An introduction to statistical inference

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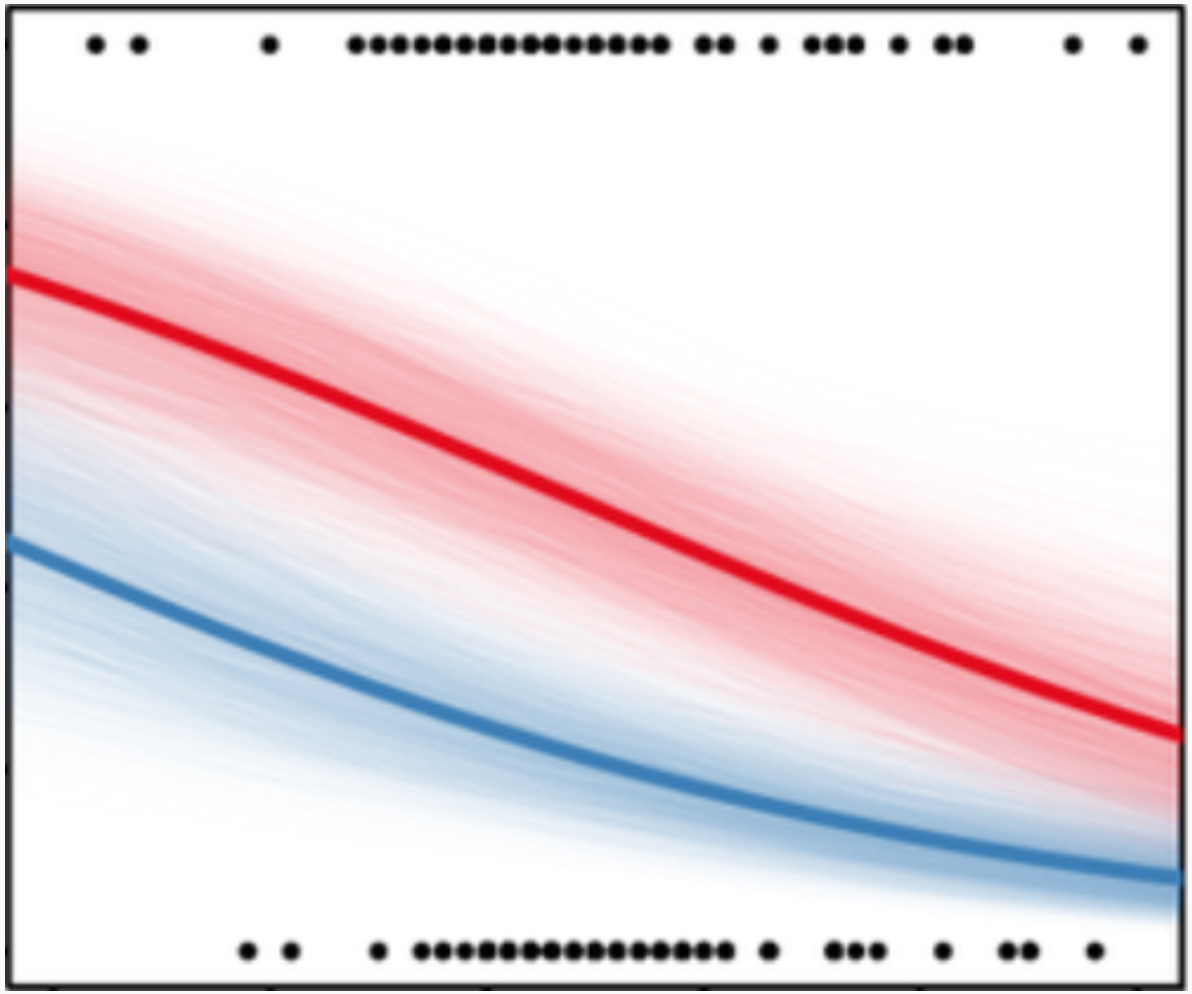
https://frodriguezsanchez.net



Why statistics?

To answer questions like...

- what's the probability that something occurs?

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- does X influence Y? How much?

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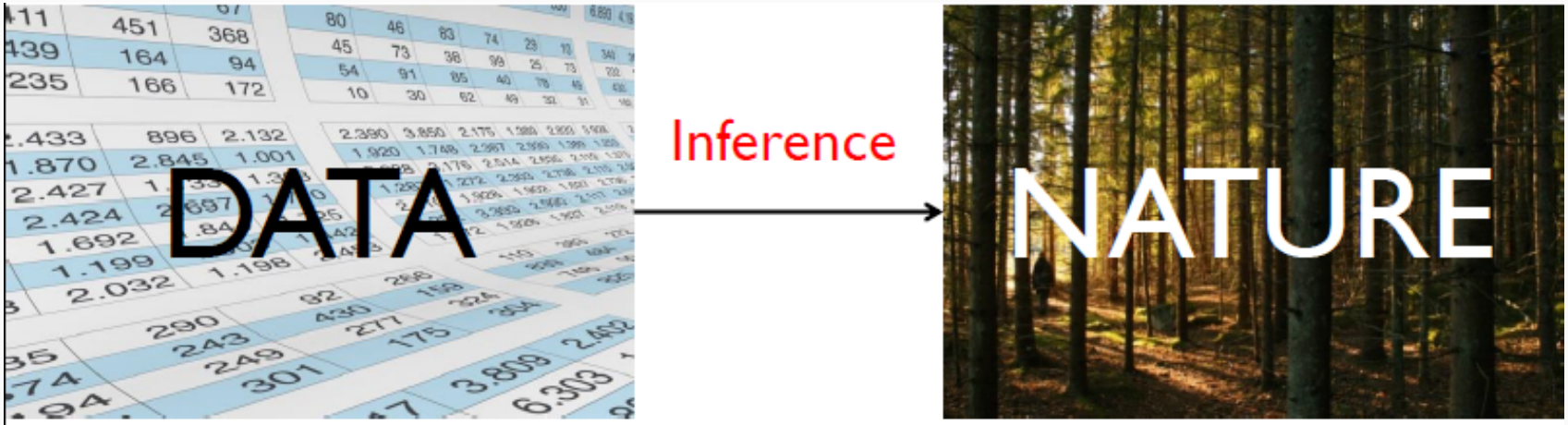
- what's the probability that something occurs?
- does X influence Y? How much?
- can we predict Y knowing X, Z... How well?

To ensure correct inferences

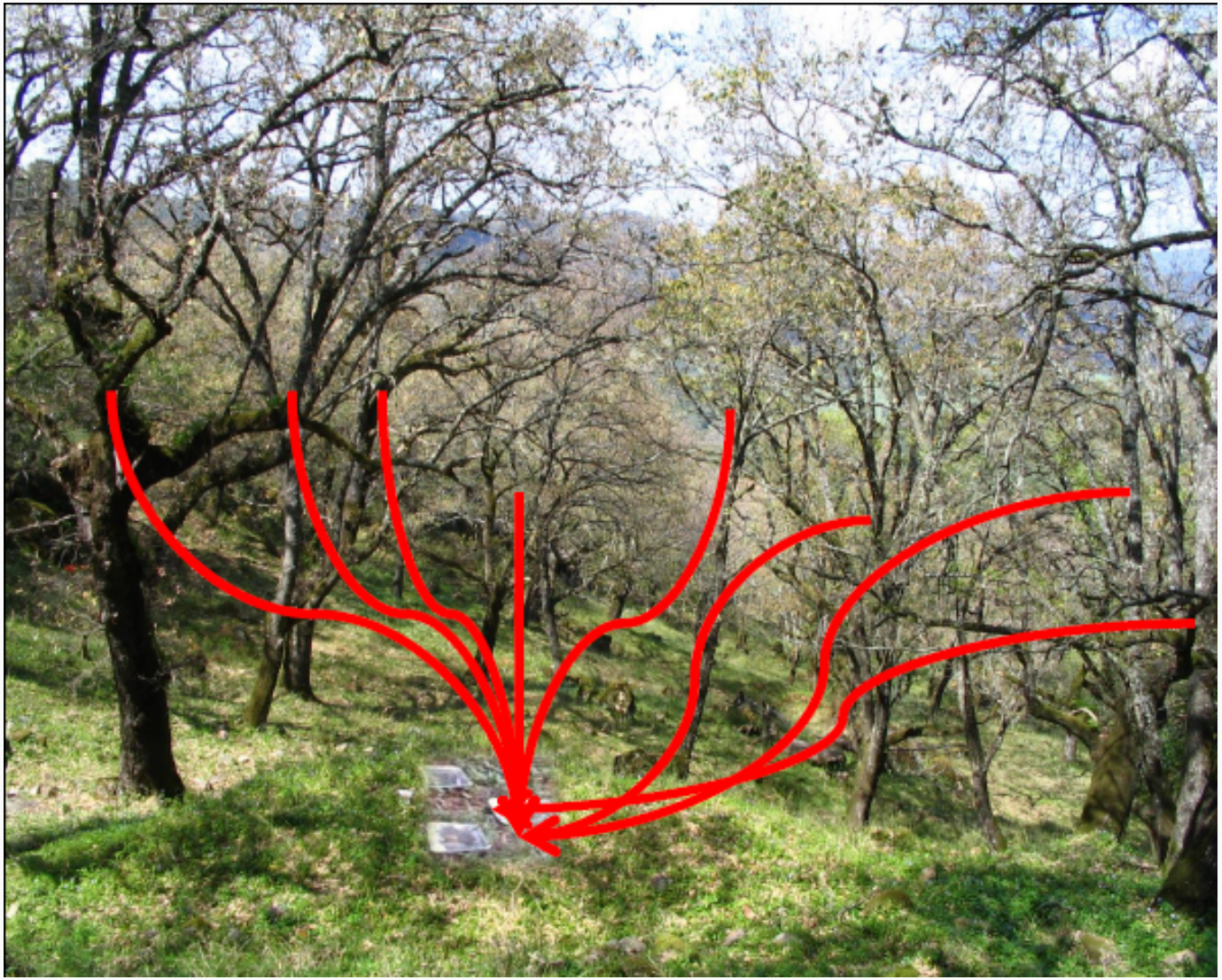


Bolker et al 2009 TREE:

'311 out of 537 GLMM analyses (58%) used these tools inappropriately'

To get answers to tough problems

Inferring tree fecundity



Course goals

- **Understand** statistical inference

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- Avoid **misconceptions**

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- **Understand** statistical inference
- Avoid **misconceptions**
- Promote **good practices**

The purpose of models is not to fit data
but to sharpen thinking

Sam Karlin

Topics

- Descriptive statistics

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- Graphics

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- Graphics
- Sampling

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- Experimental design

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- Linear models & GLMs
- Model selection

Descriptive statistics

Measure trunk diameter of 30 trees in your neighbourhood

Read data

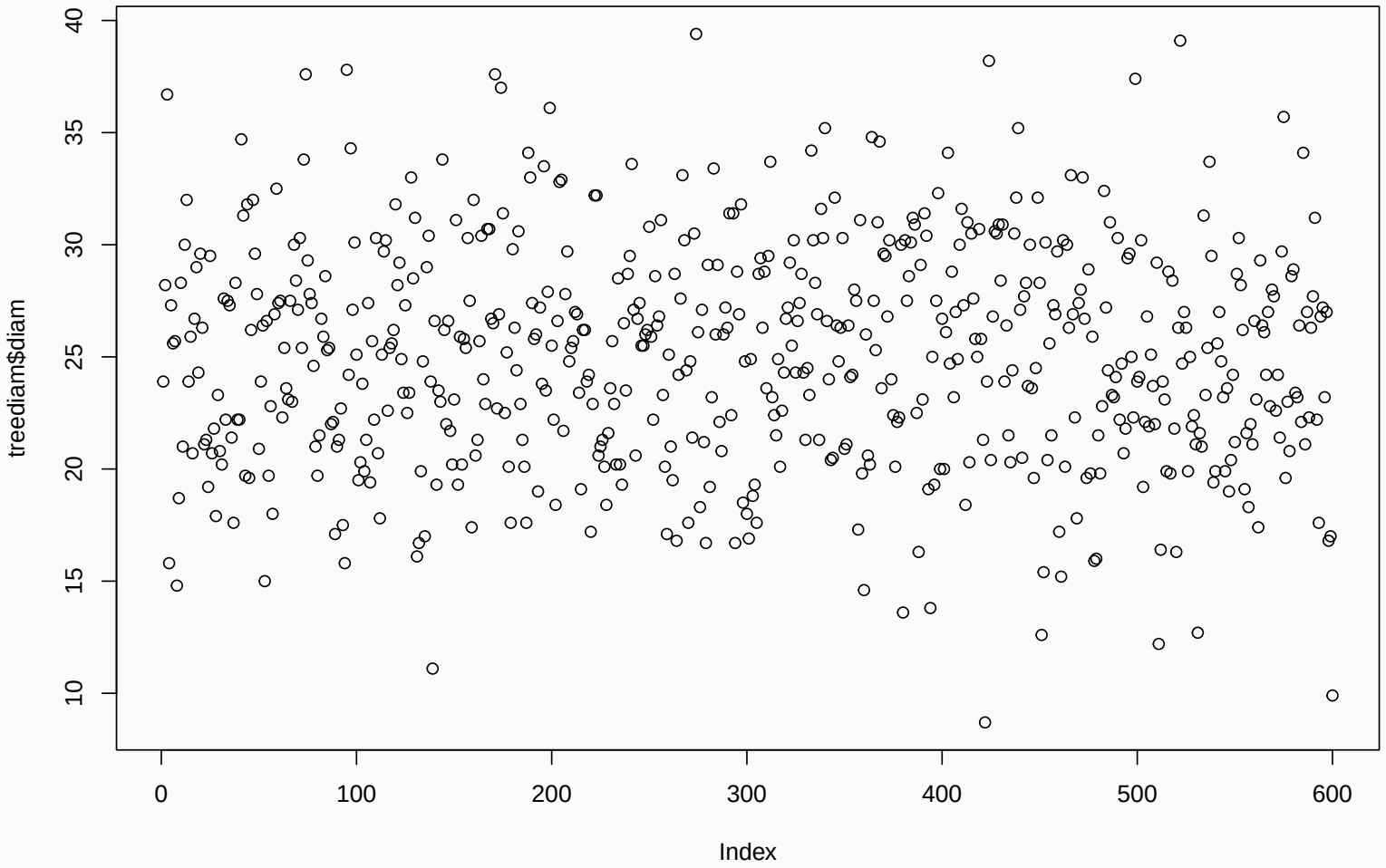
```
treediam <- read.csv("treediam.csv")
```

```
summary(treediam)
```

```
      site            tree            diam      
 Min.   : 1.00   Min.   : 1.0   Min.   : 8.70  
 1st Qu.: 5.75   1st Qu.: 8.0   1st Qu.:21.40  
 Median :10.50   Median :15.5   Median :25.25  
 Mean   :10.50   Mean   :15.5   Mean   :25.04  
 3rd Qu.:15.25   3rd Qu.:23.0   3rd Qu.:28.40  
 Max.   :20.00   Max.   :30.0   Max.   :39.40  
```

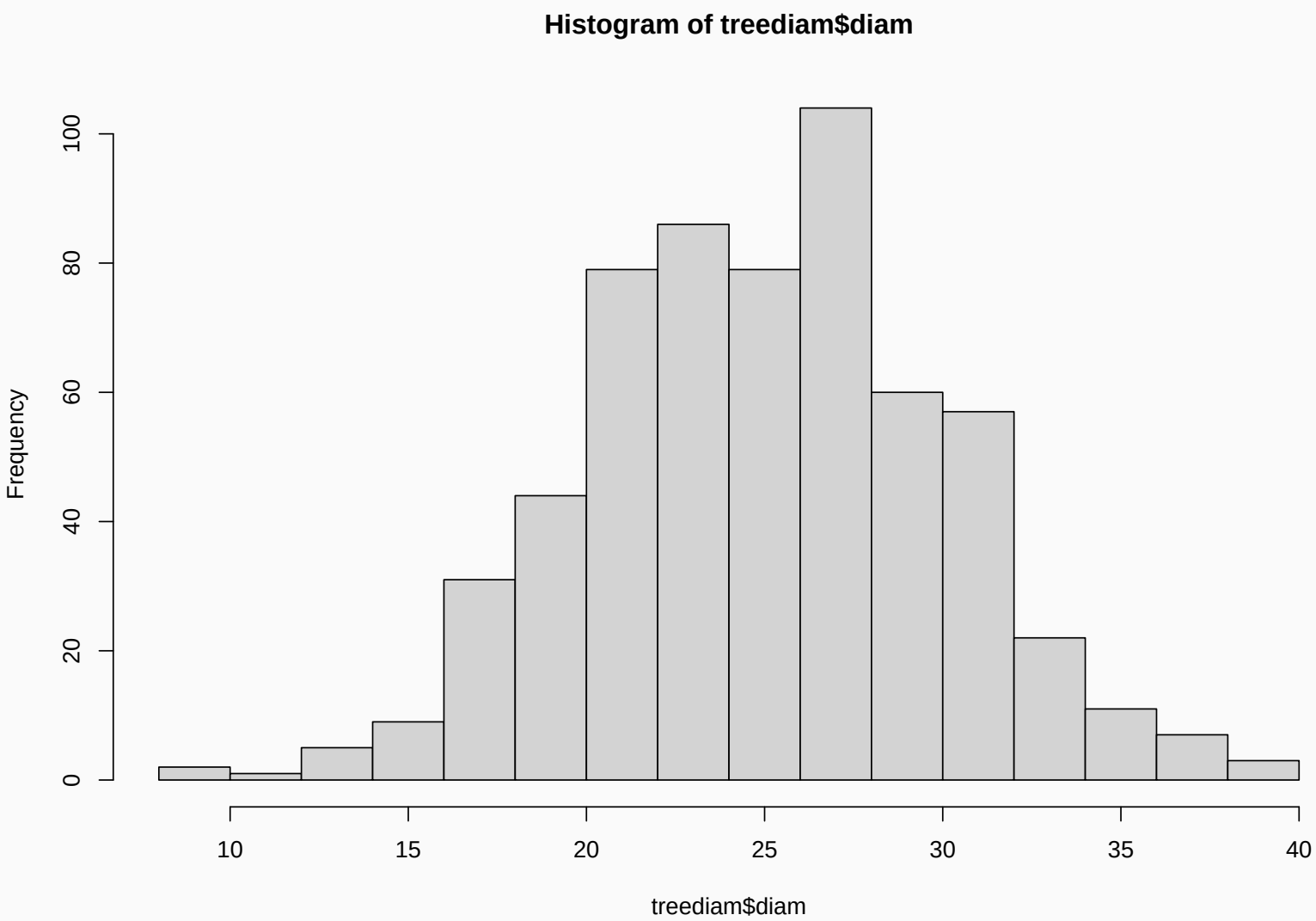
Visualisation of tree diameters

```
plot(treediam$diam)
```



Visualisation of tree diameters

```
hist(treediam$diam)
```



How well do these values

represent actual tree diameters

in your neighbourhood?

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- At what height did you measure?

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    - (trees may get thinner w/ high evapotranspiration)

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- Did you measure with tape, caliper, by eye?
- When did you measure: dawn, midday, night?
    - (trees may get thinner w/ high evapotranspiration)
- Where did you measure?
    - (differences among streets, species, etc)

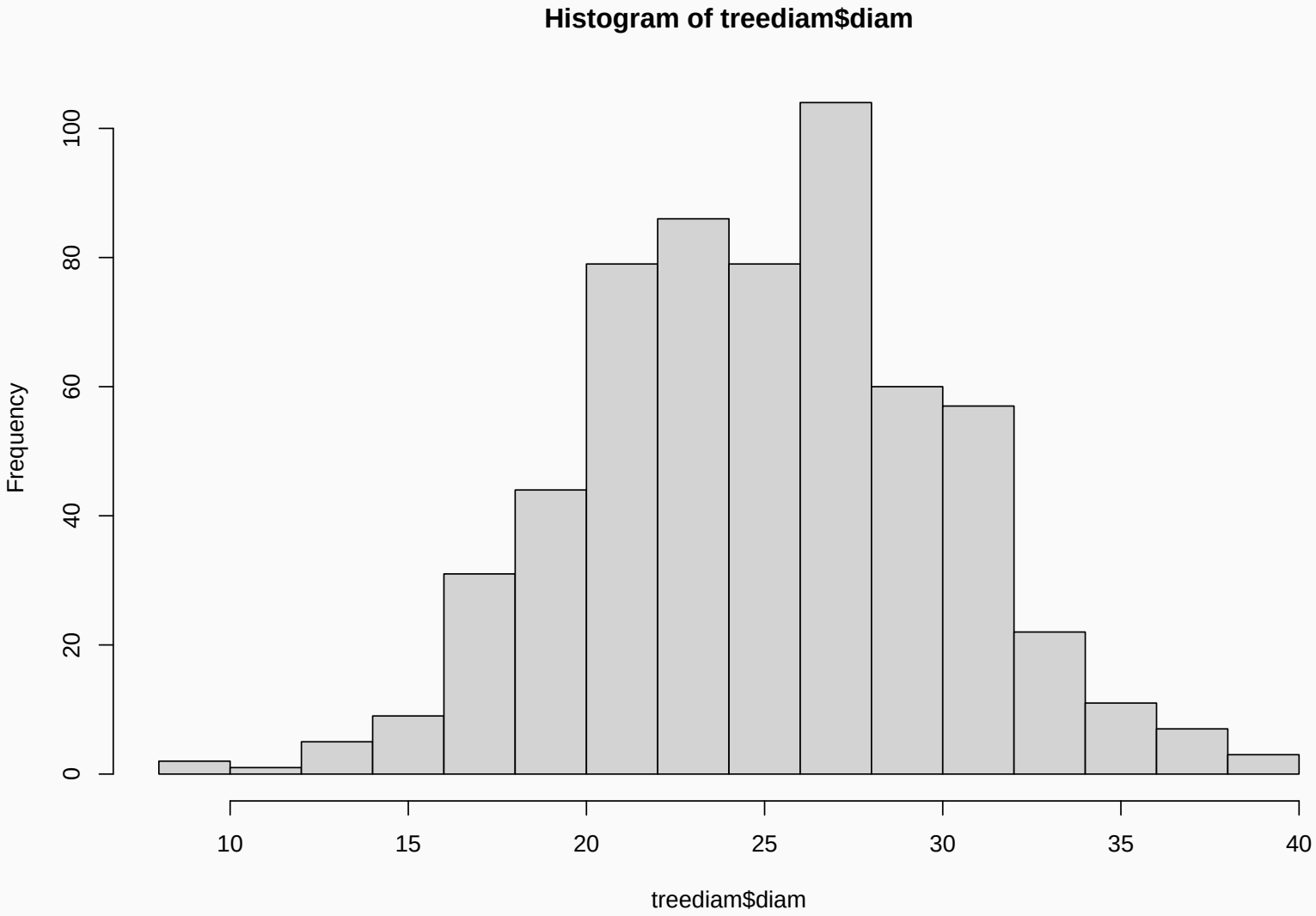
Data are hardly ever objective.

We decide **what to measure, when, where, and how**.

Always consider:

How well do data reflect what we are trying to measure?

How would you describe this distribution?

