MANCHESTER 1824

The University of Manchester

Model Based Vision 2:
Statistical Shape Models (SSMs), Active Shape Models (ASMs), Active Appearance Models (AAMs)

Spring 2021

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Handouts & Lecture Notes

- **Report in Scientific American (June 2014):**

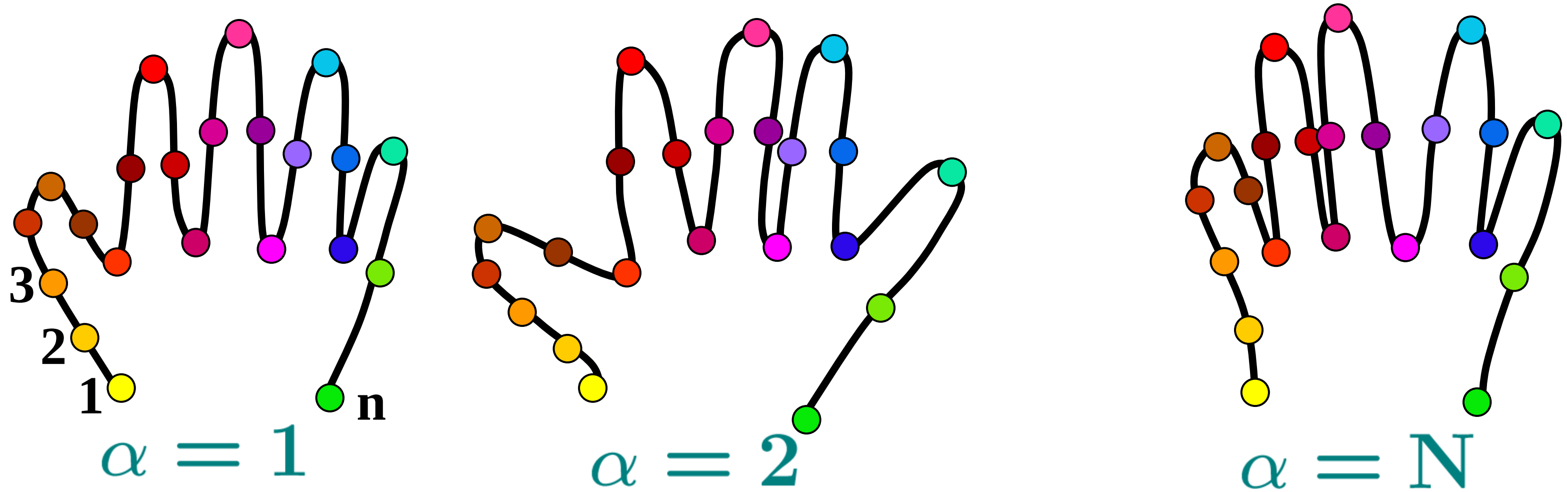
  "In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops."

  The Pen Is Mightier Than the Keyboard

  P. A. Mueller, D. M. Oppenheimer, *Psychological Science,* Vol 25,Issue 6, pp. 1159 – 1168, April-23-2014.

- **Handouts are to aid note taking, not a total replacement for note taking**
- **Podcasts, slides, pdfs etc on BlackBoard**

Statistical Shape Models (SSMs)

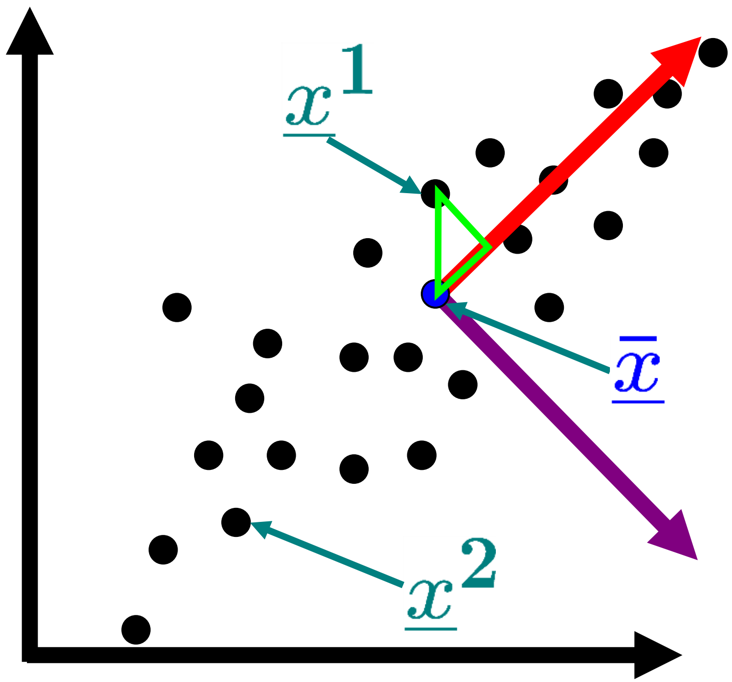


- Set of ***aligned*** shapes
- **Shape vector:** $\underline{x}^{\alpha} = (\mathbf{x}^{\alpha}_{\mathbf{1}}, \mathbf{y}^{\alpha}_{\mathbf{1}}, \mathbf{x}^{\alpha}_{\mathbf{2}}, \mathbf{y}^{\alpha}_{\mathbf{2}}, \ldots, \mathbf{x}^{\alpha}_{\mathbf{n}}, \mathbf{y}^{\alpha}_{\mathbf{n}})$
- **Corresponding points on all shapes**
- **Entire aligned training set, set of shape vectors:**

$$\{\underline{x}^{\alpha} : \alpha = \mathbf{1}, \mathbf{2}, \ldots, \mathrm{N}\}$$

Principal Component Analysis (PCA)

$\{\underline{x}^{\alpha} : \alpha = 1, 2, \ldots, N\}$



Shape Space:
axes = coordinates of every shape point

Mean shape:

$$\underline{\bar{x}} \doteq \frac{1}{N} \sum_{\alpha=1}^{N} \underline{x}^{\alpha}$$

$\underline{\hat{n}}$, unit axis vector

Maximize data projection:

$$\arg\max_{\underline{\hat{n}}} \sum_{\alpha=1}^{N} \left(\underline{\hat{n}} \bullet (\underline{x}^{\alpha} - \underline{\bar{x}})\right)^{2}$$

