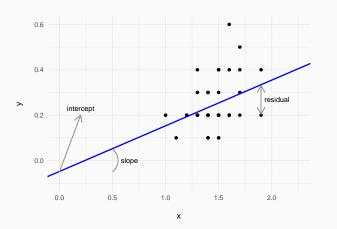
Introduction to linear models

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Our unified regression framework (GLM)

$$y_{i} = a + bx_{i} + \varepsilon_{i}$$
$$\varepsilon_{i} \sim N(0, \sigma^{2})$$



Data

y = response variable x = predictor

Parameters

a = intercept

b = slope

 σ = residual variation

arepsilon = residuals

What's the intercept?

Expected value of y when predictors (x) = 0

If
$$x = 0$$
:

•
$$y = a + b*0$$

What's the intercept?

Expected value of y when predictors (x) = 0

If x = 0:

- y = a + b*0
- y = a

What's the slope?

How much y increases (or decreases) when x increases in 1 unit

If we have model

$$y = 0.5 + 2 * x$$

• If
$$x = 10 \rightarrow y = 0.5 + 2 * 10 = 20.5$$

If x increases 1 unit, y increases 2 units

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What's the slope?

How much y increases (or decreases) when x increases in 1 unit

If we have model

$$y = 0.5 + 2 * x$$

- If $x = 10 \rightarrow y = 0.5 + 2 * 10 = 20.5$
- If x = 11 -> y = 0.5 + 2 * 11 = 22.5

If x increases 1 unit, y increases 2 units

Slopes can be negative

If we have model

$$y = 0.5 - 2 * x$$

• If
$$x = 10 \rightarrow y = 0.5 - 2 * 10 = -19.5$$

If x increases 1 unit, y decreases 2 units

Slopes can be negative

If we have model

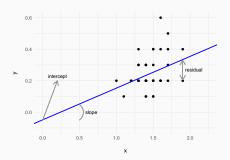
$$y = 0.5 - 2 * x$$

• If
$$x = 10 \rightarrow y = 0.5 - 2 * 10 = -19.5$$

• If
$$x = 11 \rightarrow y = 0.5 + 2 * 11 = -21.5$$

If x increases 1 unit, y decreases 2 units

What are residuals?



How far points fall from the regression line

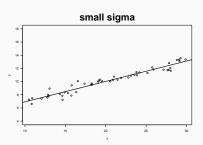
Difference between observed values and values predicted by model (regresssion line)

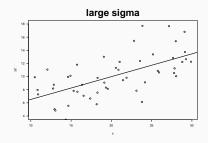
If sigma is large, residuals are larger

$$\varepsilon_{i}\sim N\left(0,\sigma^{2}\right)$$

If sigma is larger:

- · points farther from regression line
- · larger difference of observed predicted values

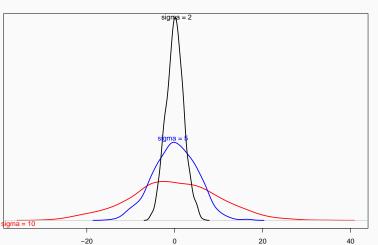




Residual variation (sigma) is the Std. Dev. of residuals

$$\varepsilon_i \sim N(0, \sigma^2)$$

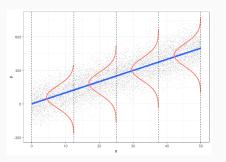
Distribution of residuals



In a general linear model we assume residuals are

$$\varepsilon_{i}\sim N\left(0,\sigma^{2}\right)$$

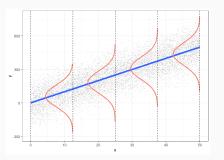
· Normal



In a general linear model we assume residuals are

$$\varepsilon_i \sim N\left(0, \sigma^2\right)$$

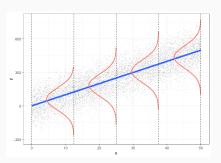
- · Normal
- · Centred on 0 (no bias)



In a general linear model we assume residuals are

$$\varepsilon_i \sim N\left(0, \sigma^2\right)$$

- · Normal
- · Centred on 0 (no bias)
- · Homogeneous variance (homoscedasticity)



Different ways to write same model

$$y_{i} = a + bx_{i} + \varepsilon_{i}$$
$$\varepsilon_{i} \sim N(0, \sigma^{2})$$

$$y_{i} \sim N(\mu_{i}, \sigma^{2})$$
$$\mu_{i} = a + bx_{i}$$
$$\varepsilon_{i} \sim N(0, \sigma^{2})$$

Quiz

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