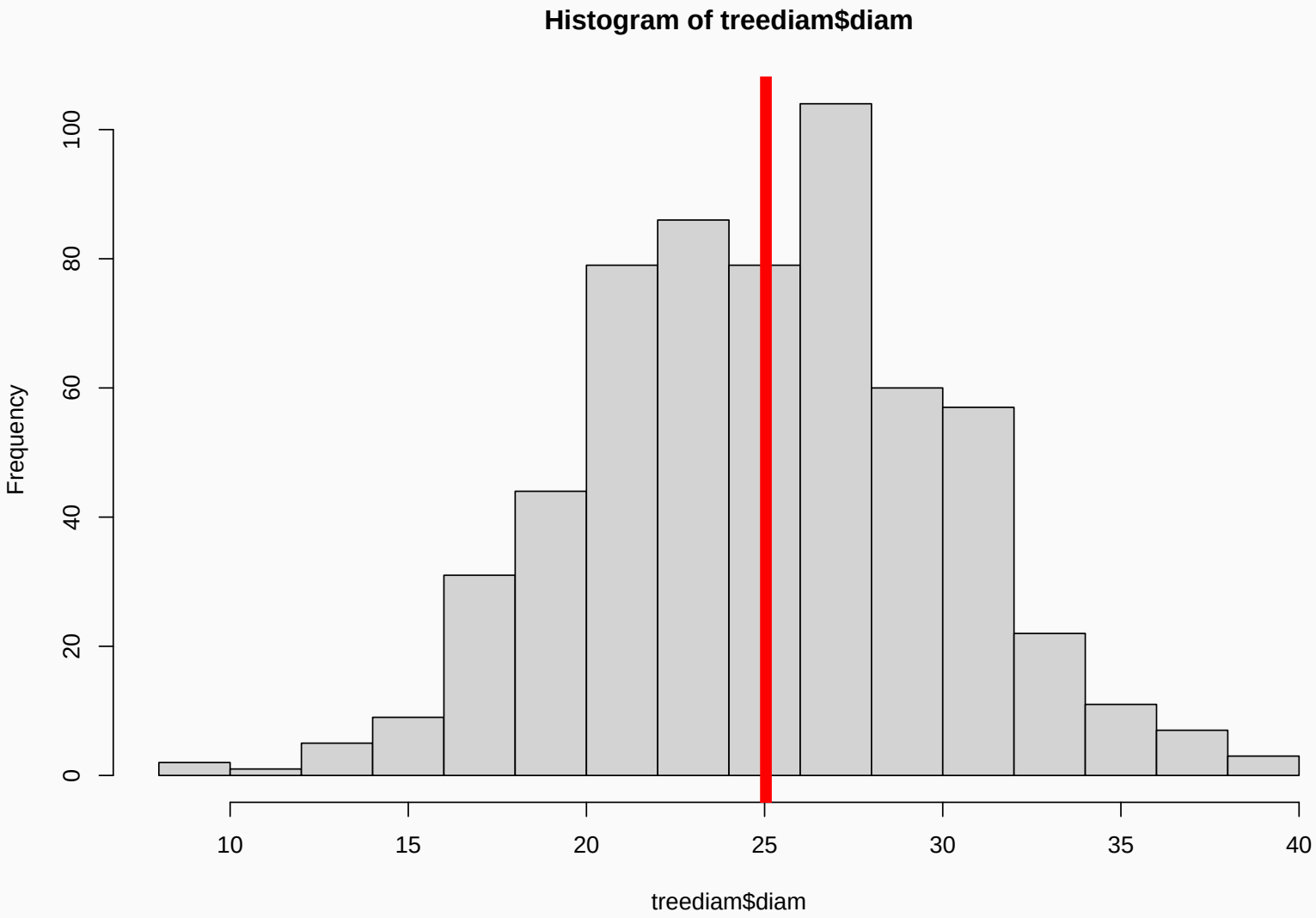


(Discuss with your partner)

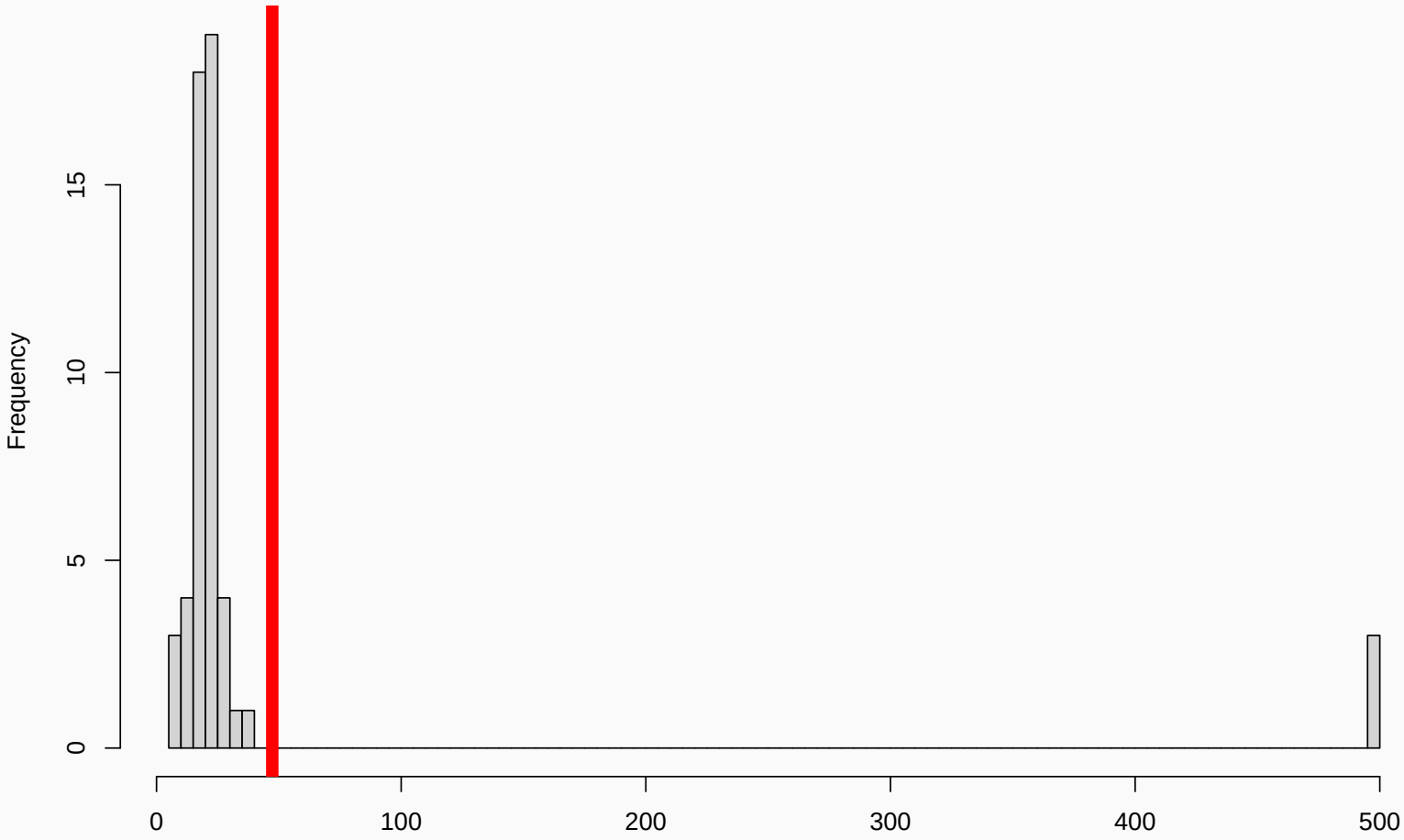
Location / Central tendency

Mean / Average

$$\text{mean} = \frac{d_1 + d_2 + d_3}{n}$$

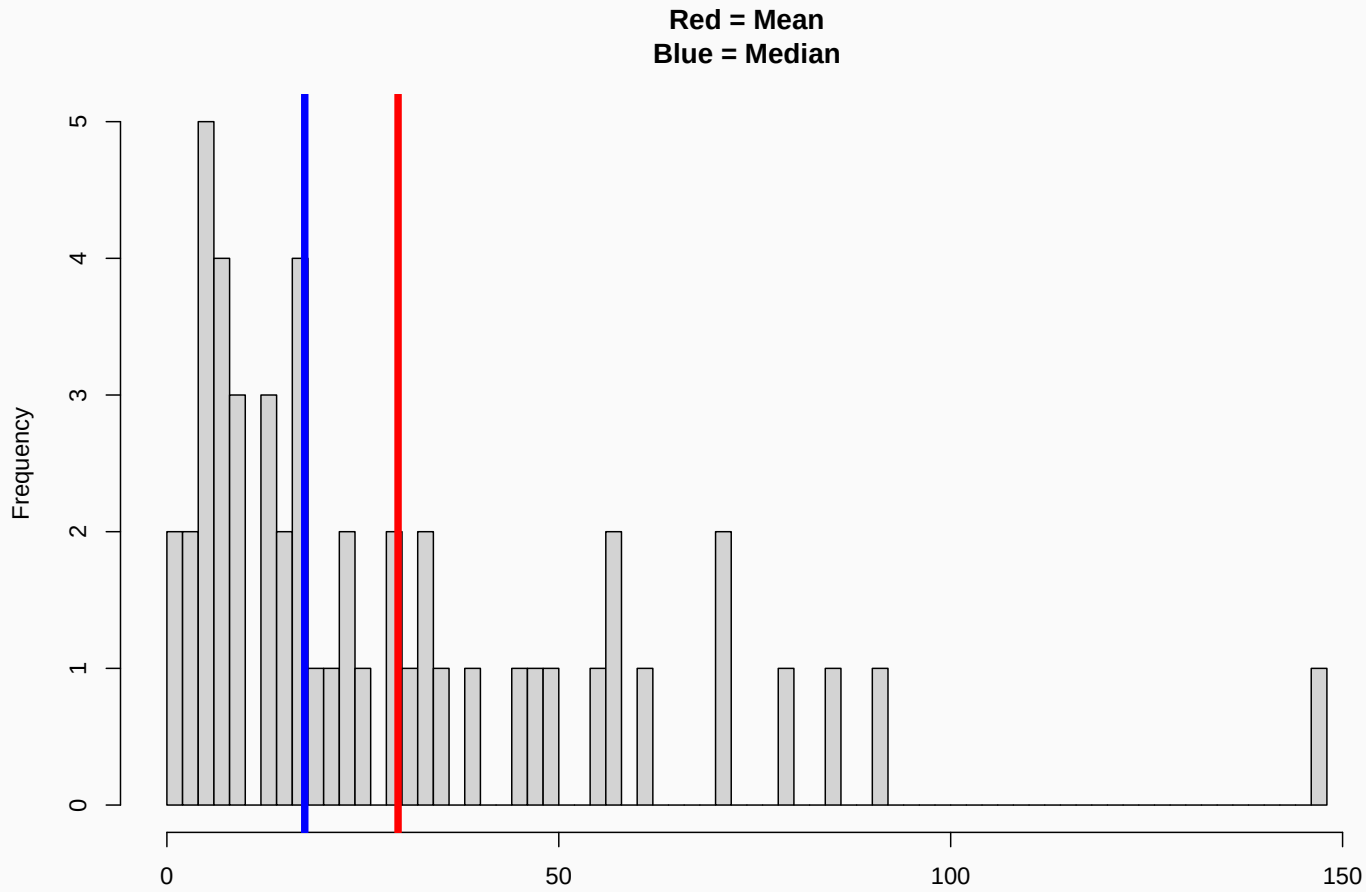


Mean is sensitive to skew/outliers



Median

50% percentile. Leaves half of the data values on each side

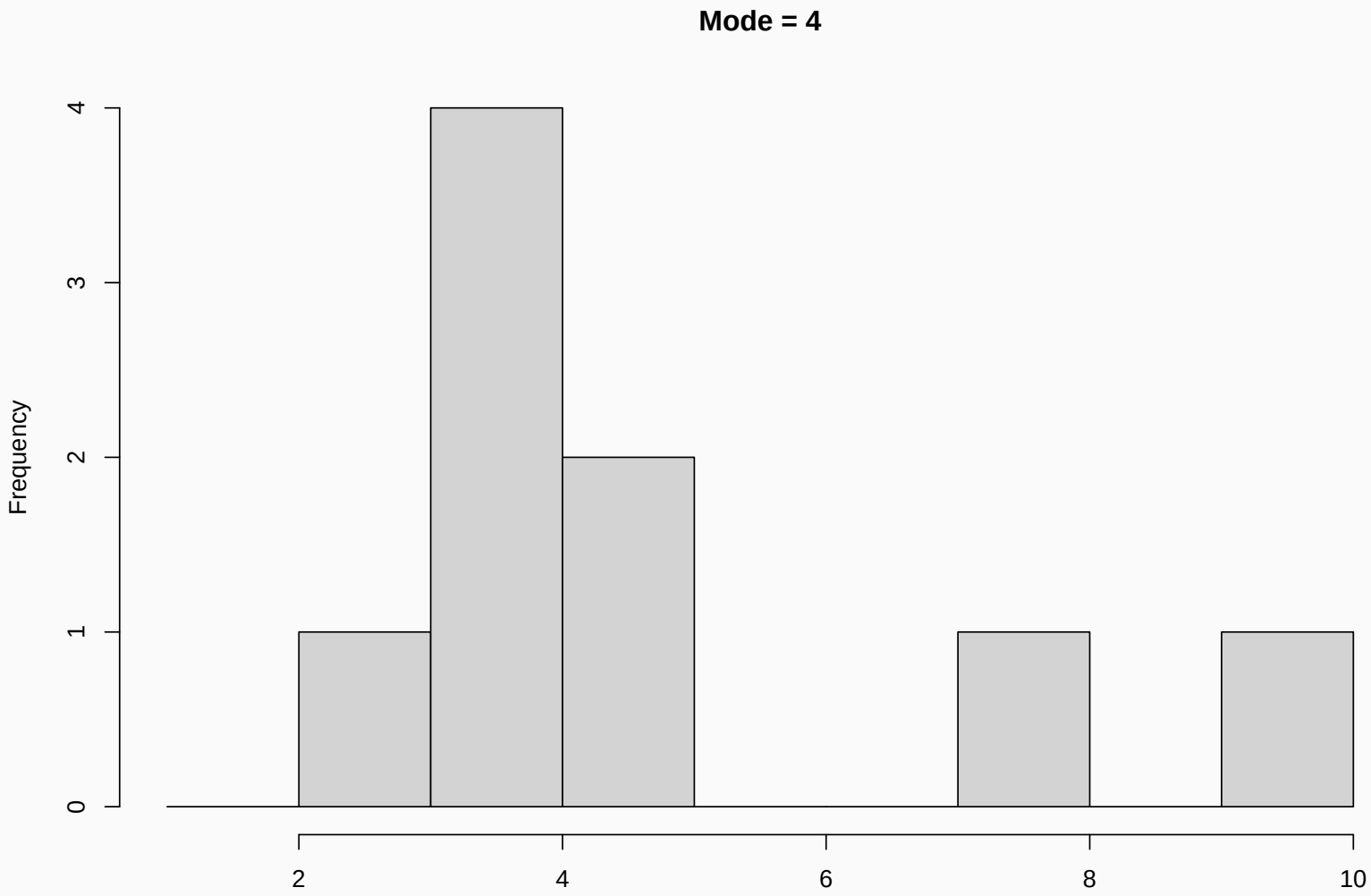


Median of c(2, 4, **6**, 8, 10) = 6

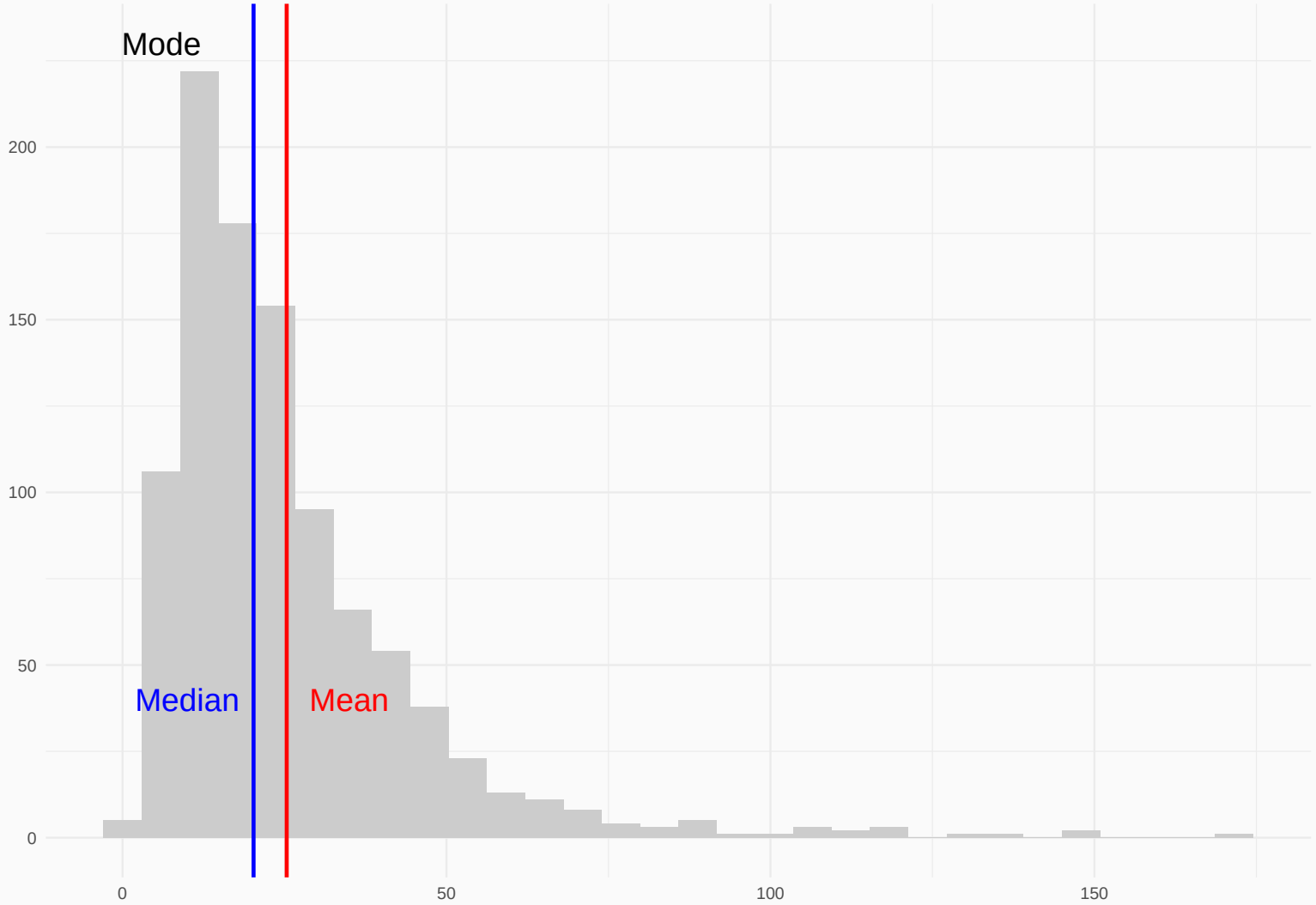
Median of c(2, 4, 6, 8) = (4 + 6) / 2 = 5

Mode

Most frequent value

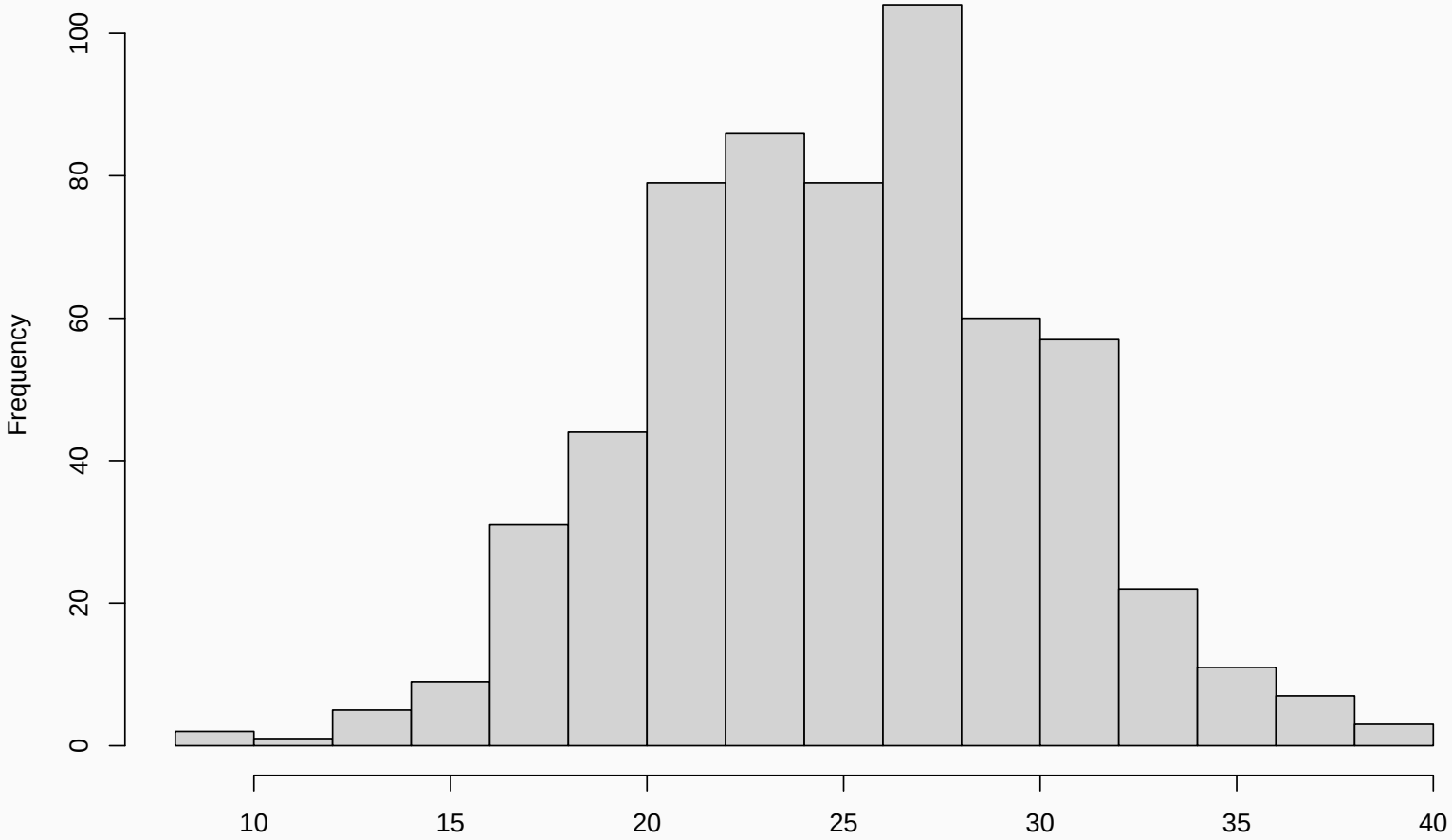


Describing the location / central tendency



Describing Variation / Spread

Minimum, Maximum, Range



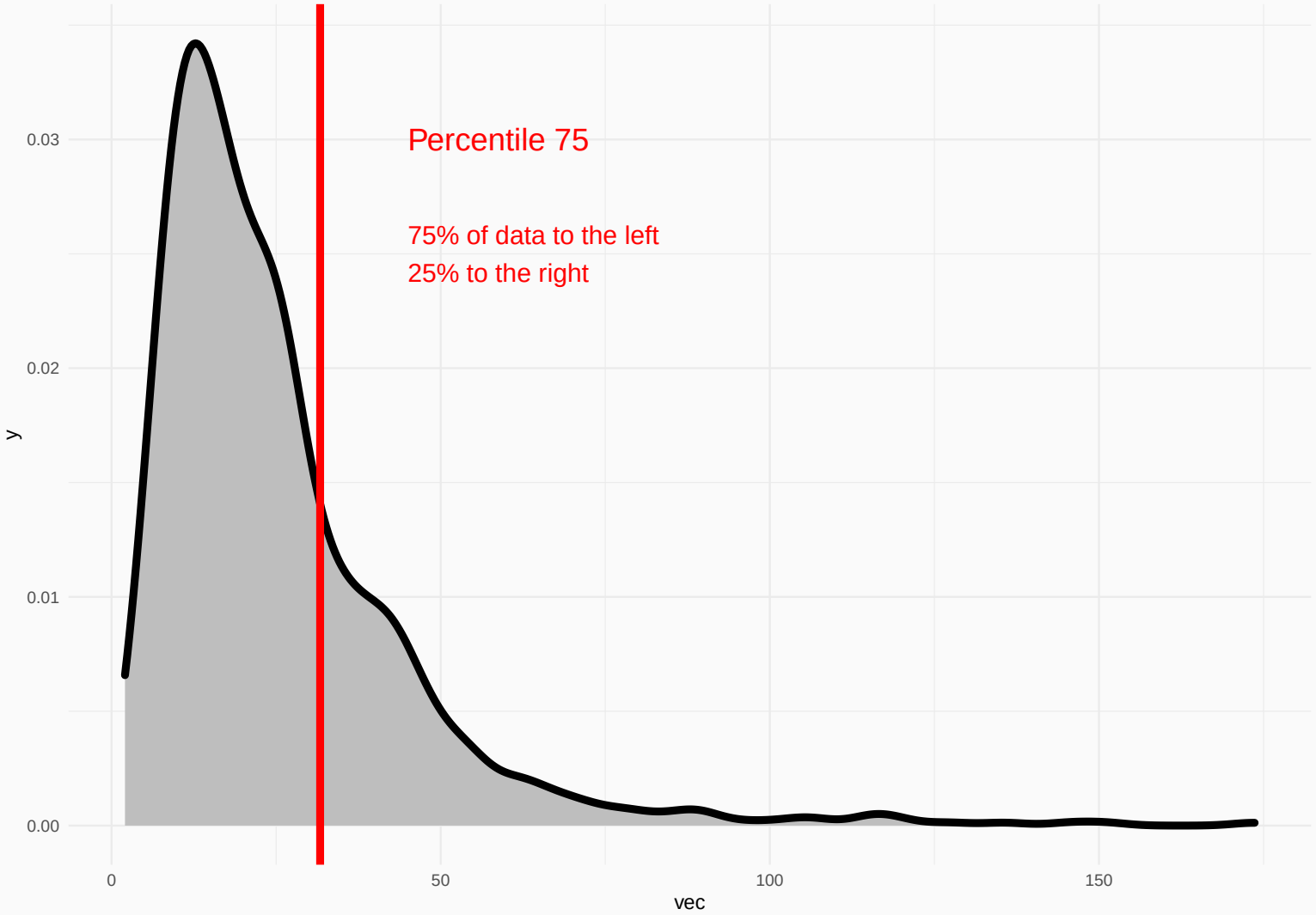
Minimum = 9.2

Maximum = 41.9

Range = 9.2, 41.9

Quantiles

Quartiles, Percentiles...