Repeat:

$$\arg\max_{\underline{\hat{m}}} \sum_{\alpha=1}^{N} \left(\underline{\hat{m}} \bullet (\underline{x}^{\alpha} - \underline{\bar{x}})\right)^{2},$$

Constraint: $\underline{\hat{m}} \bullet \underline{\hat{n}} = 0$

PCA Solution: Covariance Matrix

Mean shape: $\underline{\bar{x}} \doteq \frac{1}{\mathbf{N}} \sum_{\alpha=1}^{\mathbf{N}} \underline{x}^{\alpha}$

n: number of points on each shape
2: number of spatial dimensions

Covariance matrix:

$$\mathbf{C_{ij}} \doteq \frac{1}{\mathbf{N}} \sum_{\alpha=1}^{\mathbf{N}} (\underline{x}^{\alpha} - \underline{\bar{x}})_{\mathbf{i}} (\underline{x}^{\alpha} - \underline{\bar{x}})_{\mathbf{j}}, \ \mathbf{i}, \mathbf{j} = 1, \ldots, \mathbf{2n}$$

Solve covariance matrix eigenproblem:

$$\mathbf{C}\underline{\hat{n}}^{\mu} = \lambda^{\mu}\underline{\hat{n}}^{\mu}$$

(matrix × vector = number × vector)

Eigenvectors: $\{\underline{\hat{n}}^{\mu}\}$, directions of new axes

Ordered eigenvalues: $\{\lambda^{\mu} : \lambda^{\mathbf{1}} \geq \lambda^{\mathbf{2}} \ldots\}$,

how much variance in each direction

Generative Shape Models:

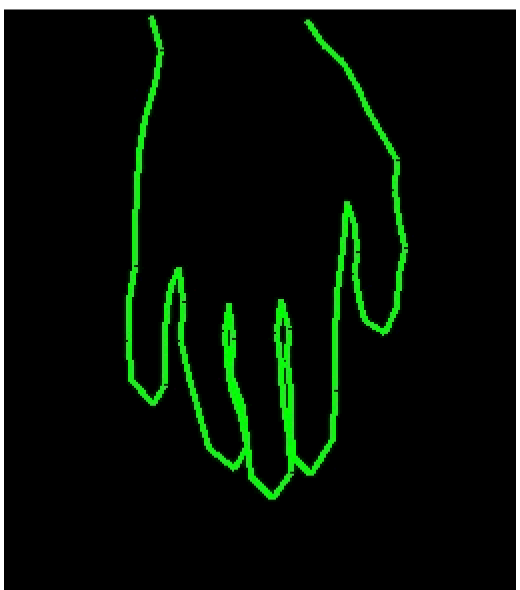
generated shape → $\underline{x} = \underline{\bar{x}} + \mathbf{P}\underline{b}$ ← shape parameters, coords wrt PCA axes

mean shape

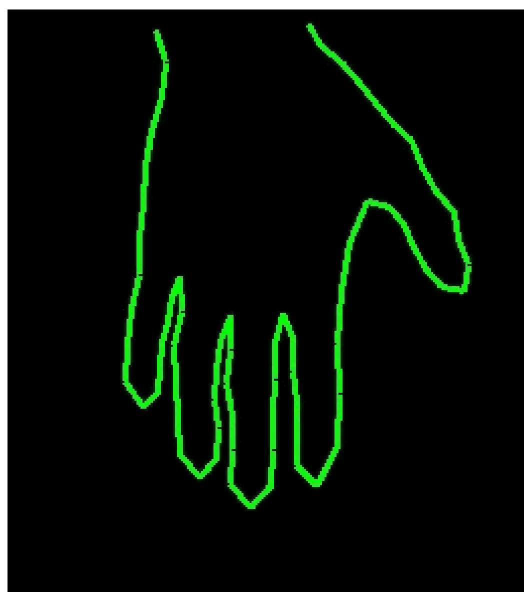
matrix, columns are eigenvectors

- New shape = mean plus weighted sum of eigenvectors
- PCA automatically finds relevant modes of variation

