Sampling, confidence intervals, likelihood and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples**.

Then we make inferences from sample to population.

If we sample 30 trees in our neighbourhood...

Can we extrapolate results to

- whole neighbourhood?
- whole city?
- whole country?
- the world?

What's the **suitable population** to make inferences given this sample?

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- To read more: Morey et al (2015), Rohrer 2024

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- but still 5% of CIs will NOT contain true mean!

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https://pollev.com/franciscorod726

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- If we repeated the experiment, 95% of the CIs would contain the true value of X
- The probability that X is greater than 0 is at least 95%
- The probability that X equals 0 is smaller than 5%

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Bayesian credible intervals

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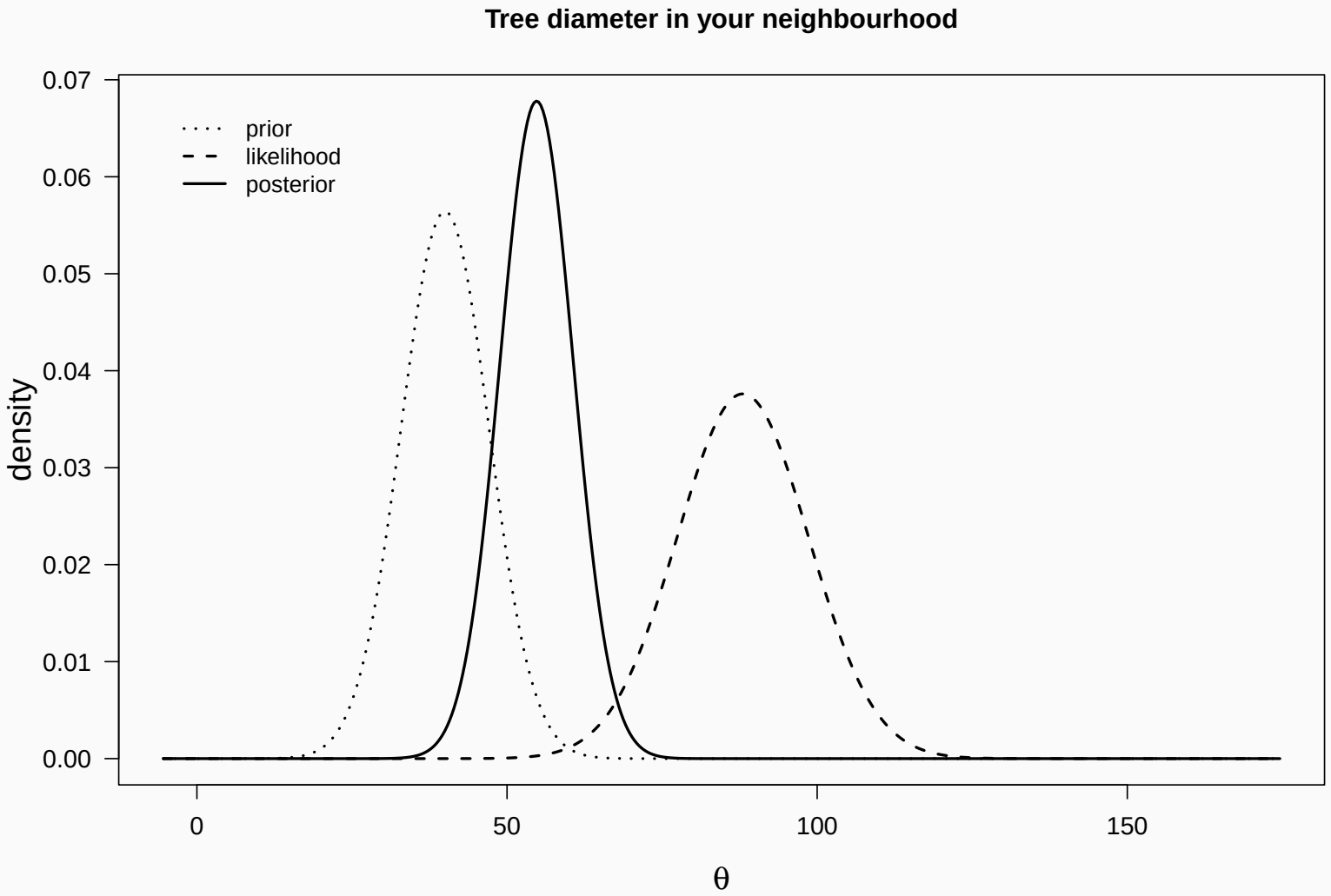
- Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