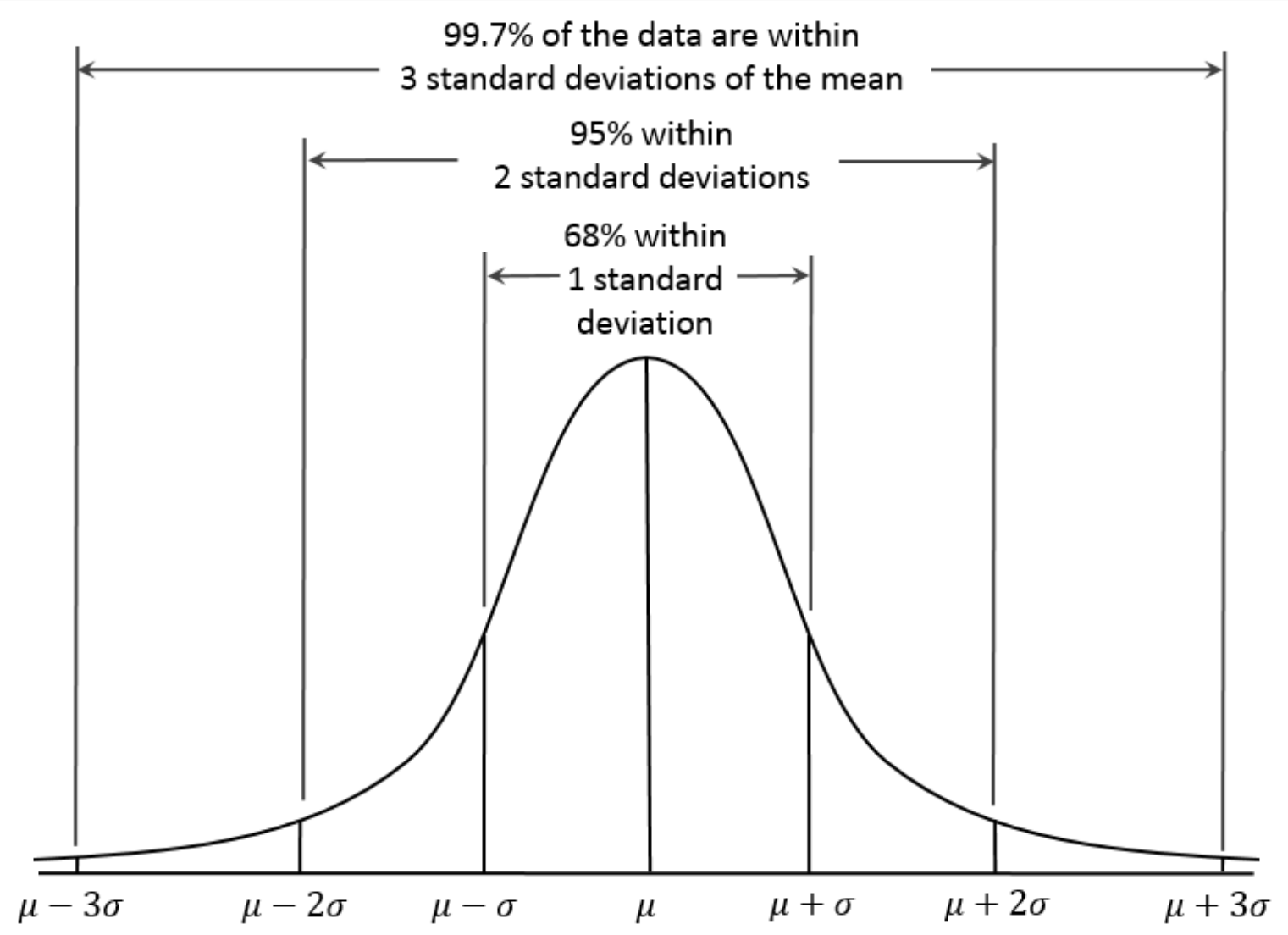


Standard Deviation

Average distance between data points and the mean

$$SD = \sqrt{\frac{\sum (x - \mu)^2}{n - 1}}$$

In a Normal distribution

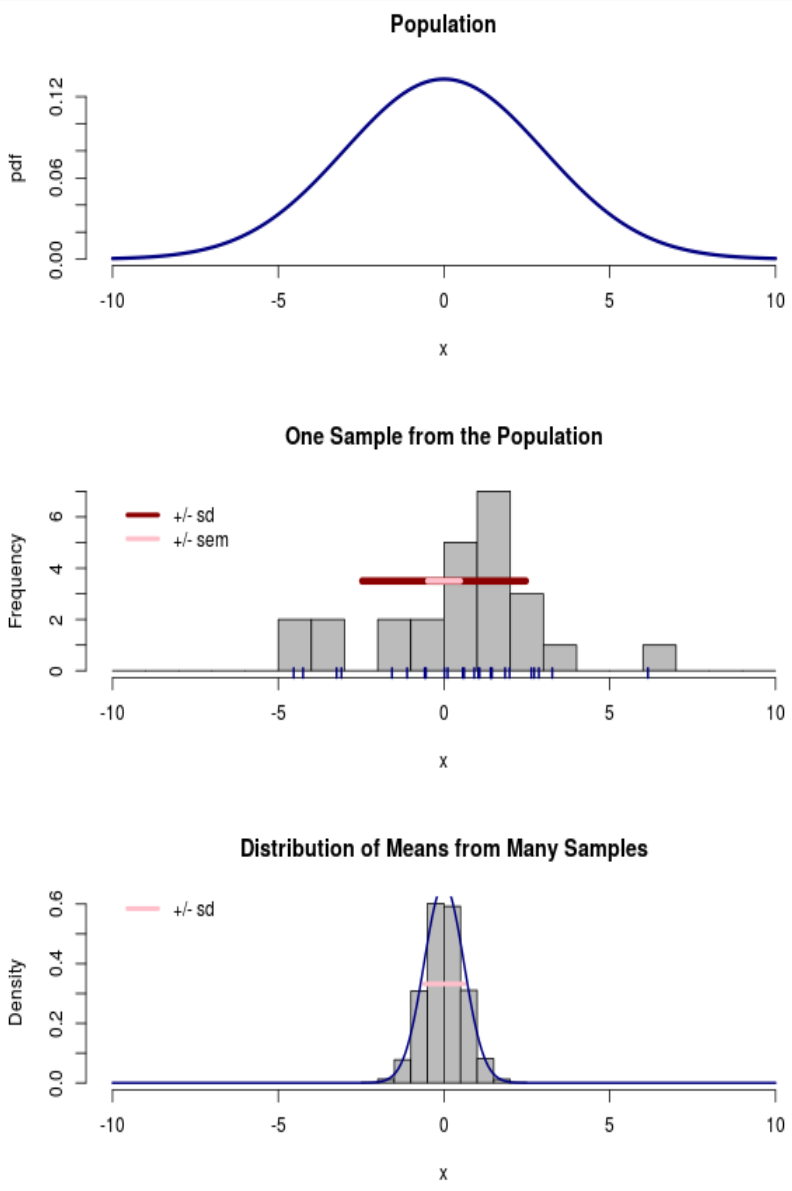


Standard Error of the Mean

$$SEM = \frac{SD}{\sqrt{n}}$$

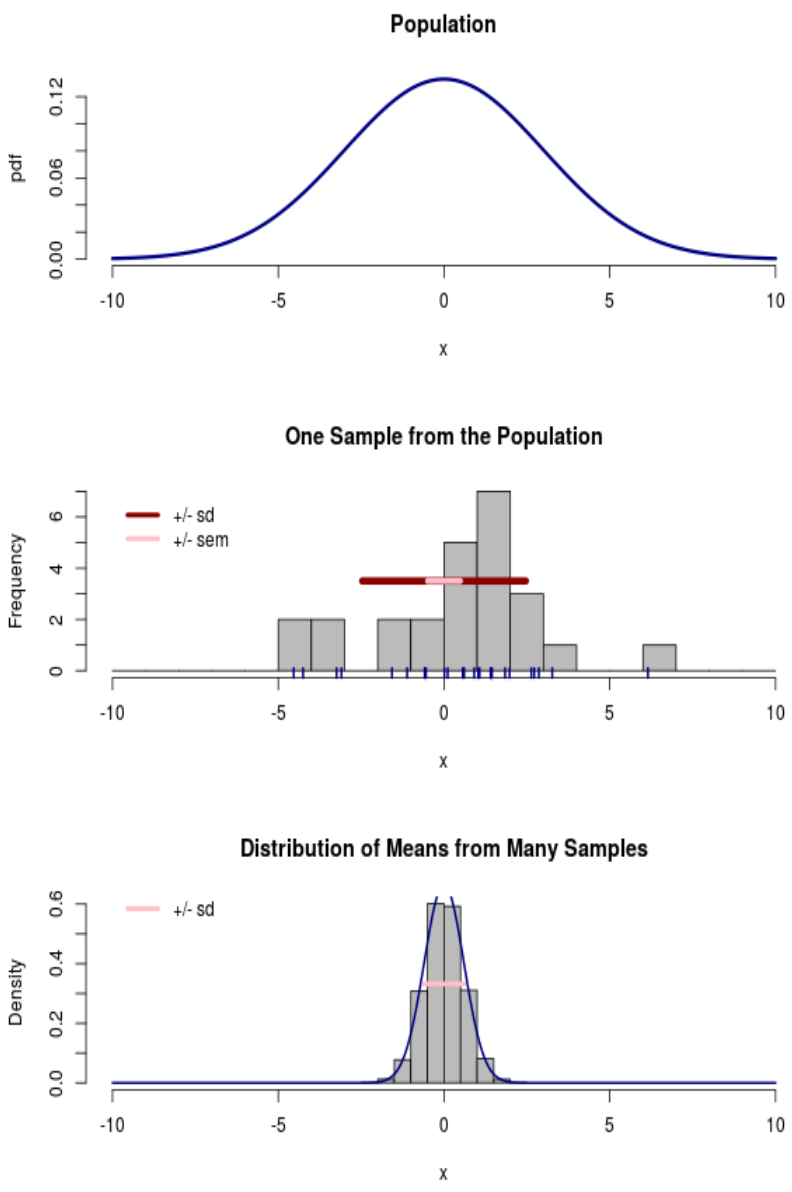
Estimates uncertainty (spread) of the parameter ‘mean’

Relationship between SD and SEM



- SD quantifies scatter in population

Relationship between SD and SEM



- SD quantifies scatter in population
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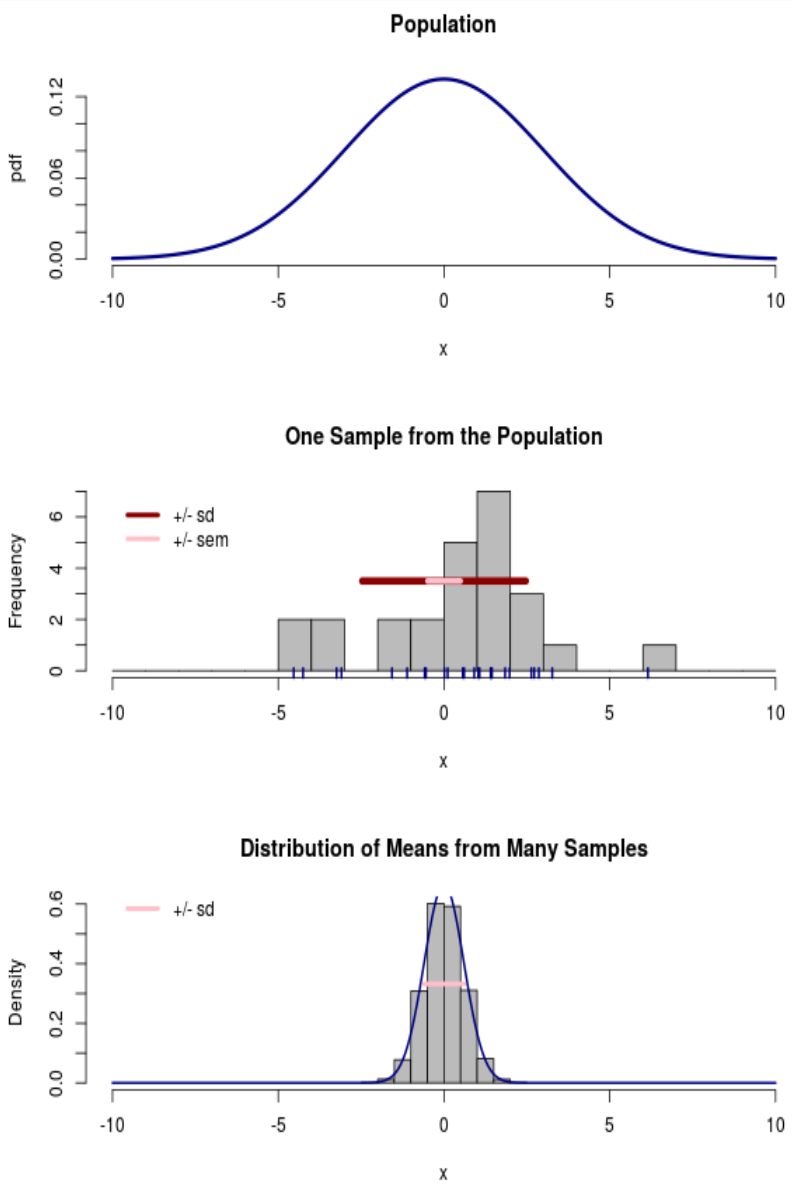
Relationship between SD and SEM



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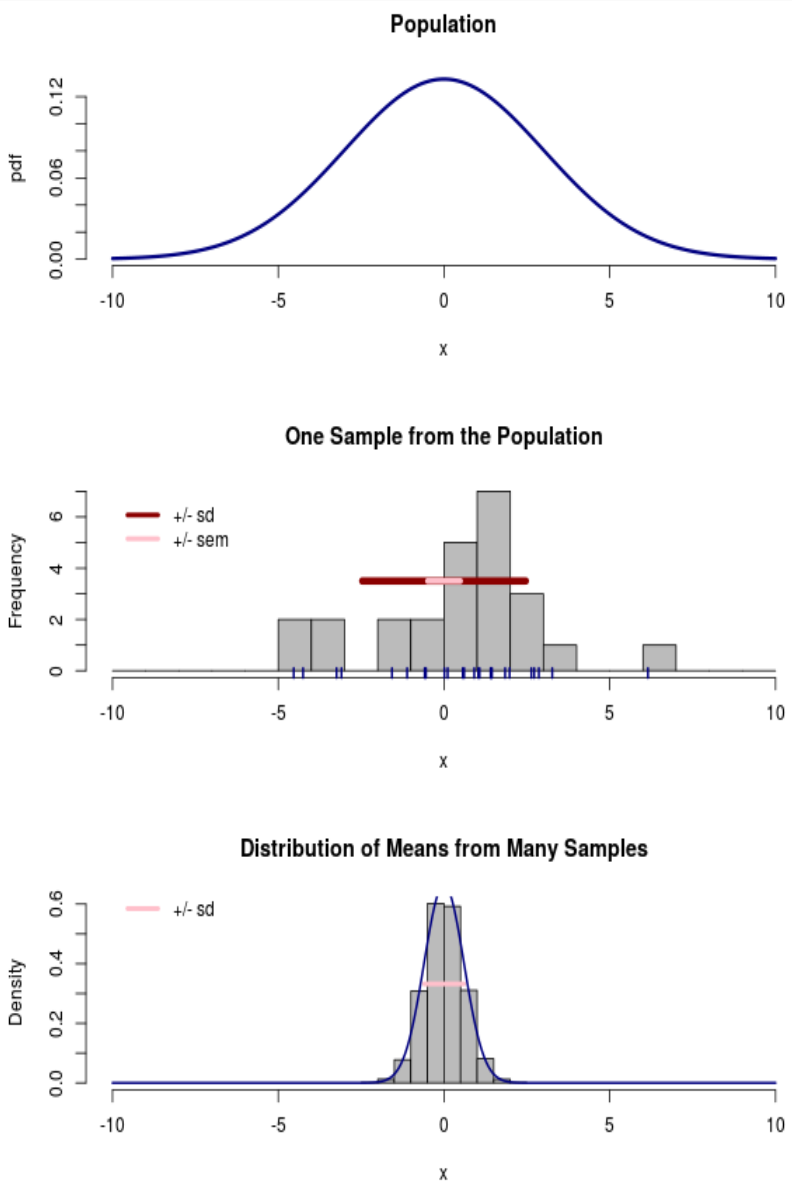
- SEM = SD/sqrt(n)

Relationship between SD and SEM



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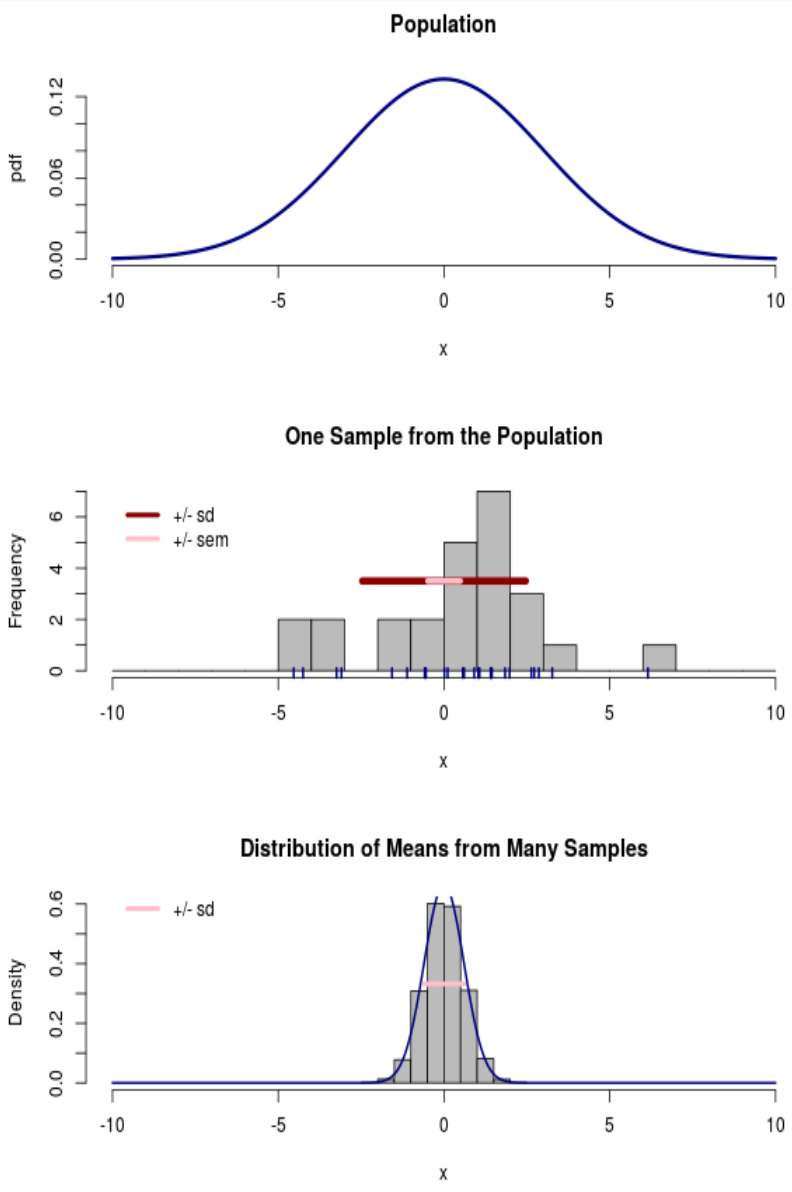

