

# An introduction to statistical inference

Francisco Rodríguez-Sánchez

[http://bit.ly/frod\\_san](http://bit.ly/frod_san)

Why statistics?

To answer questions like...

- ▶ what's the probability that something occurs?

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- ▶ what's the probability that something occurs?
- ▶ does X influence Y? How much?

## To ensure correct inferences

111	451	368	80	46	83	74	29	71	489	43
439	164	94	45	73	38	98	25	75	340	3
235	166	172	54	91	89	40	78	45	731	19
10	30	62	49	32	11	10	10	10	10	10
1.433	895	2.132	2.380	3.860	2.775	1.580	2.000	3.000	3.000	3.000
1.870	2.845	1.001	1.920	1.740	2.981	3.000	1.580	1.580	1.580	1.580
2.427	1	1.33	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
2.424	2.657	1	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
1.692	84	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
1.199	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
3	2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
35	290	92	285	164	224	224	224	224	224	224
74	243	430	277	175	334	334	334	334	334	334
84	301	249	175	334	334	334	334	334	334	334
17	3.868	2.455	6.303	6.303	6.303	6.303	6.303	6.303	6.303	6.303

**DATA**

Inference



Bolker et al 2009 TREE:

'311 out of 537 GLMM analyses (58%) used these tools  
inappropriately'

Figure 1:

To get answers to tough problems

For example. . .

# How many seeds do trees produce?



A. Torrenegra

Figure 2:

## Inferring tree fecundity

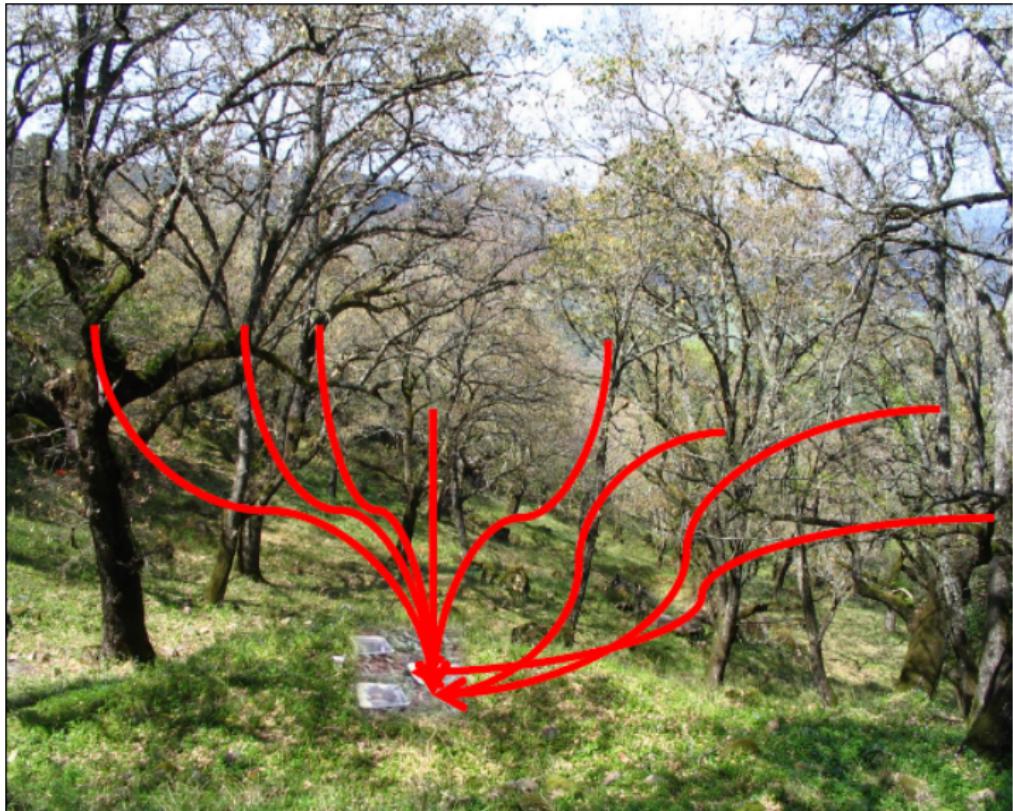


Figure 3:

# Course goals

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- ▶ Avoid **misconceptions**

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- ▶ **Understand** statistical inference
- ▶ Avoid **misconceptions**
- ▶ Promote **good practices**

*The purpose of models is not to fit data but to sharpen thinking*

Sam Karlin

# Topics

- ▶ Descriptive statistics

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- ▶ Graphics

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- ▶ Sampling

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- ▶ Linear models & GLMs

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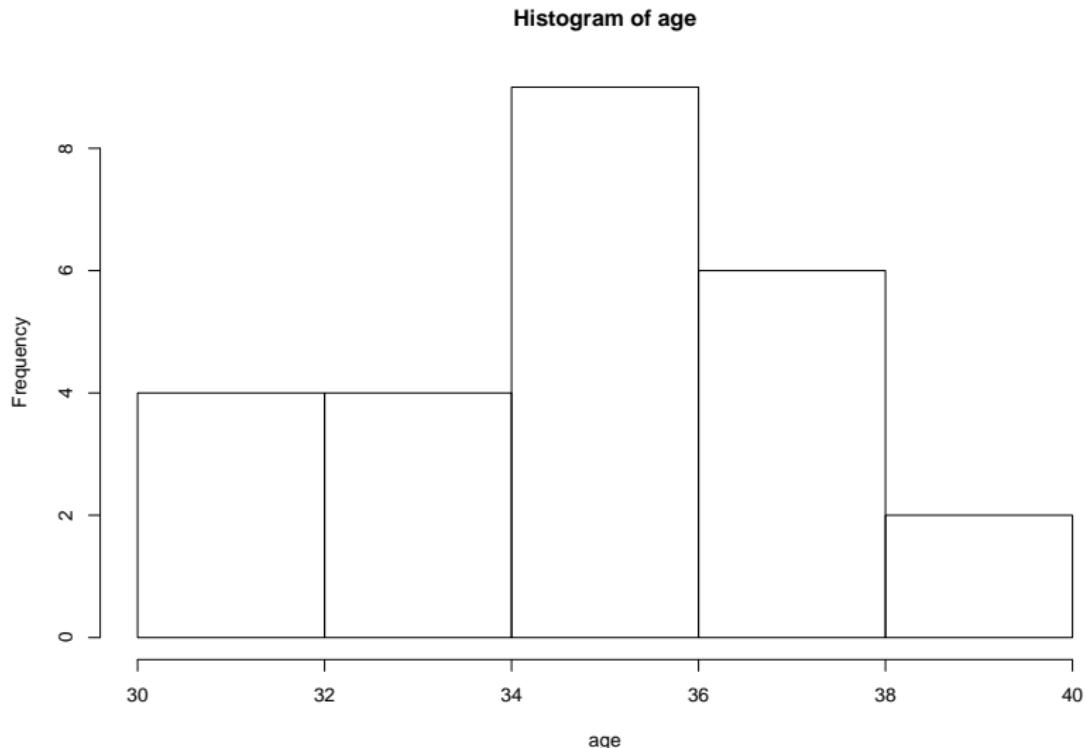
- ▶ Descriptive statistics
- ▶ Graphics
- ▶ Sampling
- ▶ Experimental design
- ▶ Hypothesis testing
- ▶ Bayesian inference
- ▶ Linear models & GLMs
- ▶ Model selection

