Lecture 4:
Edge Based Vision

Spring 2021

Slides by: Dr Carole Twining

Presented by: Terence Morley

Handouts & Lecture Notes

- **Report in Scientific American (June 2014):**

  "In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops."

  The Pen Is Mightier Than the Keyboard

  P. A. Mueller, D. M. Oppenheimer, *Psychological Science,* Vol 25,Issue 6, pp. 1159 – 1168, April-23-2014.

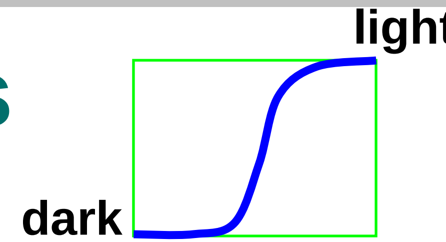
- **Handouts are to aid note taking, not a total replacement for note taking**
- **Podcasts, slides, pdfs etc on BlackBoard**

Overview:

- Why Edges Matter:
  - Edges in images correspond to physical events: edge of object, change in colour, change of surface orientation
- Edges and Derivatives
  - Convolution and filters (to detect changes)
- Edges and Scale
  - Physical edges persist across scales
- Edge Detection
  - Problem with noise, and accurate edge location
- Edge growing
  - Thresholding with hysteresis
  - Edge relaxation
- Hough Transform
  - Finding lines

Edges and Derivatives

First-Derivative Edge Filters



- What is an edge?
- To detect: look at the slope

Discrete version of ∂_{x},
Central difference

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Sobel

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Roberts

$$\begin{bmatrix} ? & & ? \\ -1 & 0 & 1 \\ ? & & ? \end{bmatrix}$$

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

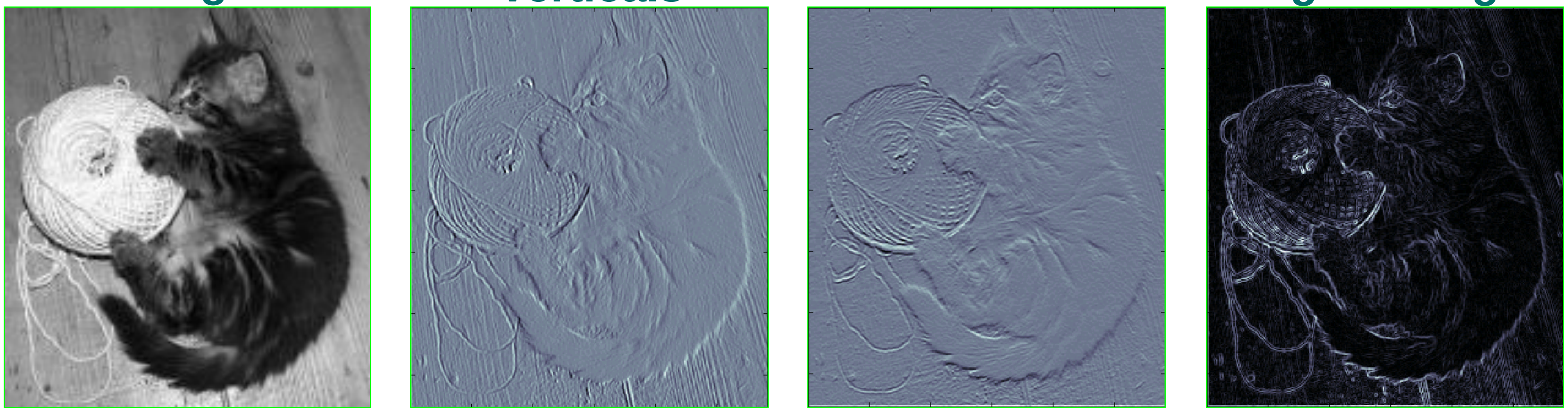
6 5 Multiplies and adds

Decomposable:
Exterior product

$$(a \otimes b) * \mathcal{I} \equiv a * (b * \mathcal{I})$$

First Derivative Filters : Sobel

Image	Verticals	Horizontals	Edge Strength

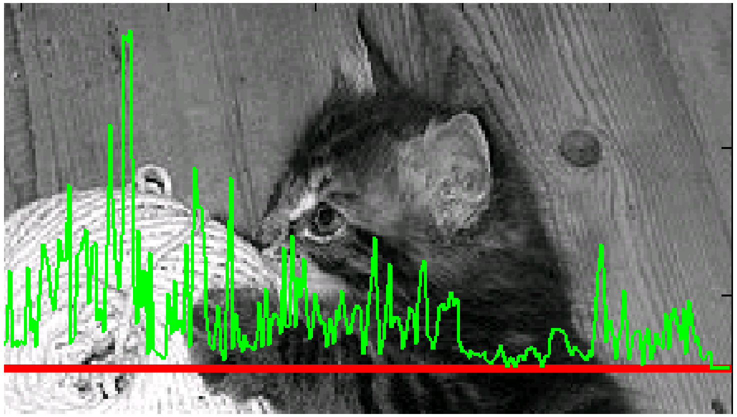


$\mathcal{I}$	$\mathcal{I}_{\mathbf{x}} \doteq \mathbf{S}_{\mathbf{x}} * \mathcal{I}$	$\mathcal{I}_{\mathbf{y}} \doteq \mathbf{S}_{\mathbf{y}} * \mathcal{I}$	$\sqrt{\mathcal{I}_x^2 + \mathcal{I}_y^2}$

Edge strength: $g = |\vec{\nabla}\mathcal{I}| = \sqrt{\mathcal{I}_x^2 + \mathcal{I}_y^2}$

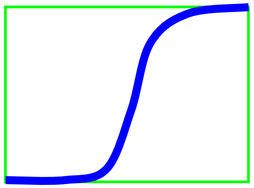
Ridges of g at edges, but noisy.

Normal to Edge: $\hat{n} = \dfrac{\vec{\nabla}\mathcal{I}}{|\vec{\nabla}\mathcal{I}|}$

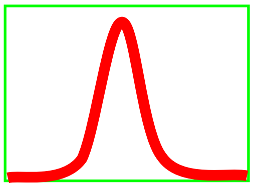


Second-Derivative Edge Filters

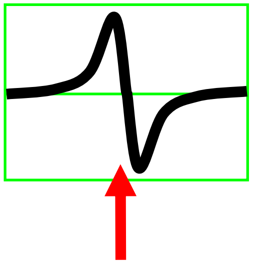
$\mathcal{I}$



$\partial_x \mathcal{I}$



$\partial_x^2 \mathcal{I}$



zero crossing

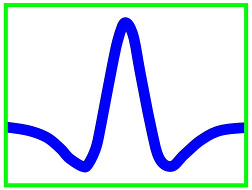
- **Laplacian: scalar operator**

$$\triangle = \nabla^2 = \partial_{\mathbf{x}}^2 + \partial_{\mathbf{y}}^2$$

- **Difference of Gaussian, Laplacian of Gaussian: includes gaussian smoother**
- **False edges: every peak/trough of gradient gives a zero-crossing, not just big peaks**
- **Doesn't tell us the direction of the edge (scalar operator)**
- **Tends to create closed loops of edges ('plate of spaghetti' effect)**

-1	-1	-1
-1	8	-1
-1	-1	-1

Laplacian

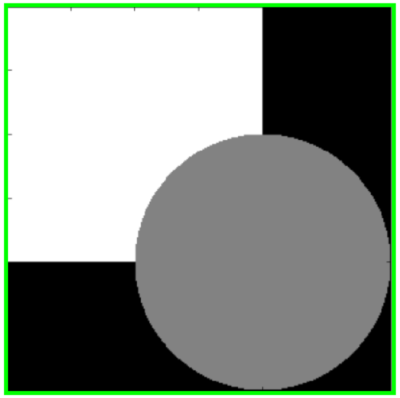


'mexican hat'

