Model selection

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- *Non-nested models*: compare fit of different models (e.g. alternative hypotheses)
  - But building larger model might be better than choosing any of them!

Overfitting and balanced model complexity

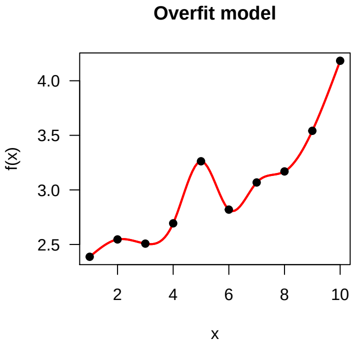

Figure 1: Overfitted model

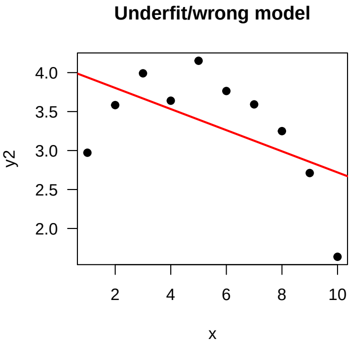

Figure 2: Wrong model

Overfitting: an example with niche modelling

Wenger & Olden (2012) Assessing transferability of ecological models: an underappreciated aspect of statistical validation. *Methods Ecol Evol*.

GLMM

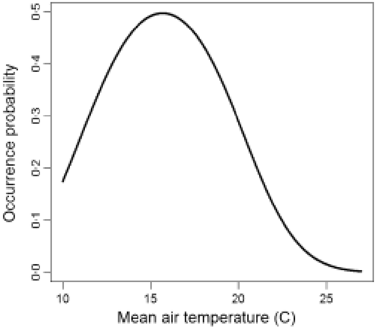

Figure 3:

Random forests (overfit)

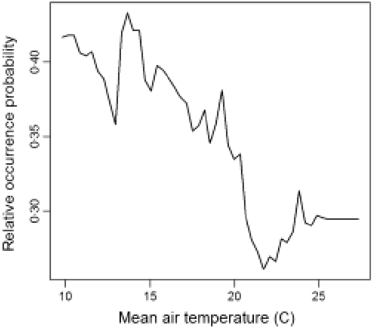

Figure 4:

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- On the other hand, we want to avoid overfitting and overly complex models.

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- All these methods have flaws!

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- Doesn't work with hierarchical models or informative priors!

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- They give average out-of-sample prediction error, but prediction errors can differ substantially within the same dataset (e.g. populations, species).
- Sometimes better models rank poorly (Gelman et al. 2013). So, combine with thorough model checks.

So which variables should enter my model?

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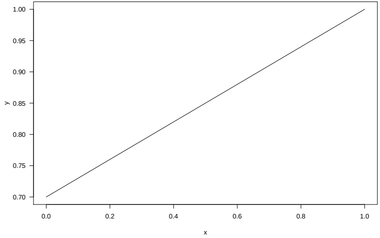
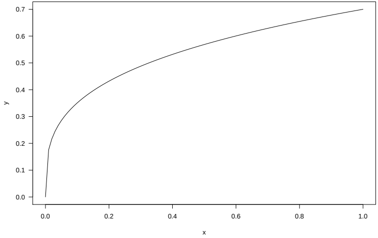
- See also Zuur et al 2010.

Think about the shape of relationships

$y \sim x + z$
Really? Not everything has to be linear! Actually, it often is not.
Think about shape of relationship. See chapter 3 in Bolker's book.





Removing predictors

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- This includes stepAIC (e.g. Dahlgren 2010; Burnham et al 2011; Hegyi & Garamszegi 2011).

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- Significant + expected sign = keep it!

The modelling process

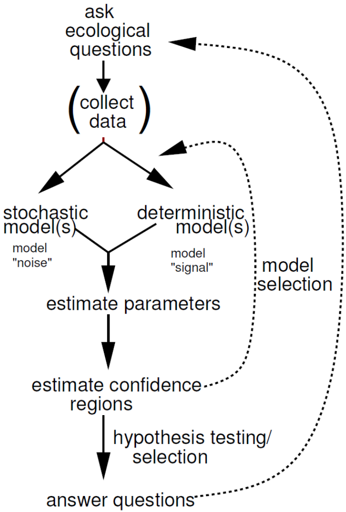

Figure 1.5 Flow of the modeling process.

Figure 6:

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