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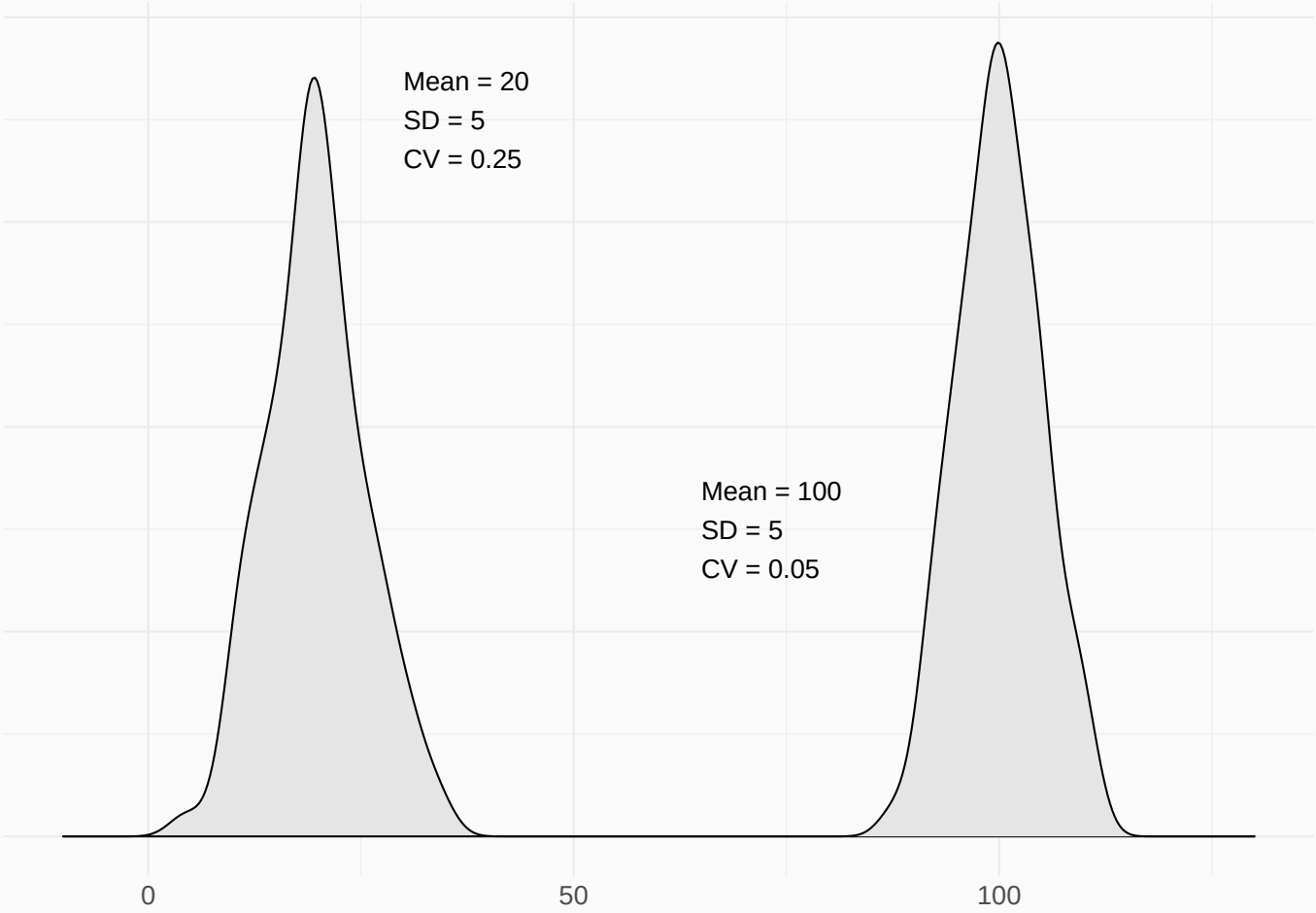
- https://gallery.shinyapps.io/sampling_and_stderr/

Coefficient of Variation

Facilitates comparing spread of distributions with different means

$$CV = \frac{SD}{mean}$$

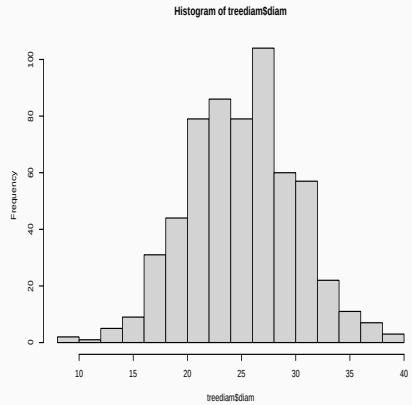


Summarise a distribution

Central tendency / location

- mean (average)

Variation / Spread

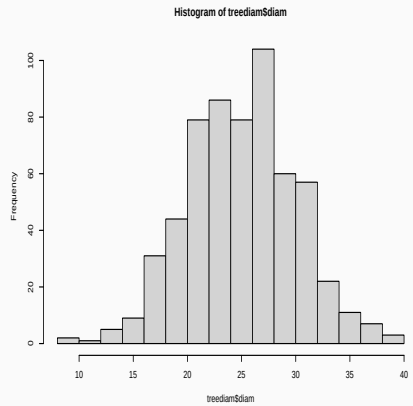


Summarise a distribution

Central tendency / location

- mean (average)
- median (50% percentile)

Variation / Spread



Summarise a distribution

Central tendency / location

- mean (average)
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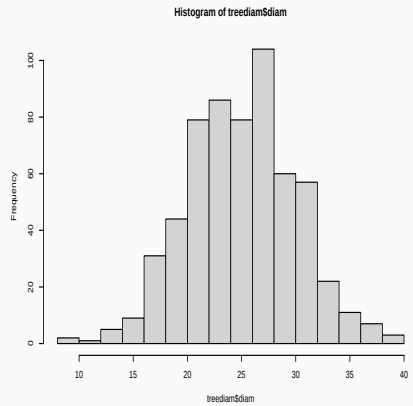
- mode (most frequent value)

Variation / Spread



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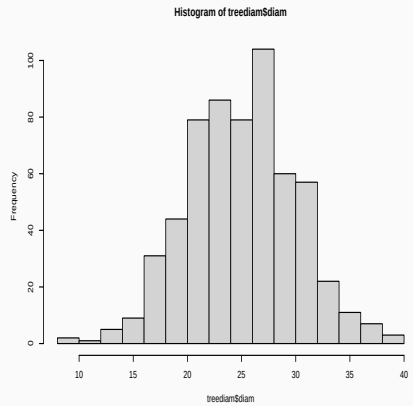
- min, max, range



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- min, max, range
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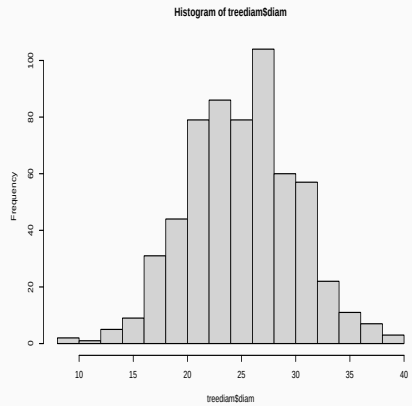

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Central tendency / location

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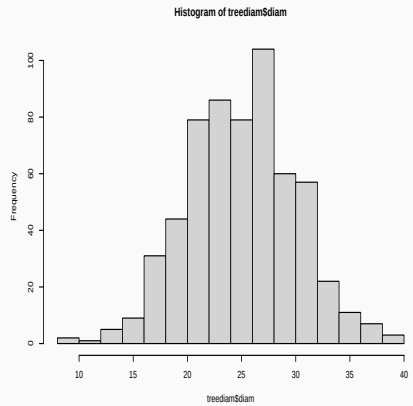
Variation / Spread

- min, max, range
- quantiles (quartiles, percentiles...)
- standard deviation



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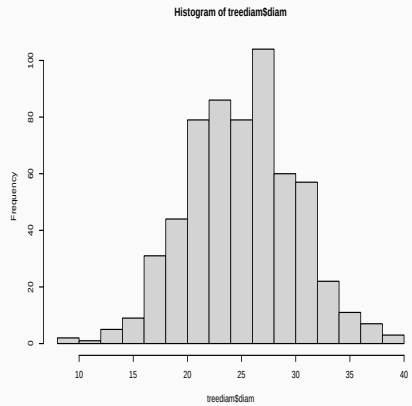
- standard error of the mean



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- min, max, range
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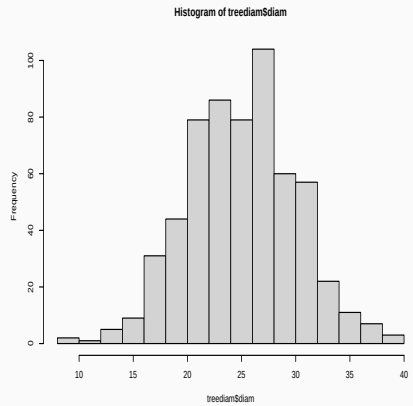
- standard deviation
- standard error of the mean
- coefficient of variation



What statistical descriptors are best? (and why)

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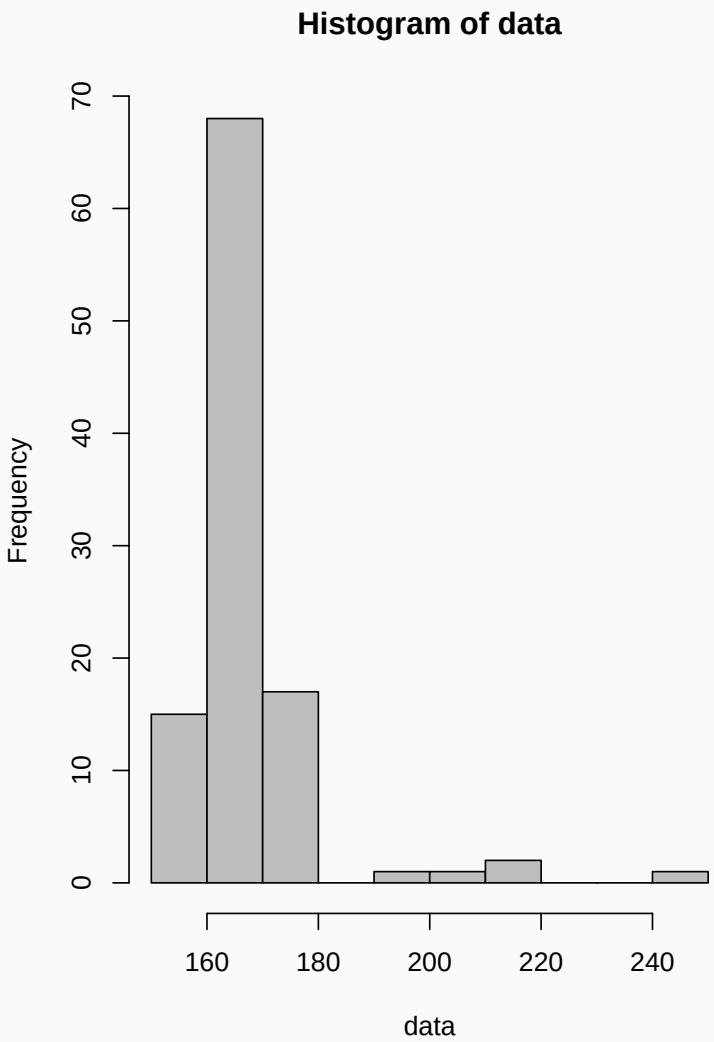

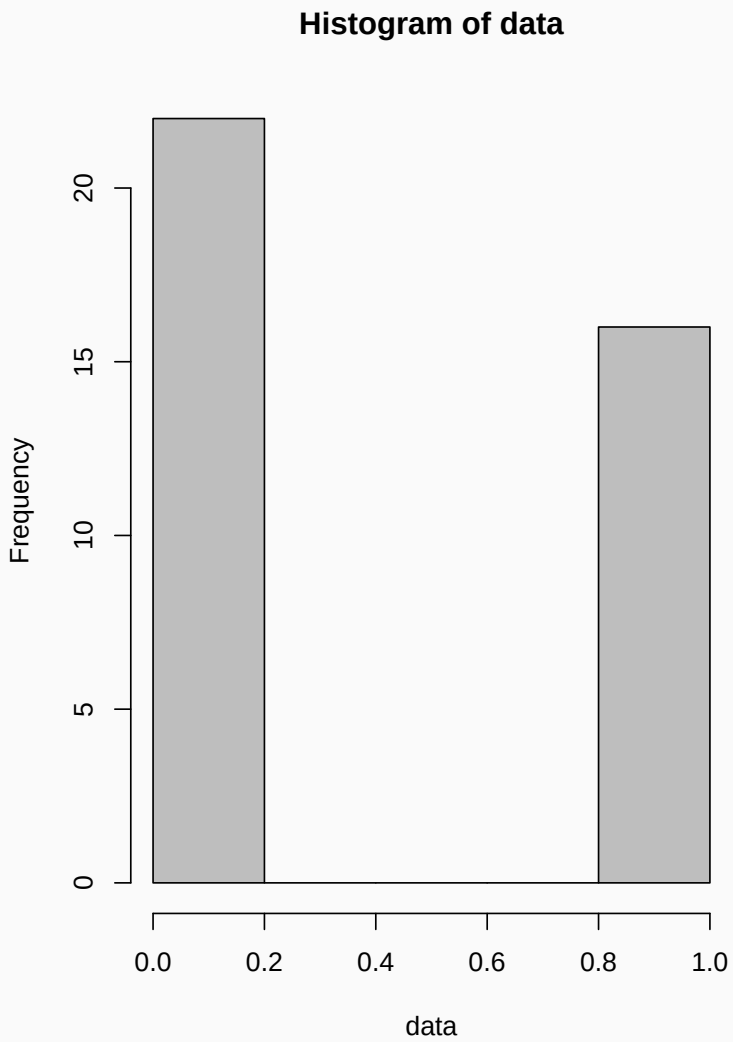

Sampling, confidence intervals, likelihood and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples**.

Then we make inferences from sample to population.

If we sample 30 trees in our neighbourhood...

Can we extrapolate results to

- whole neighbourhood?
- whole city?
- whole country?
- the world?

What's the **suitable population** to make inferences given this sample?

Understanding confidence intervals

- Summarise **uncertainty** in parameter estimates.

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- It's a frequentist, long-run property.
- To read more: Morey et al (2015)

What happens if we increase sample size?

https://rpsychologist.com/d3/CI/

- CI width *decreases*...

What happens if we increase sample size?

https://rpsychologist.com/d3/CI/

- CI width *decreases*...
- but still 5% of CIs will NOT contain true mean!

If the 95% CI of X is (120, 150)...

- There is a 95% probability that X lies between 120 and 150

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- If we repeated the experiment, 95% of the time X would fall between 120 and 150
- If we repeated the experiment, 95% of the CIs would contain the true value of X
- The probability that X is greater than 0 is at least 95%

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- If we repeated the experiment, 95% of the time X would fall between 120 and 150
- If we repeated the experiment, 95% of the CIs would contain the true value of X
- The probability that X is greater than 0 is at least 95%
- The probability that X equals 0 is smaller than 5%

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Bayesian credible intervals

- Bayesian **credible** intervals do give the probability that true parameter value is contained within them.

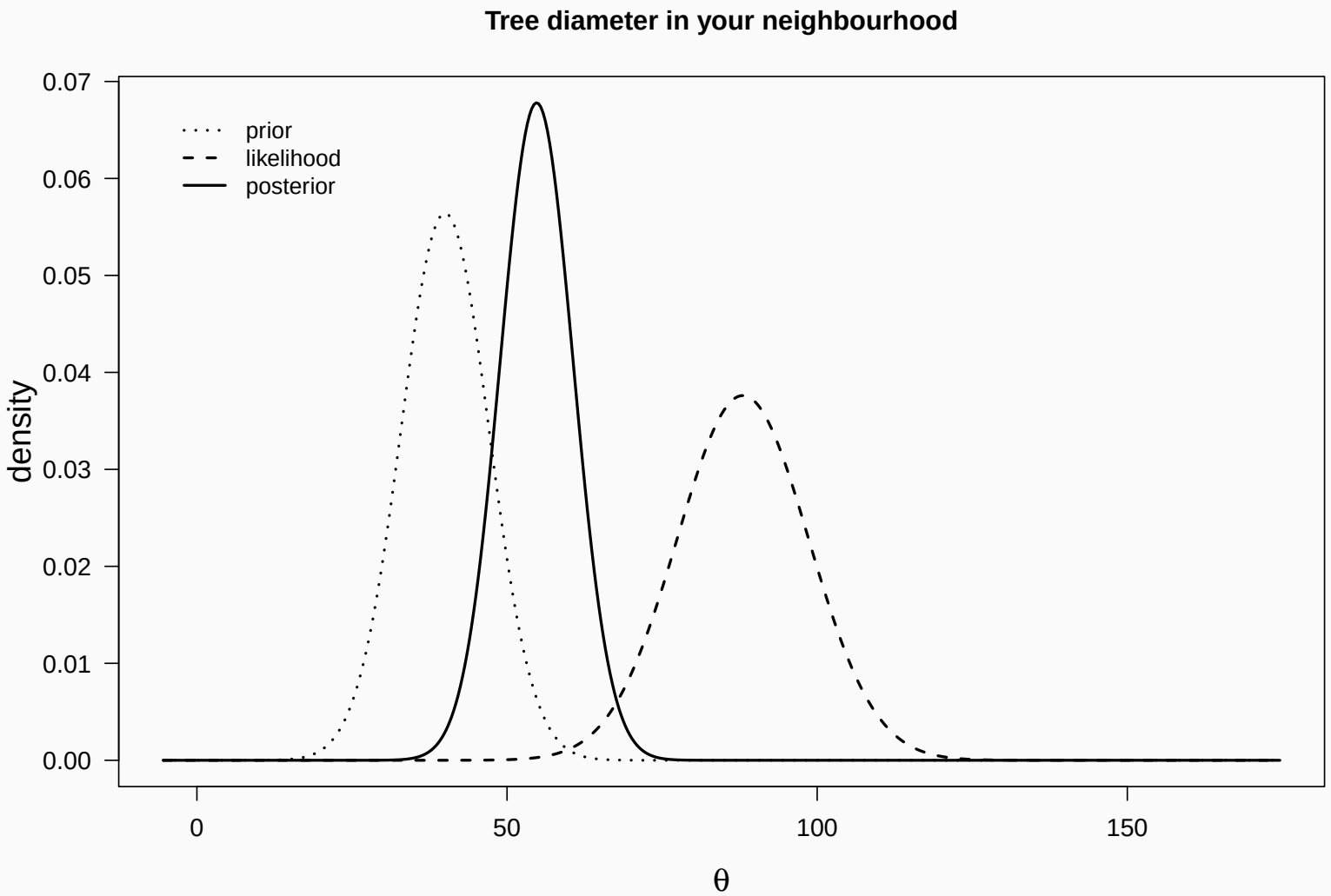
Bayesian credible intervals

- Bayesian **credible** intervals do give the probability that true parameter value is contained within them.
- Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

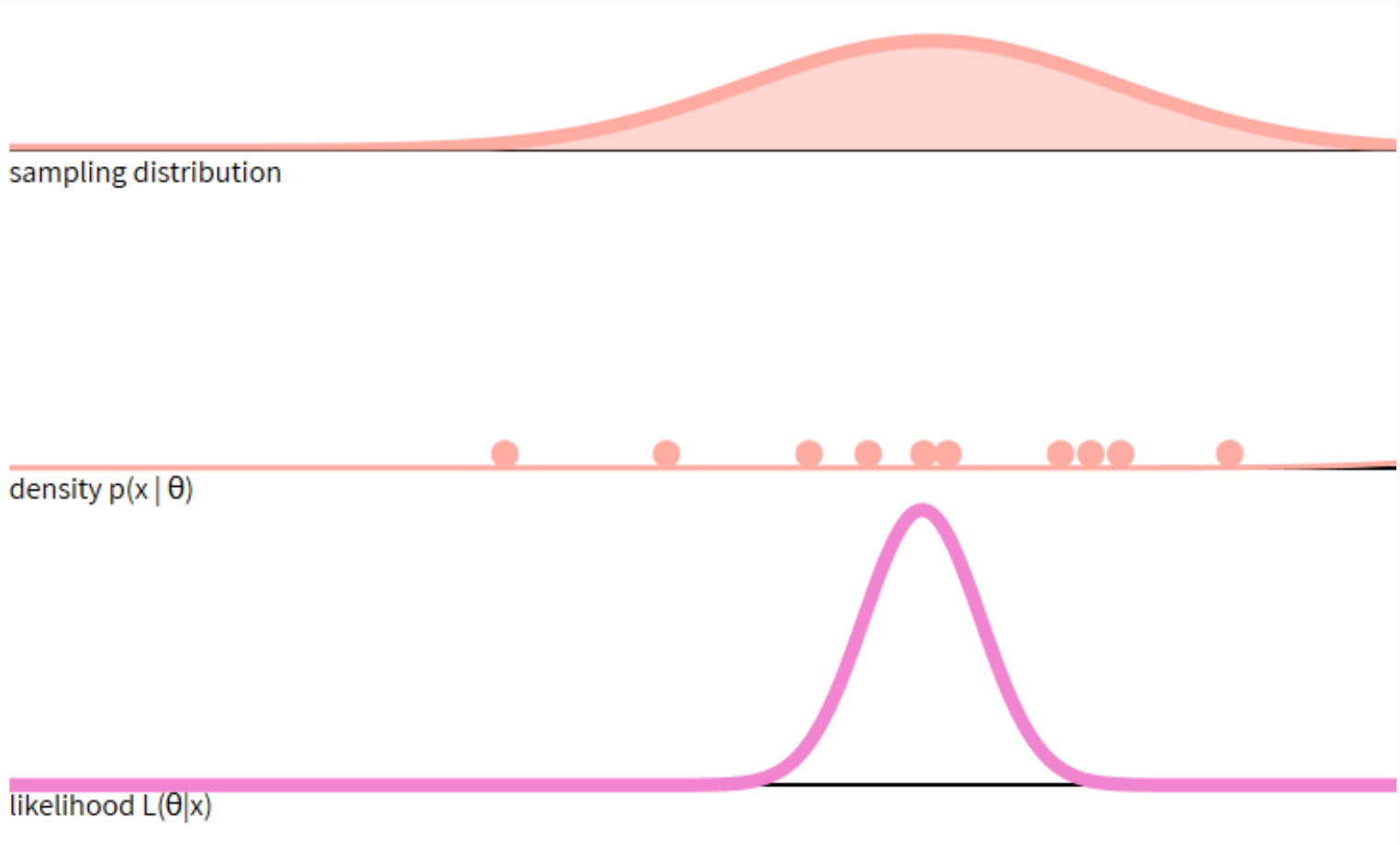
$$P(H|D) \propto P(D|H) \cdot P(H)$$

Posterior $\propto$ *Likelihood* $\cdot$ *Prior*



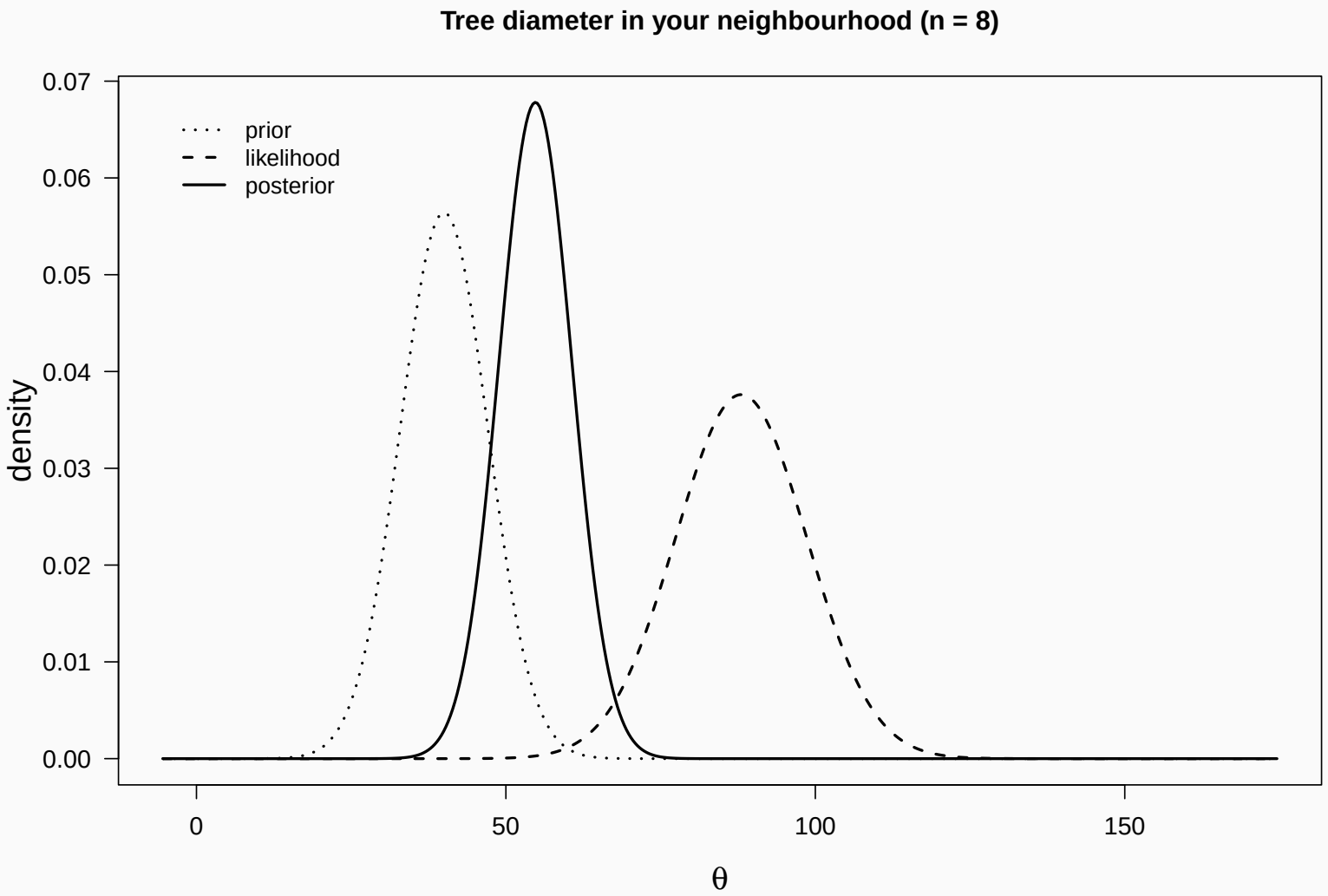
What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$