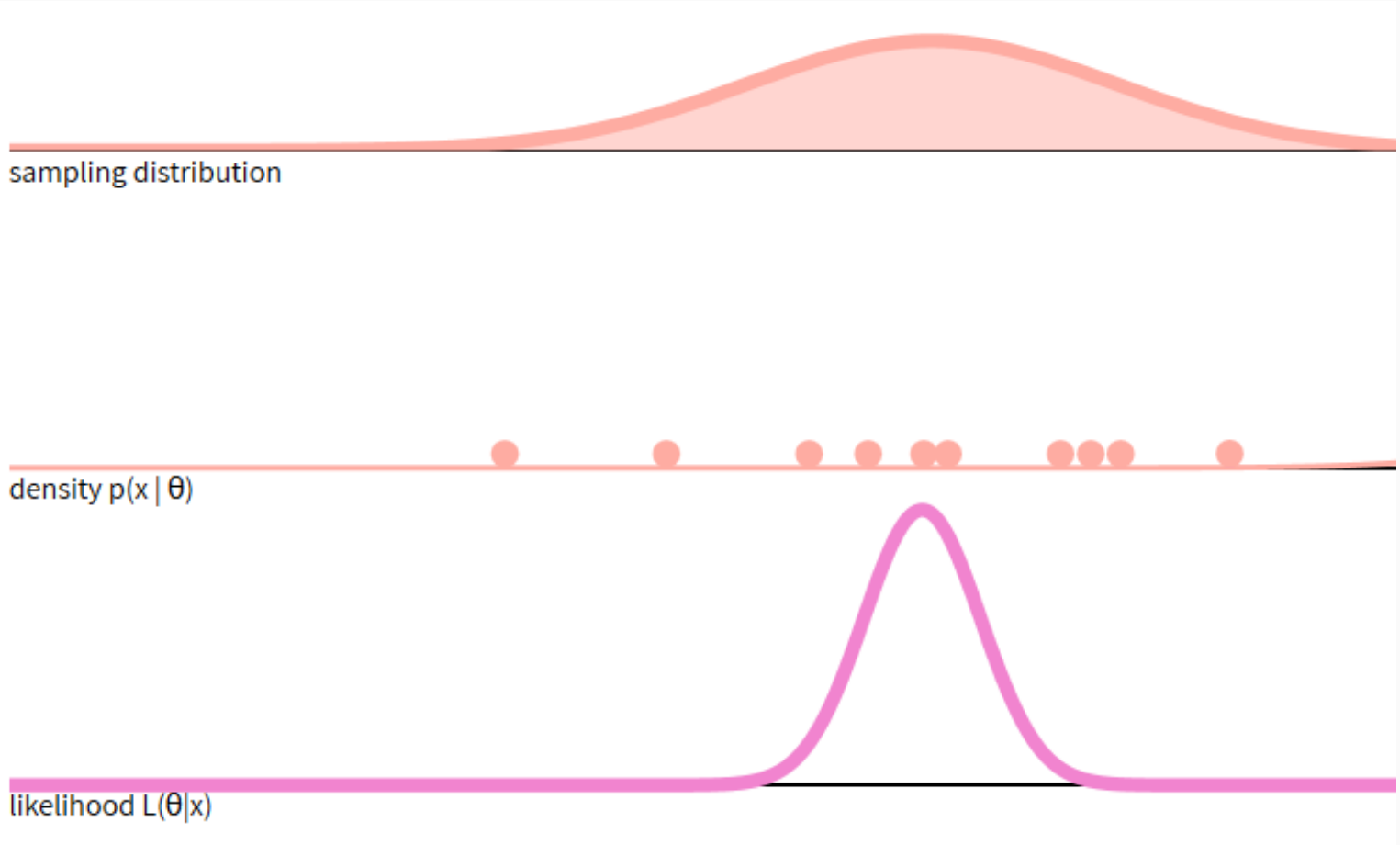
$$P(Unknown|Data) \propto P(Data|Unknown) \times P(Unknown)$$

