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template from KACTL

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# Template (1)

```
template.cpp27 lines

#pragma once

#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

using ll = long long;
using db = long double;
using vi = vector<int>;
using vl = vector<ll>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<db, db>;
const int INF = 0x3fffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const ll LINF = 0xffffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));
```

```
1int main(){
2    cin.tie(nullptr)->sync_with_stdio(false);
3}

c.sh2 lines

1g++ -std=gnu++2a -Wall $1 -o a.out
2./a.out
```

## Mathematics (2)

### 2.1 Goldbatch’s Conjecture

- Even number can be written in sum of two primes (Up to 1e12)
- Range of  $N^{th}$  prime and  $N + 1^{th}$  prime will be less than or equal to 300 (Up to 1e12)

### 2.2 Divisibility

Number of divisors of  $N$  is given by  $\prod_{i=1}^k (a_i + 1)$  where  $N = \prod_{i=1}^k p_i^{a_i}$  and  $p_i$  are prime factors of  $N$ .

## Combinatorial (3)

### 3.1 Permutations

#### 3.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

#### 3.1.2 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

#### 3.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n - 1)(D(n - 1) + D(n - 2)) = nD(n - 1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

#### 3.1.4 Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).  
If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

### 3.2 Partitions and subsets

#### 3.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

#### 3.2.2 Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

#### 3.2.3 Binomials

multinomial.h

### 3.3 General purpose numbers

#### 3.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).  
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

#### 3.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$\begin{aligned} c(n, k) &= c(n - 1, k - 1) + (n - 1)c(n - 1, k), \quad c(0, 0) = 1 \\ \sum_{k=0}^n c(n, k) x^k &= x(x + 1) \dots (x + n - 1) \end{aligned}$$

$$\begin{aligned} c(8, k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n, 2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots \end{aligned}$$

### 3.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j + 1)$ ,  $k + 1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 3.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

### 3.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

### 3.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$   
# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \cdots n_k n^{k-2}$   
# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \cdots (d_n-1)!)$

### 3.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \ C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with with  $n + 1$  leaves (0 or 2 children).
- ordered trees with  $n + 1$  vertices.
- ways a convex polygon with  $n + 2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

## Numerical (4)

### 4.1 Newton’s Method

if  $F(Q) = 0$ , then  $Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$

$$Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}$$

$$Q = \ln P = \int \frac{P'}{P} \mathrm{d}x$$

$$Q = e^P : Q_{2n} \equiv Q_n (1 + P - \ln Q_n) \pmod{x^{2n}}$$

$$Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2} (Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}$$

$$Q = P^k = \alpha^k x^{kt} e^{k \ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1$$

## Group (5)

### 5.1 Monoid

monoid/MonoidBase.hpp

### 5.2 Action

action/MonoidActionBase.hpp

action/DefaultAction.hpp

## Data Structures (6)

OrderedSet.hpp

FenwickTree.hpp

SmallSegmentTree.hpp

SegmentTree.hpp

SegmentTreeBeats.hpp

LazySegmentTree.hpp

DynamicSegmentTree.hpp

DSU.hpp

BinaryTrie.hpp

LiChaoTree.hpp

DynamicLiChaoTree.hpp

SplayTreeBase.hpp

LazyReversibleBBST.hpp

LazyReversibleSplayTree.hpp

LinkCutTreeBase.hpp

LazyLinkCutTree.hpp

StaticTopTree.hpp

CartesianTree.hpp

## Number Theory (7)

ExtendedEuclid.hpp

euclid.h

CRT.hpp

phiFunction.hpp

FloorSum.hpp

### 7.1 Prime Numbers

MillerRabin.hpp

LinearSieve.hpp

FastEratosthenes.hpp

GolbatchConjecture.hpp

## Graph (8)

### 8.1 Matching

HopcroftKarp.hpp

Kuhn.hpp

WeightedMatching.hpp

### 8.2 Network Flow

Dinic.hpp

MinCostFlow.hpp

BinaryOptimization.hpp

KaryOptimization.hpp

### 8.3 Connectivity

SCC.hpp

LowLink.hpp

## Tree (9)

HLD.hpp

CentroidDecom.hpp

## Polynomials (10)

FormalPowerSeries.hpp

FFT.hpp

NTT.hpp

Strings (11)

Manacher.hpp

SuffixArray.hpp

ZAlgo.hpp

PrefixFunction.hpp

SuffixAutomaton.hpp

Geometry (12)

12.1 Geometric primitives

Point.h

lineDistance.h

SegmentDistance.h

SegmentIntersection.h

lineIntersection.h

sideOf.h

OnSegment.h

linearTransformation.h

LineProjectionReflection.h

Angle.h

12.2 Circles

CircleIntersection.h

CircleTangents.h

CircleLine.h

CirclePolygonIntersection.h

circumcircle.h

MinimumEnclosingCircle.h

12.3 Polygons

InsidePolygon.h

PolygonArea.h

PolygonCenter.h

PolygonCut.h

ConvexHull.h

HullDiameter.h

PointInsideHull.h

LineHullIntersection.h

12.4 Misc. Point Set Problems

ClosestPair.h

ManhattanMST.h

kdTree.h

DelaunayTriangulation.h

FastDelaunay.h

12.5 3D

PolyhedronVolume.h

Point3D.h

3dHull.h

sphericalDistance.h

ComplexGeometry.hpp

DefiniteIntegral.hpp

Dynamic Programming (13)

