

# Sampling, confidence intervals, and Bayesian inference

## Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.



# What's the average height in this group?

1. Write down your height and place of origin (Sevilla or other) in a piece of paper and put it in the bag.

## What's the average height in this group?

1. Write down your height and place of origin (Sevilla or other) in a piece of paper and put it in the bag.
2. Now everyone **sample** 5 individuals from the whole **population** of heights.

## What's the average height in this group?

1. Write down your height and place of origin (Sevilla or other) in a piece of paper and put it in the bag.
2. Now everyone **sample** 5 individuals from the whole **population** of heights.
3. Calculate the mean and 95% CI for your sample (<http://graphpad.com/quickcalcs/CImean1/>).

## What's the average height in this group?

1. Write down your height and place of origin (Sevilla or other) in a piece of paper and put it in the bag.
2. Now everyone **sample** 5 individuals from the whole **population** of heights.
3. Calculate the mean and 95% CI for your sample (<http://graphpad.com/quickcalcs/CImean1/>).
4. Draw on blackboard.

## What's the average height in this group?

1. Write down your height and place of origin (Sevilla or other) in a piece of paper and put it in the bag.
2. Now everyone **sample** 5 individuals from the whole **population** of heights.
3. Calculate the mean and 95% CI for your sample (<http://graphpad.com/quickcalcs/CImean1/>).
4. Draw on blackboard.
5. Do all CIs contain true mean height?

If the 95% CI of  $X$  is  $(120, 150)$ ...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150

<https://pollev.com/franciscorod726>



If the 95% CI of  $X$  is  $(120, 150)$ ...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150
- ▶ We can be 95% confident that  $X$  lies between 120 and 150

<https://pollev.com/franciscorod726>

If the 95% CI of  $X$  is (120, 150)...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150
- ▶ We can be 95% confident that  $X$  lies between 120 and 150
- ▶ If we repeated the experiment, 95% of the time  $X$  would fall between 120 and 150

<https://pollev.com/franciscorod726>

If the 95% CI of  $X$  is (120, 150)...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150
- ▶ We can be 95% confident that  $X$  lies between 120 and 150
- ▶ If we repeated the experiment, 95% of the time  $X$  would fall between 120 and 150
- ▶ If we repeated the experiment, 95% of the CIs would contain the true value of  $X$

<https://pollev.com/franciscorod726>

If the 95% CI of  $X$  is (120, 150)...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150
- ▶ We can be 95% confident that  $X$  lies between 120 and 150
- ▶ If we repeated the experiment, 95% of the time  $X$  would fall between 120 and 150
- ▶ If we repeated the experiment, 95% of the CIs would contain the true value of  $X$
- ▶ The probability that  $X$  is greater than 0 is at least 95%

<https://pollev.com/franciscorod726>

If the 95% CI of  $X$  is (120, 150)...

- ▶ There is a 95% probability that  $X$  lies between 120 and 150
- ▶ We can be 95% confident that  $X$  lies between 120 and 150
- ▶ If we repeated the experiment, 95% of the time  $X$  would fall between 120 and 150
- ▶ If we repeated the experiment, 95% of the CIs would contain the true value of  $X$
- ▶ The probability that  $X$  is greater than 0 is at least 95%
- ▶ The probability that  $X$  equals 0 is smaller than 5%

<https://pollev.com/franciscorod726>

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)
- ▶ A 95% CI is **NOT** 95% likely to contain the true parameter value!



# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)
- ▶ A 95% CI is **NOT** 95% likely to contain the true parameter value!
- ▶ Instead, 95% of the CIs obtained with this sampling will contain the true value.

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)
- ▶ A 95% CI is **NOT** 95% likely to contain the true parameter value!
- ▶ Instead, 95% of the CIs obtained with this sampling will contain the true value.
- ▶ Like person who tells truth 95% of the time, but we can't tell if a particular statement is true.