$$Posterior \propto Likelihood \times Prior$$



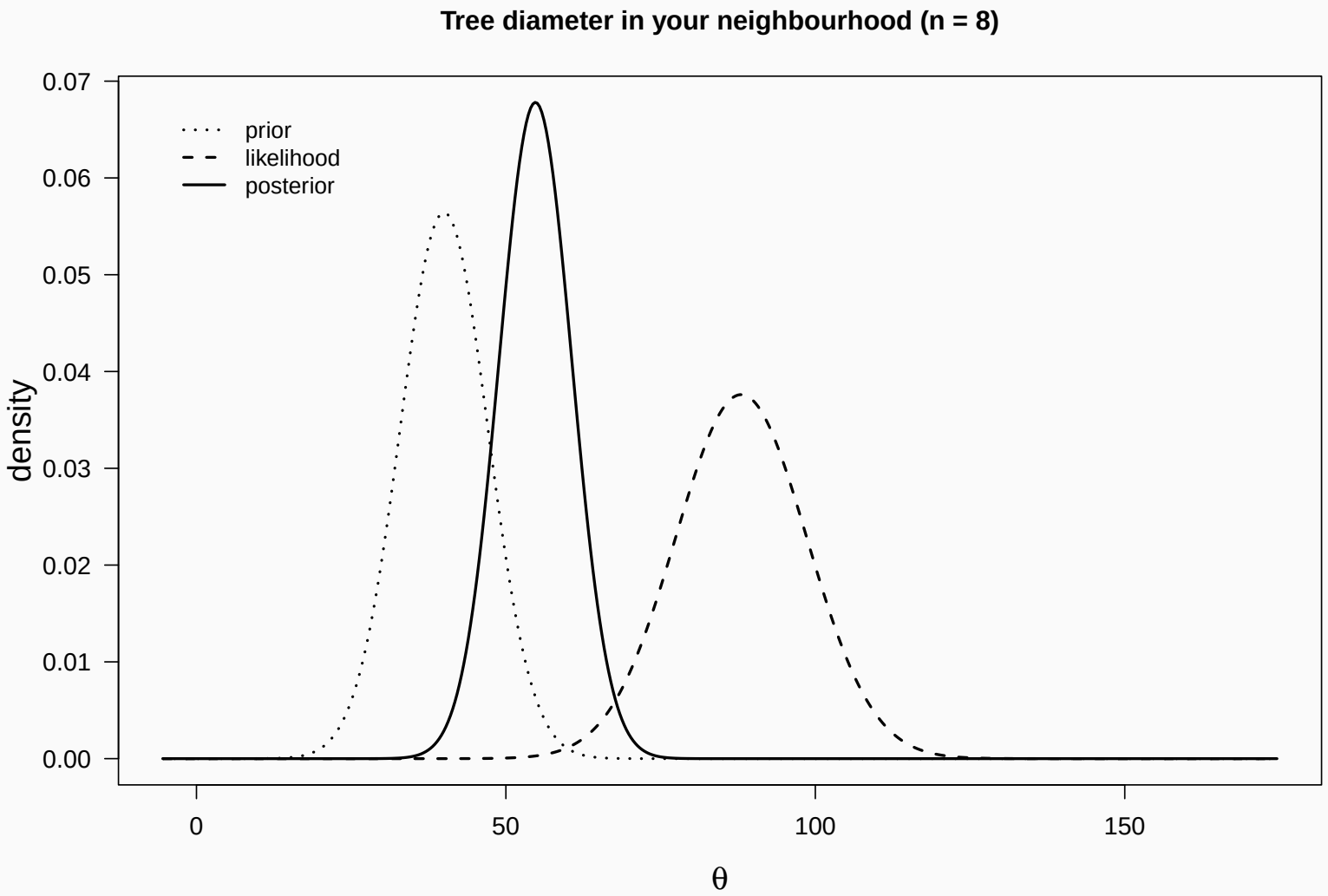
What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$