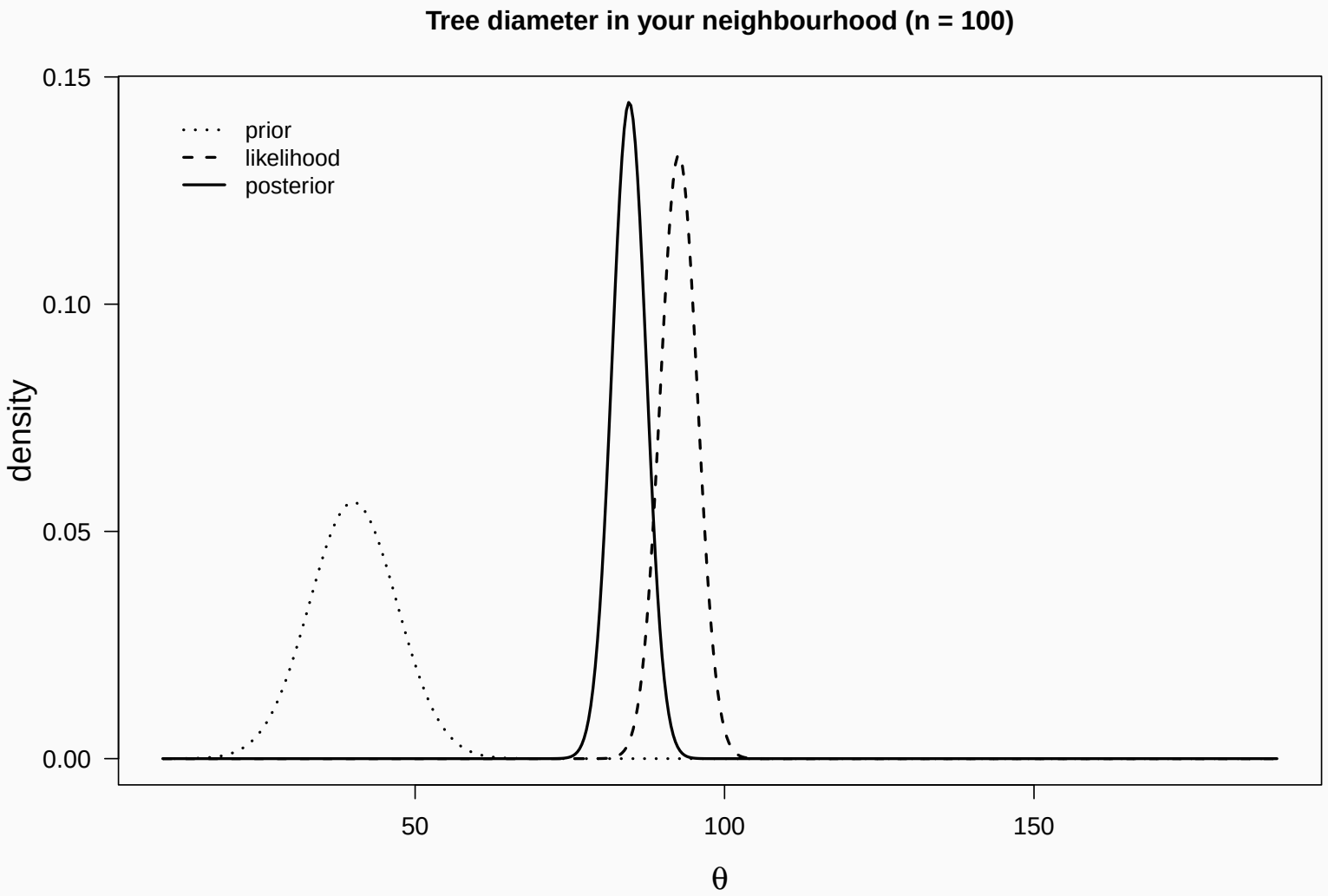


https://seeing-theory.brown.edu/bayesian-inference/index.html

Bayesian inference: prior and likelihood produce posterior



With increasing sample size, likelihood dominates prior



More apps to introduce Bayesian inference

- Bayesian Demo

More apps to introduce Bayesian inference

- Bayesian Demo
- Bayesian inference for a population mean

More apps to introduce Bayesian inference

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- Normal

More apps to introduce Bayesian inference

- Bayesian Demo
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- Own data

More apps to introduce Bayesian inference

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- Bayesian inference for a population mean
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- Binomial
- Own data
- Bayesian t-test

Bayesian statistics in practice

- Integrate information (prior)

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- Prior regularises unlikely estimates from data
- Large dataset -> prior effect diminishes
- Uncertainty / Propagate errors

Experimental design

How would you evaluate fertilizer effect?

Discuss with partner (5’)

Experimental design principles

Replication

Replication!

Replication

- Replication is key: we need several samples.

Replication

- Replication is key: we need several samples.
- How many? As much as you can! See Gelman & Carlin 2014.

H. Stern / A. Gelman

The most important aspect of a statistical analysis is not what you do with the data, it's what data you use

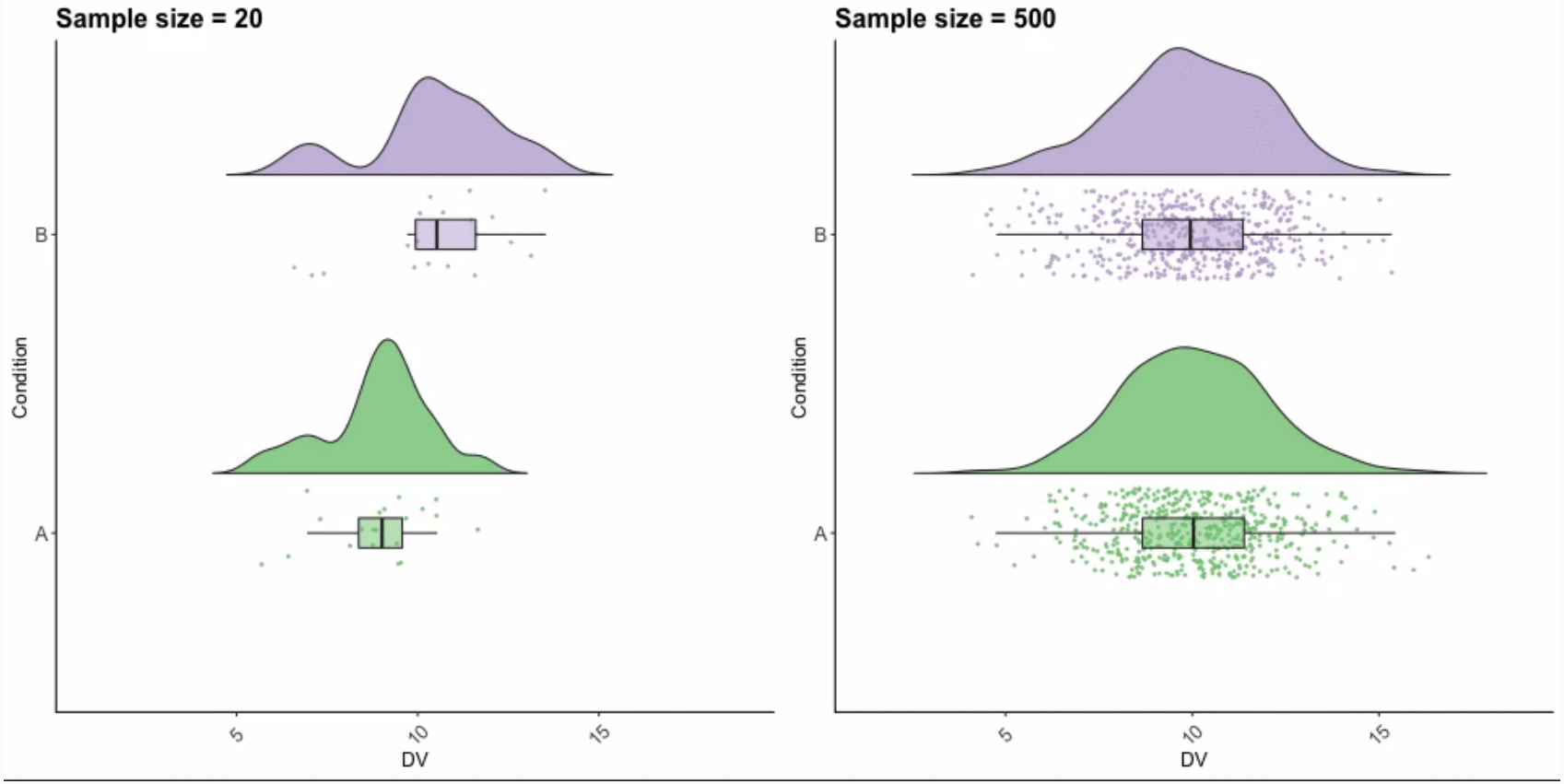
The importance of sample size

- Many studies have **too low sample sizes**.

The importance of sample size

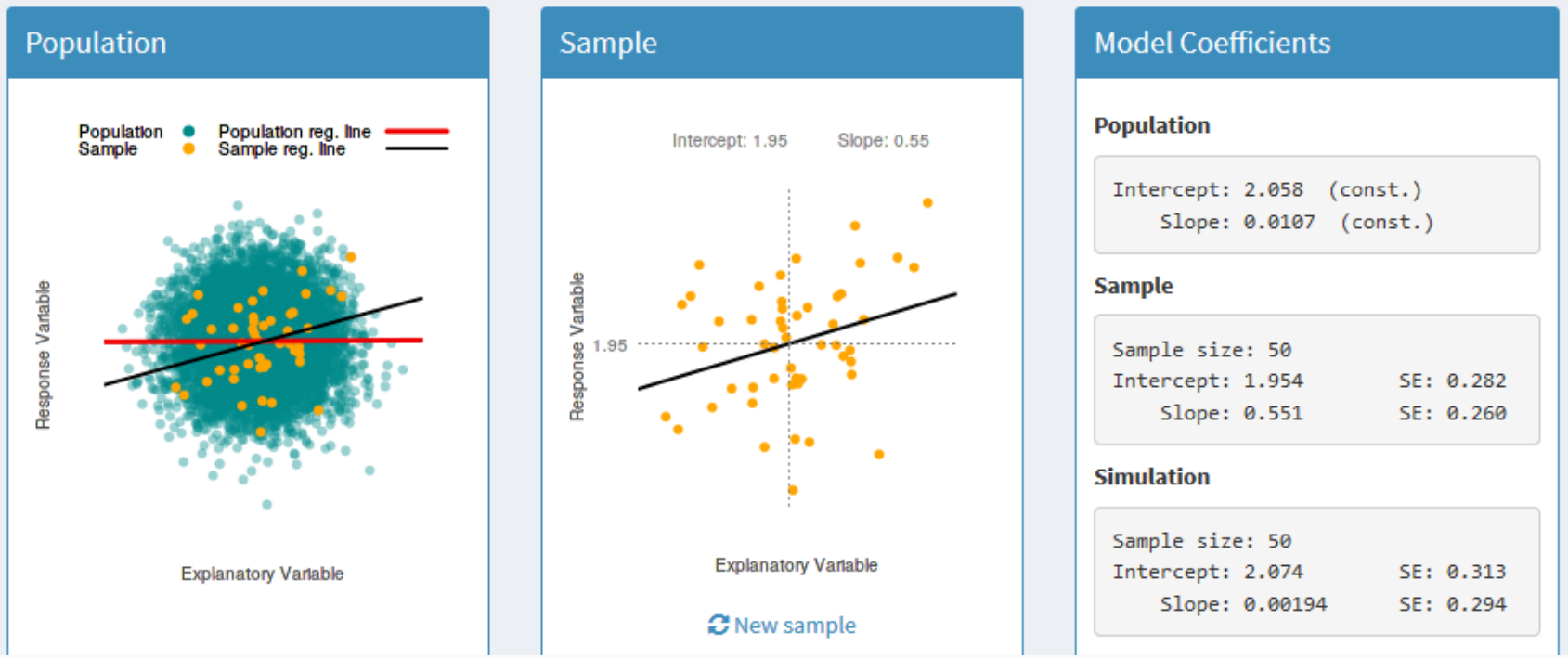
- Many studies have **too low sample sizes**.
- Low sample sizes miss subtle effects, but also **prone to bias**.

Low sample sizes very sensitive to random noise



https://twitter.com/ajstewart_lang/status/1020038488278945797

Low sample sizes may bias inferences about population



http://statisticalgate.com/regression-simulation/

Low sample sizes may bias inferences

See *The evolution of correlations*

Stopping rules

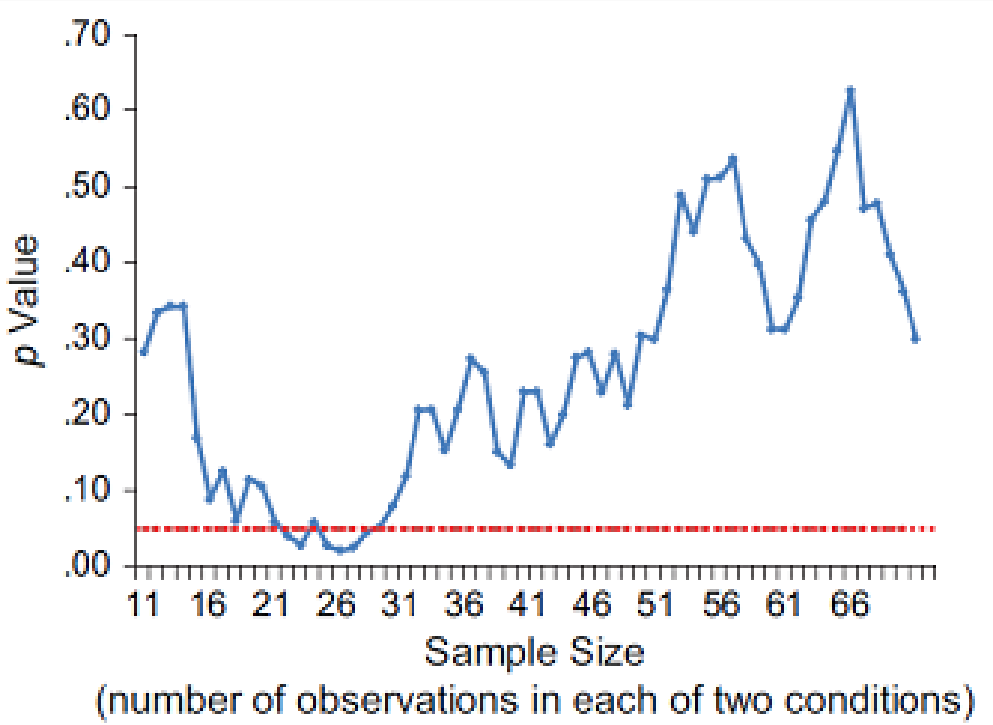


Fig. 2. Illustrative simulation of p values obtained by a researcher who continuously adds an observation to each of two conditions, conducting a t test after each addition. The dotted line highlights the conventional significance criterion of $p \leq .05$.

Sample size estimation

- Plan model/statistical analysis **before** data collection.

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Sample size estimation

- Plan model/statistical analysis **before** data collection.
- **Do simulations**. Power/Sample size/Precision analyses (e.g. see papers like this & this, or software like this & this).
- Plan to have at least **10-30 observations per predictor**.
- Complex models (w/ many predictors, interactions etc) require **high** sample sizes.

Sample size estimation

Calculating sample size for Gaussian (Normal) response model:

- expected mean: 30
- expected sd: 10
- 10 parameters (predictors)
- expected R-squared: 0.2

```
library(pmsampsize)
pmsampsize(type = "c", parameters = 10, intercept = 30, sd = 10, rsquared = 0.2)
```

```
NB: Assuming 0.05 acceptable difference in apparent & adjusted R-squared
NB: Assuming MMOE <= 1.1 in estimation of intercept & residual standard deviation
SPP - Subjects per Predictor Parameter

            Samp_size Shrinkage Parameter Rsq  SPP
Criteria 1        313     0.900        10 0.2 31.3
Criteria 2        161     0.827        10 0.2 16.1
Criteria 3        244     0.876        10 0.2 24.4
Criteria 4*       313     0.900        10 0.2 31.3
Final             313     0.900        10 0.2 31.3

 Minimum sample size required for new model development based on user inputs = 313

 * 95% CI for intercept = (29.69, 30.31), for sample size n = 313
```

Sample size estimation

Calculating sample size for binary response model:

- expected prevalence: 0.1
- 20 parameters (predictors)
- expected R-squared: 0.2

```
library(pmsampsize)
pmsampsize(type = "b", parameters = 20, prevalence = 0.1, rsquared = 0.2)
```

```
NB: Assuming 0.05 acceptable difference in apparent & adjusted R-squared
NB: Assuming 0.05 margin of error in estimation of intercept
NB: Events per Predictor Parameter (EPP) assumes prevalence = 0.1

           Samp_size Shrinkage Parameter Rsq Max_Rsq  EPP
Criteria 1       796     0.900        20 0.2    0.48 3.98
Criteria 2       738     0.893        20 0.2    0.48 3.69
Criteria 3       139     0.900        20 0.2    0.48 0.70
Final            796     0.900        20 0.2    0.48 3.98

 Minimum sample size required for new model development based on user inputs = 796,
 with 80 events (assuming an outcome prevalence = 0.1) and an EPP = 3.98
```

Randomization

Randomization

Randomization

- Haphazard $\neq$ Random

Randomization

- Haphazard $\neq$ Random
- Stratify: randomize within groups (e.g. species, soil types)

Controls

Have controls

- Untreated individuals, plots... (assigned randomly, of course).

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- **Must differ only in treatment** (i.e. homogeneous environment).

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- Measure **before & after** treatment.

Have controls

- Untreated individuals, plots... (assigned randomly, of course).
- **Must differ only in treatment** (i.e. homogeneous environment).
- Measure **before & after** treatment.
- Consider **blind designs** to avoid observer bias.