DVC.hpp

SlopeTrick.hpp

Convolutions (14)

AndConvolution.hpp

GCDCconvolution.hpp

LCMConvolution.hpp

ORConvolution.hpp

XORConvolution.hpp

MaxPlusConvolution.hpp

Various (15)

GaussianElimination.hpp

BinaryTrie.hpp

InfixPropostfix.hpp

RangeXor.hpp

15.1 LP Duality

Maximization	Minimization
Inequality constraint $\leq$	Nonnegative variable $\geq$
Inequality constraint $\geq$	Nonpositive variable $\leq$
Equality constraint $=$	Free variable
Nonnegative variable $\geq$	Inequality constraint $\geq$
Nonpositive variable $\leq$	Inequality constraint $\leq$
Free variable	Equality constraint $=$

15.2 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

15.2.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

15.2.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

# Competitive Programming Topics

## (A)

topics.txt

159 lines

Recursion  
Divide and conquer  
    Finding interesting points in  $N \log N$   
Algorithm analysis  
    Master theorem  
    Amortized time complexity  
Greedy algorithm  
    Scheduling  
    Max contiguous subvector sum  
    Invariants  
    Huffman encoding  
Graph theory  
    Dynamic graphs (extra book-keeping)  
    Breadth first search  
    Depth first search  
        \* Normal trees / DFS trees  
    Dijkstra's algorithm  
    MST: Prim's algorithm  
    Bellman-Ford  
    Konig's theorem and vertex cover  
    Min-cost max flow  
    Lovasz toggle  
    Matrix tree theorem  
    Maximal matching, general graphs  
    Hopcroft-Karp  
    Hall's marriage theorem  
    Graphical sequences  
    Floyd-Warshall  
    Euler cycles  
    Flow networks  
        \* Augmenting paths  
        \* Edmonds-Karp  
    Bipartite matching  
    Min. path cover  
    Topological sorting  
    Strongly connected components  
    2-SAT  
    Cut vertices, cut-edges and biconnected components  
    Edge coloring  
        \* Trees  
    Vertex coloring  
        \* Bipartite graphs ( $\Rightarrow$  trees)  
        \*  $3^n$  (special case of set cover)  
    Diameter and centroid  
    K'th shortest path  
    Shortest cycle  
Dynamic programming  
    Knapsack  
    Coin change  
    Longest common subsequence  
    Longest increasing subsequence  
    Number of paths in a dag  
    Shortest path in a dag  
    Dynprog over intervals  
    Dynprog over subsets  
    Dynprog over probabilities  
    Dynprog over trees  
     $3^n$  set cover  
    Divide and conquer  
    Knuth optimization  
    Convex hull optimizations  
    RMQ (sparse table a.k.a  $2^k$ -jumps)  
    Bitonic cycle

    Log partitioning (loop over most restricted)  
Combinatorics  
    Computation of binomial coefficients  
    Pigeon-hole principle  
    Inclusion/exclusion  
    Catalan number  
    Pick's theorem  
Number theory  
    Integer parts  
    Divisibility  
    Euclidean algorithm  
    Modular arithmetic  
        \* Modular multiplication  
        \* Modular inverses  
        \* Modular exponentiation by squaring  
    Chinese remainder theorem  
    Fermat's little theorem  
    Euler's theorem  
    Phi function  
    Frobenius number  
    Quadratic reciprocity  
    Pollard-Rho  
    Miller-Rabin  
    Hensel lifting  
    Vieta root jumping  
Game theory  
    Combinatorial games  
    Game trees  
    Mini-max  
    Nim  
    Games on graphs  
    Games on graphs with loops  
    Grundy numbers  
    Bipartite games without repetition  
    General games without repetition  
    Alpha-beta pruning  
Probability theory  
Optimization  
    Binary search  
    Ternary search  
    Unimodality and convex functions  
    Binary search on derivative  
Numerical methods  
    Numeric integration  
    Newton's method  
    Root-finding with binary/ternary search  
    Golden section search  
Matrices  
    Gaussian elimination  
    Exponentiation by squaring  
Sorting  
    Radix sort  
Geometry  
    Coordinates and vectors  
        \* Cross product  
        \* Scalar product  
    Convex hull  
    Polygon cut  
    Closest pair  
    Coordinate-compression  
    Quadtrees  
    KD-trees  
    All segment-segment intersection  
Sweeping  
    Discretization (convert to events and sweep)  
    Angle sweeping  
    Line sweeping  
    Discrete second derivatives  
Strings

    Longest common substring  
    Palindrome subsequences  
    Knuth-Morris-Pratt  
    Tries  
    Rolling polynomial hashes  
    Suffix array  
    Suffix tree  
    Aho-Corasick  
    Manacher's algorithm  
    Letter position lists  
Combinatorial search  
    Meet in the middle  
    Brute-force with pruning  
    Best-first (A\*)  
    Bidirectional search  
    Iterative deepening DFS / A\*  
Data structures  
    LCA ( $2^k$ -jumps in trees in general)  
    Pull/push-technique on trees  
    Heavy-light decomposition  
    Centroid decomposition  
    Lazy propagation  
    Self-balancing trees  
    Convex hull trick (wcipeg.com/wiki/Convex\_hull\_trick)  
    Monotone queues / monotone stacks / sliding queues  
    Sliding queue using 2 stacks  
    Persistent segment tree