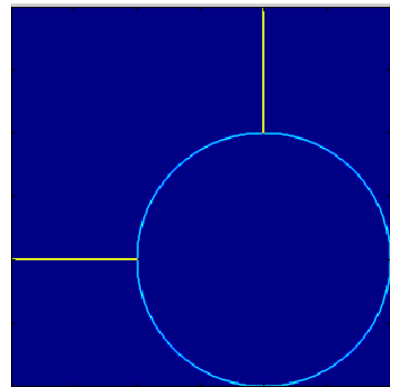
Laplacian Filter

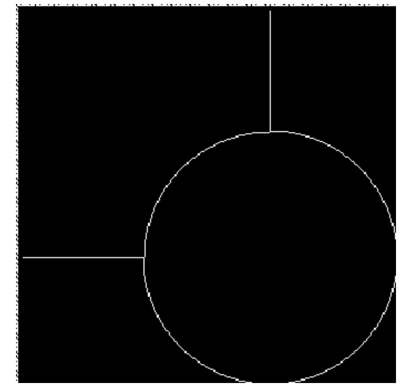
Zero Crossings

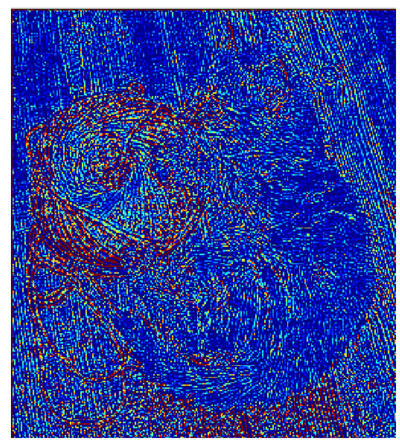
$\mathcal{I}$

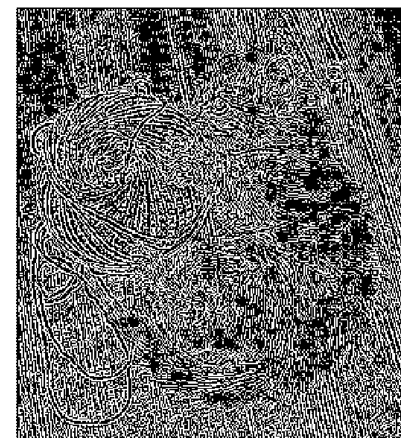


$|\Delta \mathcal{I}|$







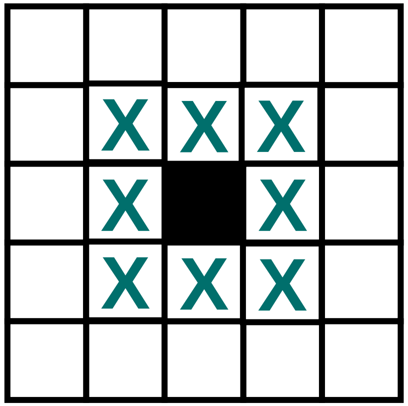


- **Need to consider smoothing and noise**
- **Need to consider scale**
- **Need to consider edge detection**

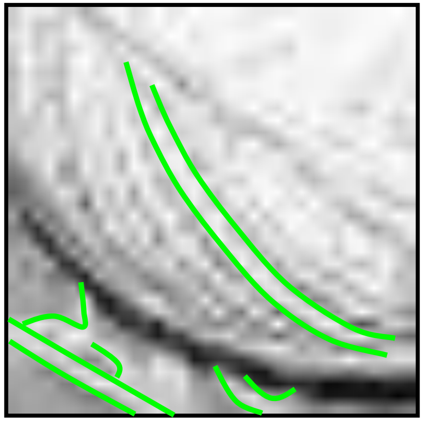
-1	-1	-1
-1	8	-1
-1	-1	-1

Edges and Scale

Edges and Scale



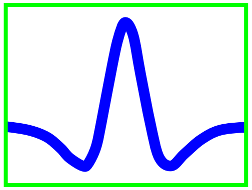
- Edge filters enhance noise
- What is a ‘real’ edge and what noise?
- Edges exist at many different scales
- What scales matter depends on application
- Sensible approach: use many different scales
  - Edges persist across scales, allows fusion across scales
- Gaussian gives scale & smoothing separable filter



Edges and Scale

Marr-Hildreth:

- Convolve with gaussian $\mathcal{G}$
- Take Laplacian ∇^2 of result:
  - combine into single stage LoG
- Edges at zero-crossings
- Edges move with scale if curved
- No information on direction
- ‘Plate of spaghetti’ problem



‘mexican hat’

Canny:

- Convolve with gaussian $\mathcal{G}$
- Take gradient $\vec{\nabla}$ of result

$$\vec{\nabla}(\mathcal{G} * \mathcal{I}),\ g = |\vec{\nabla}(\mathcal{G} * \mathcal{I})|$$

- Find gradient direction:

$$\underline{\hat{n}} = \vec{\nabla}(\mathcal{G} * \mathcal{I})/g$$

- Create gaussian-smoothed derivative tuned to this direction
- Take another derivative in that direction to find local maximum, zero-crossing
- Stable across scales

Marr-Hildreth vs Canny

- Both involve pre-smoothing with gaussian
- Both involve second-derivative BUT:

Marr-Hildreth:

- No information on direction
- By adding second-derivative in other direction, increases effect of noise

Canny:

- Create tuned derivative given estimated gradient direction
- Only compute second derivative in gradient direction
- Check that it really is local maximum of edge strength in that direction (see non-maximum suppression)