## Descriptive statistics

Guess my age

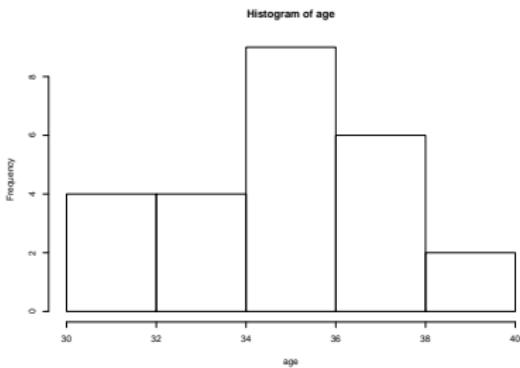
# Graph your estimates

```
hist(age)
```



# Summarise that distribution

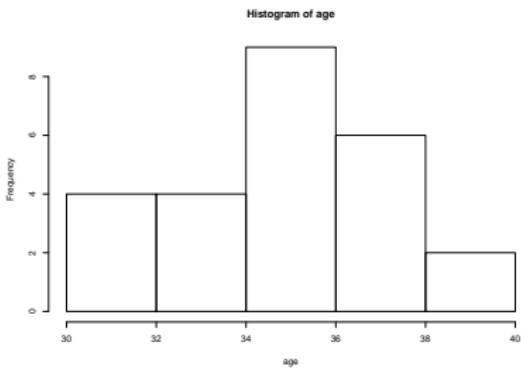
- ▶ Central tendency / location



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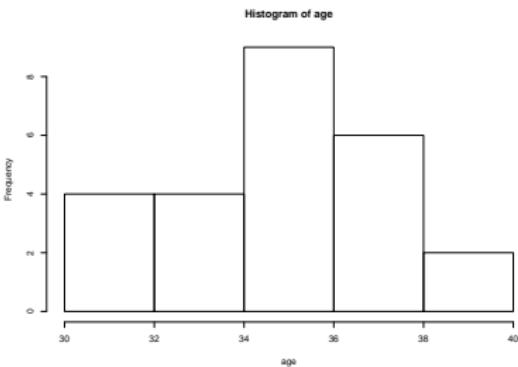
- ▶ mean



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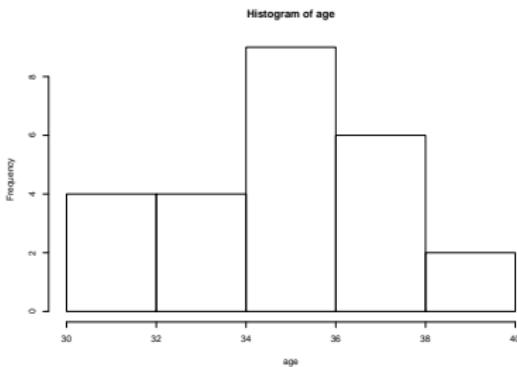
- ▶ mean
- ▶ median



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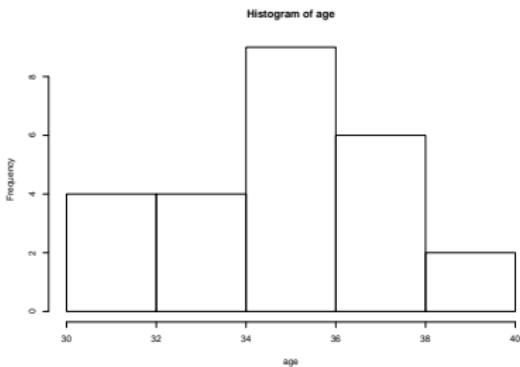
## ► Central tendency / location

- ▶ mean
- ▶ median
- ▶ mode



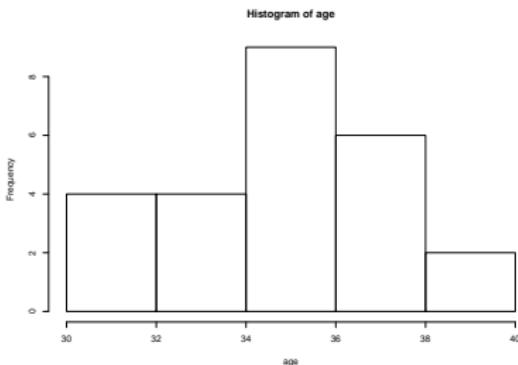
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  - ▶ mode
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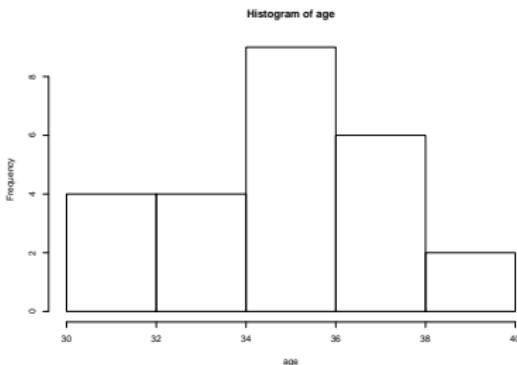
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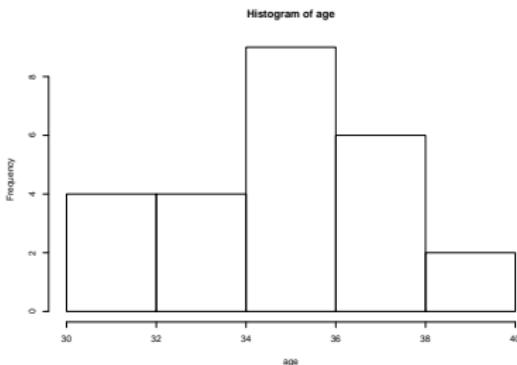
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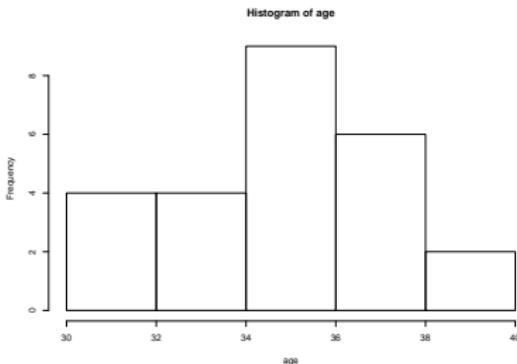
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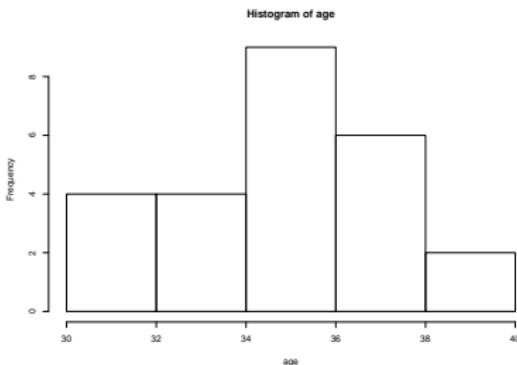
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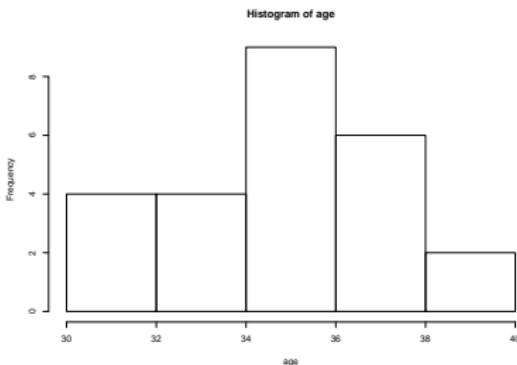
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- ▶ coefficient of variation
- ▶ confidence intervals



