Sampling, confidence intervals, and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.



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- 5. Do all CIs contain true mean height?

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- The probability that X is greater than 0 is at least 95%
- The probability that X equals 0 is smaller than 5%

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- To read more: Morey et al (2015)

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- but still 5% of CIs will NOT contain true mean!

Bayesian credible intervals

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Bayesian credible intervals

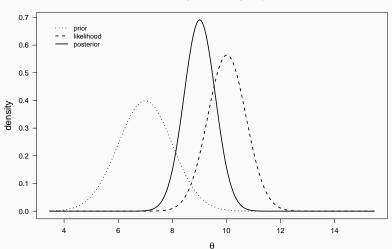
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- Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

$$P(H|D) \propto P(D|H) \cdot P(H)$$

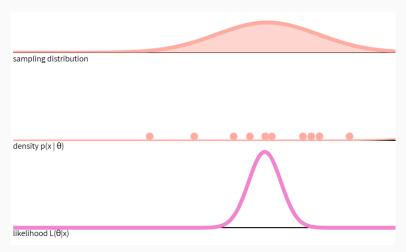
Posterior ∝ Likelihood · Prior





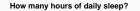
What is the likelihood?

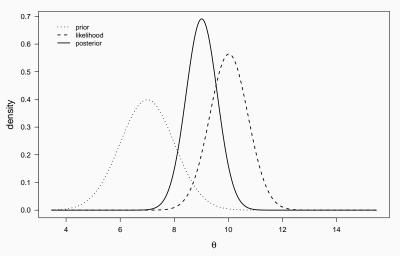
$$L(\theta|x) = P(x|\theta)$$



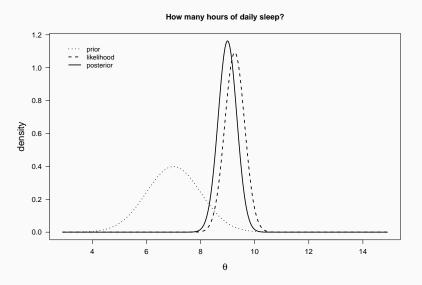
https://seeing-theory.brown.edu/bayesianinference/index.html

Bayesian inference: prior and likelihood produce posterior





With increasing sample size, likelihood dominates prior



· Bayesian Demo

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- · Bayesian inference for a population mean

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- Normal

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- · Bayesian t-test

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- Uncertainty / Propagate errors