

# Image Registration 1:

**Spring 2021**

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**Presented by: Terence Morley**

# Handouts & Lecture Notes

- Report in Scientific American (June 2014):  
*“In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops.”*

## **The Pen Is Mightier Than the Keyboard**

P. A. Mueller, D. M. Oppenheimer, *Psychological Science*, Vol 25, Issue 6, pp. 1159 – 1168, April-23-2014.

- Handouts are to aid note taking, not a total replacement for note taking
- Podcasts, slides, pdfs etc on BlackBoard



# Overview

- **What** is registration?
- **Why** registration?
- Example biomedical image data
- Breakdown of image registration problem

**Image matching**

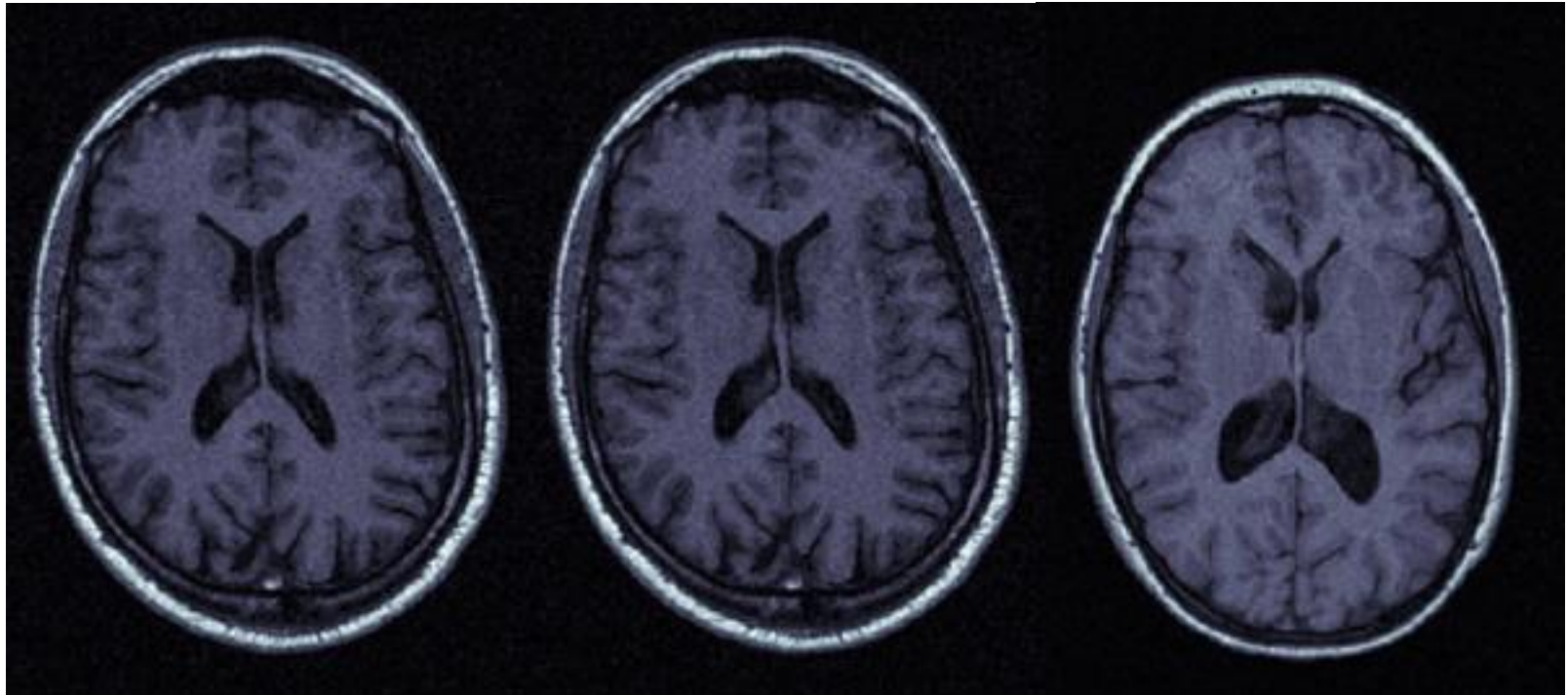
**Representing warps & regularisation**

**Optimisation**

# What is Non-Rigid Registration? (NRR)

Source Image

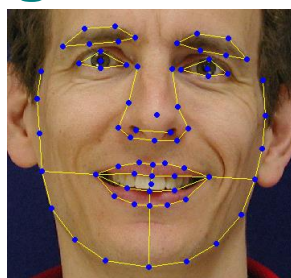
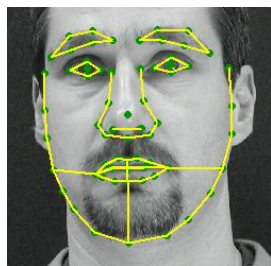
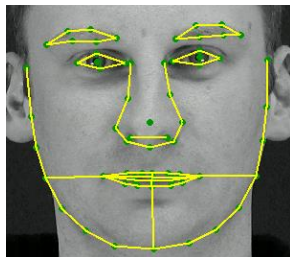
Target Image



- **Warp** source image to look like target image
- **Warp**: spatial deformation

# Landmark Based Registration

## Annotated Images



Frame of  
mean  
shape



Original  
image

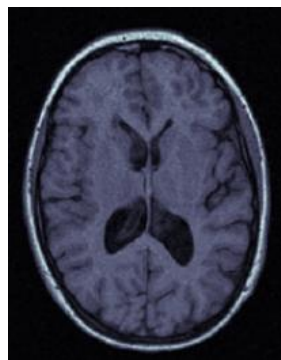
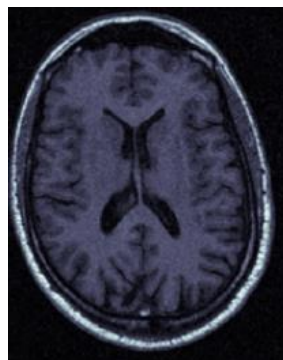
Frame of  
mean shape

- Interpolate movement of landmarks to warp image patch  