## In a Normal distribution

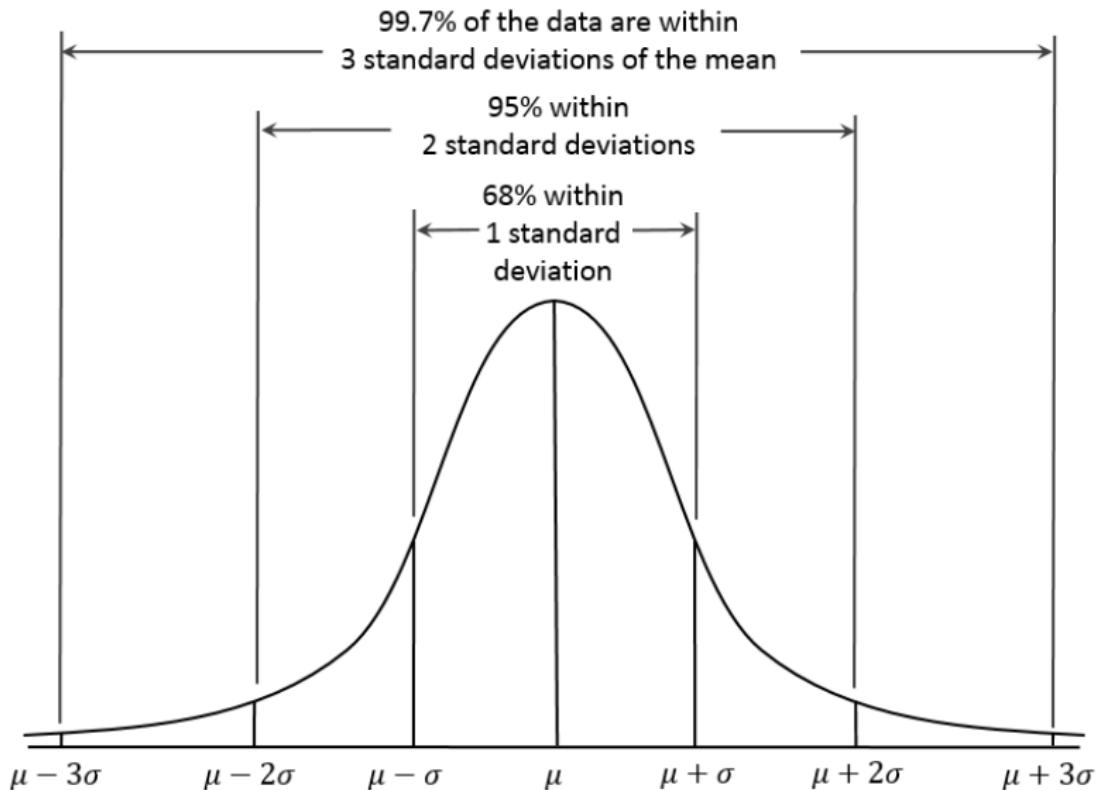
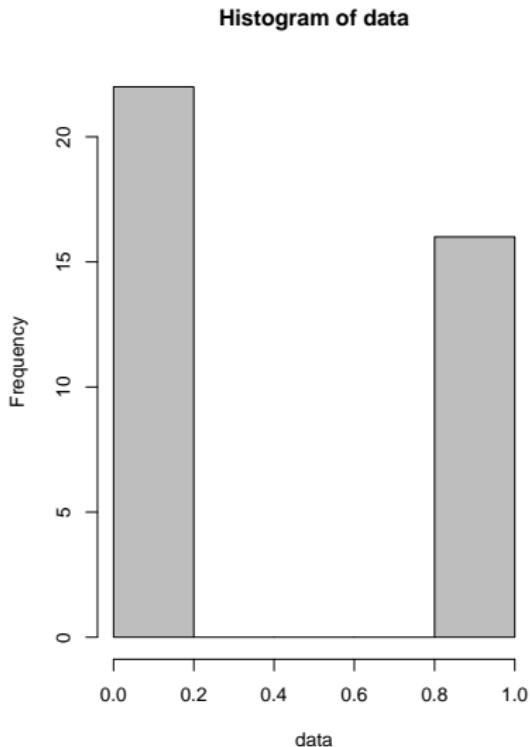
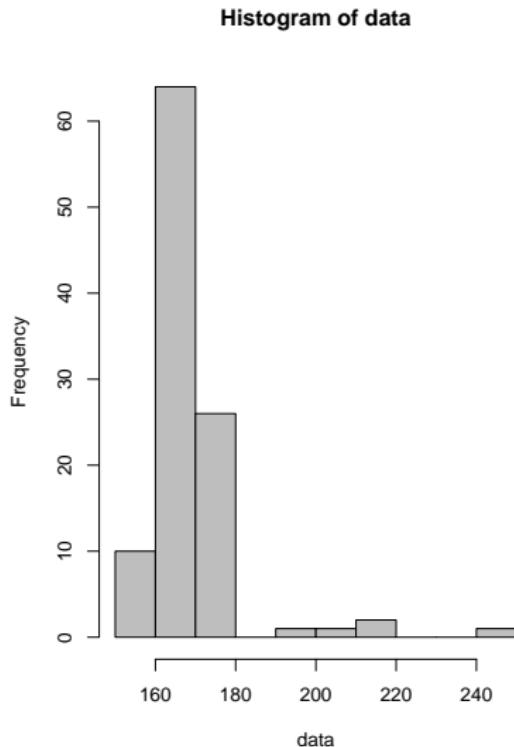


Figure 4:

# What statistical descriptors are best? (and why)

<https://pollev.com/franciscorod726>



# Sampling, confidence intervals, and Bayesian inference

## Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.



Figure 5:

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5. Do all CIs contain true mean height?

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- ▶ The probability that  $X$  is greater than 0 is at least 95%
- ▶ The probability that  $X$  equals 0 is smaller than 5%

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- ▶ To read more: Morey et al (2015)

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- ▶ but still 5% of CIs will NOT contain true mean!

## Bayesian credible intervals

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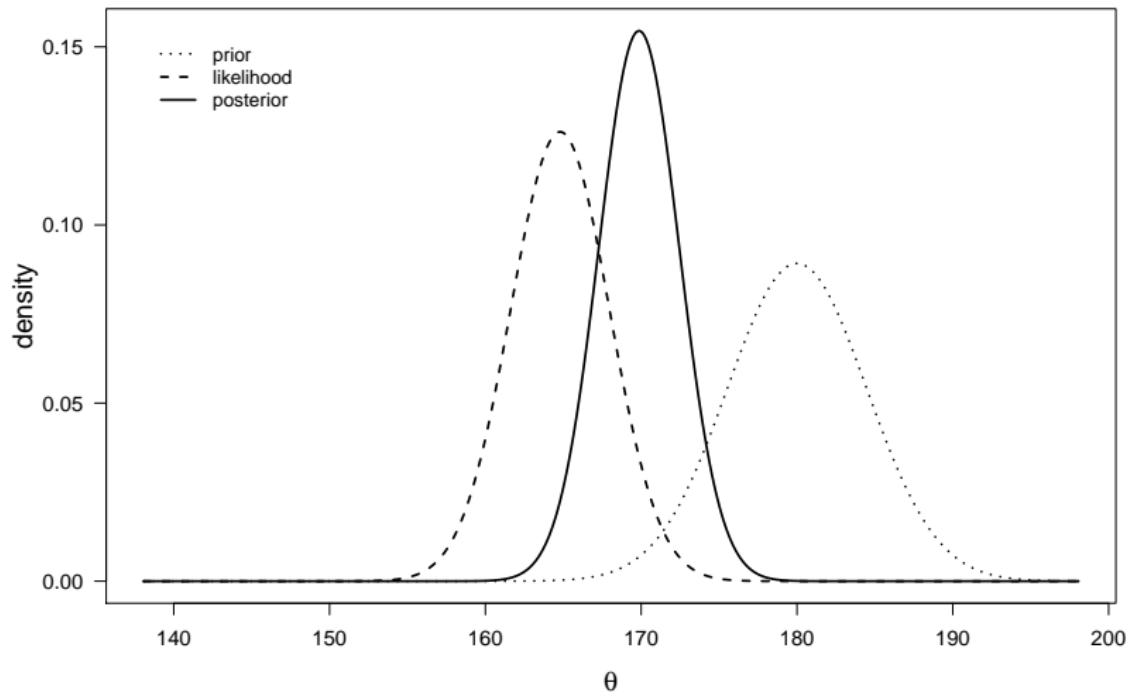
## Bayesian credible intervals

- ▶ Bayesian **credible** intervals do give the probability that true parameter value is contained within them.
- ▶ Frequentist CIs and Bayesian credible intervals can be similar, but not always.