$b_1 \neq 0$ only



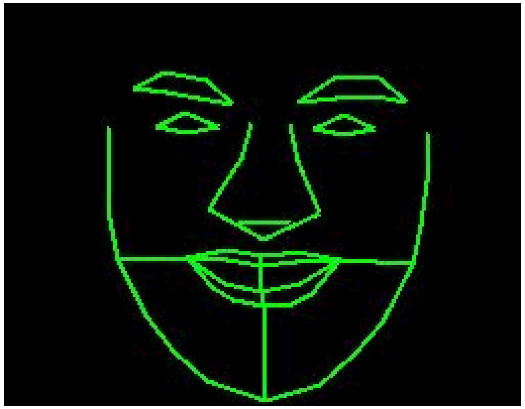
$b_2 \neq 0$ only



$b_3 \neq 0$ only

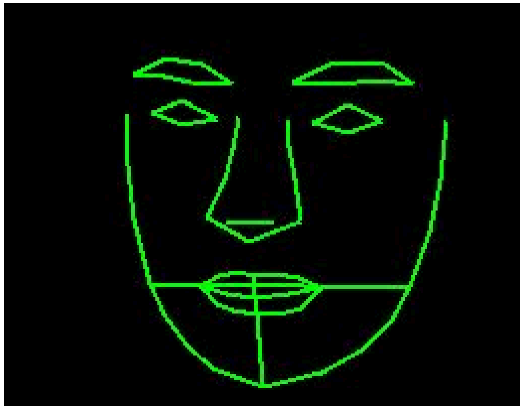
Generative Shape Models: Faces

First Mode



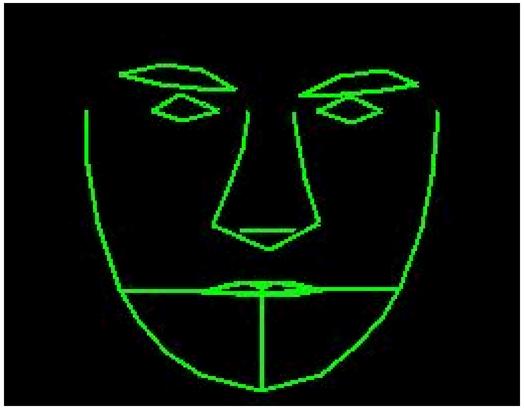
$b_1 \neq 0$ only

Second Mode



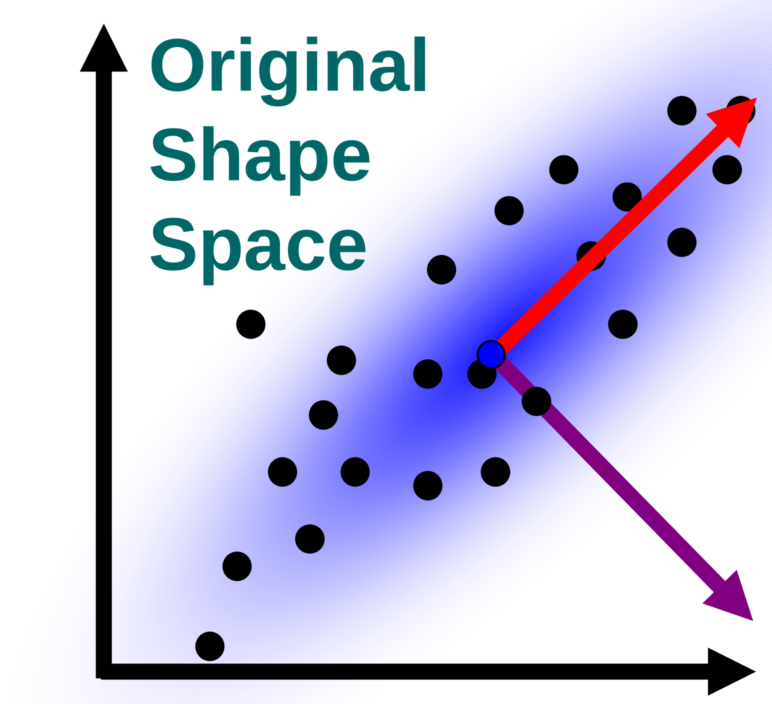
$b_2 \neq 0$ only

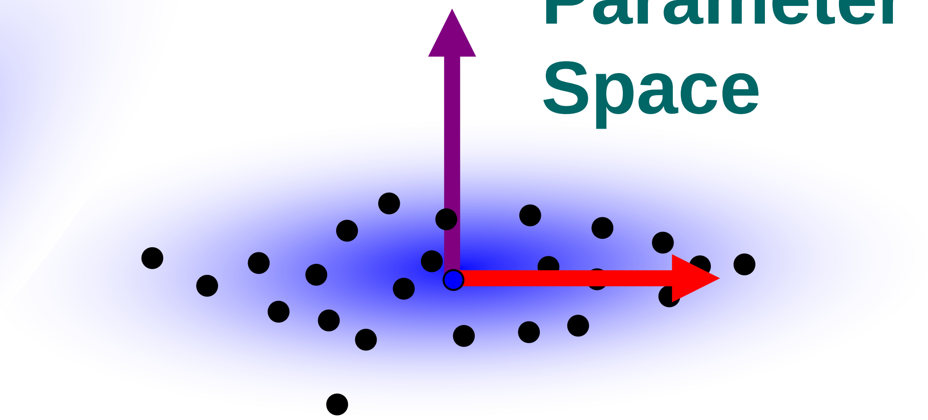
Third Mode



$b_3 \neq 0$ only

Statistical Shape Models:





no of shape modes retained

$$p(\underline{b}) \propto \prod_{\mu=1}^{\boxed{n_m}} \exp\left(-\frac{b_\mu^2}{2\lambda^\mu}\right)$$

- Multivariate Gaussian probability density function
- Aligned with PCA directions (eigenvectors)
- Matches variance seen in training set (eigenvalues)
- Product of Gaussians in parameter space

SSMs: Summary

Construction:

- Training set of shapes, corresponding landmarks
- Procrustes align shapes and compute mean shape
- Covariance matrix and solve PCA eigenproblem
- Shape parameters, modes of variation
- Construct gaussian probabilistic model

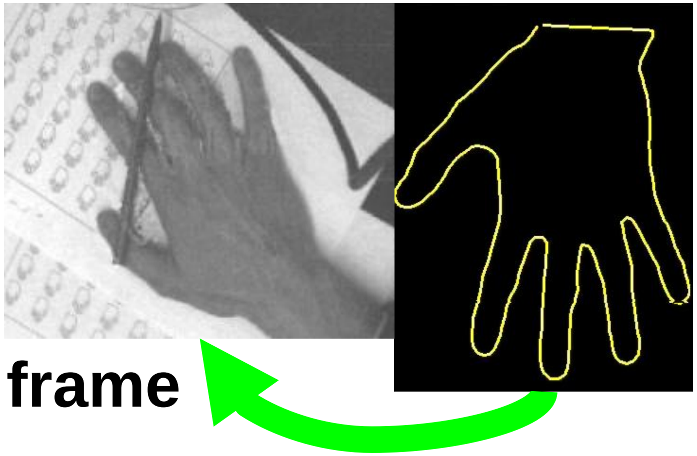
Results:

- General: modes of variation capture full variation
- Specific: modes capture only variation actually seen
- Assign probabilities to generated shapes

From SSMs to Active Shape Models

Task:

- Find shape in unseen image

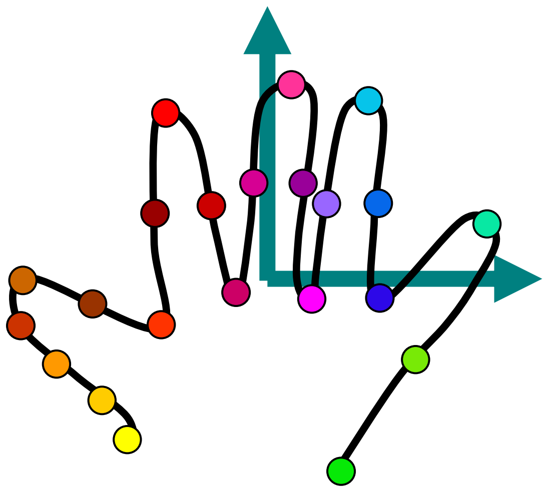


Solution:

- Map from model frame to image frame
- Iterative localised search:
  - Search in neighbourhood of current points for new points
  - Fit model to suggested new shape
  - Apply constraints to shape based on learnt variation
  - Repeat until convergence
- Like ACM, moves towards edges, lines etc
- Unlike ACM, remains valid shape as it does so