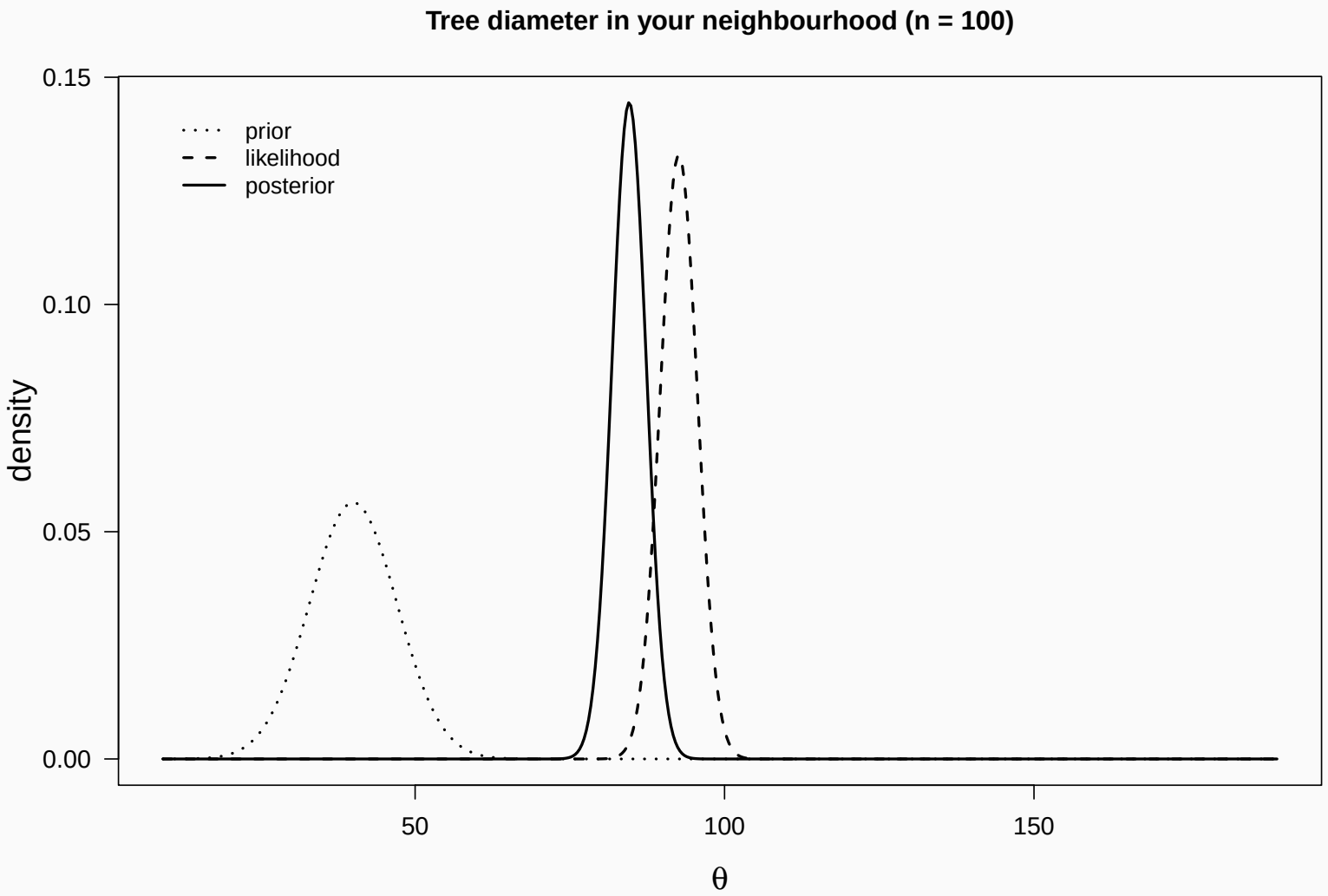


https://seeing-theory.brown.edu/bayesian-inference/index.html

Bayesian inference: prior and likelihood produce posterior



With increasing sample size, likelihood dominates prior



More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference

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Bayesian statistics in practice

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- Uncertainty / Propagate errors

The base rate fallacy

Do you have COVID?