Experimental design principles

1. Replication

Experimental design principles

1. Replication
2. Randomization

Experimental design principles

1. Replication
2. Randomization
3. Controls

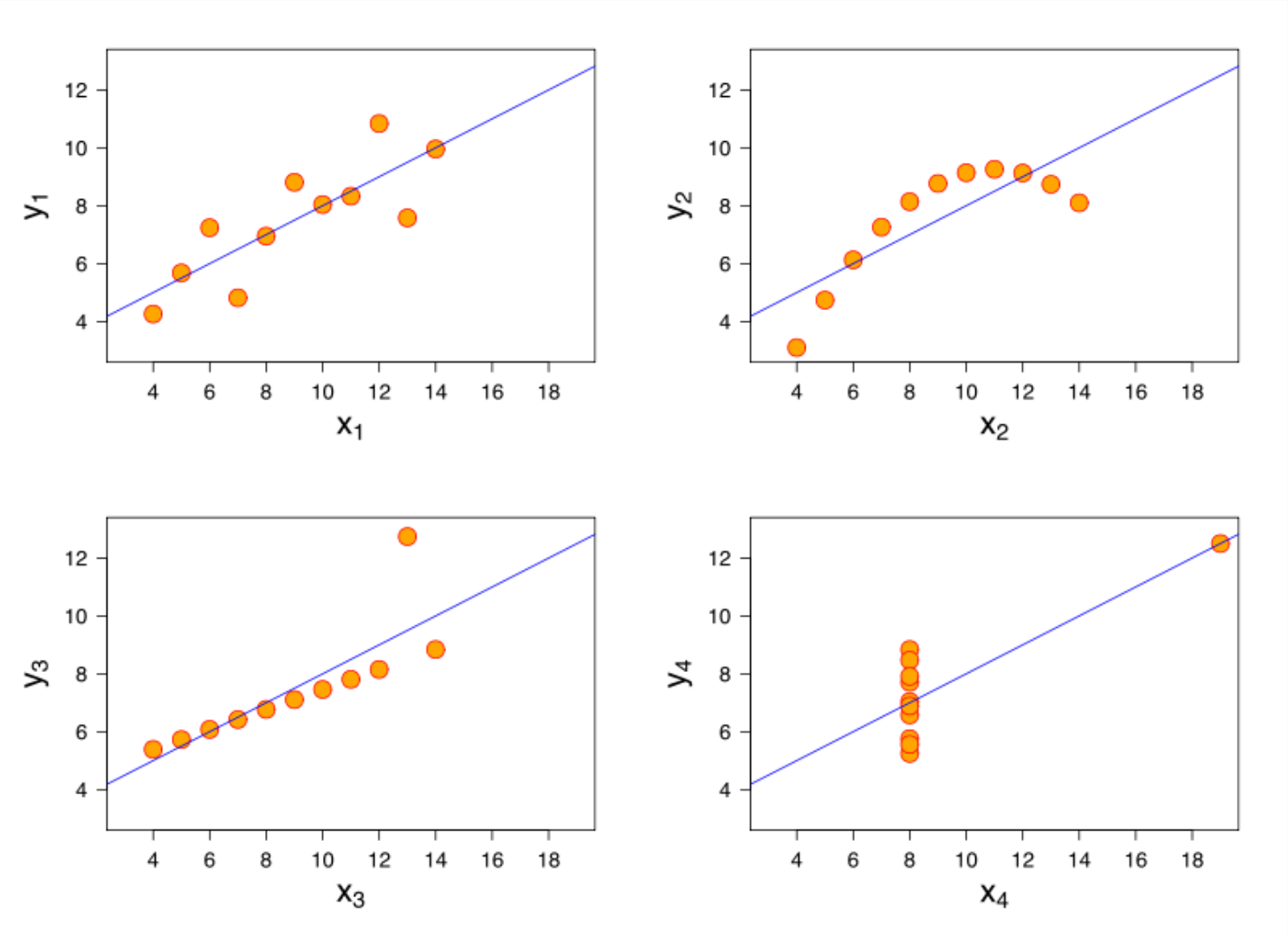
To read more

- Ruxton & Colegrave. Experimental Design for the Life Sciences. OUP

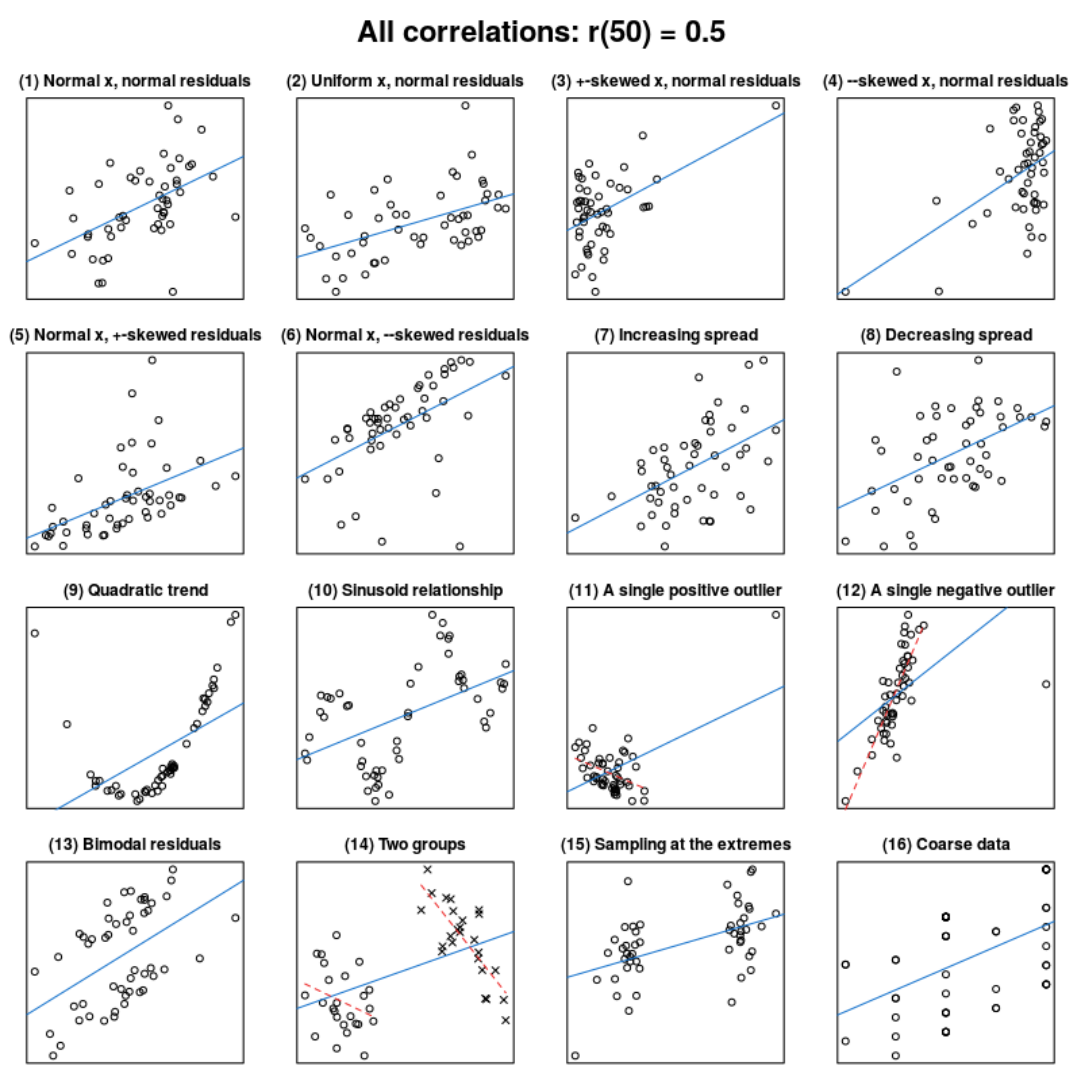
The importance of visualisation

Visualisation of data & models is key

Always plot data and models

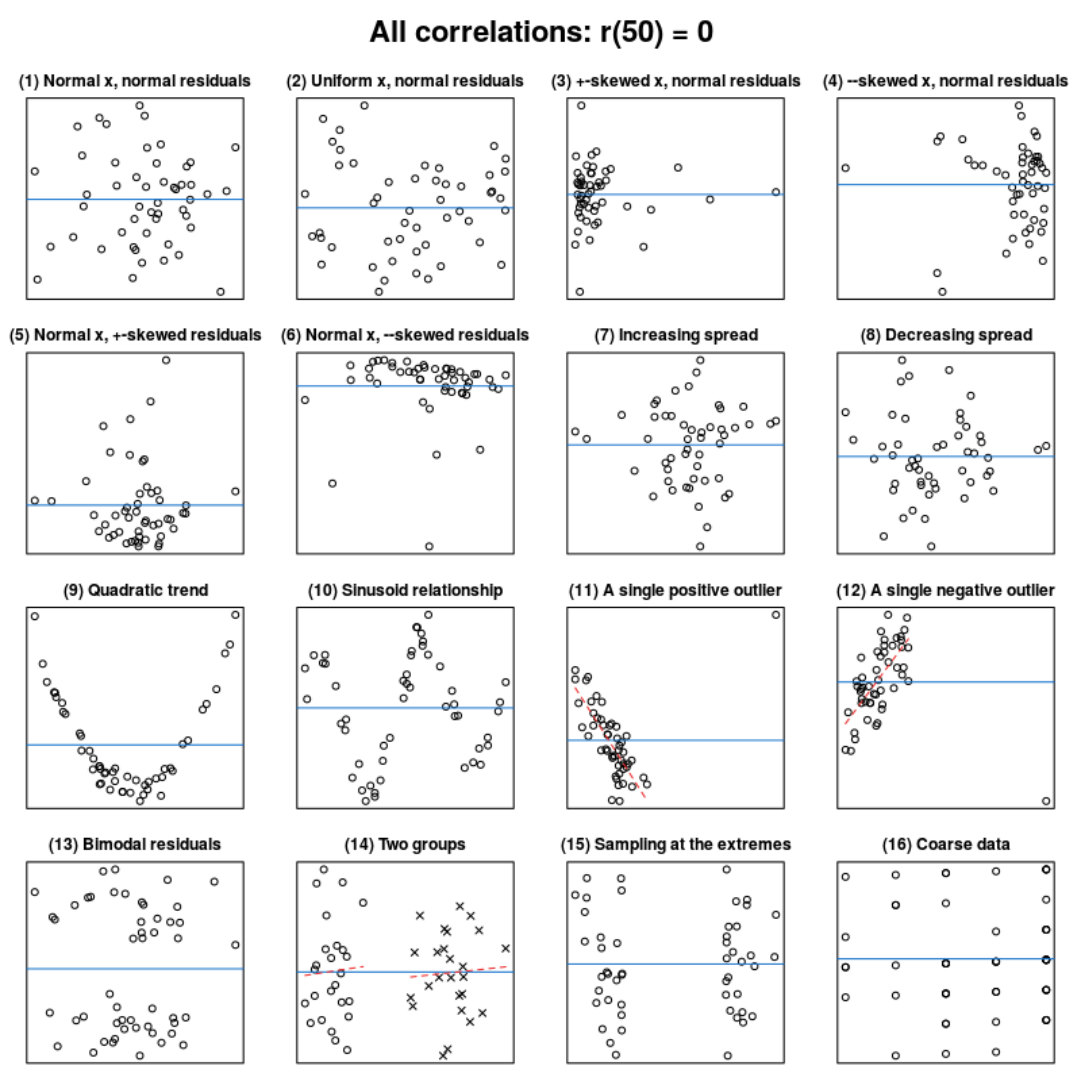


Don’t use statistics blindly: *Visualise*



https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like

Don’t use statistics blindly: *Visualise*



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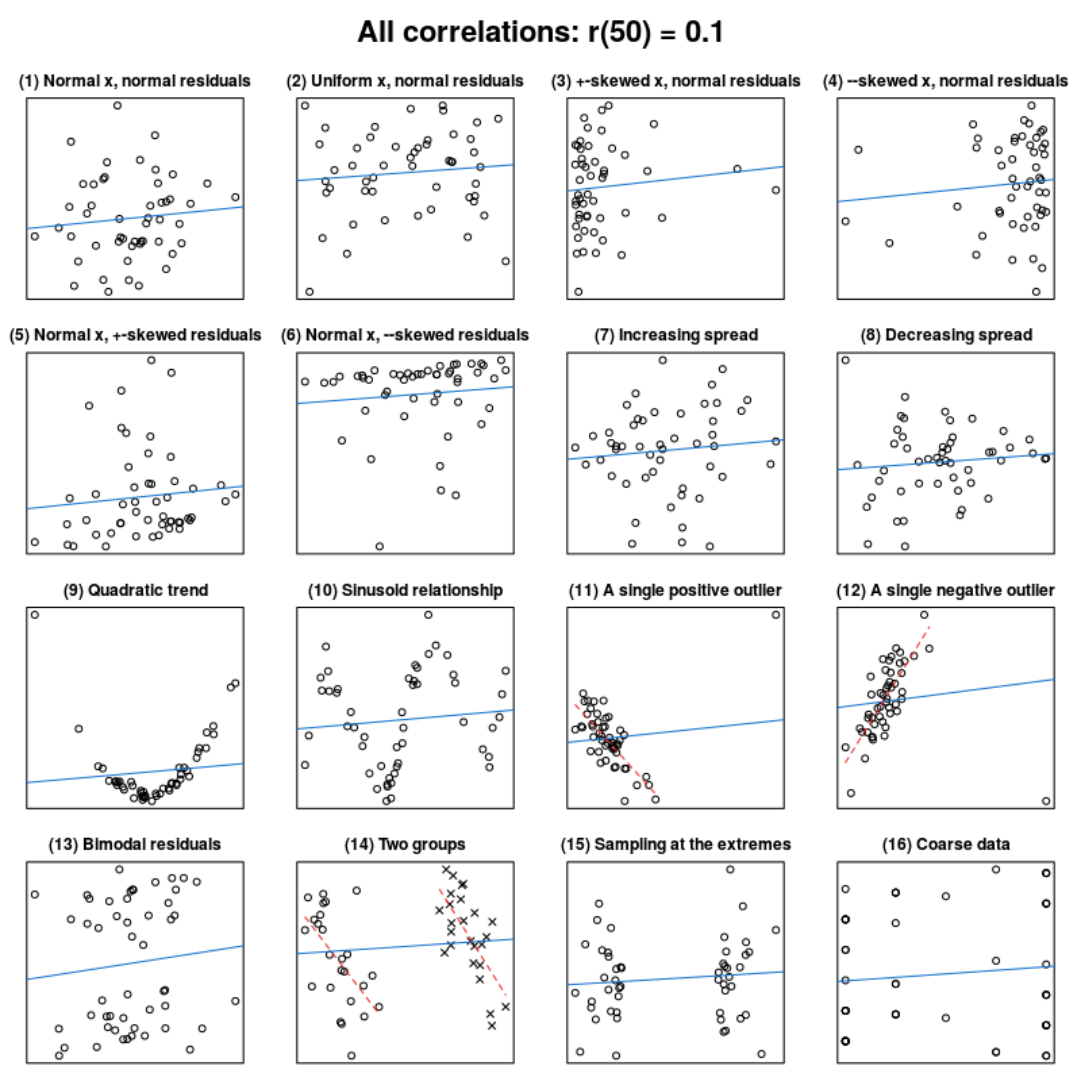

https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like

Plot. Check models. Plot. Check assumptions. Plot.

Lavine 2014 *Ecology*

Inference from observational studies

News: Hamburgers increase risk of heart attack

- In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.

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- **Do hamburgers increase heart attacks?**

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- https://pollev.com/franciscorod726

Bigger flowers increase reproductive success

- We found that plants with big flowers produced 30% more seeds...

Bigger flowers increase reproductive success

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Bigger flowers increase reproductive success

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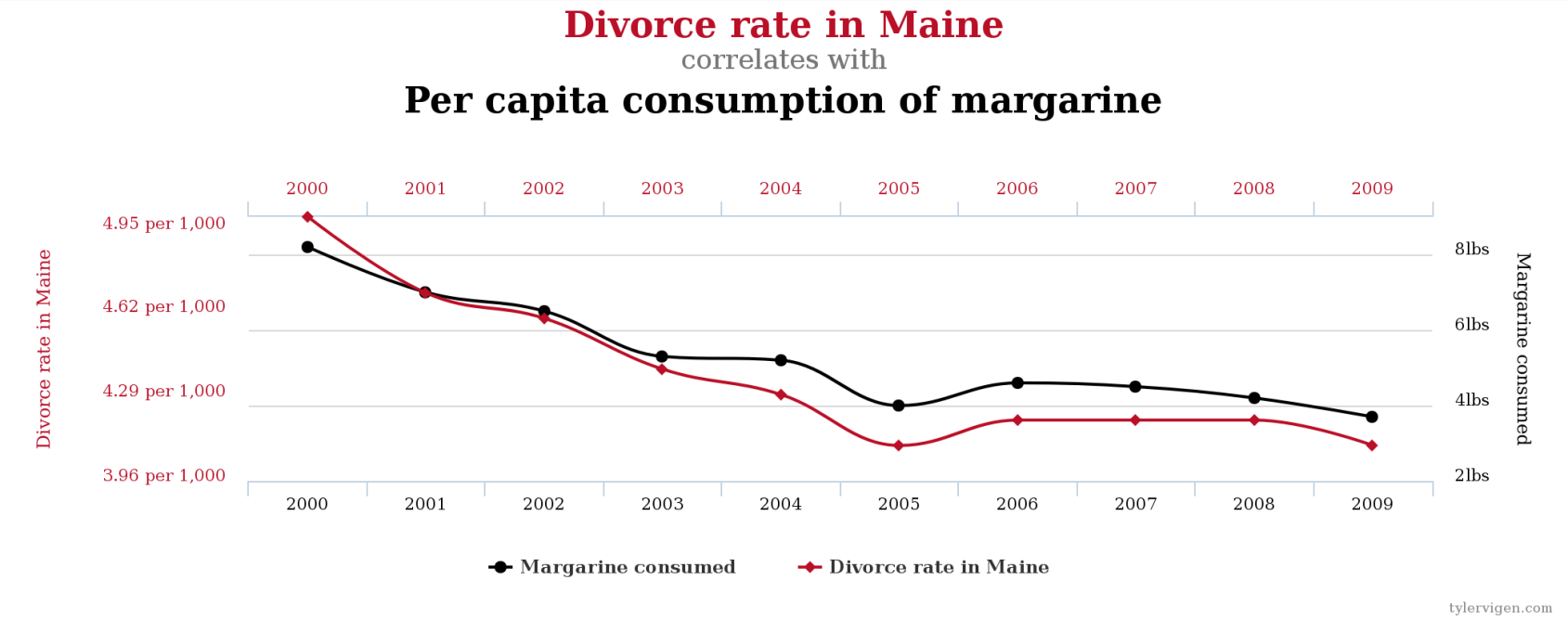
Correlation vs Causation



http://tylervigen.com/spurious-correlations

Hypothesis testing

NHST concepts

Null and alternative hypotheses

- Tell me...

Null and alternative hypotheses

- Tell me...
- **Null hypothesis**: there is no difference between groups.

Null and alternative hypotheses

- Tell me...
- **Null hypothesis**: there is no difference between groups.
- **Alternative hypothesis**: groups are different.

In biology, everything is somewhat different

Are there any differences? A non-sensical question in ecology

Alejandro Martínez-Abraín

IMEDEA (CSIC-UIB), C/Miquel Marquès 21, 07190 Esporles, Majorca, Spain

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ABSTRACT

One of the main questions that ecologists pose in their investigations includes the analysis of differences in some trait between two or more populations. I argue here that asking whether there are differences or not between populations is biologically irrelevant, since no two livings things are ever equal. On the contrary the appropriate question to pose is how large differences are between populations. That is, we urge a shift in interest from statistical significance to biological relevance for proper knowledge accumulation. I empha-

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- The probability that the observed data were produced by chance

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- What assumptions?
    - Null hypothesis is true
    - No uncontrolled sources of bias (measurement or programming error, p-hacking, etc)

How to interpret P-values

- **Low P-value**: data unlikely if every model assumption (including H0) were true.

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- See Greenland et al 2016

For example

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- p-value < 10^{-7} -> reject null hypothesis of equal speed
- In reality, measurement error (loose cable)

If p-value > 0.05

- the null hypothesis is false, i.e. the alternative hypothesis must be true

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If p-value > 0.05

- the null hypothesis is false, i.e. the alternative hypothesis must be true
- the alternative hypothesis is false, i.e. the null hypothesis must be true
- it's unclear if there are differences between groups
- there is no difference between groups

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Are differences *significant*?

Common practice:

- If $p < 0.05$, we **reject** H0.

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https://doi.org/10.1038/d41586-019-00857-9

Are these two groups different?

```
t.test(group.A, group.B)
```

```
    Welch Two Sample t-test

data:  group.A and group.B
t = -0.85334, df = 6.8795, p-value = 0.4222
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -19.282564   9.082564
sample estimates:
mean of x mean of y
    170.2     175.3
```

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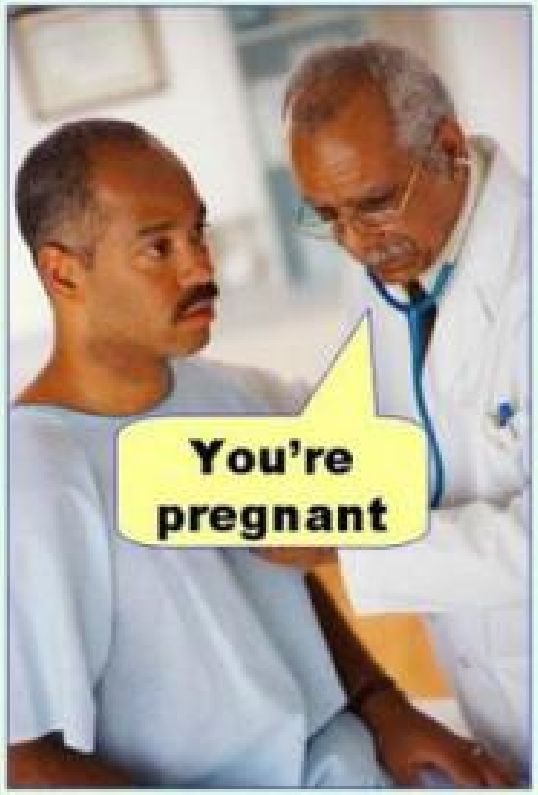
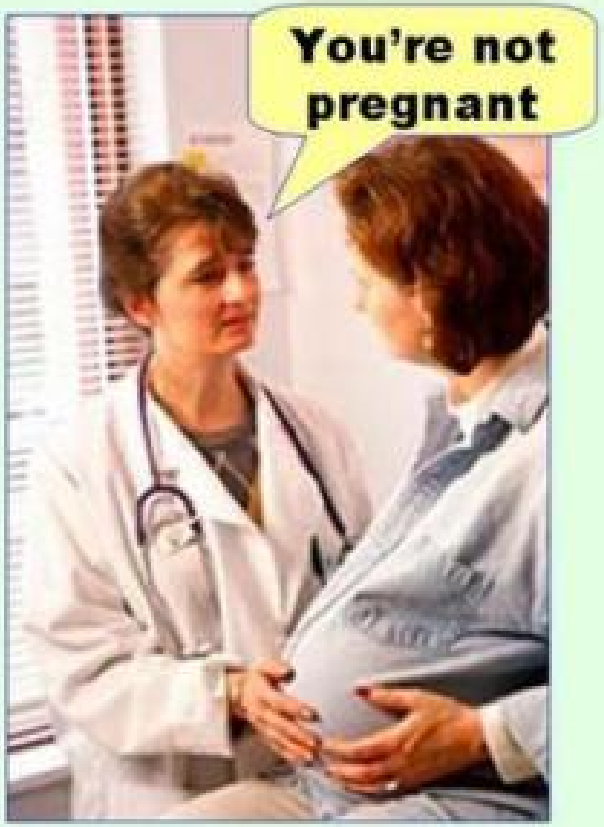
Rejecting hypotheses: two types of error

Type I error
(false positive)



Type II error
(false negative)



Rejecting hypotheses: two types of error

Statistics: Hypothesis Test	Null Hypothesis is True	Null Hypothesis is False
Reject Null Hypothesis	Type I Error	Correct
Fail to Reject Null Hypothesis	Correct	Type II Error

POWER: Probability of detecting true difference (rejecting H0 when it's false).

Is this coin biased?

```
 [1] 1 1 0 0 1 1 0 0 0 0

    1-sample proportions test with continuity correction

data:  sum(coin) out of ntrials, null probability 0.5
X-squared = 0.1, df = 1, p-value = 0.7518
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.1369306 0.7263303
sample estimates:
  p
0.4
```

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Understanding NHST

http://rpsychologist.com/d3/NHST/

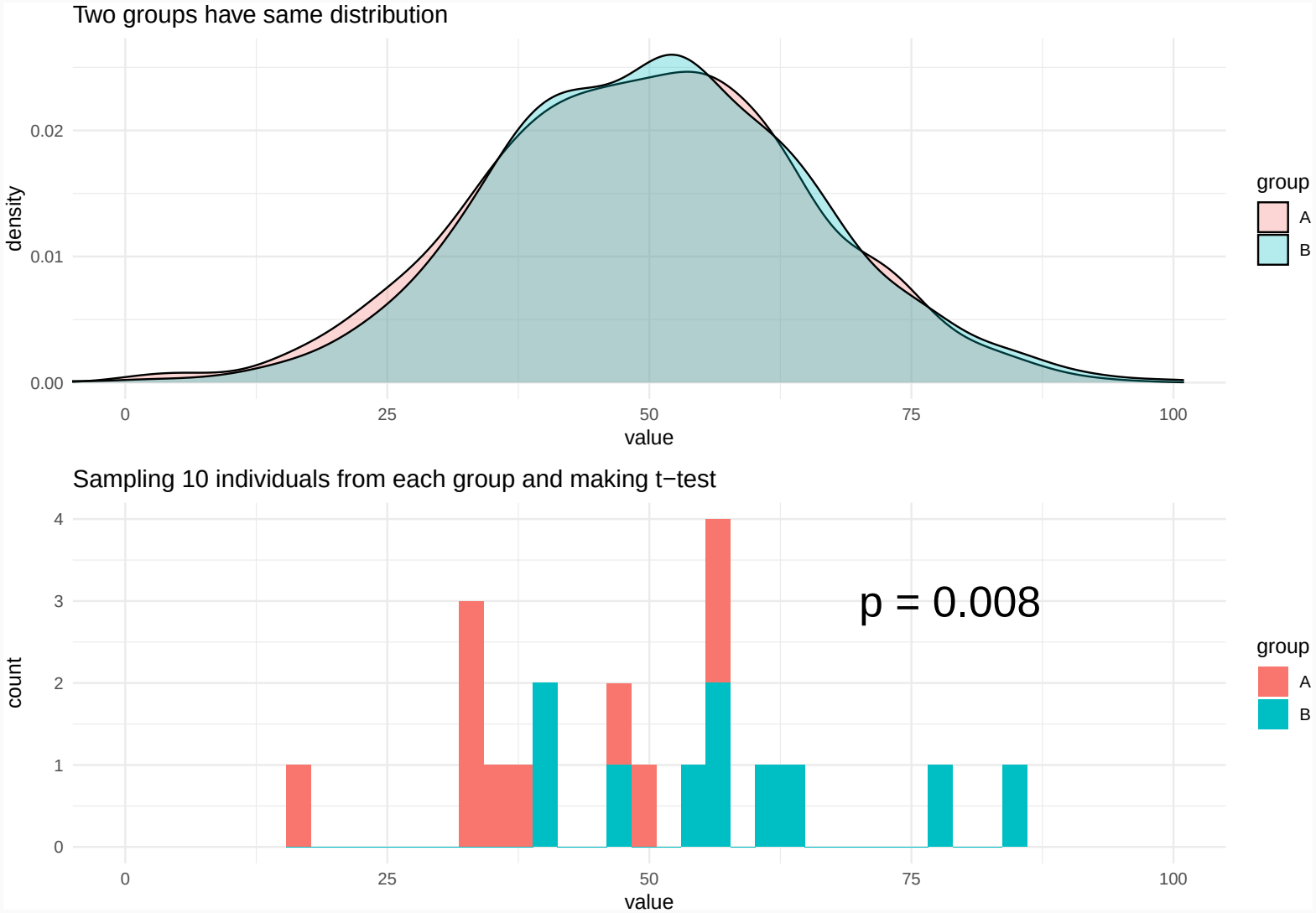
http://daniellakens.blogspot.com/2017/12/understanding-common-misconceptions.html