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)
- ▶ A 95% CI is **NOT** 95% likely to contain the true parameter value!
- ▶ Instead, 95% of the CIs obtained with this sampling will contain the true value.
- ▶ Like person who tells truth 95% of the time, but we can't tell if a particular statement is true.
- ▶ It's a frequentist, long-run property.

# Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.
- ▶ <https://rpsychologist.com/d3/CI/> (or here)
- ▶ A 95% CI is **NOT** 95% likely to contain the true parameter value!
- ▶ Instead, 95% of the CIs obtained with this sampling will contain the true value.
- ▶ Like person who tells truth 95% of the time, but we can't tell if a particular statement is true.
- ▶ It's a frequentist, long-run property.
- ▶ To read more: Morey et al (2015)

# What happens if we increase sample size?

<https://rpsychologist.com/d3/CI/>

- ▶ CI width *decreases*. . .

# What happens if we increase sample size?

<https://rpsychologist.com/d3/CI/>

- ▶ CI width *decreases*. . .
- ▶ but still 5% of CIs will NOT contain true mean!

# Bayesian credible intervals

- ▶ Bayesian **credible** intervals do give the probability that true parameter value is contained within them.

# Bayesian credible intervals

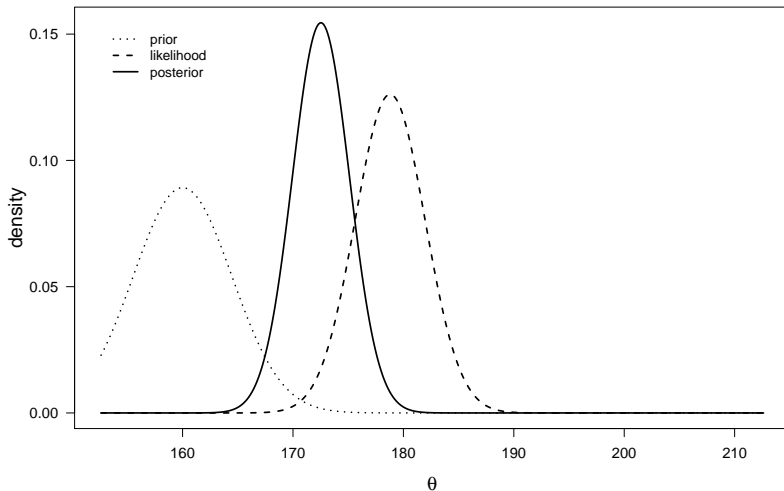
- ▶ Bayesian **credible** intervals do give the probability that true parameter value is contained within them.
- ▶ Frequentist CIs and Bayesian credible intervals can be similar, but not always.