- You test positive

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Do you have COVID?

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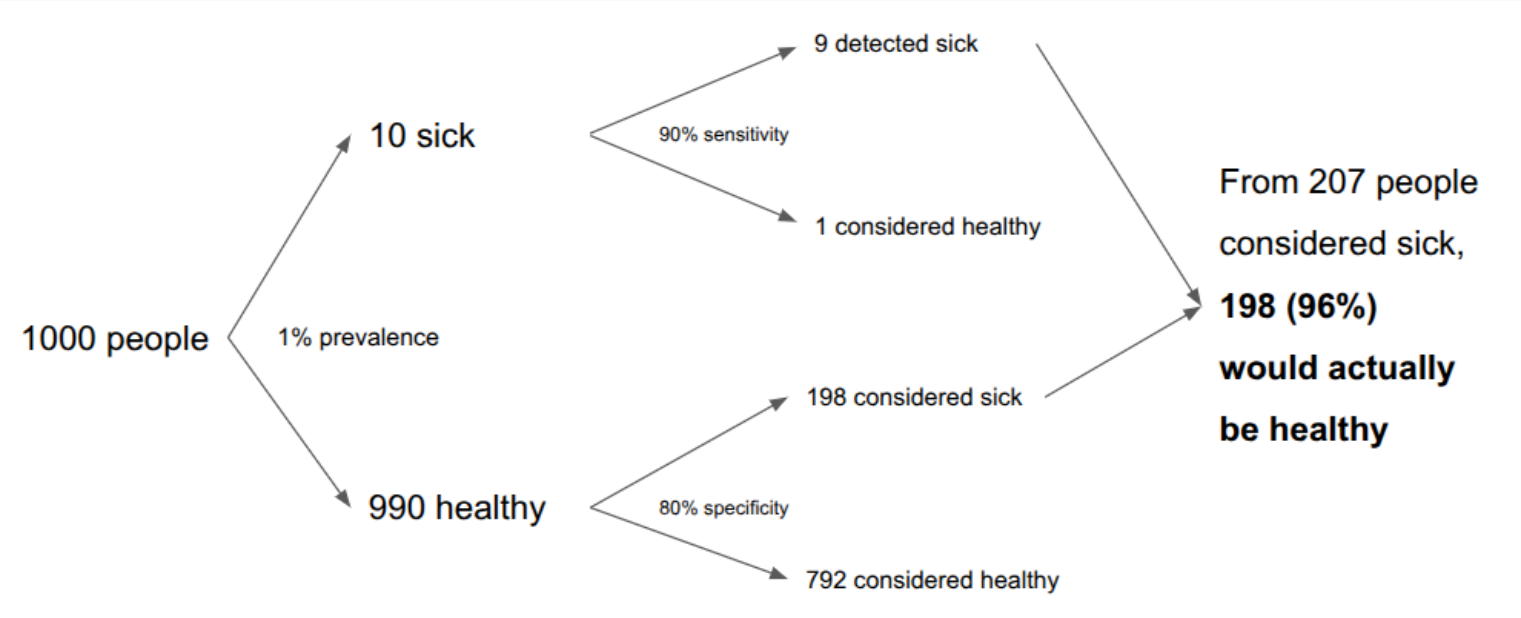
- Test can detect positive COVID 90% of times (sensitivity)
- Test correctly rejects COVID 80% of times (specificity)
- Currently 1% people have COVID
- What's the probability that you have COVID?

The probability that you have COVID is 0.06!