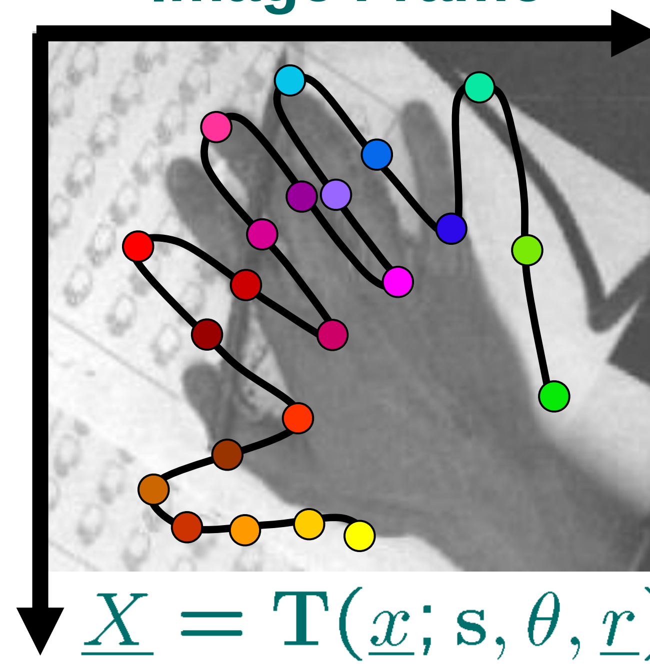
Placing the Model in an Image

Model Frame



$$\underline{x} = \bar{\underline{x}} + \mathrm{P}\underline{b}$$

Image Frame



$$\underline{X} = \mathrm{T}(\underline{x}; \mathrm{s}, \theta, \underline{r})$$

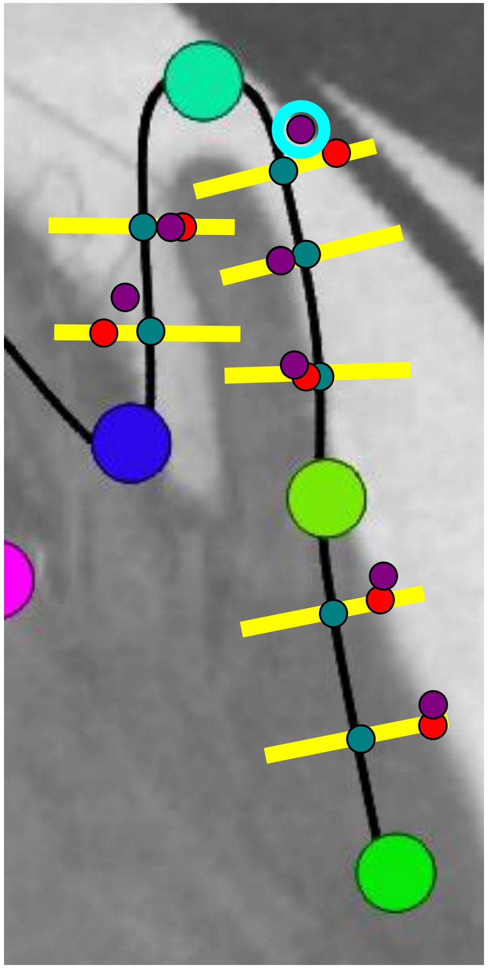
- **Scale, rotate, and translate to get to image frame:**

Pose transformation: $\mathrm{T}(\underline{x}; \mathrm{s}, \theta, \underline{r}) = \mathrm{s}\mathbf{R}(\theta)\underline{x} + \underline{r}$

- **Total set of parameters to define shape in an image:**

Pose parameters: $\mathrm{s}, \theta, \underline{r}$, Shape parameters: $\underline{b}$

ASM Local Iterative Search



- **Local search**
- **Initialise near target**
- **Search along normals**
- **Look for strongest edge**
- **Gives new set of suggested shape points**

  $\underline{X}'$
- **Best-Fit model to shape:**

  $\underline{X}' \approx \mathbf{T}(\underline{x}(\underline{b}); \mathrm{s}, \theta, \underline{r})$
- **Candidate shape:**

  $\underline{X}'' = \mathbf{T}(\underline{x}(\underline{b}); \mathrm{s}, \theta, \underline{r})$
- **Still leaves some errors**

Problems:

Shape model step: $\underline{X}'' = \mathbf{T}(\underline{x}(\underline{b}); \mathbf{s}, \theta, \underline{r})$

- Even within model, can get extreme shapes

Edge-Finding:

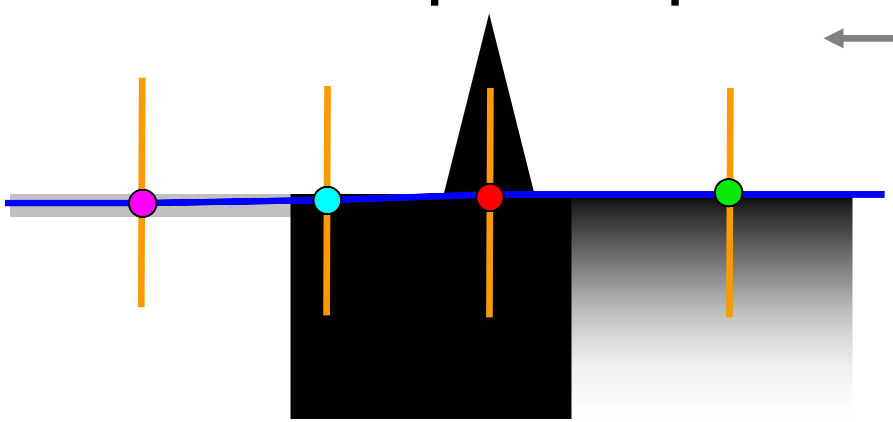
- Actual position not edge, or not strongest edge

Solution:

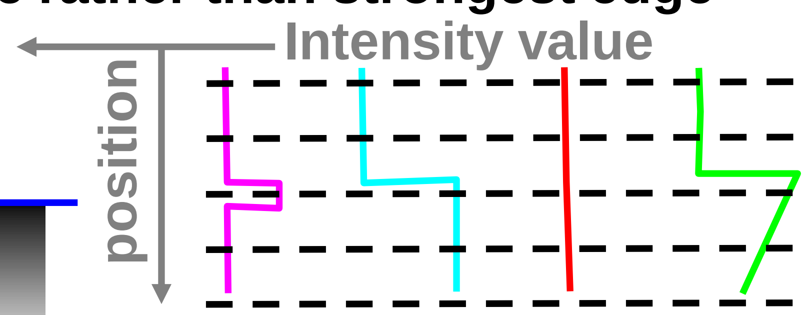
- Model profiles/appearance near points
- Apply shape and profile probability to control search

Profile Models

- **For each shape point in each training image:**
  - **Sample image values along normals to shape**
  - **Normalise to eliminate illumination effects**
- **Build statistical model as for shape**
  - **Profile vector like shape vector**
- **Model assigns probability to each possible profile**
  - **Select most probable profile rather than strongest edge**



sampling the image



profile sample points

collected profiles

Applying Shape Constraints in Search

- **Sample as before, find best fit to profile model at each point**
- **Suggested new set of shape points:** $\underline{X}'$

Hard Constraints:

$$\text{Minimize}\ :\ |\underline{X}' - \mathbf{T}(\underline{x}(\underline{b}); \mathrm{s}, \theta, \underline{r})|^2$$

$$\text{with}\ \ \mathrm{p}(\underline{b}) \geq \mathrm{p}_{\text{thresh}} \qquad \underline{X}'' = \mathbf{T}(\underline{x}(\underline{b}); \mathrm{s}, \theta, \underline{r})$$

Soft Constraints:

$$\text{Minimize}\ :\ |\underline{X}' - \mathbf{T}(\underline{x}(\underline{b}); \mathrm{s}, \theta, \underline{r})|^2 - \alpha \log(\mathrm{p}(\underline{b}))$$

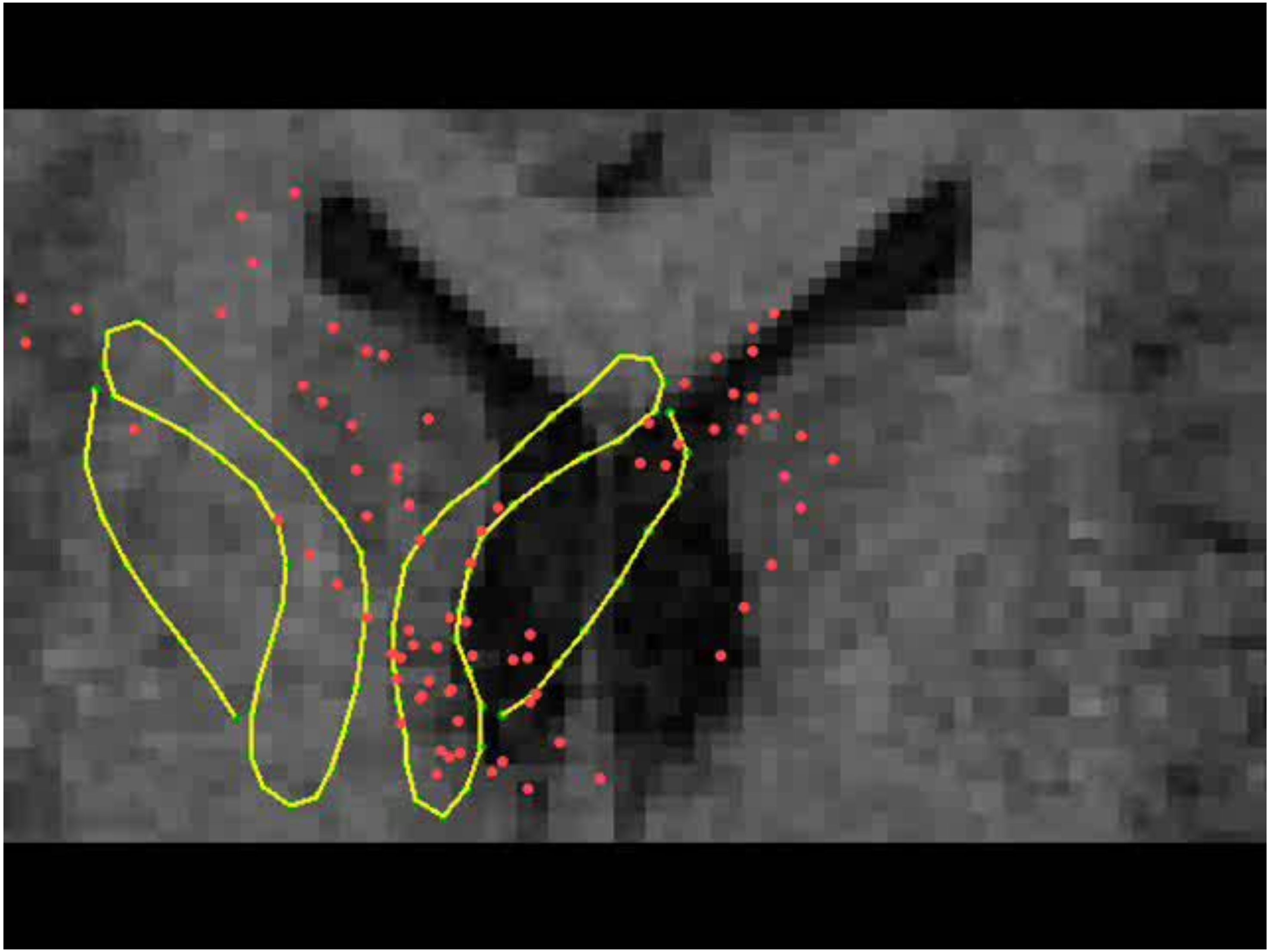
- **Can also include profile probability as local quality of match into process – more extreme shapes provided quality of profile match merits it**

ASM Multi-Resolution Search

- To increase basin of attraction, use multi-resolution
  - Gaussian pyramid of training images
  - Same shape points, but different profile models at each level
- Start search at coarse level, refine at finer level
- Similar to multi-resolution gaussian smoothing for edge-based Active Contours (previous lecture Slide 14)