## Bayesian inference: prior, posterior, and likelihood

$$P(H|D) \propto P(D|H) \times P(H)$$

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$



## More apps to introduce Bayesian inference

- ▶ Normal

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- ▶ Normal
- ▶ Binomial

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## More apps to introduce Bayesian inference

- ▶ Normal
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- ▶ Bayesian t-test

# Experimental Design

## How would you evaluate fertilizer effect?

Discuss with partner (5')



Figure 6:

Replication!



Figure 7:

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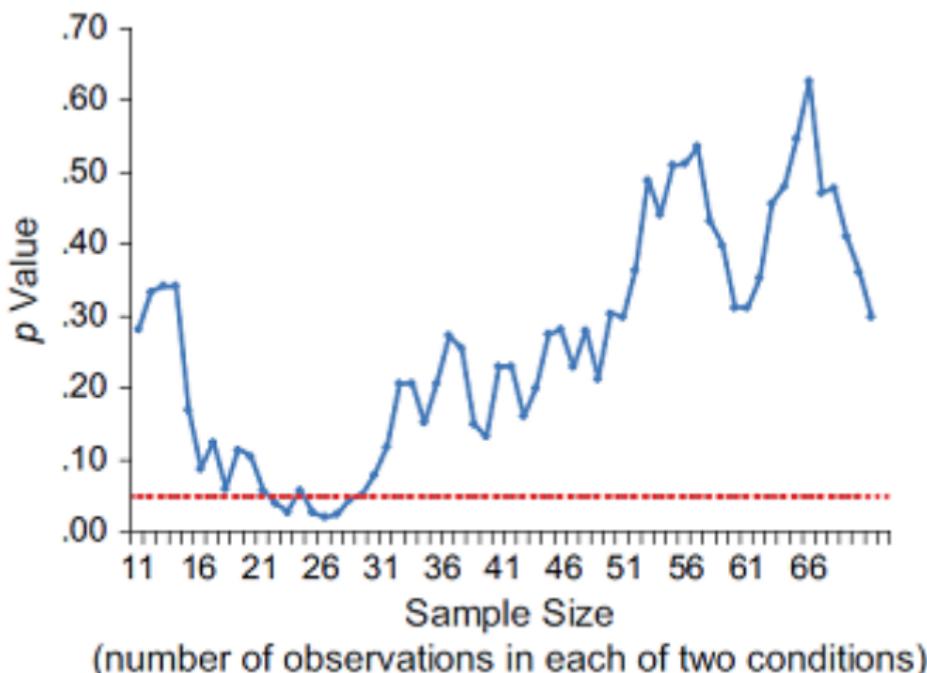
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- ▶ Traditionally, ecology studies have had too low sample sizes.
- ▶ Hence missing many subtle effects, and prone to bias.
- ▶ Complex models (w/ many predictors, interactions etc) require **high** sample sizes.

# Sample size is very important

See *The evolution of correlations*

Stopping rules:



**Fig. 2.** Illustrative simulation of  $p$  values obtained by a researcher who continuously adds an observation to each of two conditions, conducting a  $t$  test after each addition. The dotted line highlights the conventional  $p < 0.05$  threshold. (Source: Fig. 1.25)

## Randomization



Figure 9:

# Randomization

- ▶ Haphazard  $\neq$  Random

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- ▶ Stratify: randomize within groups (e.g. species, soil types)

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- ▶ Measure before & after treatment.
- ▶ Consider blind designs to avoid observer bias.

## To read more

- ▶ Ruxton & Colegrave. Experimental Design for the Life Sciences. OUP

## Hypothesis testing

## Does height differ between local and foreign students?

- ▶ Local people heights:

171 168 182 164 160

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
160	164	168	169	171	182

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- ▶ We know what happens in **our samples**, but want to extrapolate to the whole **population**.

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- ▶ What's the **suitable population** to make inferences given this sample?

## NHST concepts

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- ▶ **Null hypothesis:** there is no difference between groups.
- ▶ **Alternative hypothesis:** groups are different.

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- ▶ Low P-value: data unlikely if H<sub>0</sub> was true.
- ▶ Large P-value: data not unusual if H<sub>0</sub> was true.

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- ▶ More on this later.

## Let's do the test

```
t.test(h.sevi, h.out)
```

Welch Two Sample t-test

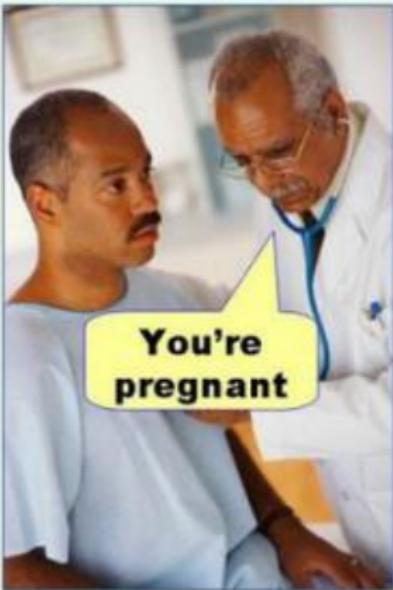
```
data: h.sevi and h.out
t = -0.20558, df = 6.346, p-value = 0.8436
alternative hypothesis: true difference in means is not equal to
95 percent confidence interval:
-14.0216 11.8216
sample estimates:
mean of x mean of y
172.2      173.3
```

**Are heights different then?**

## Rejecting hypotheses: two types of error

### Type I error

(false positive)



### Type II error

(false negative)



Figure 10:

## Rejecting hypotheses: two types of error

Statistics: Hypothesis Test	Null Hypothesis is True	Null Hypothesis is False
Reject Null Hypothesis	Type I Error	Correct
Fail to Reject Null Hypothesis	Correct	Type II Error

Figure 11:

# Understanding NHST

<http://rpsychologist.com/d3/NHST/>

## Example: biased coin

```
[1] 0 1 1 1 0 1 1 0 1 0
```

```
1-sample proportions test with continuity correction
```

```
data: sum(coin) out of ntrials, null probability 0.5  
X-squared = 0.1, df = 1, p-value = 0.7518
```

```
alternative hypothesis: true p is not equal to 0.5
```

```
95 percent confidence interval:
```

```
0.2736697 0.8630694
```

```
sample estimates:
```

```
p  
0.6
```

## Correlation between variables

<http://rpsychologist.com/d3/correlation/>

## Common pitfalls and good practice

# A must read

Eur J Epidemiol (2016) 31:337–350  
DOI 10.1007/s10654-016-0149-3



ESSAY

## **Statistical tests, *P* values, confidence intervals, and power: a guide to misinterpretations**

Sander Greenland<sup>1</sup> · Stephen J. Senn<sup>2</sup> · Kenneth J. Rothman<sup>3</sup> · John B. Carlin<sup>4</sup> ·  
Charles Poole<sup>5</sup> · Steven N. Goodman<sup>6</sup> · Douglas G. Altman<sup>7</sup>