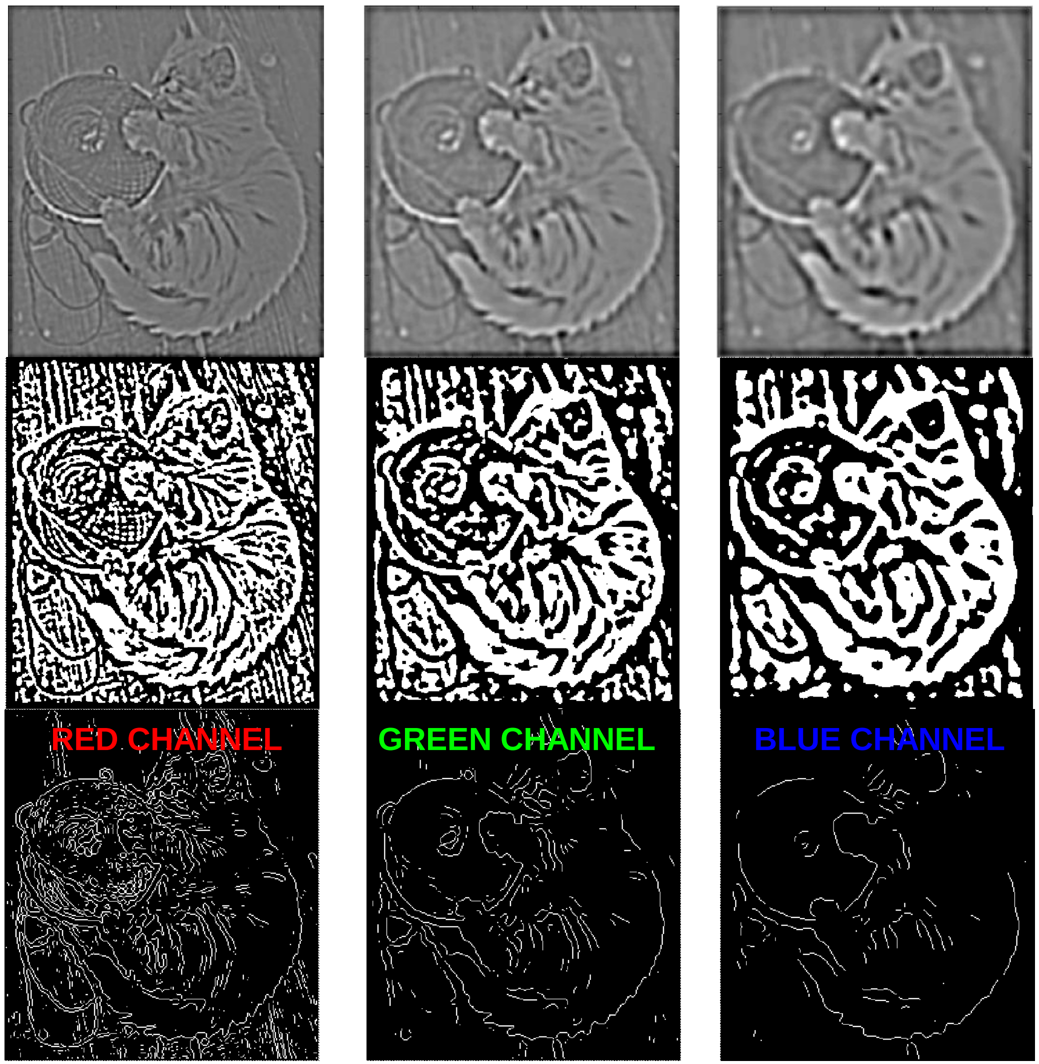
Marr-Hildreth Edge Detection

$\sigma = 2$ $\sigma = 3$ $\sigma = 4$

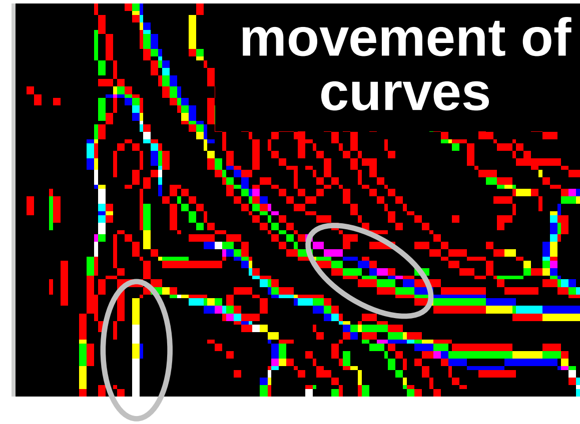
LoG

LoG > 0

zero crossings



RGB PLOT

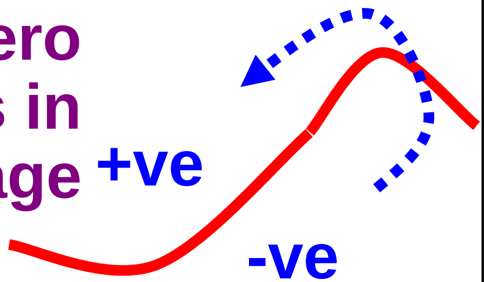


white, all 3 scales

Marr-Hildreth Edge Detection

- Some edges not well localized
- ‘Plate of spaghetti’ effects

Trace zero crossings in image



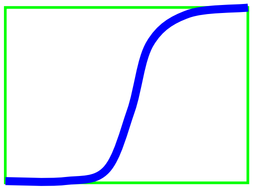
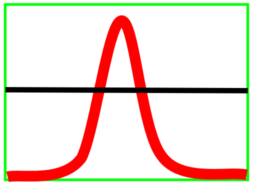
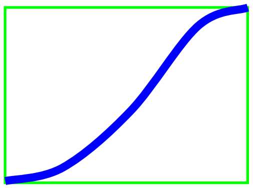
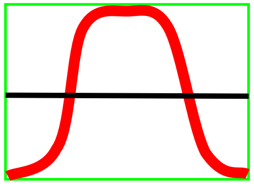
Keeps going until meets edge or closes the loop

$\sigma = 10$

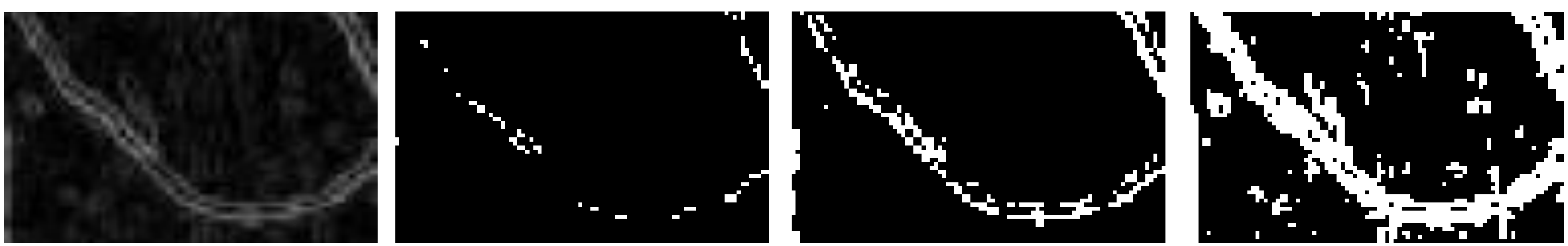
The University of Manchester

Edge Detection

Edge Detection: First Derivatives

$\mathcal{I}$  $\partial_x \mathcal{I}$  $\mathcal{I}$  $\partial_x \mathcal{I}$ 

- **Position of maximum can be difficult to locate:**
  - **second-derivative, zero crossing more precise**
- **Simple threshold:**
  - **thick edges, need to apply thinning**
  - **missed edges, streaking (see thresholding with hysteresis)**



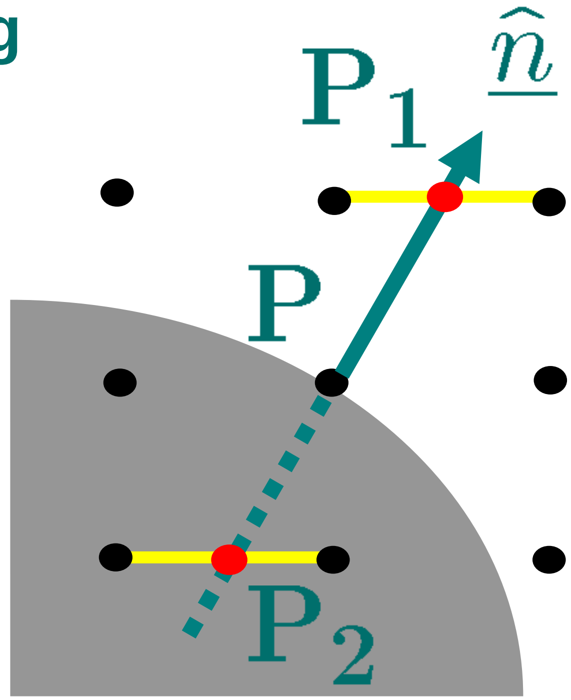
Edge Detection:

- **Zero-crossing more precisely located than maximum**
  - **Sub-pixel accuracy?**
- **Thresholding in Marr-Hildreth (LoG):**
  - **Threshold at ~zero, but what about noise?**
  - **Doesn’t use directional information**
  - **Other second derivative increases noise**
- **‘Plate of spaghetti’:**
  - **continuity => closed loops or meets boundary**
- **Thresholding & Thinning 1st Derivative**
  - **Incorporates neighbourhood information**
  - **Still doesn’t use all available information**
- **If we had the edge direction as well…….**

Non-Maximum Suppression

- Start from edge-strength signal g
- Locate possible edge point P
- Identify gradient direction $\underline{\widehat{n}}$
- Interpolate g at P_1 and P_2
- P is local maximum provided:

  g(P) > g(P_1) & g(P) > g(P_2)
- Only accepts as edge if proper maximum, rejects if not
- In practise, only allow a set of discrete possible directions