ASM Search Example: Hip

ASM Search Example: Brain



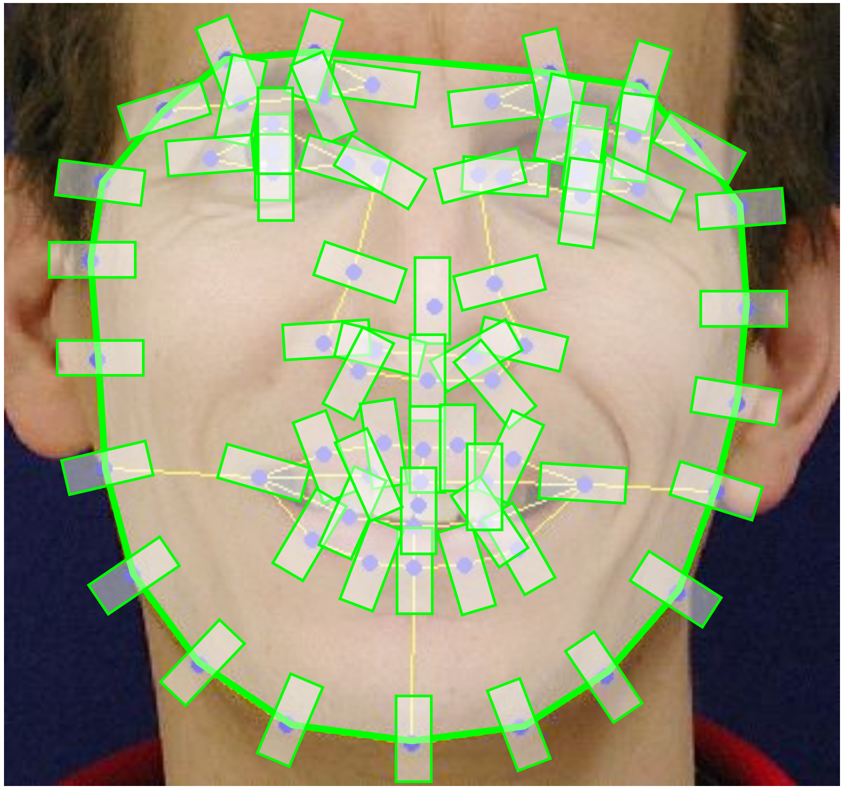
ASM: Summary

Advantages:

- **Fast, simple, accurate**
- **Efficient extension to 3D**

Disadvantages:

- **Only sparse use of image information**
- **Treats local profiles as independent**
- **AAM: models all the texture**