[In AAM case, it was done using barycentric coordinates and triangulation]
- If we could do it **without** landmarks, only have to annotate one image to build shape & appearance model
- Manual annotation not always possible or feasible
- Registration just using images => **automatic model building**

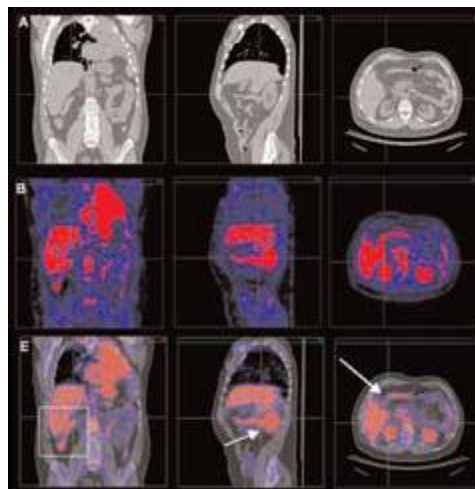
# Why do we need to do registration?

- Automatic model building
- Registering two images:
  - Aligns common structures
  - Dense (pixel by pixel or voxel to **voxel**) correspondence
- **Compare** information from two different images
- **Combine** information from two different images

3D equivalent  
of the 2D pixel



Comparing MR  
brain images



3D CT image

3D PET image

Combined

# Biomedical Image Data

- Vast amount of 2D, 3D, 4D (3D+time) data available

Routine patient scans (diagnosis & disease progression)

Research studies (multimodality & population studies)

Human NeuroScanning Project (whole brain, sectioned)

## Various Registration Scenarios:

- Same individual, same modality (Intra-subject)

Patient movement, organ movement, pre & post intervention

- Same individual, different modality  e.g. PET vs CT

Movement between scans, functional versus structural imaging

- Population studies, many individuals (Inter-subject)

Atlas construction, variation/similarity between subjects

- Histological sections (2D slices from 3D object)