Object & pixel positions

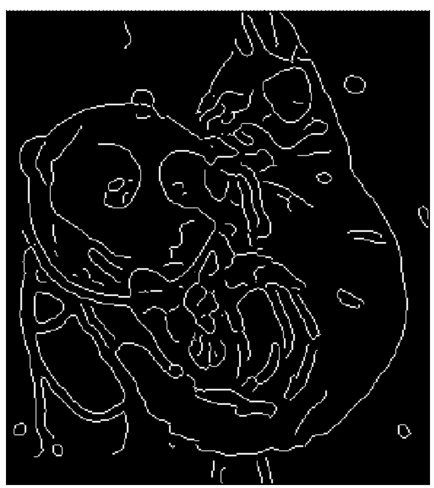
Canny Edge Detector

$\sigma = 1$

$\sigma = 1.5$

white, all 3 scales

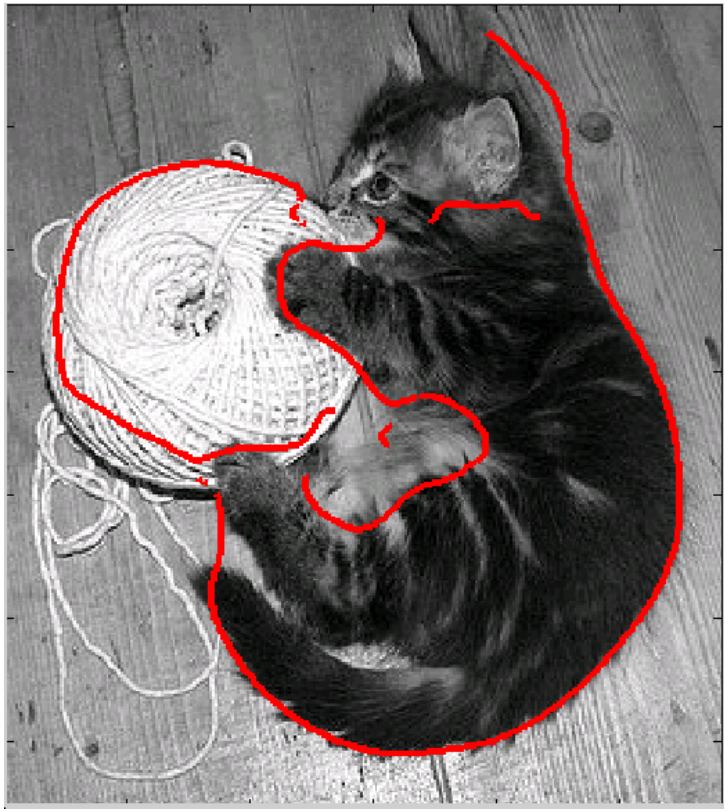
$\sigma = 2$



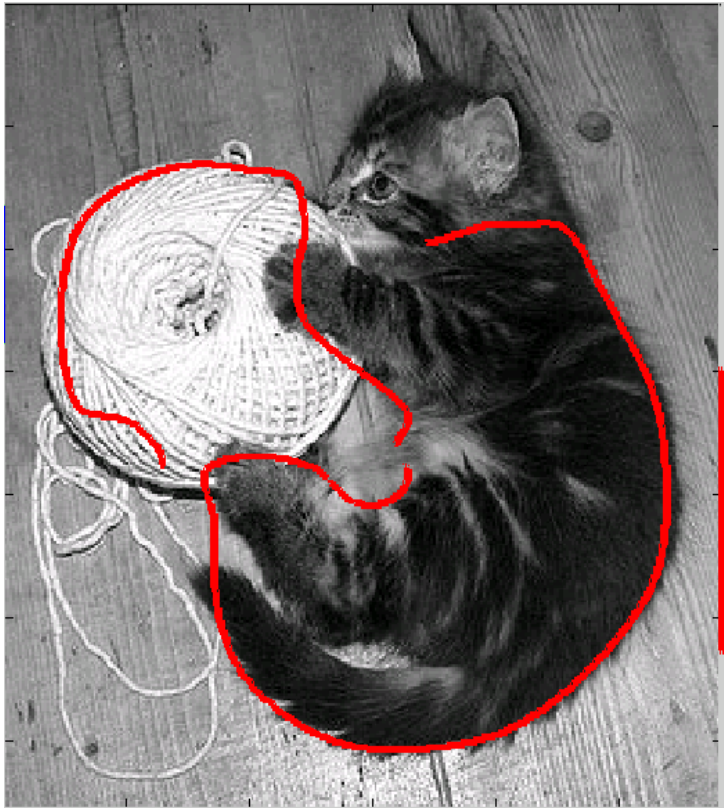
The University of Manchester

Canny Edge Detector:

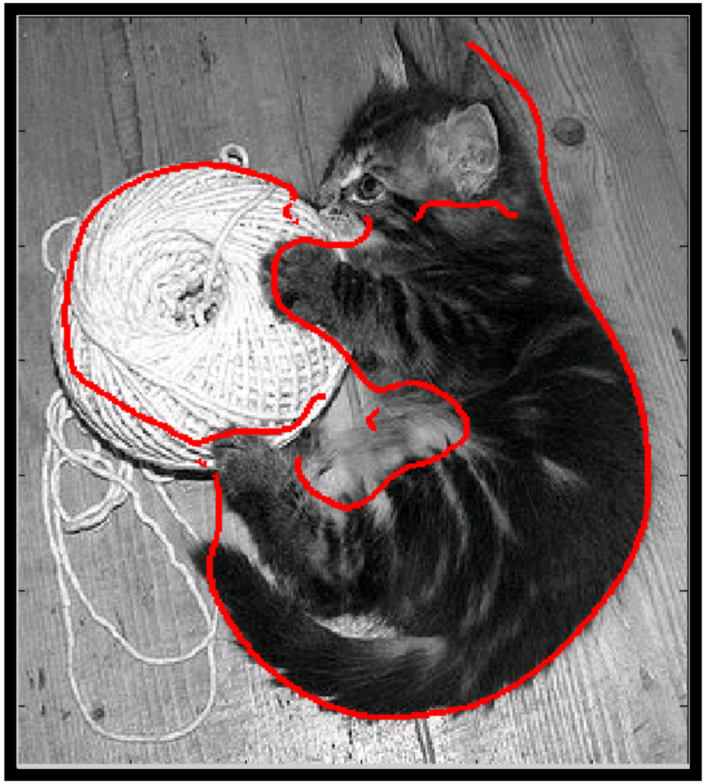
$\sigma = 10$



$\sigma = 20$



Marr-Hildreth vs Canny at σ=10



From Edge Pixels to Edges

- Have candidate edge pixels
- Have information on edge direction and strength
- Want connected edges:

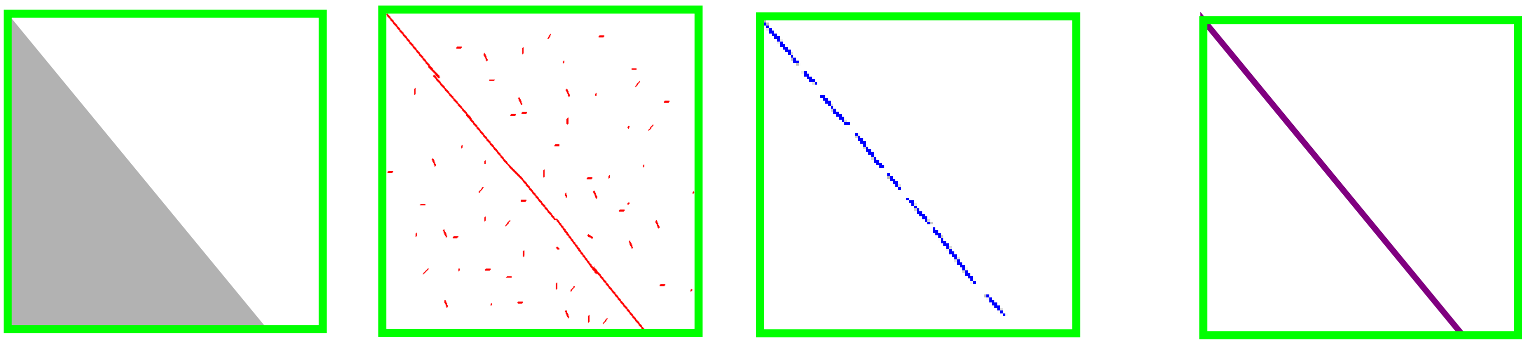
  Edge growing

- Going from individual edge pixels, to entire, connected edges – curves that are boundaries of objects

Edge Growing

Edge Thresholding with Hysteresis

- Edge strength image, two thresholds T_H & T_L
- Only edges have points g> T_H
- Edges have all points g> T_L
- Start at point g>T_H, and trace connected points with g>T_L



$\mathcal{I}$ $\quad g > T_L \quad g > T_H \quad Result$

The University of Manchester

Edge Relaxation

- **Use context to resolve ambiguity (as in segmentation)**

$g(i)$: Edge strength at pixel i

$\underline{e}(i)$: Edge direction at pixel i

Normalise edge strengths $g(i) \Rightarrow P(\underline{e}, i) \leq 1$

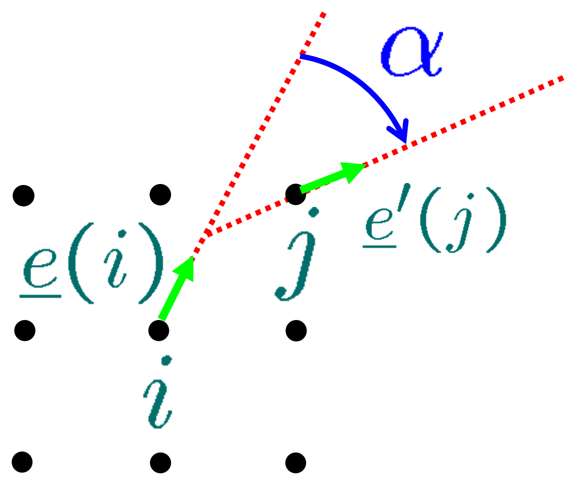
- **Compatibility**

Pixels i and j,

edge directions $\underline{e}$ and $\underline{e}'$:

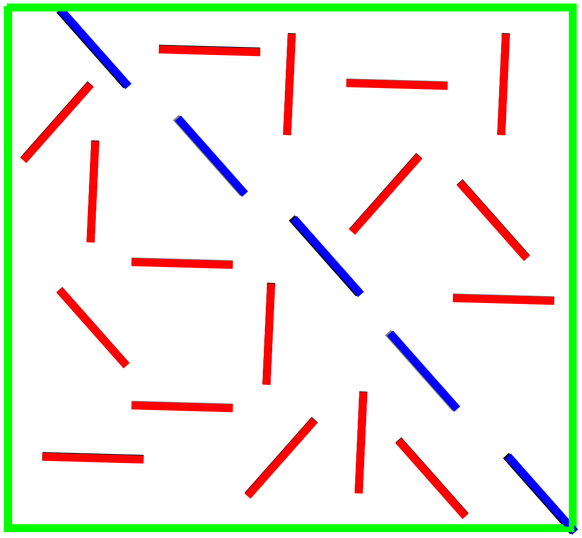
$\mathbf{c_{i,j}}(\underline{e}, \underline{e}') = 0$ **not neighbours**