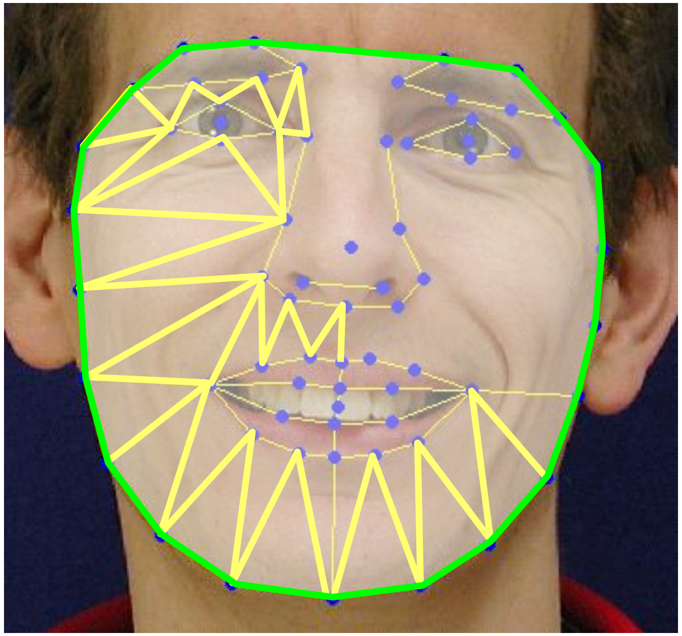
Active Appearance Models

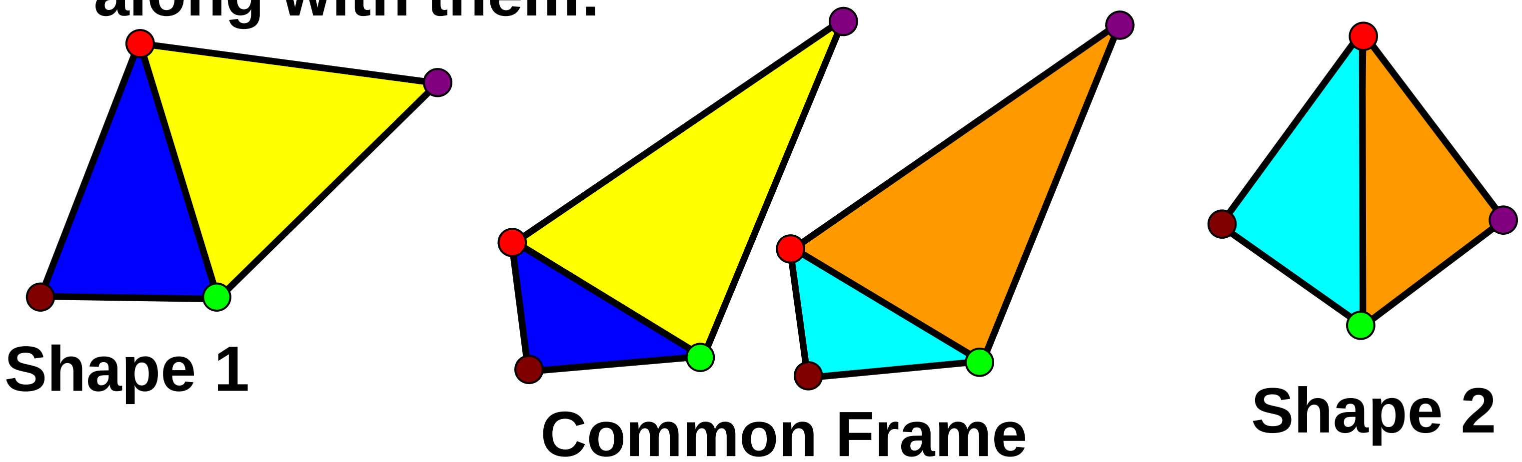
Active Appearance Models

- Build SSM as before

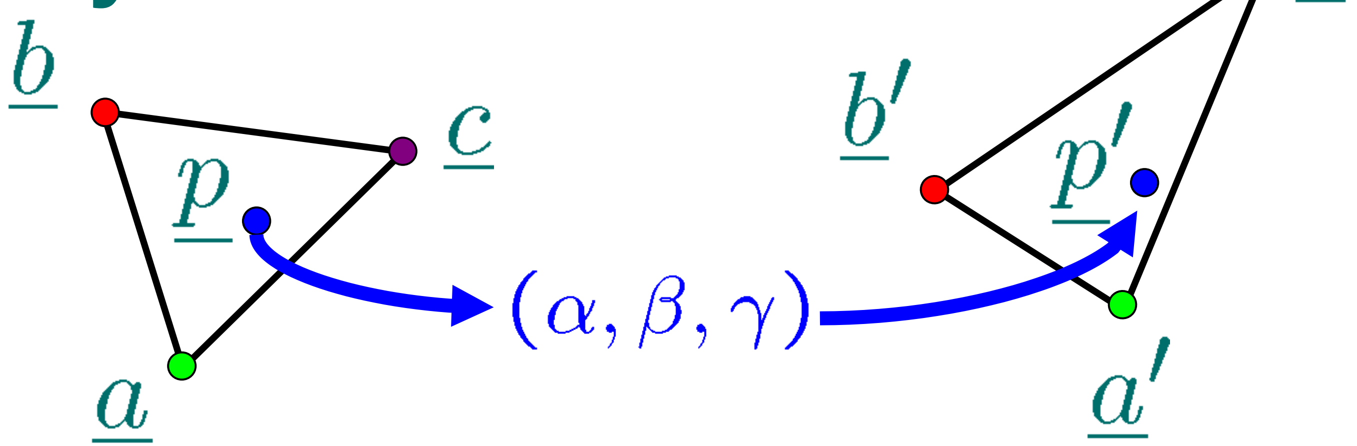
$$\underline{x} = \underline{\bar{x}} + \mathrm{P}\underline{b}$$

- Triangulate shapes
- Warp all shapes to mean shape
- Take image texture along with them:





Barycentric Coordinates



$$\underline{p} = \alpha\underline{a} + \beta\underline{b} + \gamma\underline{c}$$

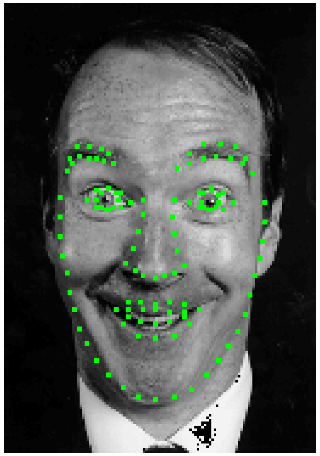
$$\alpha + \beta + \gamma = 1$$

$$\underline{p}' = \alpha\underline{a}' + \beta\underline{b}' + \gamma\underline{c}'$$

Inside: $\alpha, \beta, \gamma \geq 0$

- **Map any point from training image to point within mean shape**
- **Map all textures: set of shape-free texture samples**

Shape-Free Texture Model



Original image

Frame of mean shape

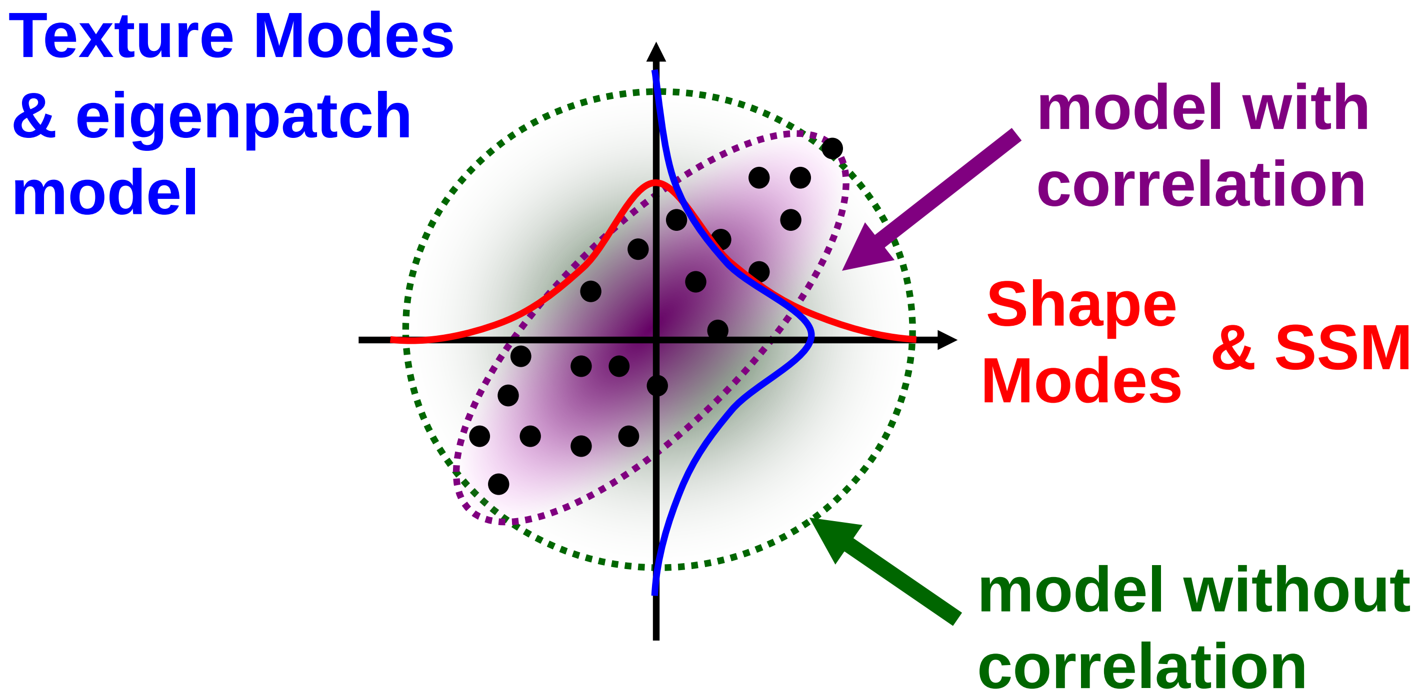
PCA Eigenmodel from shape-free textures

First texture mode

Second mode

Third mode

Combined Models:

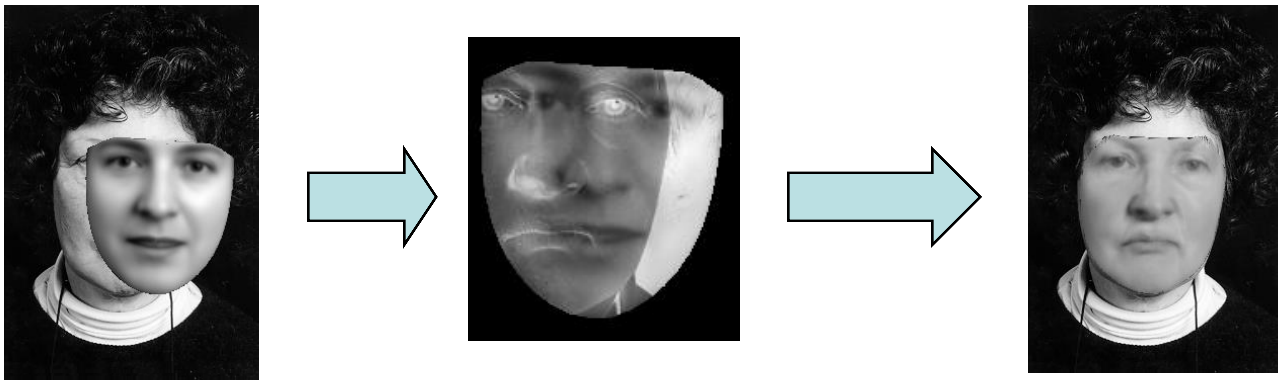


- Texture and shape don't vary independently
- Change of expression, shape changes & shadows
- Model with shape/texture correlation more specific & more compact (fewer parameters)

AAM Combined Models

- Texture changes as expression (shape) changes

AAM Search:



Place model in image

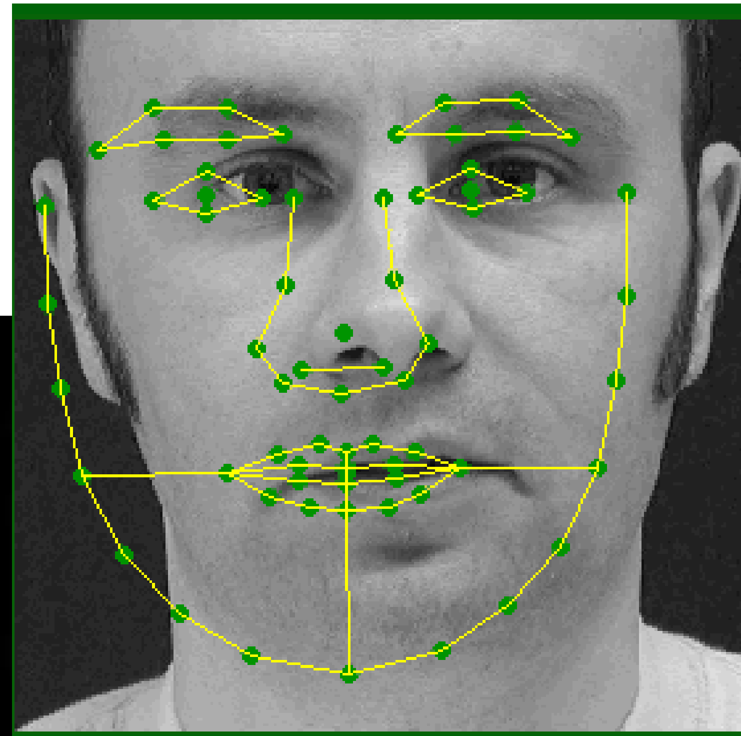
Measure Difference

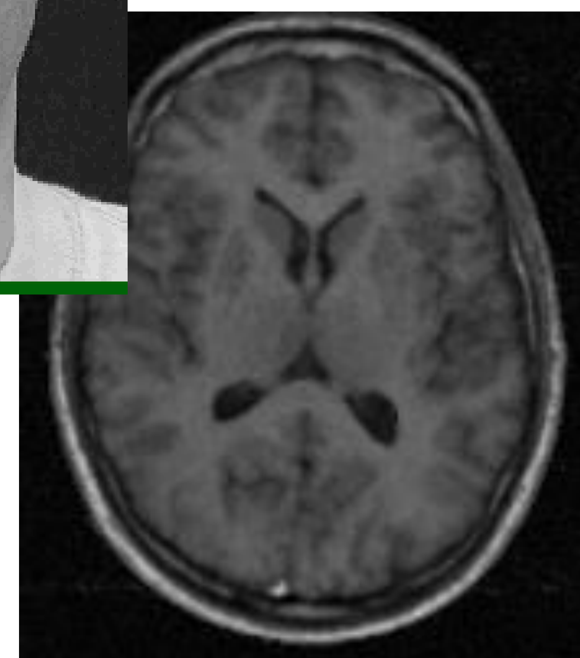
Update Model

Iterate

- **Synthesize model**
- **Measure difference**
- **Difference gives clues as to how to update model**
- **Learn from training set by displacement**
- **Iterate**

AAM Example:





Summary:

ASMs and AAMs:

- General: can encompass full range of variation
- Specific: generate only valid instances
- Fast and efficient search algorithms
- Applied in many different contexts
    - Faces (identity/expression/sex/age/ethnicity)
    - Medical images (brains/knees/hips/spines/hands etc)
- Extended to 3D (brains, kidneys, knees etc)

Research Issues:

- Annotation & Ordering not always possible
- Automatic location of correspondence
    - Geometry (e.g. curvature)
    - Automatic methods like MDL for surfaces (see our book!)
    - Image registration – Next set of lectures

Further Information:

- **Mathematical details of PCA, correspondence problem**

Electronic access via CAS/library to Springer ebooks:

Davies, Twining & Taylor, *Statistical Models of Shape*

- **ASM, AAM etc**
- **Tim Cootes personal website**
- **Wikipedia: articles & links**
- **YouTube: many videos of ASM/AAM search & variants**