# Correspondence Problem

- **Intra(same)-subject case:**

Identify **real** correspondence between **different** images

- **Inter(different)-subject case:**

**Meaningful** anatomical correspondence between **different** individuals

Correspondence between individual and some **anatomical atlas**

- **Histological case:**

Reconstruct **real** correspondence distorted by preparation



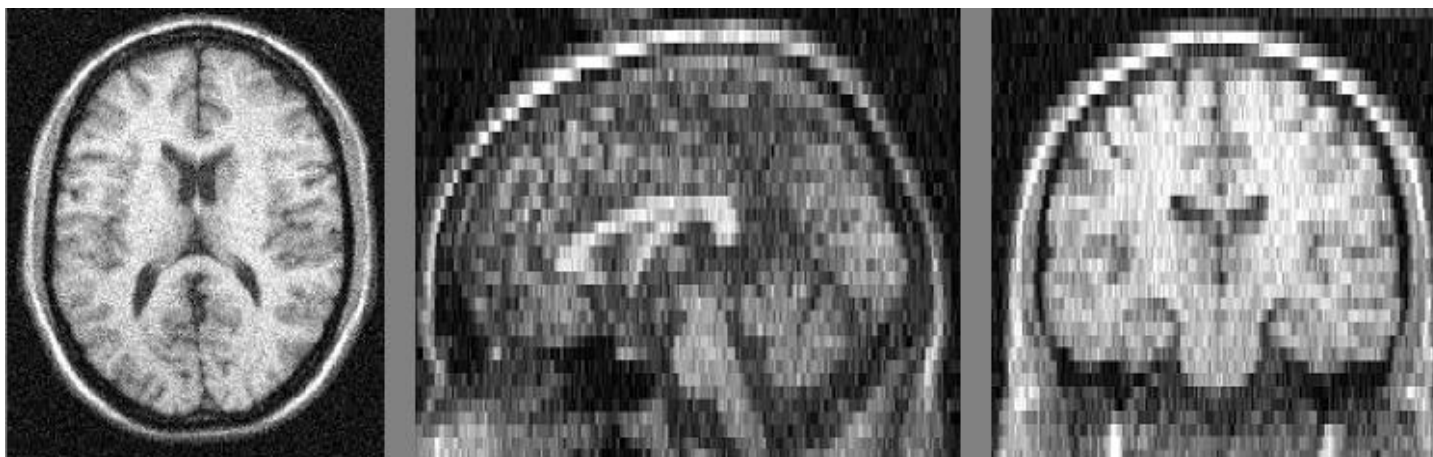
**Unslicing  
the loaf!**

# Example Biomedical Images

# Same individual, same modality



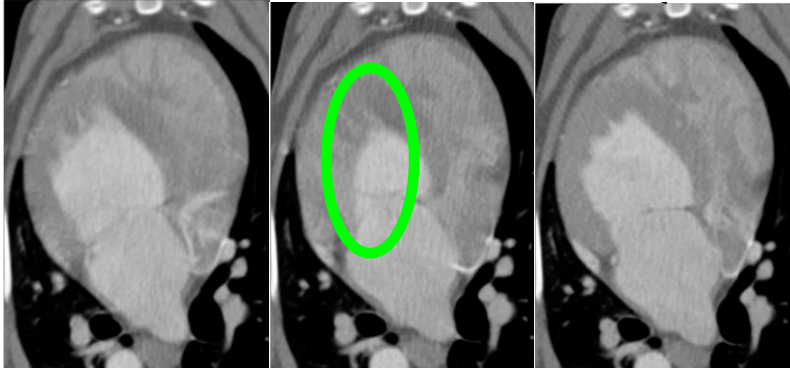
**Pre-Operative MR, 1.5 Tesla**



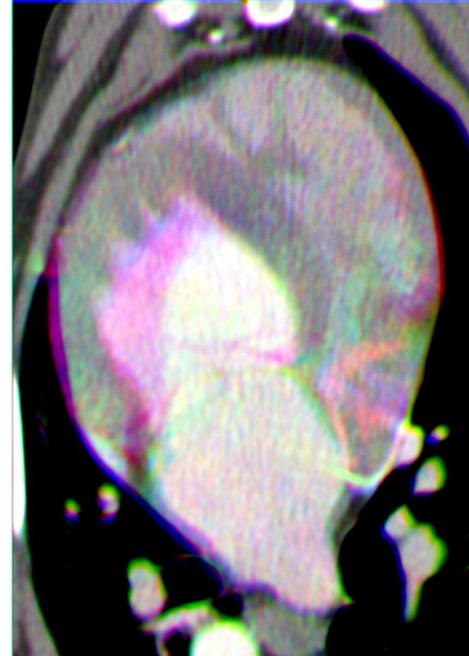