$\mathbf{c_{i,j}}(\underline{e}, \underline{e}') = |\cos(\alpha)|$

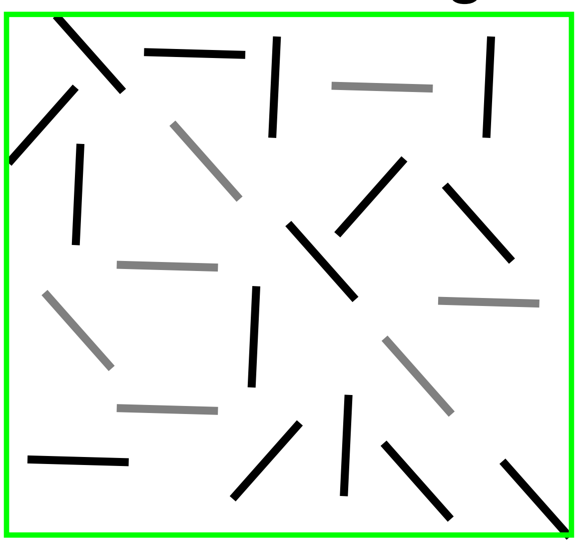


- **As before, update probabilities based on support**

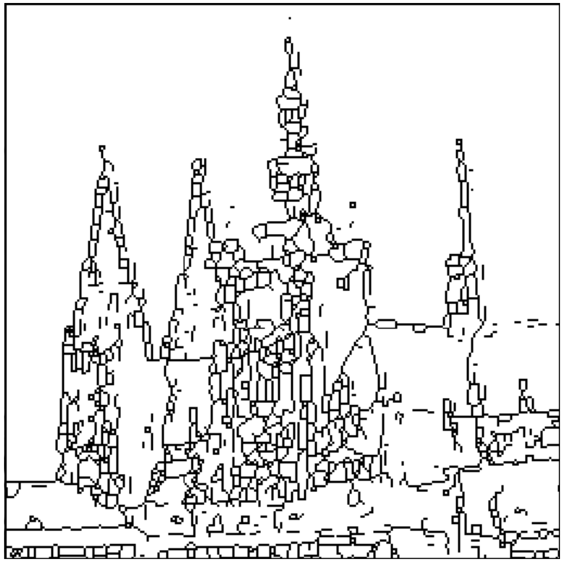
Edge Relaxation

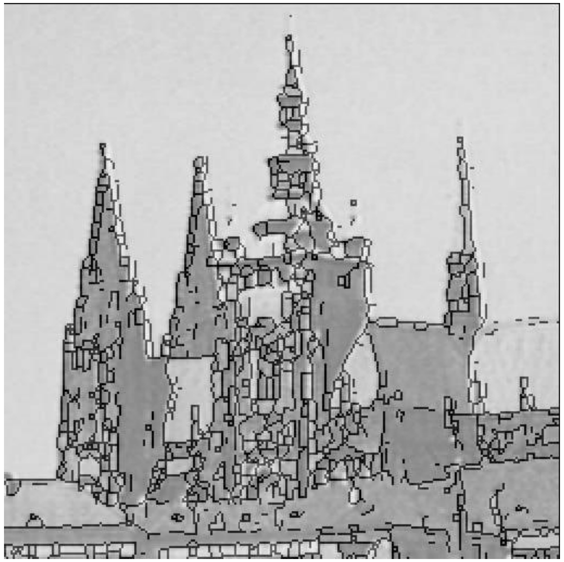


weak and strong edges



- **Supporting each other**
- **Many refinements and alternatives in the literature, but all applying same basic ideas**

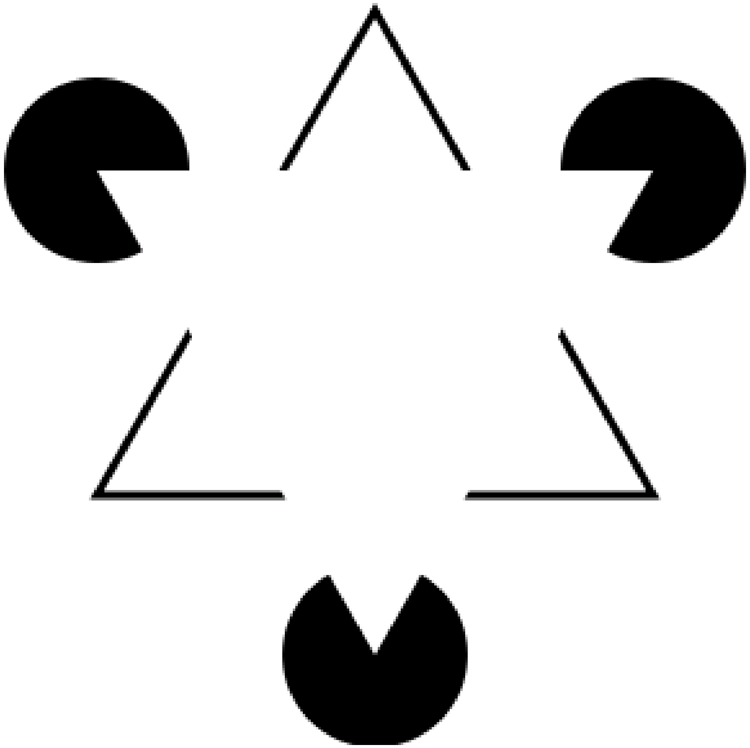


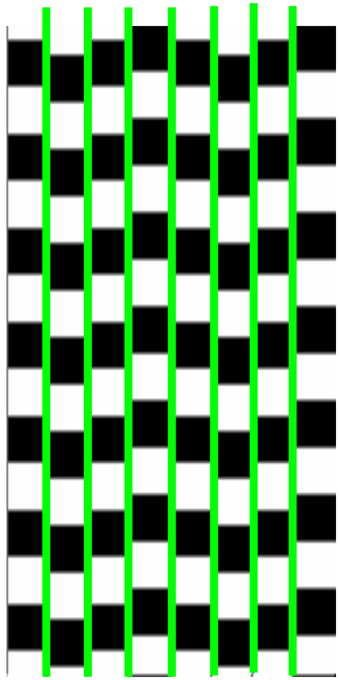


Hough Transform

The University of Manchester

Aside: Lines in human vision





See lines where we have only minimal information

Actually straight, but we don't see them as that!

Hough Transform (1)

- Have some set of points, parts of edges etc
- Want to put them together into continuous lines
- Strategy:
  - Transform to parameter space
  - Let points vote for lines that could pass through them
  - Look for clusters
- Finding the right parameter space
- Can be extended if you can find such a space for shape of interest

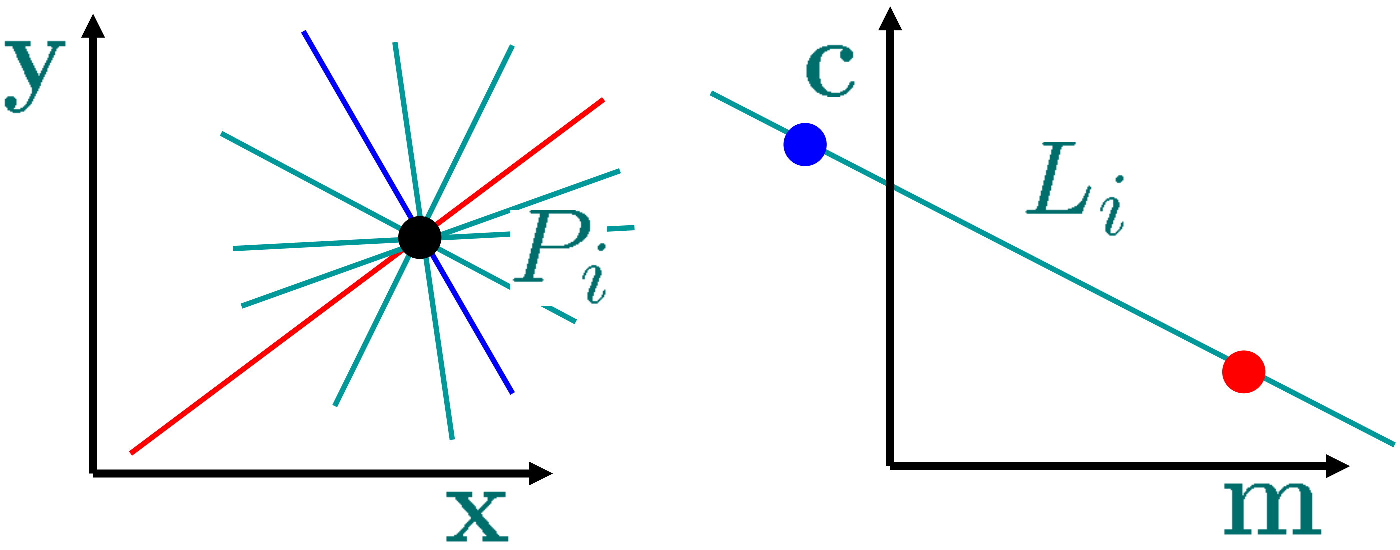
Hough Transform (2)

Set of points $\{P_i = (x_i, y_i)\}$ in image plane.