NHST and p-values: common pitfalls

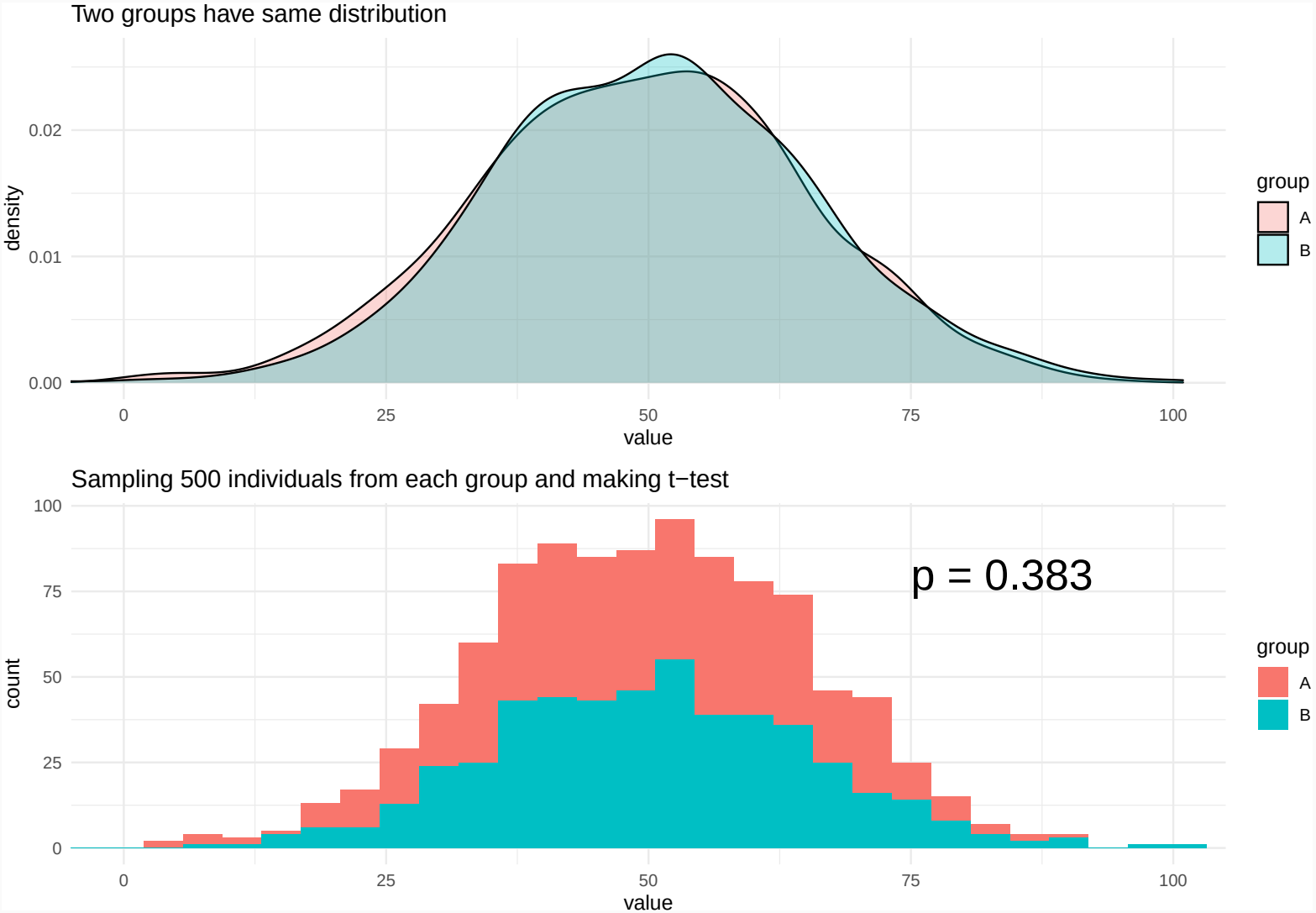
A significant p-value
does NOT mean
we found a true difference

A significant p-value does not mean we found a true difference

Particularly with low sample sizes



If sample size was larger...



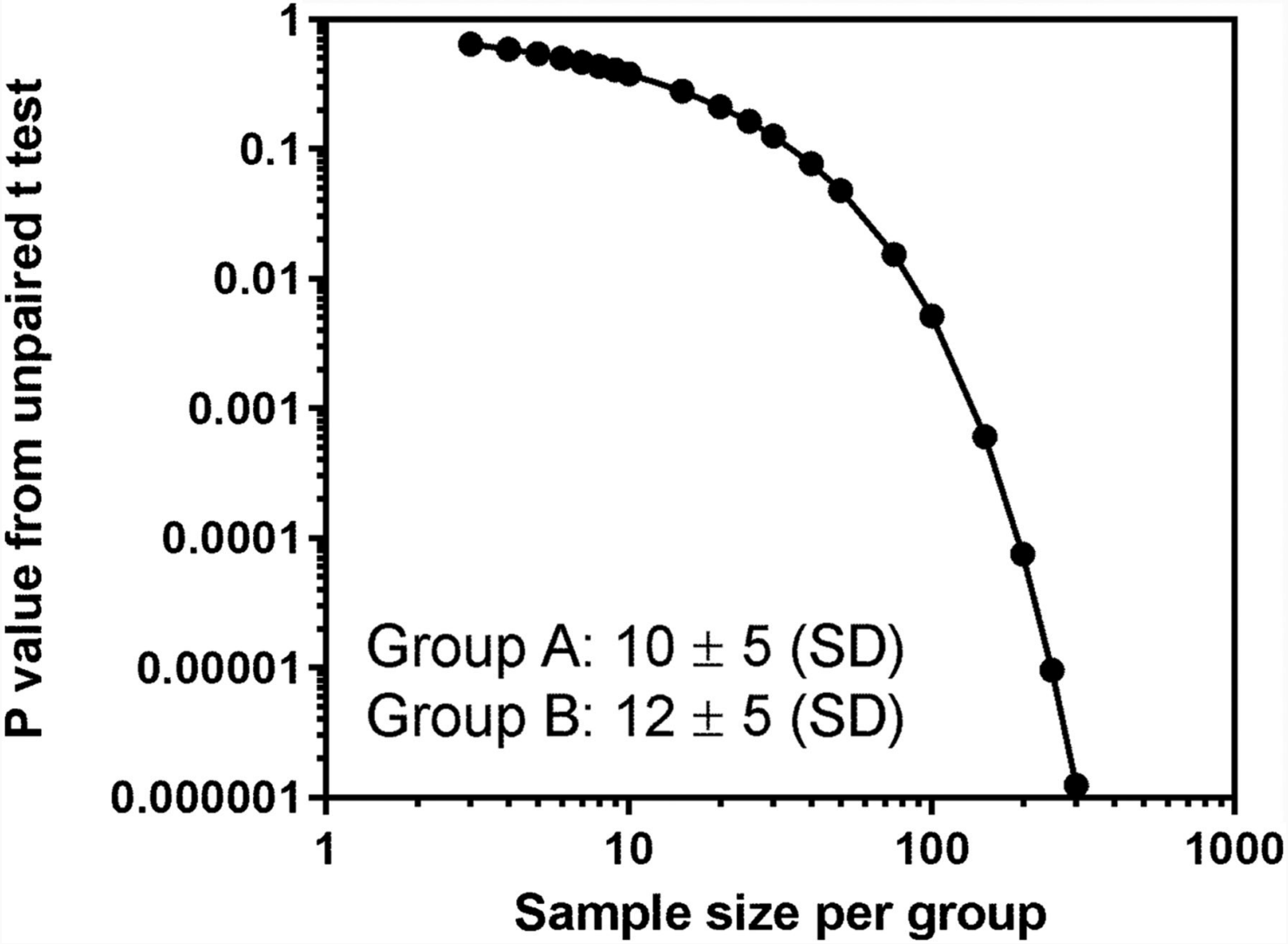
With low sample size (power),

significant p-values

are most likely overestimates

Loken & Gelman 2014, Vasisth et al. 2018

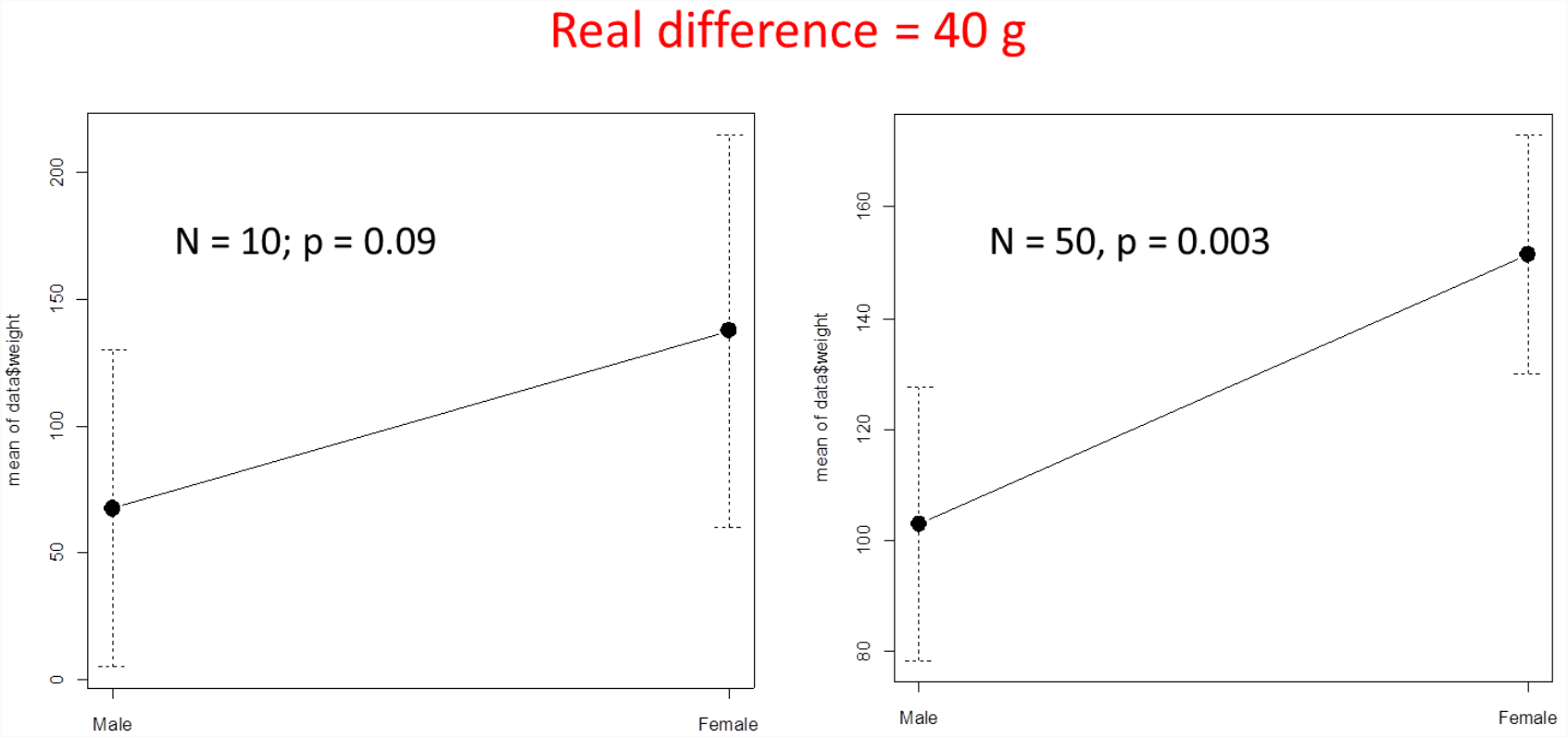
P-value depends on sample size



https://doi.org/10.1002/prp2.93

P-value depends on sample size

Same real difference is detected as **significant or not depending on sample size**



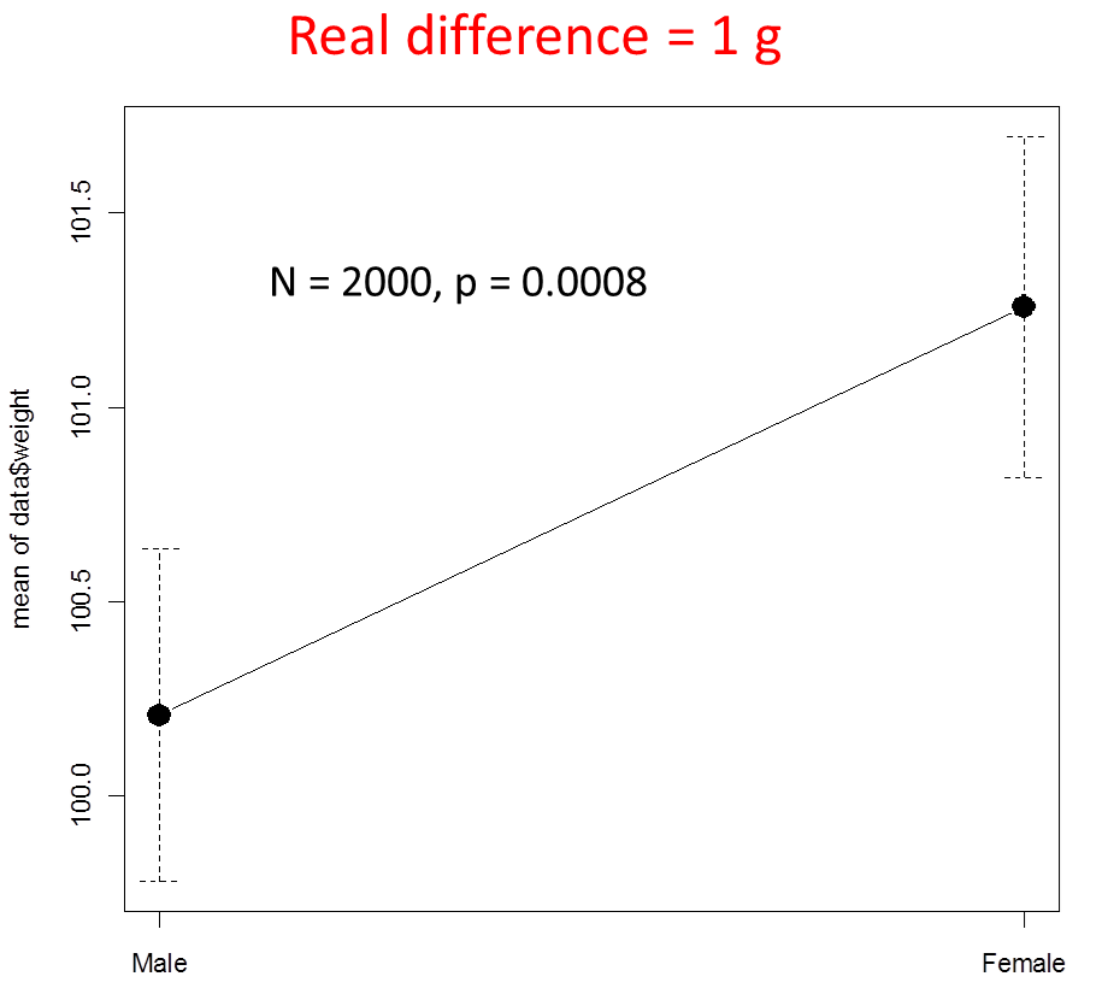
Statistically significant

!=

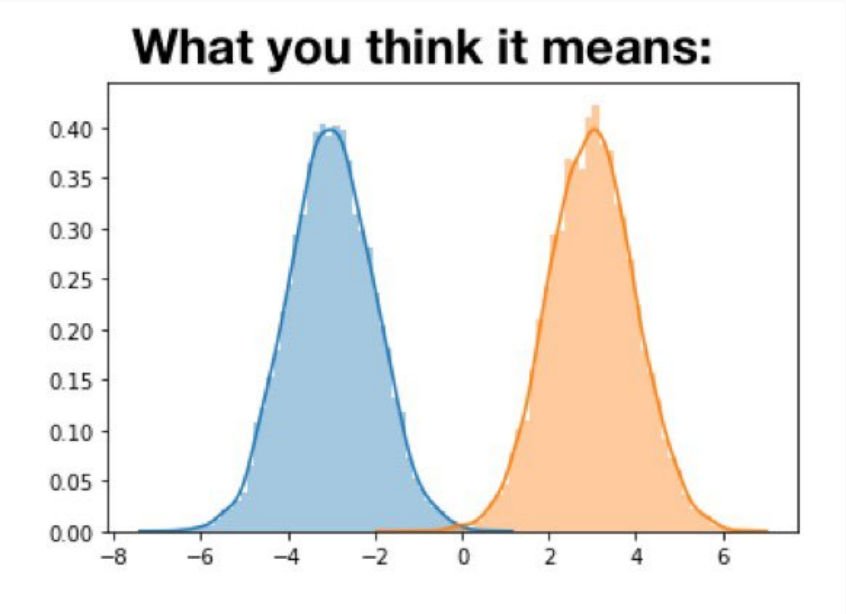
biologically important

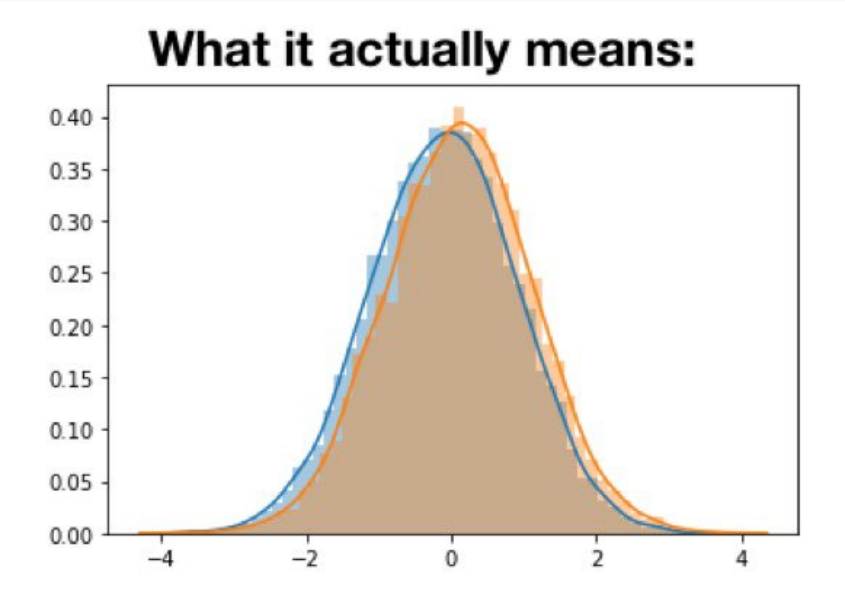
Statistically significant != biologically important

With big sample size, we can find **highly significant but biologically unimportant** differences.



Statistically significant != biologically important





https://twitter.com/weinberz/status/1422405165236178947?s=20

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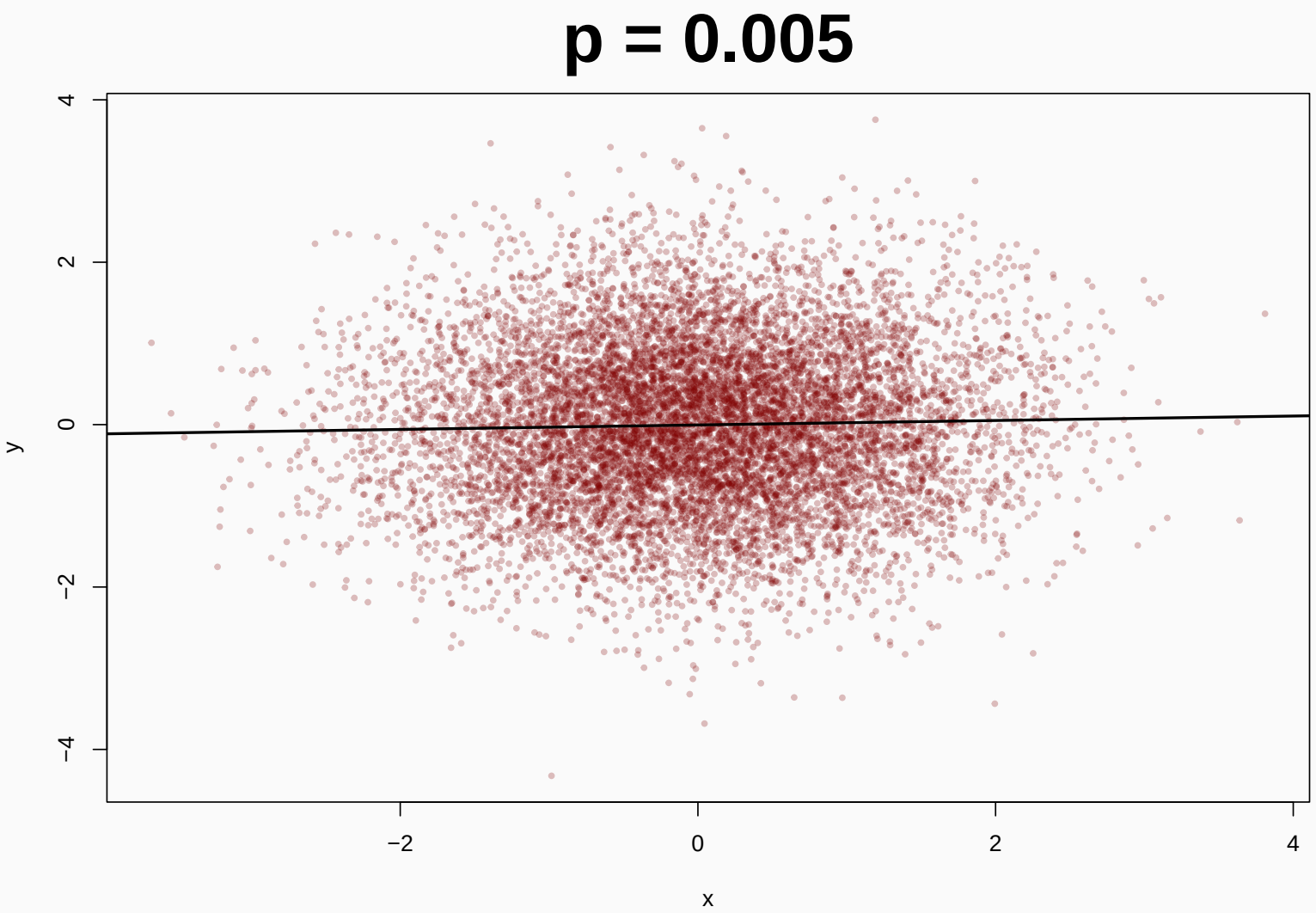

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- Statistically significant = unlikely to be zero
- Good read: *significantly misleading*
- Beyond significant/not significant, look at **effect sizes and their uncertainty**.