<https://doi.org/10.1007/s10654-016-0149-3>

# Good reading

esa

ECOSPHERE

## Applied statistics in ecology: common pitfalls and simple solutions

E. ASHLEY STEEL,<sup>1,†</sup> MAUREEN C. KENNEDY,<sup>2</sup> PATRICK G. CUNNINGHAM,<sup>3</sup> AND JOHN S. STANOVICK<sup>4</sup>

Figure 12:

<http://dx.doi.org/10.1890/ES13-00160.1>

Also <http://www.statisticsonewrong.com/>

## First things first

- ▶ Always

## First things first

- ▶ Always
- ▶ Always

## First things first

- ▶ Always
- ▶ Always
- ▶ Always

## Plot data and models

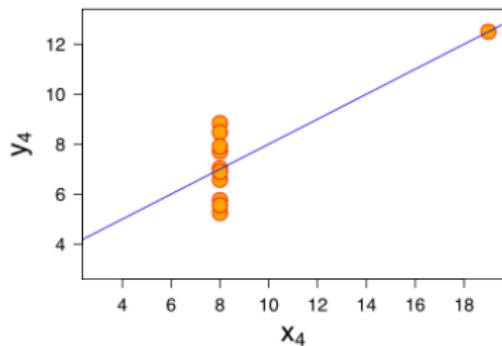
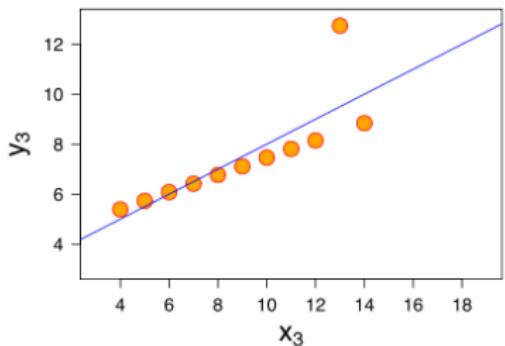
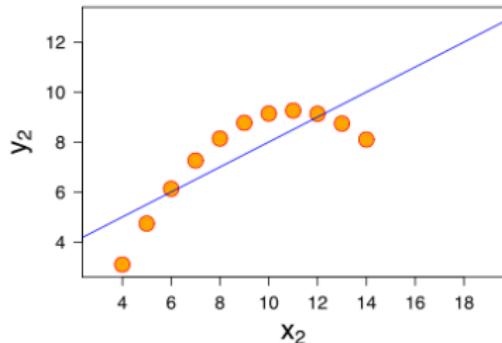
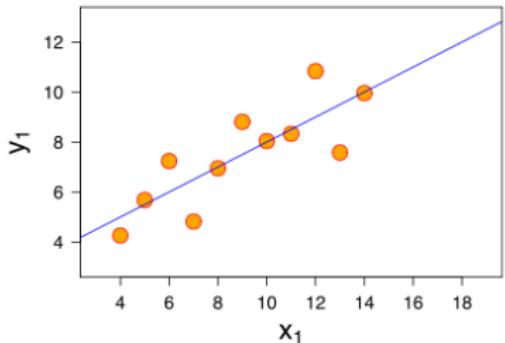
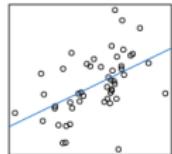


Figure 13:

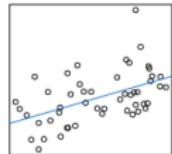
# Don't use statistics blindly: Visualise

All correlations:  $r(50) = 0.5$

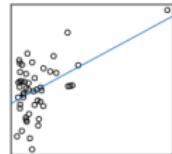
(1) Normal x, normal residuals



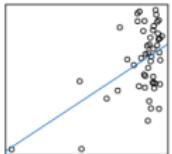
(2) Uniform x, normal residuals



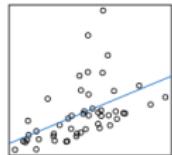
(3) +-skewed x, normal residuals



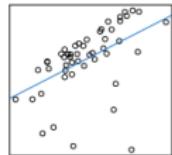
(4) --skewed x, normal residuals



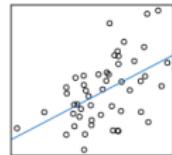
(5) Normal x, +-skewed residuals



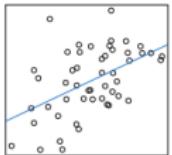
(6) Normal x, --skewed residuals



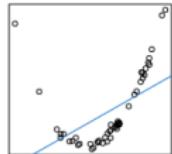
(7) Increasing spread



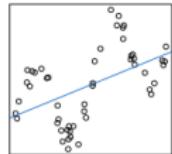
(8) Decreasing spread



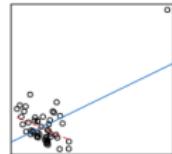
(9) Quadratic trend



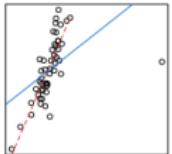
(10) Sinusoid relationship



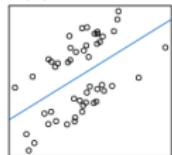
(11) A single positive outlier



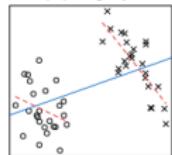
(12) A single negative outlier



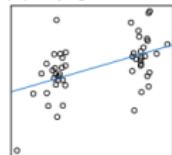
(13) Bimodal residuals



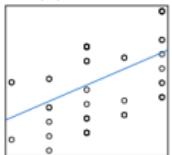
(14) Two groups



(15) Sampling at the extremes



(16) Coarse data

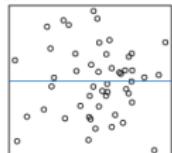


<https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like>

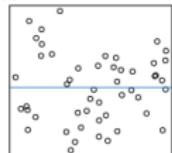
# Don't use statistics blindly: Visualise

All correlations:  $r(50) = 0$

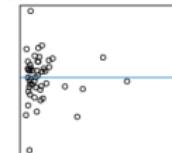
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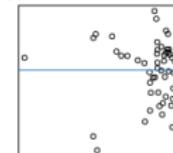
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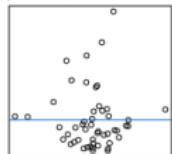
(3) +-skewed x, normal residuals