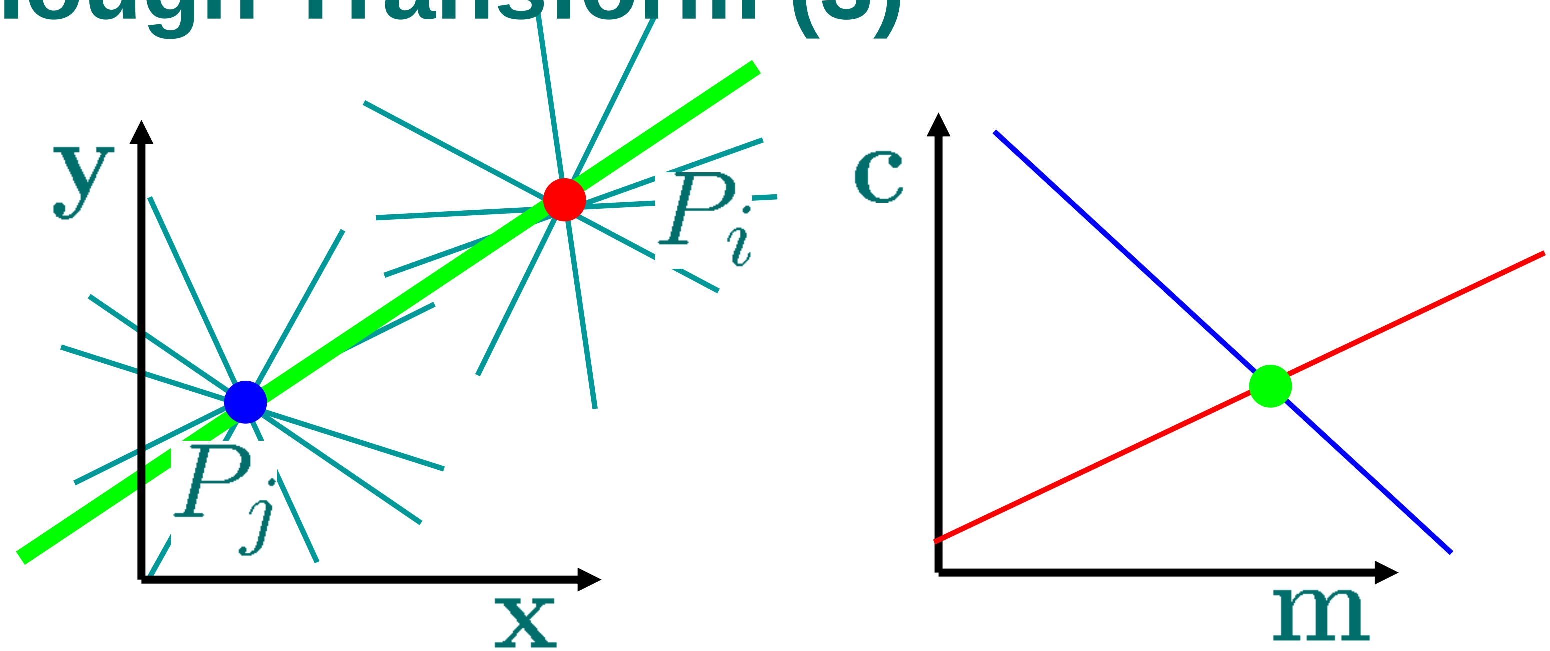
Any and all straight lines thro' P_i:

$\boxed{y_i = mx_i + c} \quad \Rightarrow \quad \boxed{c = -x_i m + y_i}$

L_i: line in (c, m) plane, intercept y_i, gradient $-x_i$

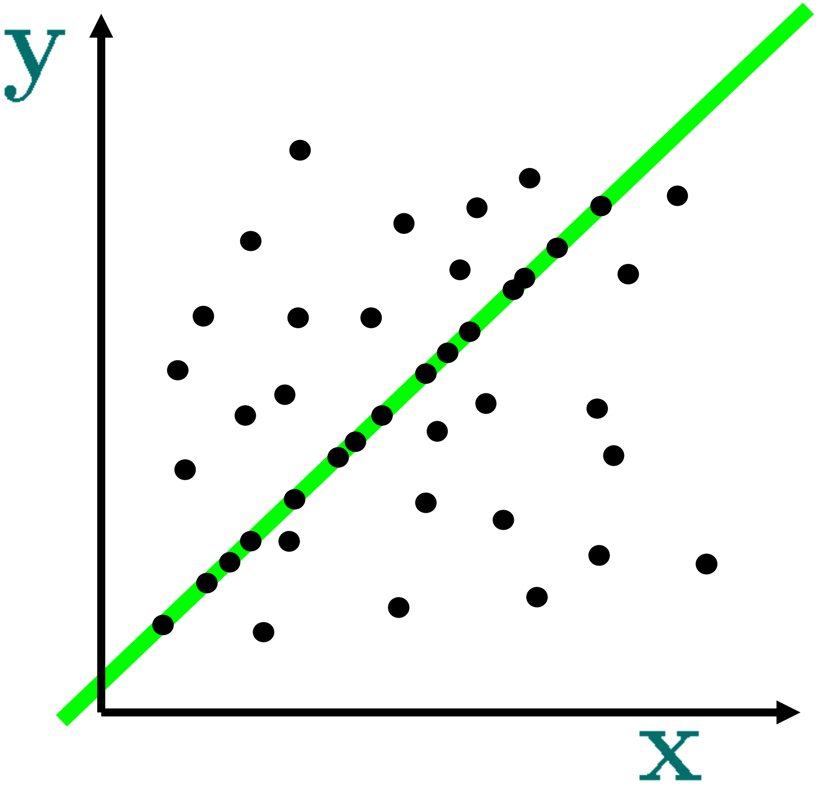


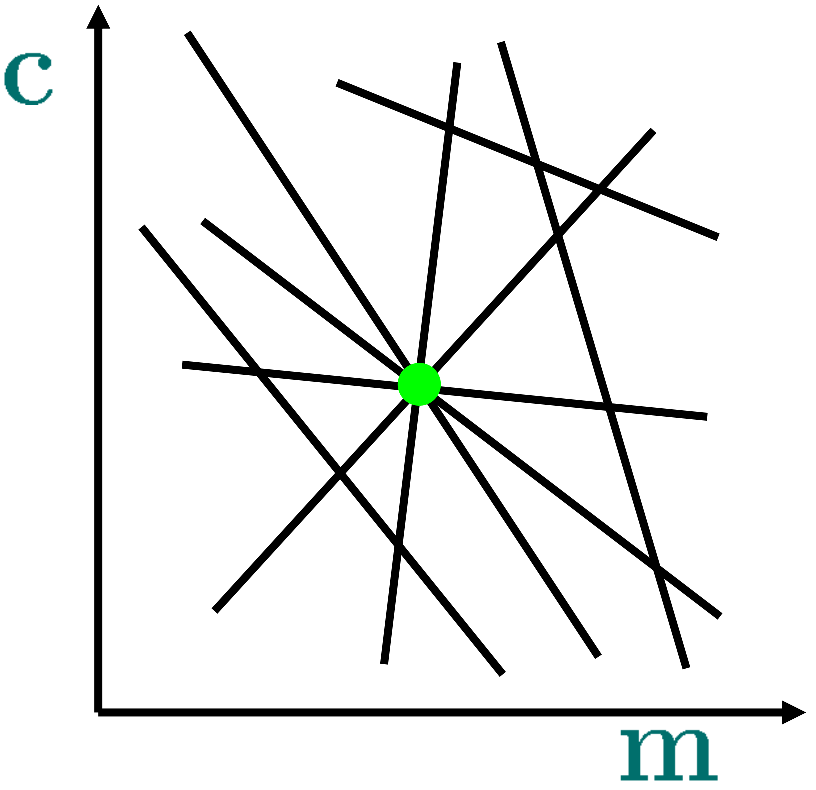
Hough Transform (3)



- Repeat for all points $\{P_i = (x_i, y_i)\}$ in image plane
- Look for points in (c,m) plane where lots of lines cross
- Lines which pass thro' lots of points in image plane

