Sampling, confidence intervals, and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.

Figure 1:

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5. Do all CIs contain true mean height?

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https://pollev.com/franciscorod726

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▶ The probability that X is greater than 0 is at least 95%
▶ The probability that X equals 0 is smaller than 5%

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▶ To read more: Morey et al (2015)

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- ▶ but still 5% of CIs will NOT contain true mean!

Bayesian credible intervals

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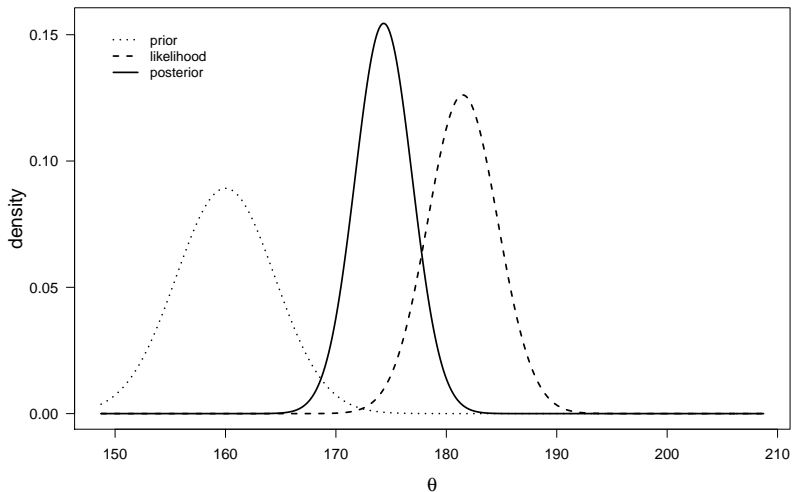
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- Frequentist CIs and Bayesian credible intervals can be similar, but not always.
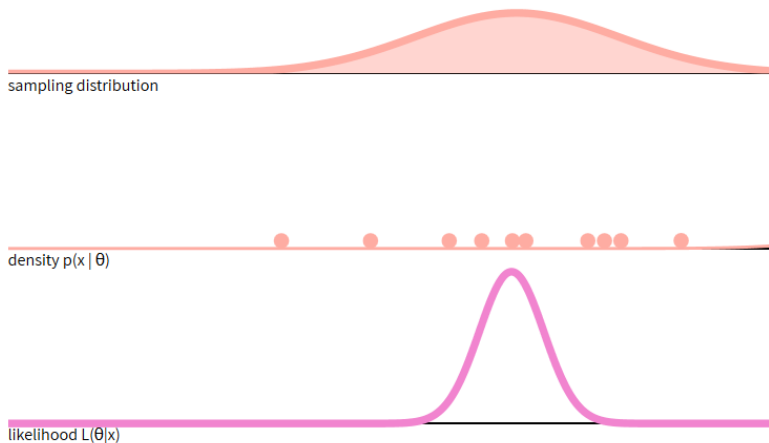
Bayesian inference: prior, posterior, and likelihood

$P(H|D) \propto P(D|H) \times P(H)$

Posterior $\propto$ Likelihood $\cdot$ Prior

What is the likelihood?
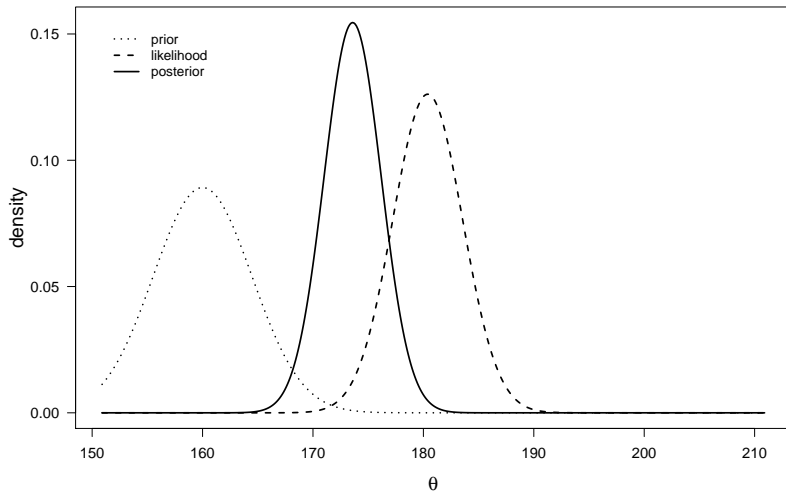
$$L(\theta|x) = P(x|\theta)$$

sampling distribution

density p(x | θ)

likelihood L(θ|x)

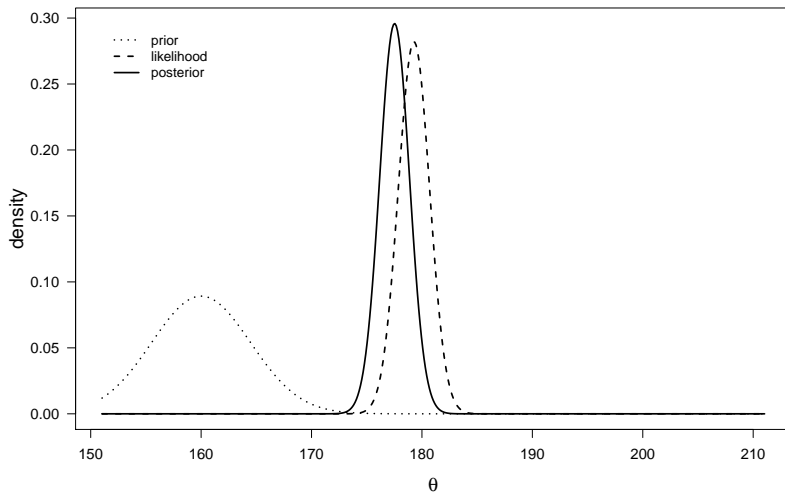https://seeing-theory.brown.edu/bayesian-inference/index.html

Bayesian inference: prior and likelihood produce posterior


```
$posterior.mean
```

With increasing sample size, likelihood dominates prior


```
$posterior.mean
```

More apps to introduce Bayesian inference

- Normal

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- ▶ Normal
- ▶ Binomial

More apps to introduce Bayesian inference

- Normal
- Binomial
- Own data

More apps to introduce Bayesian inference

- Normal
- Binomial
- Own data
- Bayesian t-test