

Sampling, confidence intervals, and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.



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5. Do all CIs contain true mean height?

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<https://pollev.com/franciscorod726>

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- ▶ If we repeated the experiment, 95% of the CIs would contain the true value of X
- ▶ The probability that X is greater than 0 is at least 95%
- ▶ The probability that X equals 0 is smaller than 5%

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- ▶ To read more: Morey et al (2015)

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- ▶ but still 5% of CIs will NOT contain true mean!

Bayesian credible intervals

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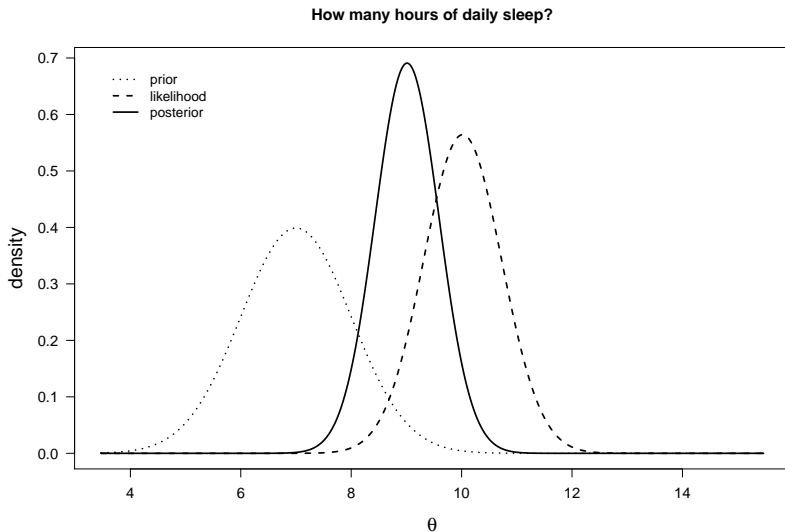
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- ▶ Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

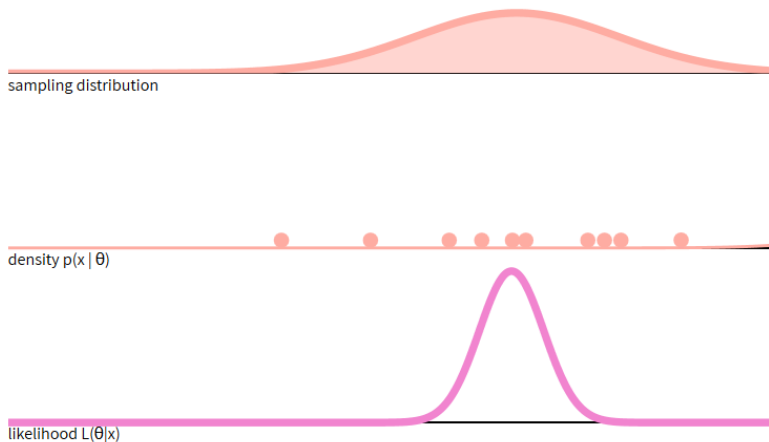
$$P(H|D) \propto P(D|H) \cdot P(H)$$

$$\text{Posterior} \propto \text{Likelihood} \cdot \text{Prior}$$



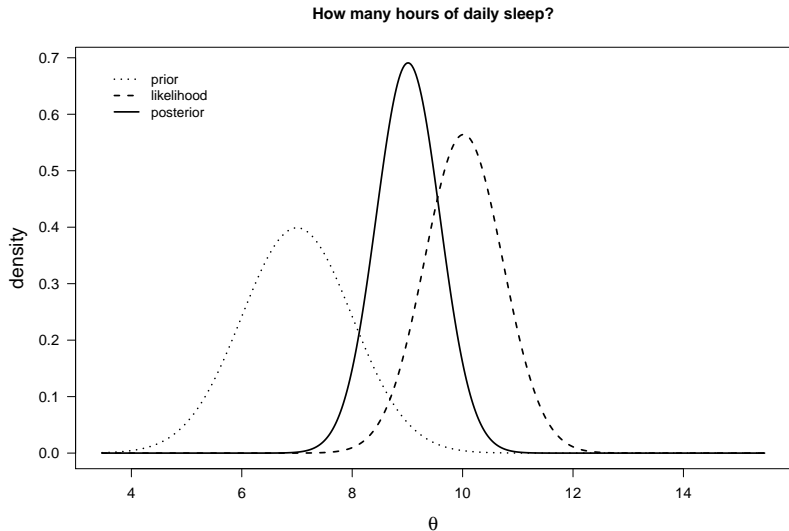
What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$

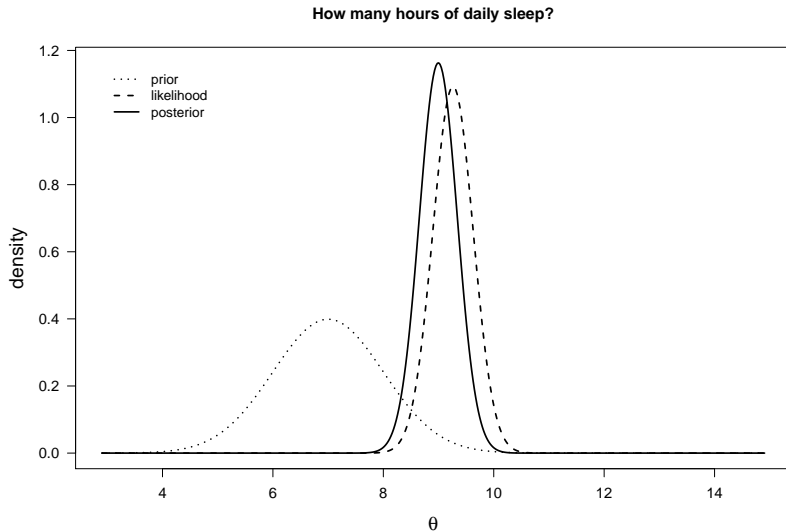


<https://seeing-theory.brown.edu/bayesian-inference/index.html>

Bayesian inference: prior and likelihood produce posterior



With increasing sample size, likelihood dominates prior



More apps to introduce Bayesian inference

- ▶ Bayesian Demo

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- ▶ Bayesian inference for a population mean

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- ▶ Bayesian t-test

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- ▶ Uncertainty / Propagate errors