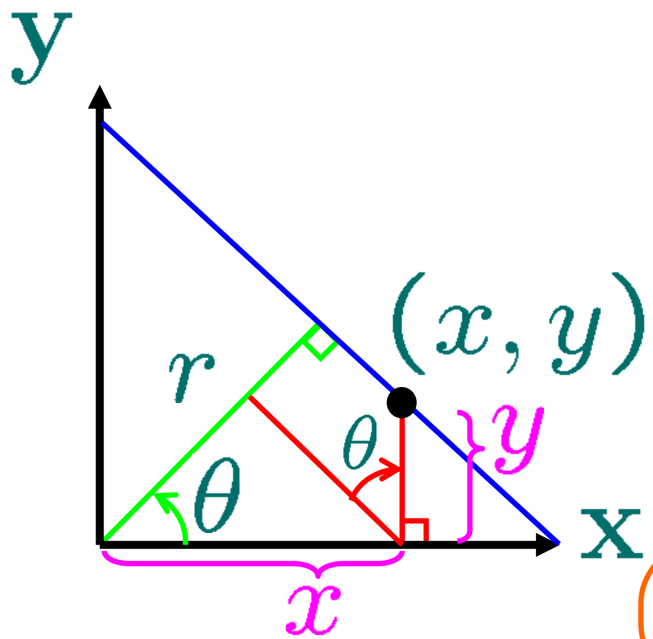
Hough Transform (4)





- Verticals, m is infinite! Need better parameter space

Hough Transform (5)



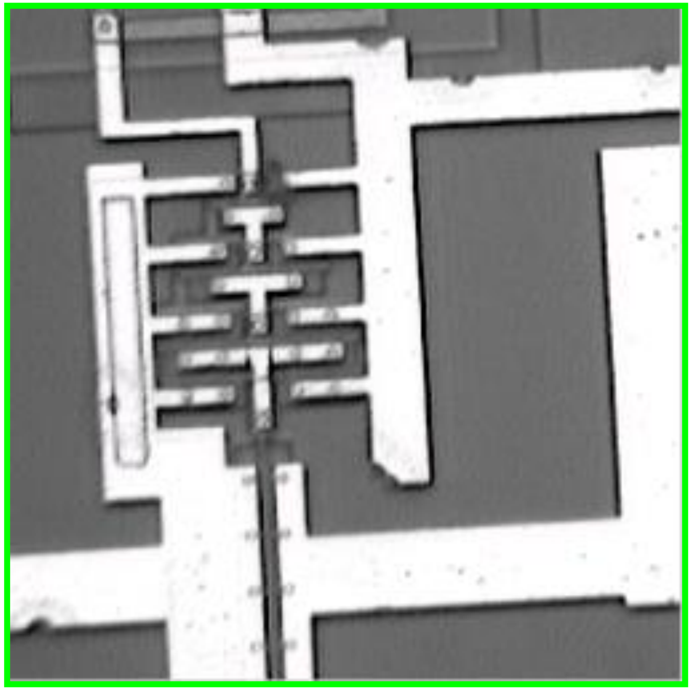
$$y = mx + c$$

$$(c, m) \Rightarrow (r, \theta)$$

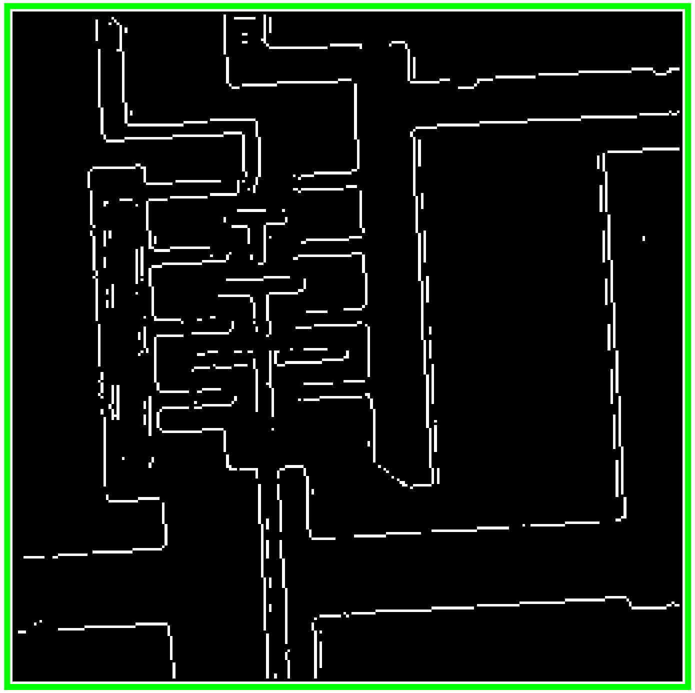
$$\boxed{r = x\cos\theta + y\sin\theta}$$

- **Single point** $P_i = (x_i, y_i)$
- **All possible** θ **: allowed values of r, sinusoid curve**
- **Extend to other than lines, generalised Hough transform**