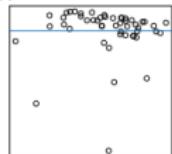
(4) --skewed x, normal residuals



(5) Normal x, +-skewed residuals



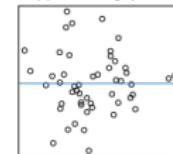
(6) Normal x, --skewed residuals



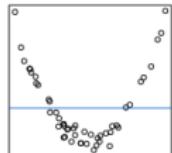
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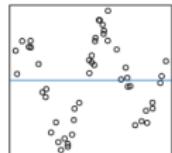
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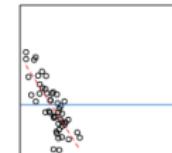
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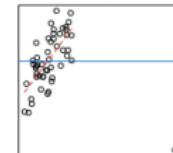
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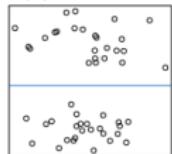
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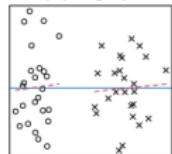
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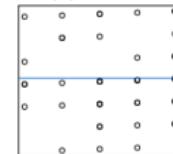
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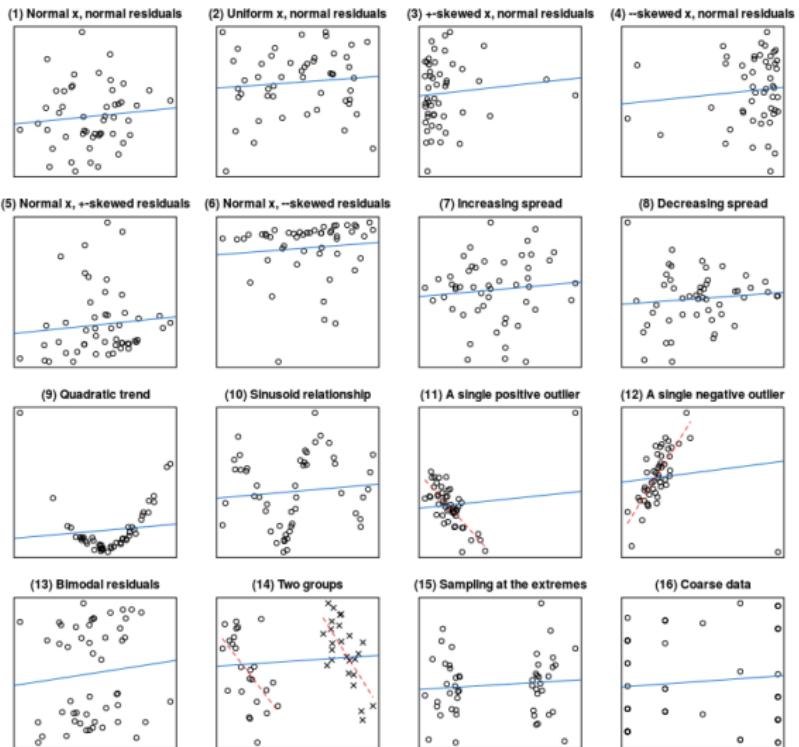
(16) Coarse data



<https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like>

# Don't use statistics blindly: Visualise

All correlations:  $r(50) = 0.1$



[https:](https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like)

//janhove.github.io/teaching/2016/11/21/what-correlations-look-like

***Plot. Check models. Plot. Check assumptions. Plot.***

Lavine 2014 *Ecology*

## News: Hamburgers increase risk of heart attack

- ▶ In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.

## News: Hamburgers increase risk of heart attack

- ▶ In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.
- ▶ **Do hamburgers increase heart attacks?**

## News: Hamburgers increase risk of heart attack

- ▶ In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.
- ▶ **Do hamburgers increase heart attacks?**
- ▶ <https://pollev.com/franciscorod726>

## Bigger flowers increase reproductive success

- ▶ We found that plants with big flowers produced 30% more seeds...

## Bigger flowers increase reproductive success

- ▶ We found that plants with big flowers produced 30% more seeds...
- ▶ **Do big flowers increase reproductive success?**

## Bigger flowers increase reproductive success

- ▶ We found that plants with big flowers produced 30% more seeds...
- ▶ **Do big flowers increase reproductive success?**
- ▶ <https://pollev.com/franciscorod726>

# Correlation vs Causation

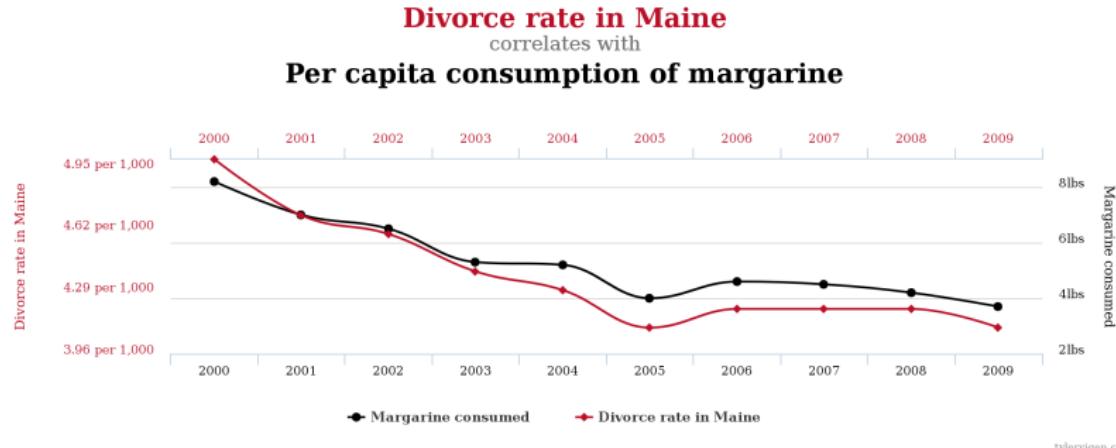


Figure 14:

<http://tylervigen.com/spurious-correlations>

# Learning statistics through xkcd



# P-value depends on sample size

- ▶ Same real difference is detected as significant or not depending on sample size:

Real difference = 40 g

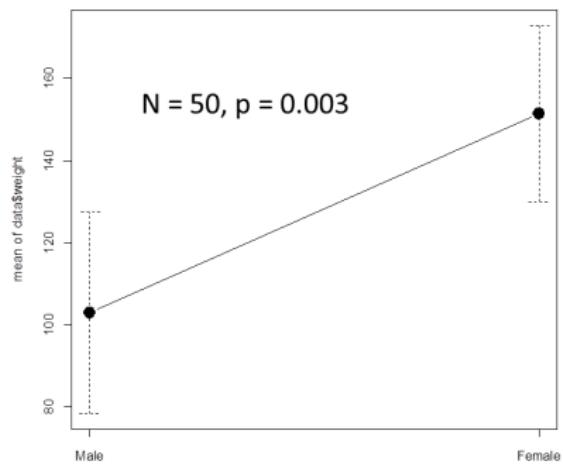
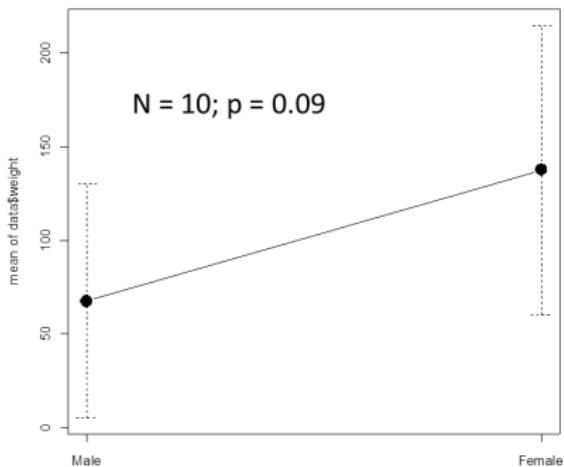
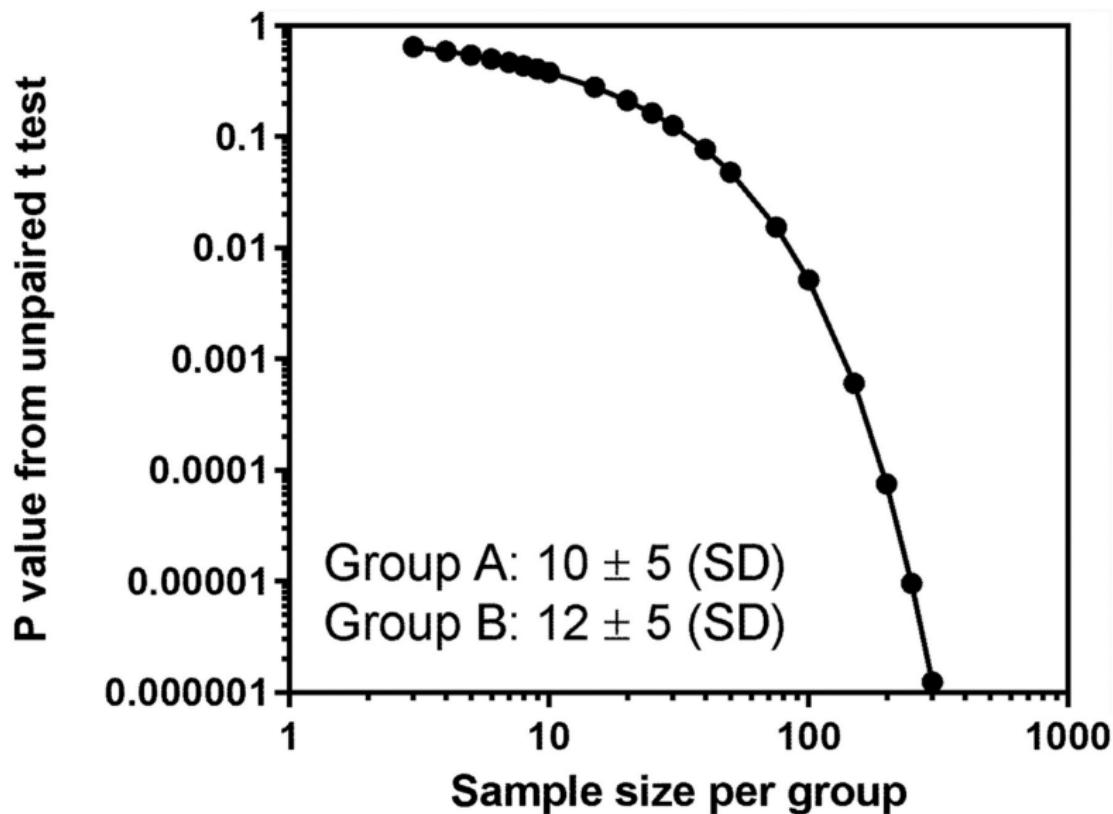


