



Chulalongkorn University

# What Name ?

Pakapim E., Pasin P., Sarit P.

template from KACTL

2025-07-20

1	Template
2	Mathematics
3	Combinatorial
4	Data Structures
5	Various
6	Competitive Programming Topics

Template (1)

```
template.cpp31 lines

#pragma once
#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

typedef long long ll;
typedef double db;
typedef long double ld;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;

template<typename T> bool ckmin(T &a, const T &b) { return b < a ? a = b, 1 : 0; }
template<typename T> bool ckmax(T &a, const T &b) { return a < b ? a = b, 1 : 0; }

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

const char nl = '\n';
const int INF = 0x3fffffff;
const int MOD=1000000007;
// const int MOD = 998244353;
const ll LINF = 0xffffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));

signed main(){
    ios_base::sync_with_stdio(0); cin.tie(NULL);
    return 0;
}
```

```
c.sh2 lines

g++ -std=gnu++2a -Wall $1 -o a.out
./a.out
```

Mathematics (2)

2.1 Goldbatch’s Conjecture

- Even number can be written in sum of two primes (Up to 1e12)

- Range of  $N^{th}$  prime and  $N + 1^{th}$  prime will be less than or equal to 300 (Up to 1e12)

2.2 Divisibility

Number of divisors of  $N$  is given by  $\prod_{i=1}^k (a_i + 1)$  where  $N = \prod_{i=1}^k p_i^{a_i}$  and  $p_i$  are prime factors of  $N$ .

Combinatorial (3)

3.1 Permutations

3.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

3.1.2 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

3.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.1.4 Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

3.2 Partitions and subsets

3.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

3.2.2 Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

3.2.3 Binomials

multinomial.h

3.3 General purpose numbers

3.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).

$$B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

3.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

3.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j:s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$  j:s s.t.  $\pi(j) \geq j$ ,  $k$  j:s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 3.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n,k)=S(n-1,k-1)+kS(n-1,k)$$

$$S(n,1)=S(n,n)=1$$

$$S(n,k)=\frac{1}{k!}\sum_{j=0}^k(-1)^{k-j}\binom{k}{j}j^n$$

### 3.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n)=1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m+n)\equiv mB(n)+B(n+1)\pmod{p}$$

### 3.3.6 Labeled unrooted trees

- # on  $n$  vertices:  $n^{n-2}$
- # on  $k$  existing trees of size  $n_i$ :  $n_1n_2\cdots n_kn^{k-2}$
- # with degrees  $d_i$ :  $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

### 3.3.7 Catalan numbers

$$C_n=\frac{1}{n+1}\binom{2n}{n}=\binom{2n}{n}-\binom{2n}{n+1}=\frac{(2n)!}{(n+1)!n!}$$

$$C_0=1,\;C_{n+1}=\frac{2(2n+1)}{n+2}C_n,\;C_{n+1}=\sum C_iC_{n-i}$$

$C_n=1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an  $n\times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

## Data Structures (4)

OrderedSet.hpp

SparseTable.hpp

FenwickTree.hpp

2DFenwickTree.hpp

PersistentSegmentTree.hpp

LiChaoTree.hpp

BinaryTrie.hpp

StaticTopTree.hpp

Treap.hpp

## Various (5)

GaussianElimination.hpp

XORBasis.hpp

RangeXor.hpp

### 5.1 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

#### 5.1.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1<<K)) if (i & 1<<b) D[i] += D[i^(1<<b)];` computes all sums of subsets.

#### 5.1.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

## Competitive Programming Topics (6)

topics.txt	159 lines
Recursion	
Divide and conquer	
Finding interesting points in N log N	
Algorithm analysis	
Master theorem	
Amortized time complexity	
Greedy algorithm	
Scheduling	
Max contiguous subvector sum	
Invariants	
Huffman encoding	
Graph theory	
Dynamic graphs (extra book-keeping)	
Breadth first search	
Depth first search	
* Normal trees / DFS trees	
Dijkstra's algorithm	
MST: Prim's algorithm	

Bellman-Ford
Konig's theorem and vertex cover
Min-cost max flow
Lovasz toggle
Matrix tree theorem
Maximal matching, general graphs
Hopcroft-Karp
Hall's marriage theorem
Graphical sequences
Floyd-Warshall
Euler cycles
Flow networks
* Augmenting paths
* Edmonds-Karp
Bipartite matching
Min. path cover
Topological sorting
Strongly connected components
2-SAT
Cut vertices, cut-edges and biconnected components
Edge coloring
* Trees
Vertex coloring
* Bipartite graphs (=> trees)
* 3^n (special case of set cover)
Diameter and centroid
K'th shortest path
Shortest cycle
Dynamic programming
Knapsack
Coin change
Longest common subsequence
Longest increasing subsequence
Number of paths in a dag
Shortest path in a dag
Dynprog over intervals
Dynprog over subsets
Dynprog over probabilities
Dynprog over trees
3^n set cover
Divide and conquer
Knuth optimization
Convex hull optimizations
RMQ (sparse table a.k.a 2^k-jumps)
Bitonic cycle
Log partitioning (loop over most restricted)
Combinatorics
Computation of binomial coefficients
Pigeon-hole principle
Inclusion/exclusion
Catalan number
Pick's theorem
Number theory
Integer parts
Divisibility
Euclidean algorithm
Modular arithmetic
* Modular multiplication
* Modular inverses
* Modular exponentiation by squaring
Chinese remainder theorem
Fermat's little theorem
Euler's theorem
Phi function
Frobenius number
Quadratic reciprocity
Pollard-Rho
Miller-Rabin
Hensel lifting

Vieta root jumping
Game theory
Combinatorial games
Game trees
Mini-max
Nim
Games on graphs
Games on graphs with loops
Grundy numbers
Bipartite games without repetition
General games without repetition
Alpha-beta pruning
Probability theory
Optimization
Binary search
Ternary search
Unimodality and convex functions
Binary search on derivative
Numerical methods
Numeric integration
Newton's method
Root-finding with binary/ternary search
Golden section search
Matrices
Gaussian elimination
Exponentiation by squaring
Sorting
Radix sort
Geometry
Coordinates and vectors
* Cross product
* Scalar product
Convex hull
Polygon cut
Closest pair
Coordinate-compression
Quadtrees
KD-trees
All segment-segment intersection
Sweeping
Discretization (convert to events and sweep)
Angle sweeping
Line sweeping
Discrete second derivatives
Strings
Longest common substring
Palindrome subsequences
Knuth-Morris-Pratt
Tries
Rolling polynomial hashes
Suffix array
Suffix tree
Aho-Corasick
Manacher's algorithm
Letter position lists
Combinatorial search
Meet in the middle
Brute-force with pruning
Best-first (A*)
Bidirectional search
Iterative deepening DFS / A*
Data structures
LCA (2^k-jumps in trees in general)
Pull/push-technique on trees
Heavy-light decomposition
Centroid decomposition
Lazy propagation
Self-balancing trees
Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)

Monotone queues / monotone stacks / sliding queues
Sliding queue using 2 stacks
Persistent segment tree
troubleshooting.txt
52 lines
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.
Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.
Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).
Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?
Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?