditions of Case Study 2 and have assigned zero head loss coefficient, suggesting a water surge of undamped condition. For $\Delta t = 0.01$ s all the four method shows identical undamped oscillation as shown in Fig 6.1.

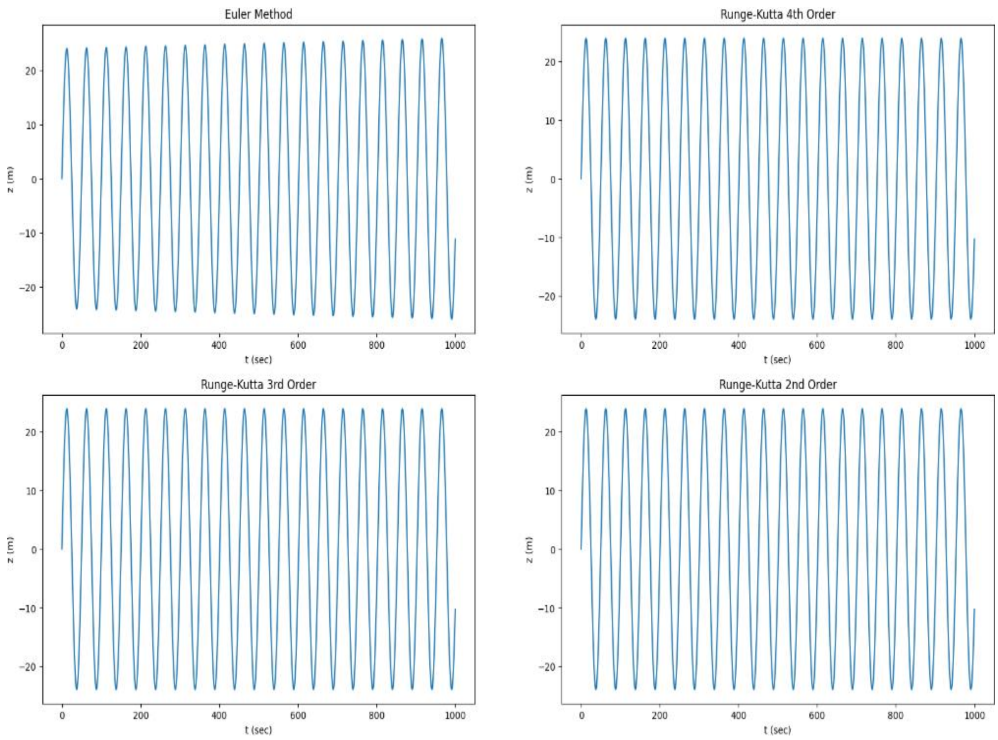


Fig. 6.1 Variation of surge tank level for c = 0, A_S = 100 m^2 and Δt = 0.01 s

But for small change in time-step size such as for $\Delta t = 0.1$ s Euler Method exhibits unstable results suggesting integration of error in each iteration.

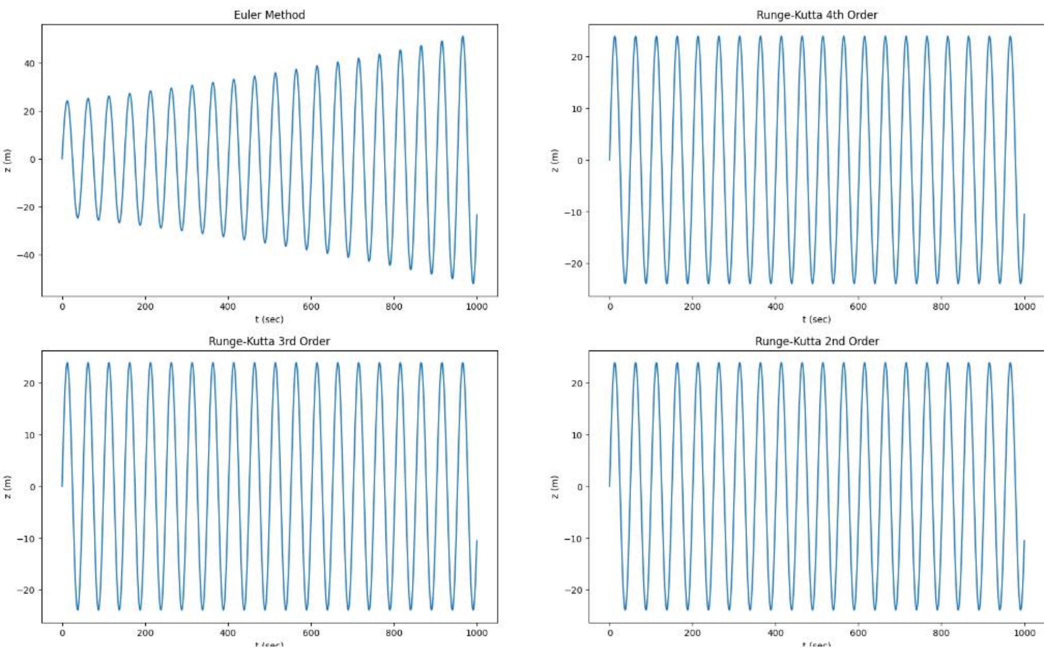


Fig. 6.2 Variation of surge tank level for c = 0, A_S = 100 m^2 and Δt = 0.1 s

For any time-step size all the Runge-Kutta methods have shown the same surge of ± 20 m, whereas Euler continues to exhibit unstable result. Thereby, with negligence of ‘c’ and converting the set of equations to Linear ODE Euler method is not suitable for the case 2 setup.

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