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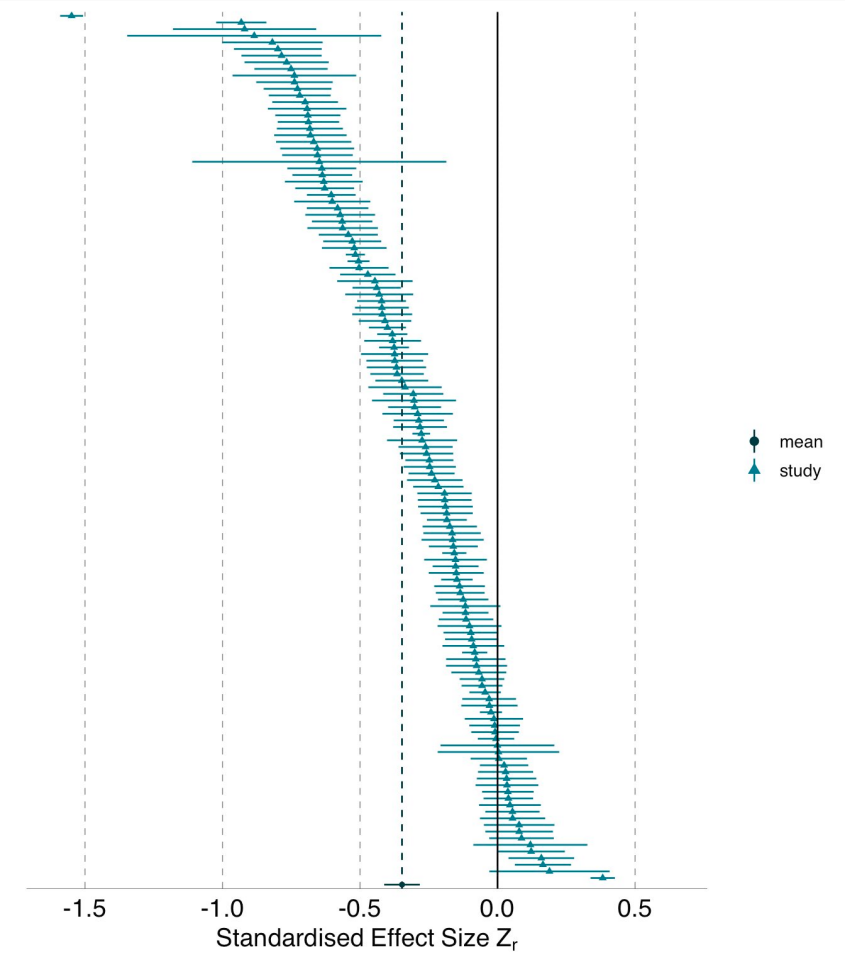
Uncertainty!

Even with same data, different teams reach different conclusions!

How does sibling competition affect nestling growth in blue tits?

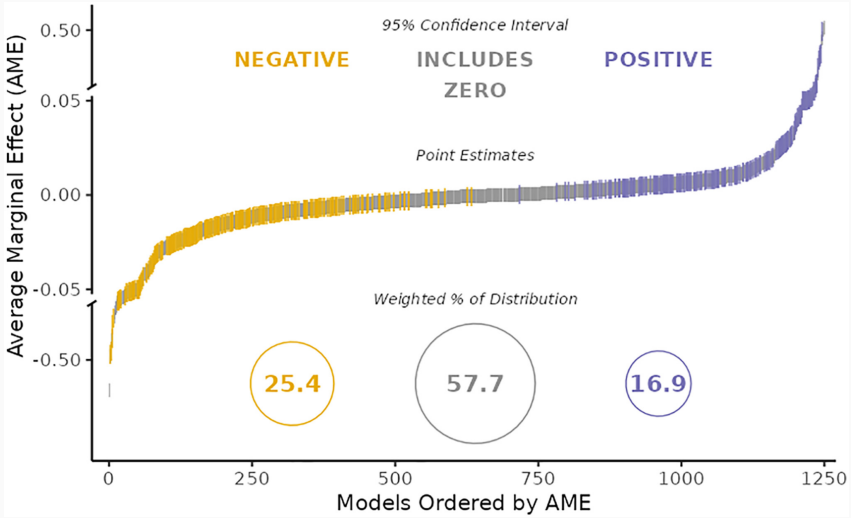
Most teams found negative effect, but large variation in effect size. Some teams no or positive effect



73 teams testing the same hypothesis with the same data

PNAS RESEARCH ARTICLE | SOCIAL SCIENCES OPEN ACCESS

Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty



"This reveals a **universe of uncertainty** that remains hidden when considering a single study in isolation"

"These results call for greater **epistemic humility** and **clarity in reporting** scientific findings"

Breznau et al 2022