

# Sampling, confidence intervals, likelihood and Bayesian inference

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# Inference: from samples to population

We rarely measure the whole **population**, but take **samples**.

Then we make inferences from sample to population.



## If we sample 30 trees in our neighbourhood...

Can we extrapolate results to

- whole neighbourhood?
- whole city?
- whole country?
- the world?

What's the **suitable population** to make inferences given this sample?

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- To read more: [Morey et al \(2015\)](#)

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- but still 5% of CIs will NOT contain true mean!

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<https://pollev.com/franciscorod726>

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- If we repeated the experiment, 95% of the CIs would contain the true value of X
- The probability that X is greater than 0 is at least 95%
- The probability that X equals 0 is smaller than 5%

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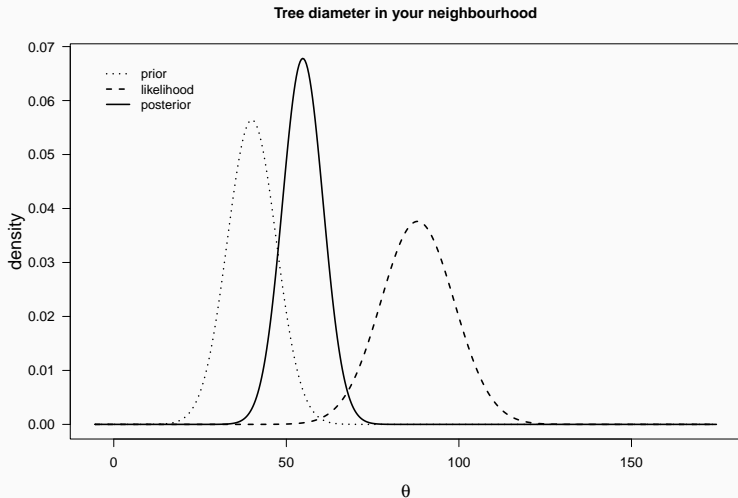
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- Frequentist CIs and Bayesian credible intervals can be similar, but not always.

# Bayesian inference: prior, posterior, and likelihood

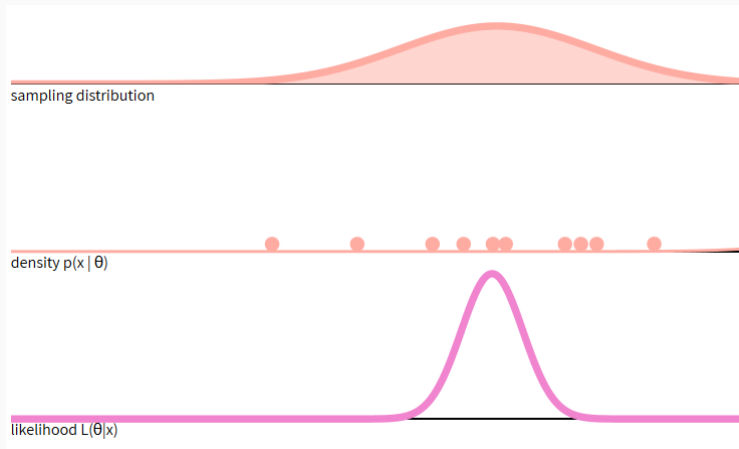
$$P(\text{Unknown}|\text{Data}) \propto P(\text{Data}|\text{Unknown}) \times P(\text{Unknown})$$

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$



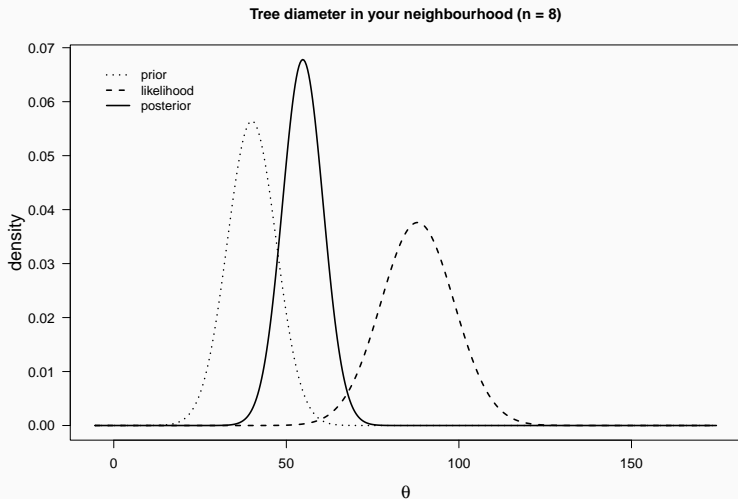
# What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$

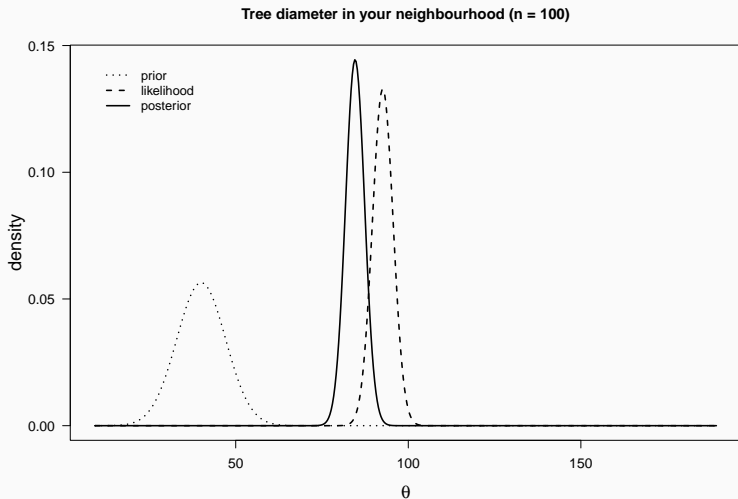


<https://seeing-theory.brown.edu/bayesian-inference/index.html>

# Bayesian inference: prior and likelihood produce posterior



## With increasing sample size, likelihood dominates prior





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- Bayesian t-test

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- Uncertainty / Propagate errors