Example: Integrated Circuit

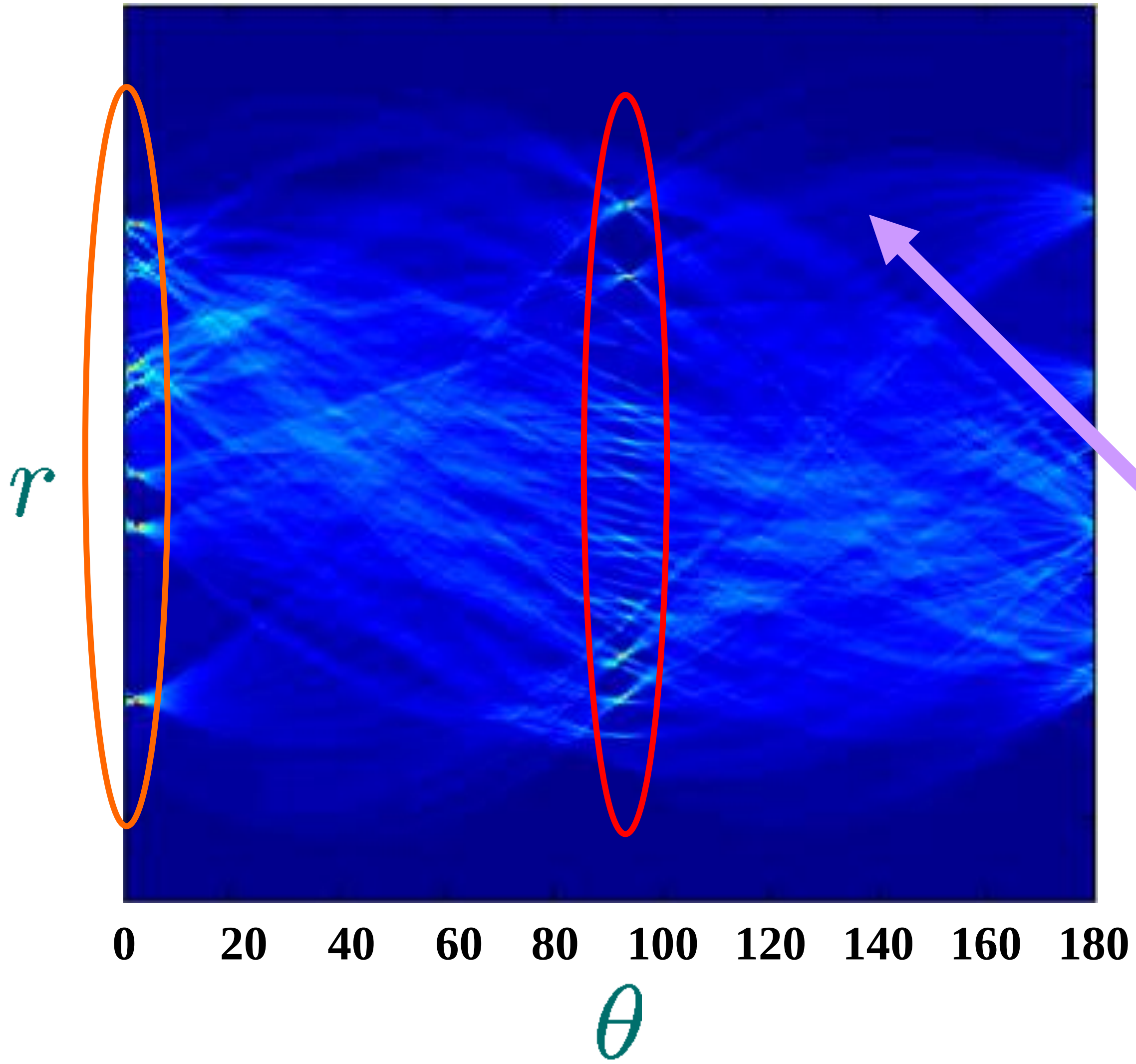


Image



Edge Pixels

Example: Integrated Circuit



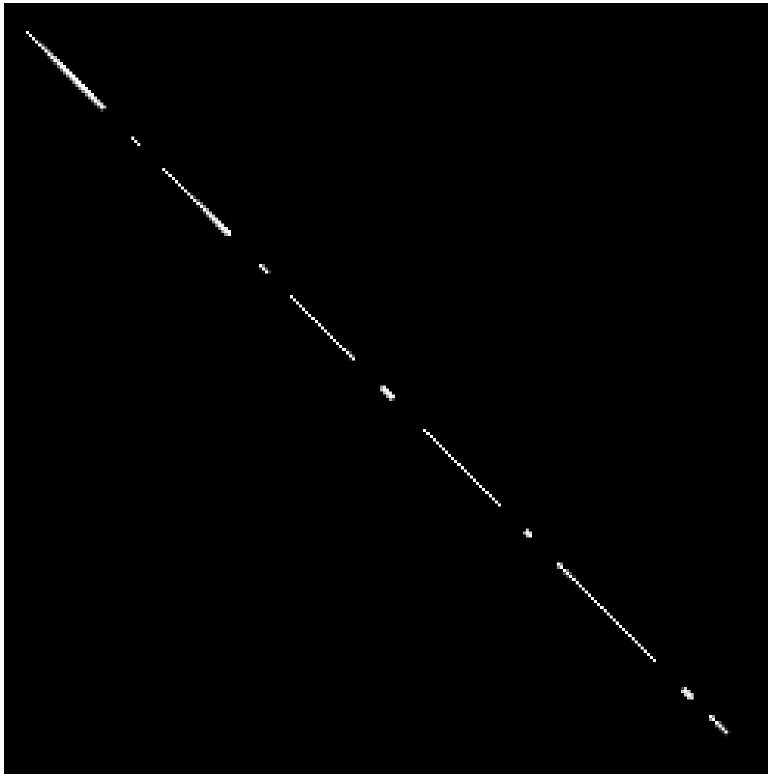
Each edge pixel = 1 sinusoid

Each peak = 1 line in image

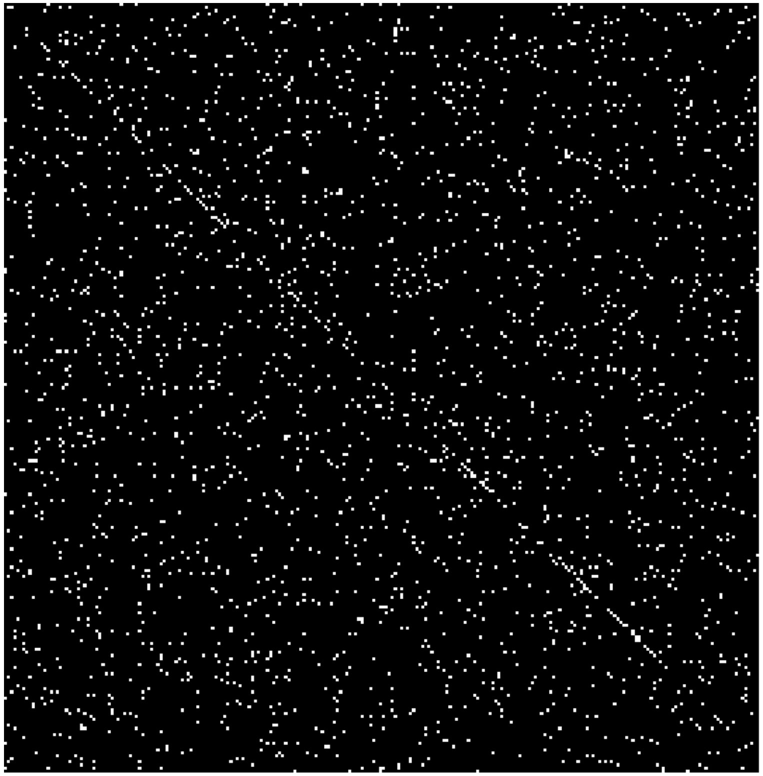
Set of peaks at approx 90^0

Another at approx 0^0

Example: Finding Lines under Noise



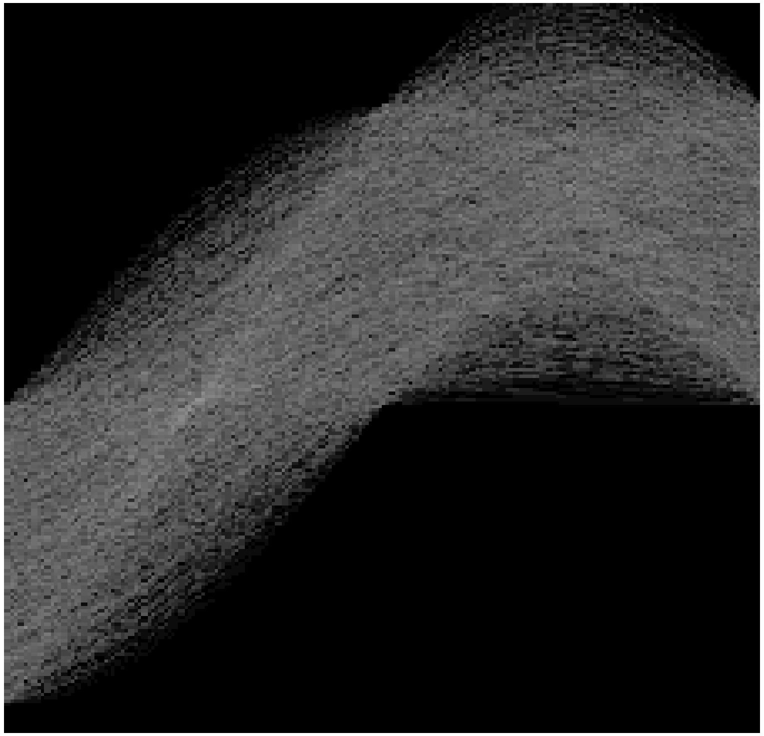
Broken Line



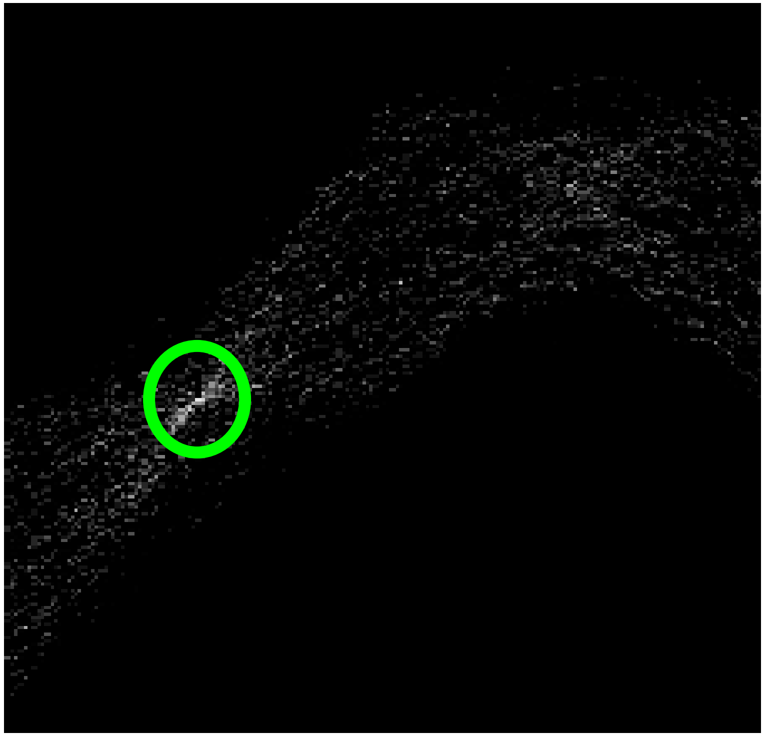
Hidden under noise

Edge Strength Image

Example: Finding Lines under Noise



Hough Space



…thresholded

Summary:

- Edges and Derivatives
  - Convolution and filters (first & second derivatives, gaussians)
- Edges and Scale
  - Physical edges persist across scales
- Edge Detection
  - Problems with noise, and accurate edge location
  - Non-maximum suppression
- Edge Growing
  - Thresholding with hysteresis
  - Edge relaxation
- Hough Transform
  - Finding lines/circles etc even when occluded