Figure 15:

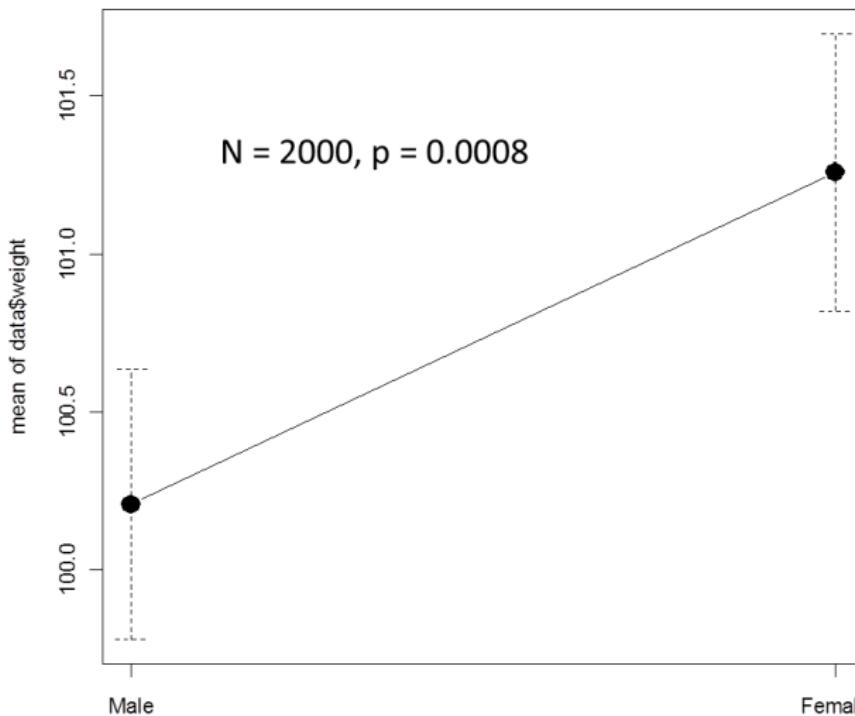
## P-value depends on sample size



## Statistically significant != biologically important

- With big sample size, we can find **highly significant but biologically unimportant** differences.

Real difference = 1 g



## Statistically significant != biologically important

- ▶ Statistically significant = unlikely to be zero

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- ▶ Suggestion: Try to avoid 'significant' all together

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- ▶ Suggested reading: *significantly misleading*

## Statistically significant != biologically important

- ▶ Statistically significant = unlikely to be zero
- ▶ Suggestion: Try to avoid 'significant' all together
- ▶ Suggested reading: *significantly misleading*
- ▶ Beyond significance, look at *effect sizes*.

'Not significant' does NOT mean 'there is no effect'

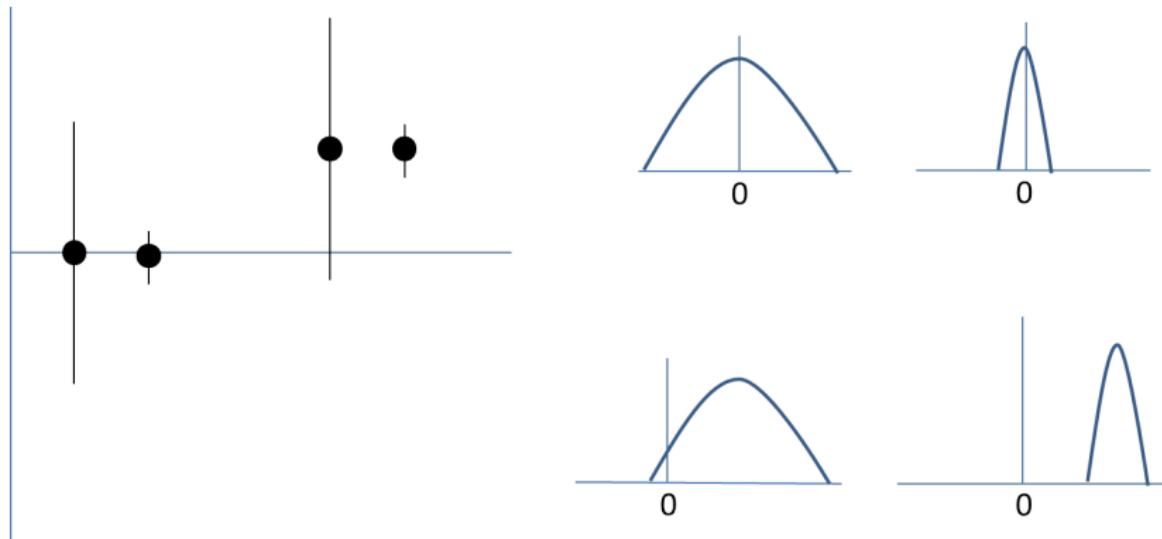


Figure 17:

- ▶ **Absence of evidence  $\neq$  Evidence of absence**

Failure to reject  $H_0$   $\neq H_0$  is true

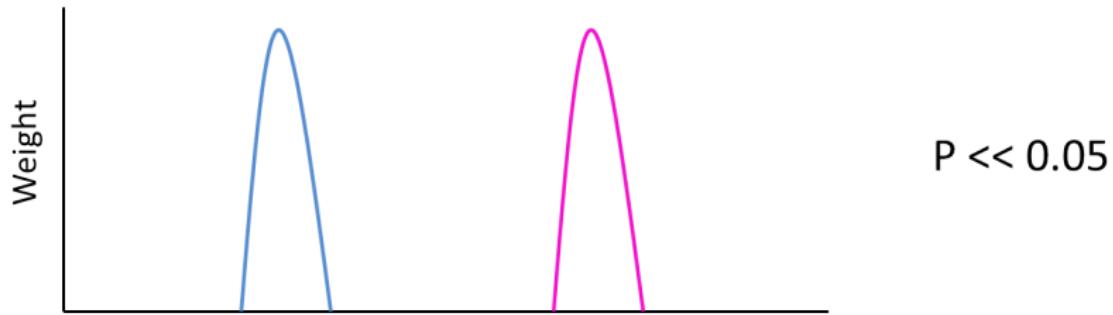
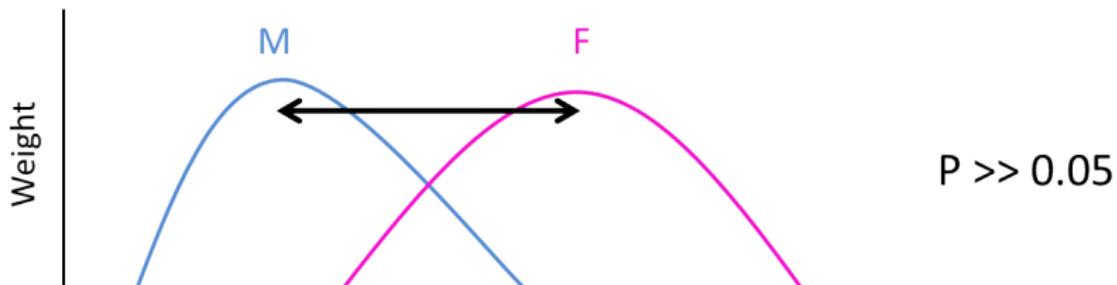