# Bayesian inference: prior, posterior, and likelihood

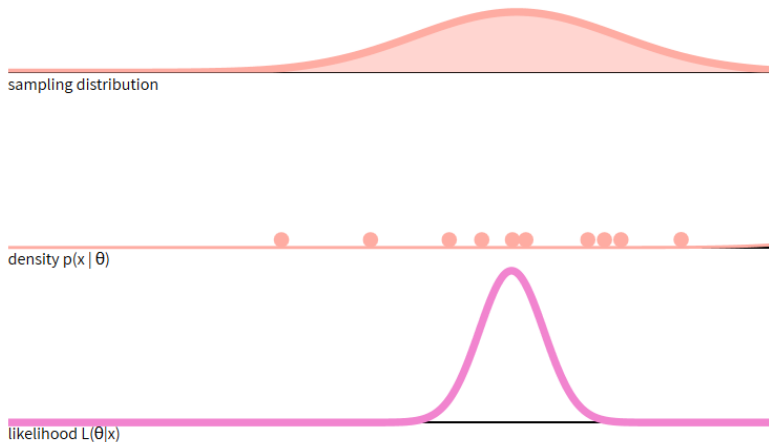
$$P(H|D) \propto P(D|H) \times P(H)$$

$$\text{Posterior} \propto \text{Likelihood} \cdot \text{Prior}$$



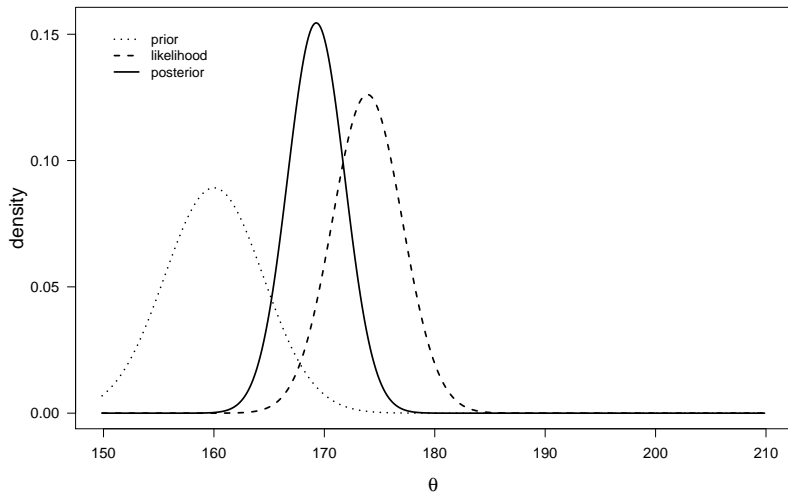
# What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$



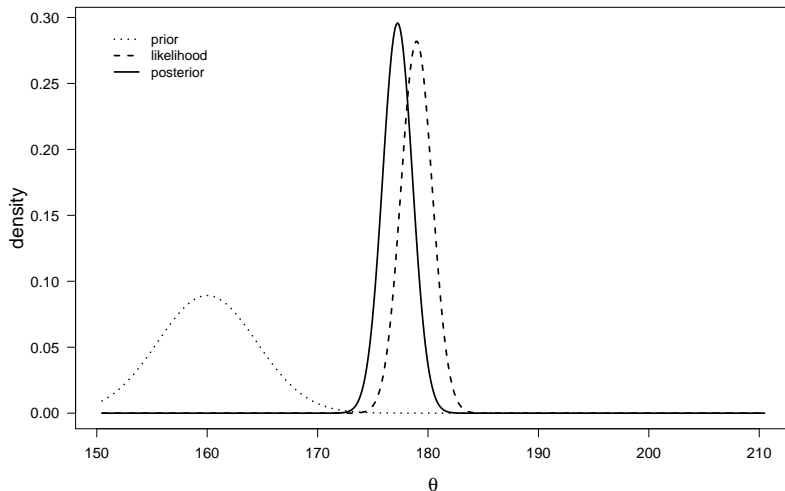
<https://seeing-theory.brown.edu/bayesian-inference/index.html>

# Bayesian inference: prior and likelihood produce posterior



`$posterior.mean`

With increasing sample size, likelihood dominates prior



`$posterior.mean`

# More apps to introduce Bayesian inference

- ▶ Bayesian Demo

# More apps to introduce Bayesian inference

- ▶ Bayesian Demo
- ▶ Bayesian inference for a population mean

# More apps to introduce Bayesian inference

- ▶ Bayesian Demo
- ▶ Bayesian inference for a population mean
- ▶ Normal

# More apps to introduce Bayesian inference

- ▶ Bayesian Demo
- ▶ Bayesian inference for a population mean
- ▶ Normal
- ▶ Binomial



# More apps to introduce Bayesian inference

- ▶ Bayesian Demo
- ▶ Bayesian inference for a population mean
- ▶ Normal
- ▶ Binomial
- ▶ Own data

# More apps to introduce Bayesian inference

- ▶ Bayesian Demo
- ▶ Bayesian inference for a population mean
- ▶ Normal
- ▶ Binomial
- ▶ Own data
- ▶ Bayesian t-test