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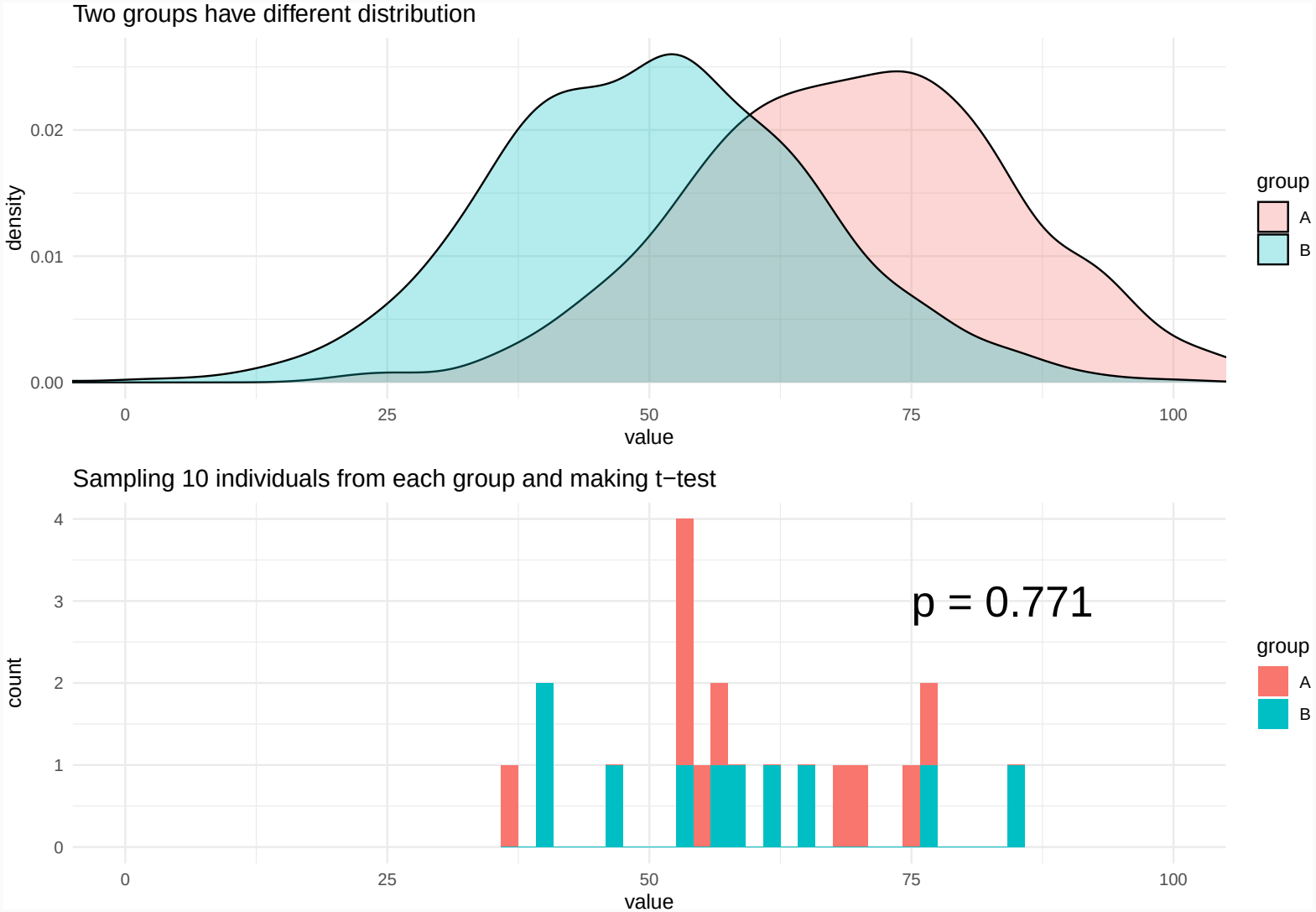

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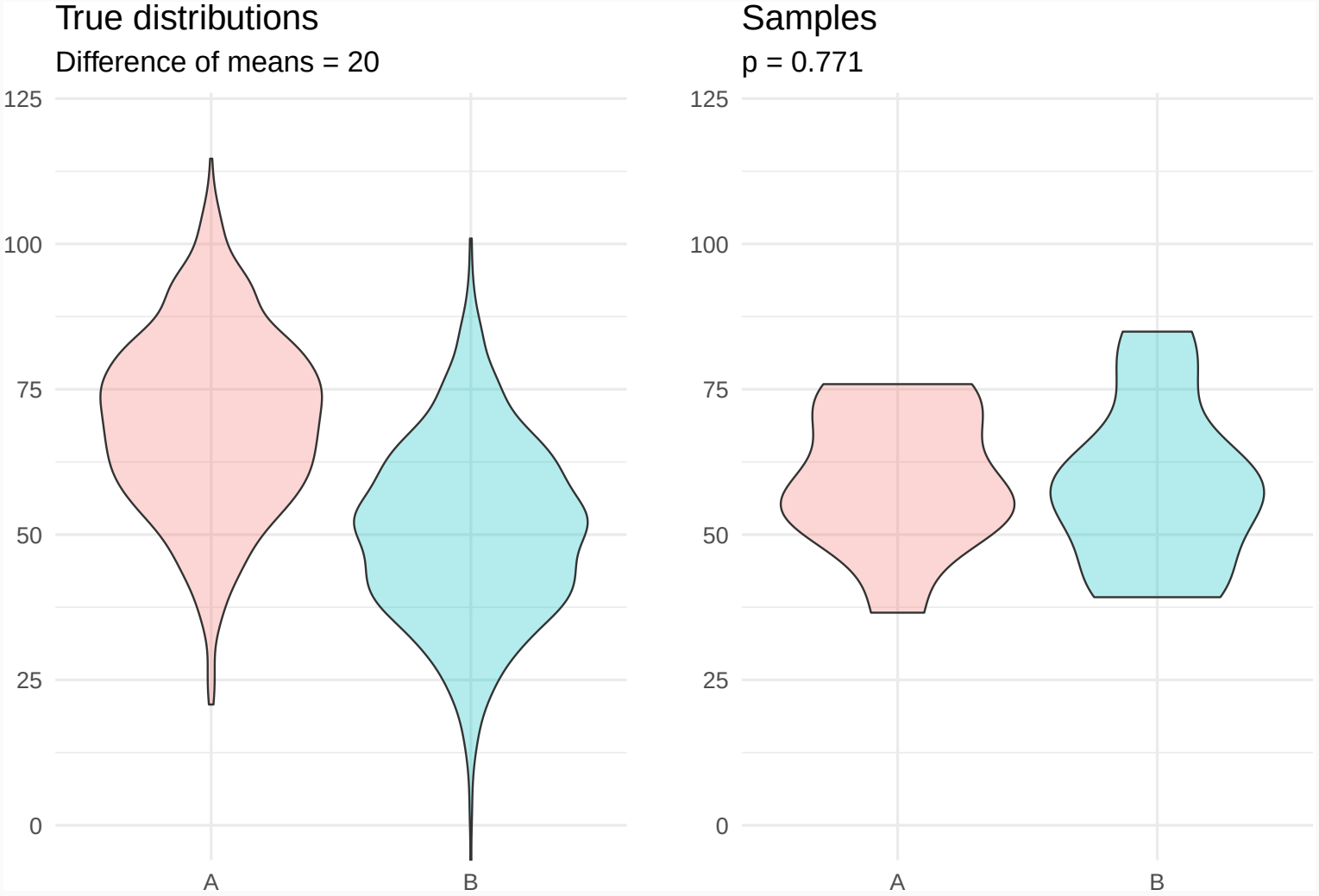

Failure to reject H0 != H0 is true

Absence of evidence != Evidence of absence

p-value > 0.05?

- "We were **unable to find evidence** against the hypothesis that A = B **with the current sample size**" (Harrell)

p-value > 0.05?

- “We were **unable to find evidence** against the hypothesis that A = B **with the current sample size**” (Harrell)
- “Differences between groups were **not statistically clear**” (Dushoff et al)

Is it safe to allow right turn with red lights?

- Right turn not allowed: 308 accidents

https://www.statisticsdonewrong.com/power.html#the-wrong-turn-on-red

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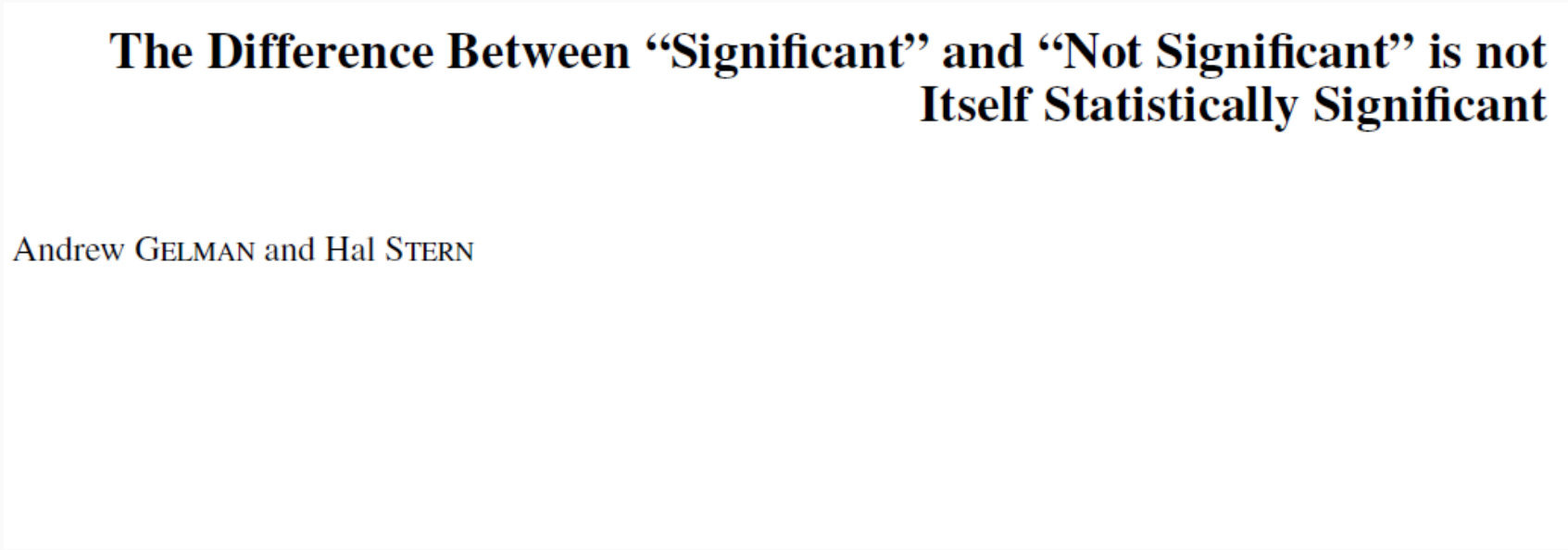
0.05 is an arbitrary threshold

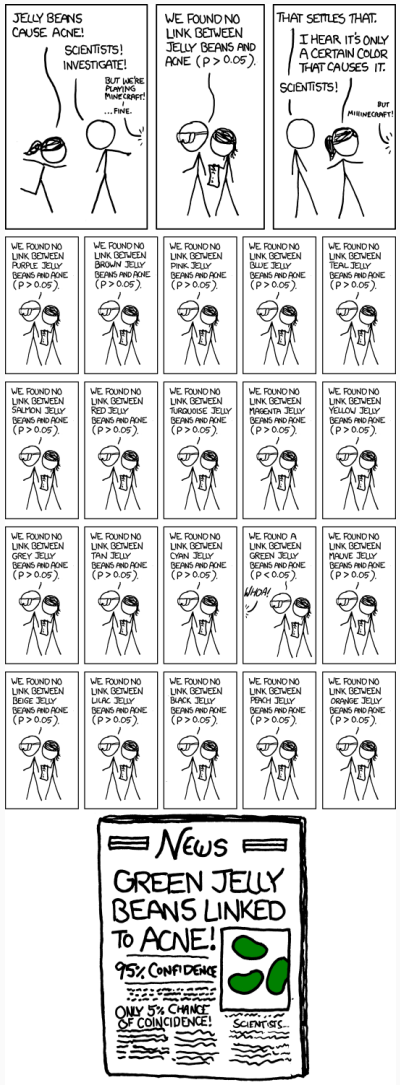


The Difference Between "Significant" and "Not Significant" is not Itself Statistically Significant

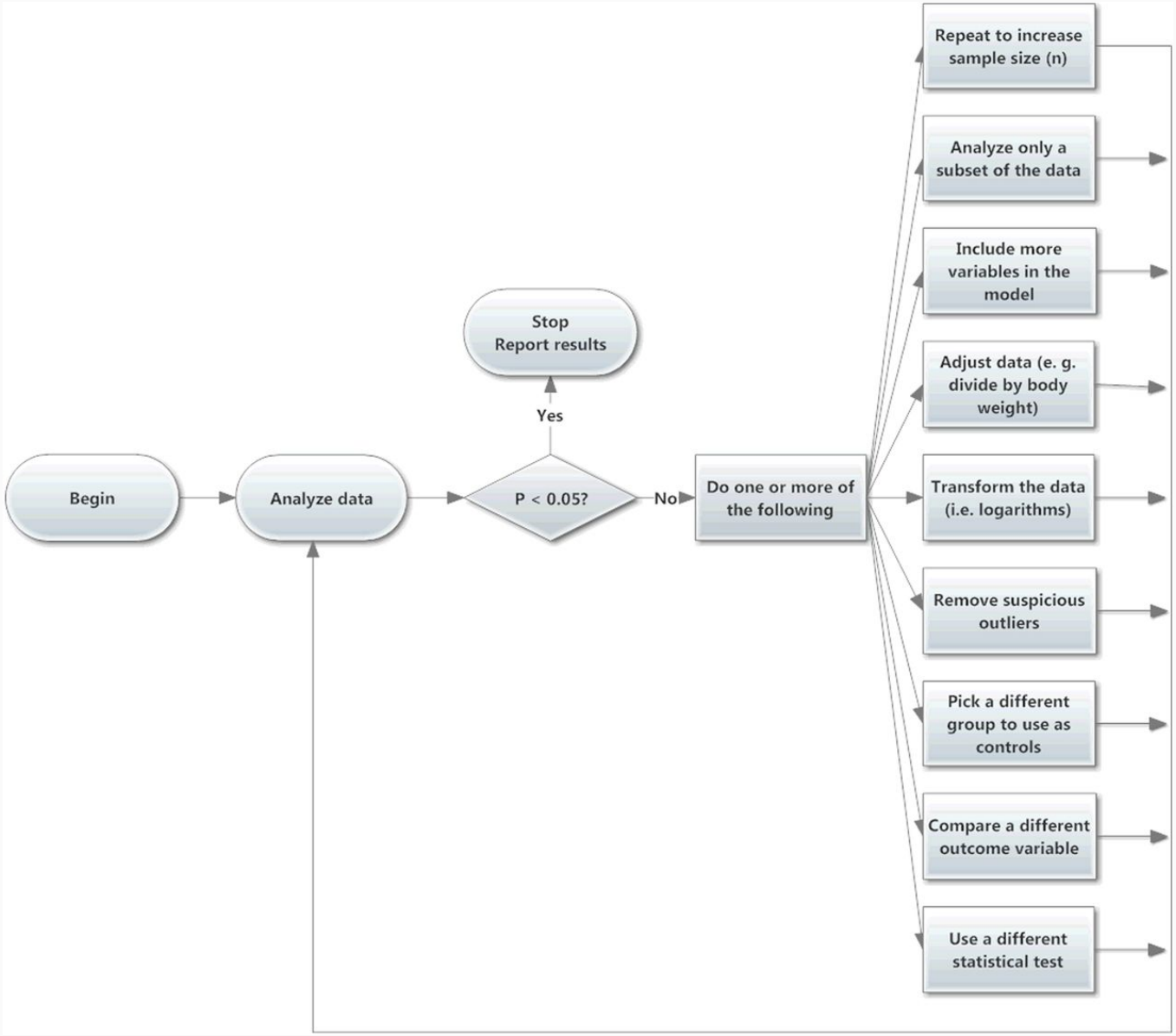
Andrew GELMAN and Hal STERN

http://dx.doi.org/10.1198/000313006X152649

Multiple hypothesis testing



How to make your results significant: *p-hacking*



http://dx.doi.org/10.1002/prp2.93

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- To read more: Simmons et al 2011

p-hacking: try it yourself

https://www.shinyapps.org/apps/p-hacker/

ASA statement on p-values

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https://doi.org/10.1080/00031305.2016.1154108

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Good practice

A must read

Eur J Epidemiol (2016) 31:337–350
DOI 10.1007/s10654-016-0149-3

CrossMark

ESSAY

Statistical tests, *P* values, confidence intervals, and power: a guide to misinterpretations

Sander Greenland[1] · Stephen J. Senn[2] · Kenneth J. Rothman[3] · John B. Carlin[4] · Charles Poole[5] · Steven N. Goodman[6] · Douglas G. Altman[7]

https://doi.org/10.1007/s10654-016-0149-3

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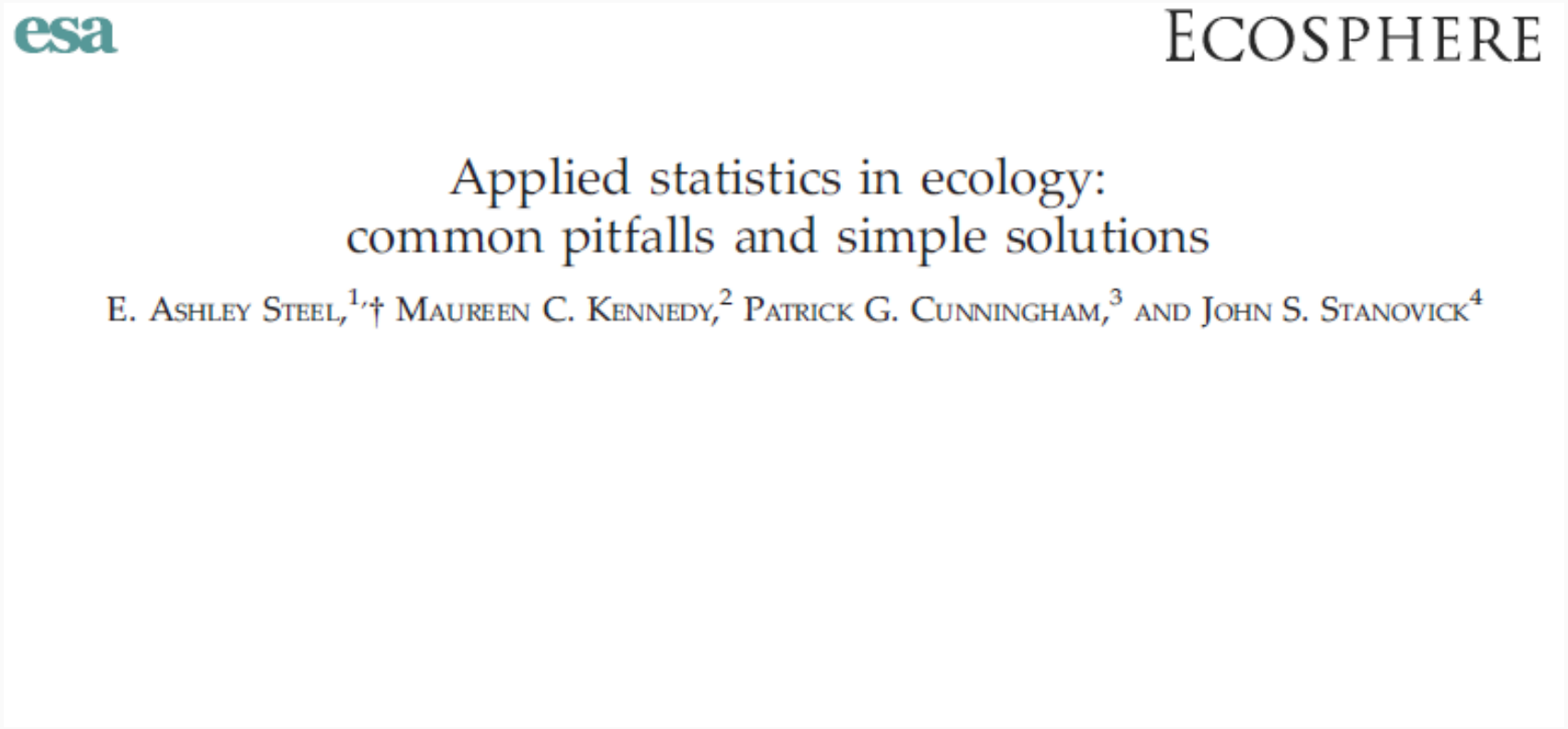
esa

ECOSPHERE

Applied statistics in ecology:
common pitfalls and simple solutions

E. Ashley Steel,[1,†] Maureen C. Kennedy,[2] Patrick G. Cunningham,[3] and John S. Stanovick[4]

https://doi.org/10.1890/ES13-00160.1

Also http://www.statisticsdonewrong.com/

Good read

Twenty tips for interpreting scientific claims

https://doi.org/10.1038/503335a

The New Statistics

Aim for estimation of effects and their uncertainty (SE, CI…)

General Article

aps ASSOCIATION FOR PSYCHOLOGICAL SCIENCE

The New Statistics: Why and How

Geoff Cumming
La Trobe University

Psychological Science
2014, Vol. 25(1) 7–29

DOI: 10.1177/0956797613504966
pss.sagepub.com
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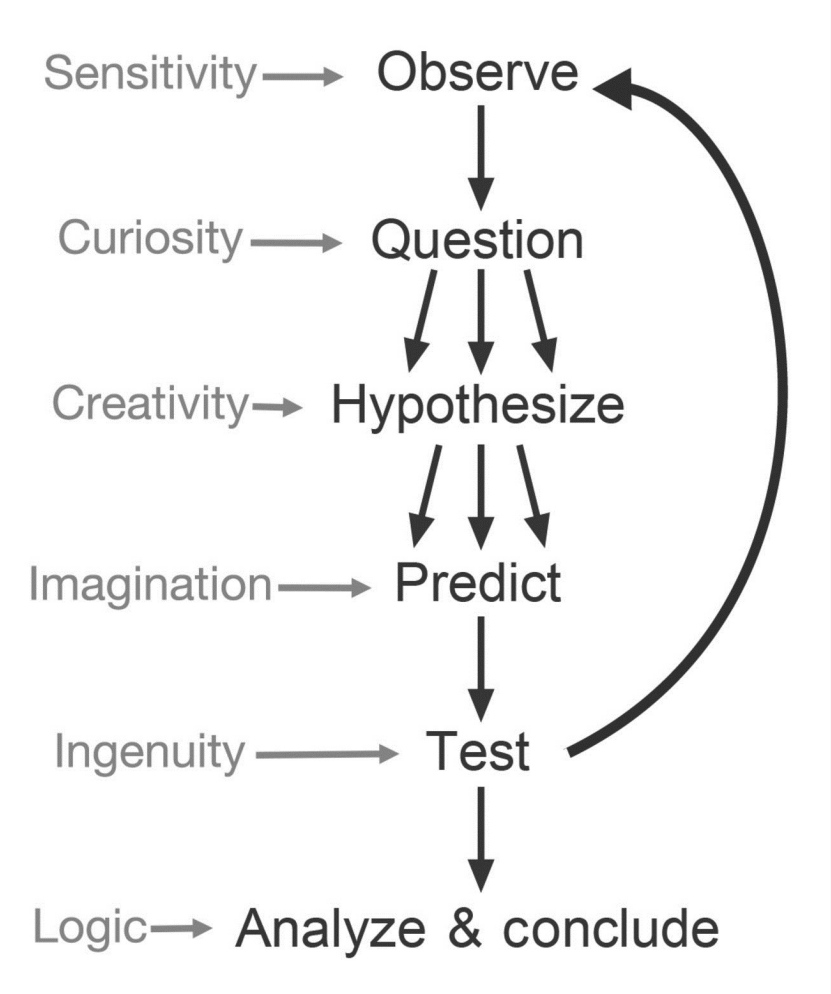
http://dx.doi.org/10.1177/0956797613504966

Instead of falsifying null model, compare meaningful models



https://doi.org/10.1242/jeb.104976

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