Figure 18:

0.05 is an arbitrary threshold

**The Difference Between “Significant” and “Not Significant” is not  
Itself Statistically Significant**

Andrew GELMAN and Hal STERN

Figure 19:

<http://dx.doi.org/10.1198/000313006X152649>

# Multiple hypothesis testing

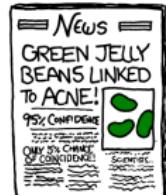
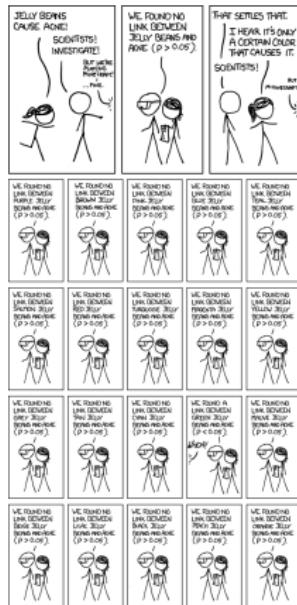
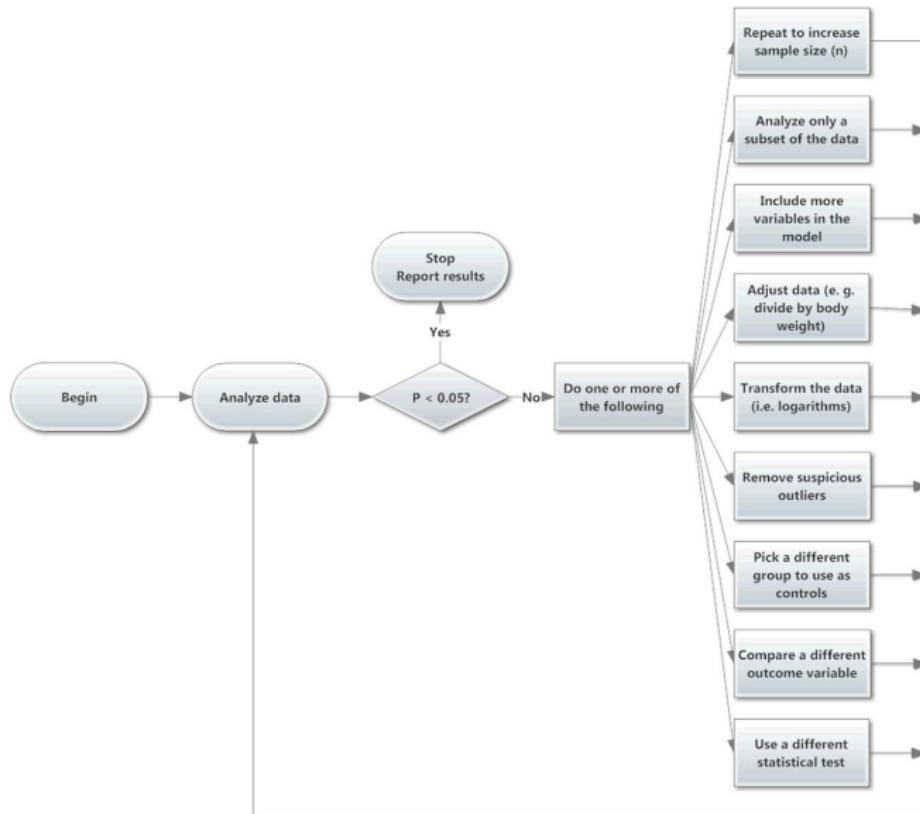


Figure 20:

# How to make your results significant: *p-hacking*



## How to make your results significant: *p-hacking*

1. Test multiple variables, then report the ones that are significant.

## How to make your results significant: *p-hacking*

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2. Artificially choose when to end your experiment.

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3. Add covariates until effects are significant.

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4. Test different conditions (e.g. different levels of a factor) and report the ones you like.

## How to make your results significant: *p-hacking*

1. Test multiple variables, then report the ones that are significant.
  2. Artificially choose when to end your experiment.
  3. Add covariates until effects are significant.
  4. Test different conditions (e.g. different levels of a factor) and report the ones you like.
- To read more: Simmons et al 2011

## How to make your results significant: *p-hacking*

<https://www.youtube.com/watch?v=ZaNtz76dNSI>

## ASA statement on p-values

- ▶ P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.

<https://doi.org/10.1080/00031305.2016.1154108>

## ASA statement on p-values

- ▶ P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.
- ▶ Scientific conclusions or policy decisions should NOT be based only on **whether a p-value passes a specific threshold**.

<https://doi.org/10.1080/00031305.2016.1154108>

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- ▶ P-value, or statistical significance, does not measure the **size of an effect** or the **importance** of a result.

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- ▶ P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.
- ▶ Scientific conclusions or policy decisions should NOT be based only on **whether a p-value passes a specific threshold**.
- ▶ P-value, or statistical significance, does not measure the **size of an effect** or the **importance** of a result.
- ▶ By itself, a p-value does NOT provide a good **measure of evidence** regarding a model or hypothesis.

<https://doi.org/10.1080/00031305.2016.1154108>

# The New Statistics

Aim for estimation of effects and their uncertainty.

*General Article*

## The New Statistics: Why and How

**Geoff Cumming**

La Trobe University



Psychological Science  
2014, Vol. 25(1) 7–29  
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DOI: 10.1177/0956797613504966  
[pss.sagepub.com](http://pss.sagepub.com)



Figure 21:

<http://dx.doi.org/10.1177/0956797613504966>

## How many types of errors?

- ▶ **Type I:** False positive (incorrect rejection of null hypothesis).

## How many types of errors?

- ▶ **Type I:** False positive (incorrect rejection of null hypothesis).
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- ▶ **Type S (Sign):** estimating effect in opposite direction.

## How many types of errors?

- ▶ **Type I:** False positive (incorrect rejection of null hypothesis).
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- ▶ **Type S (Sign):** estimating effect in opposite direction.
- ▶ **Type M (Magnitude):** Misestimating magnitude of the effect (under or overestimating).

## How many types of errors?

- ▶ **Type I:** False positive (incorrect rejection of null hypothesis).
- ▶ **Type II:** False negative (failure to reject false null hypothesis).
- ▶ **Type S (Sign):** estimating effect in opposite direction.
- ▶ **Type M (Magnitude):** Misestimating magnitude of the effect (under or overestimating).
- ▶ Beyond Power Calculations: Assessing Type S (Sign) and Type M (Magnitude) Errors

END



Figure 22:

Source code and materials: <https://github.com/Pakillo/stats-intro>