**Intra-Operative MR, 0.15 Tesla**

# Same individual, same modality

Marcin Wierzbicki, Maria Drangova, Gerard Guiraudon, Terry Peters, *Validation of dynamic heart models obtained using non-linear registration for virtual reality training, planning, and guidance of minimally invasive cardiac surgeries*, Medical Image Analysis, 8(3),2004, Pages 387-401.



2D slices from sequence of 3D images



3 frames overlaid and colour-coded to show movement between frames

- Innate organ motion, or motion between scans

# Multimodal Imaging

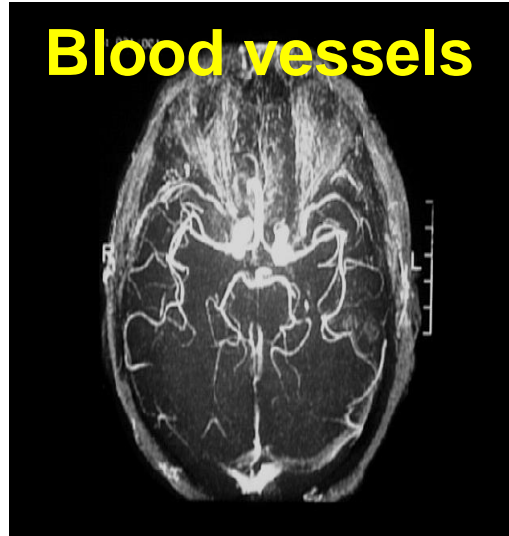
ANATOMICAL/  
STRUCTURAL

Tissues



Magnetic Resonance

Blood vessels



Angiography

Tissues

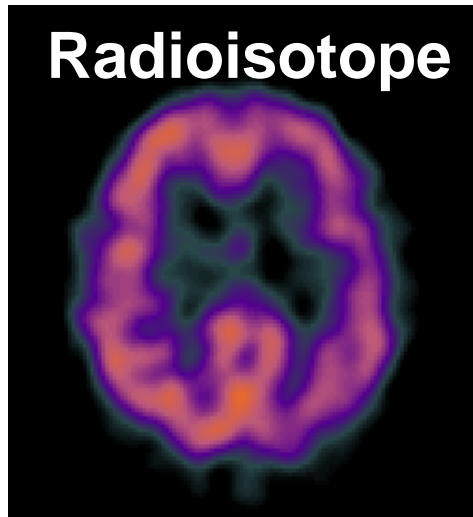


Computed Tomography

FUNCTIONAL

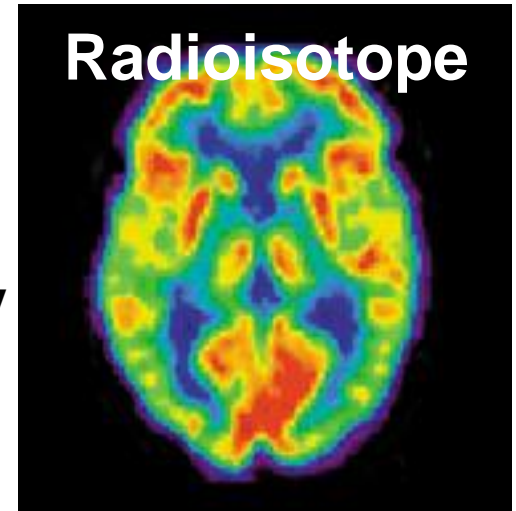
**SPECT:**  
Single  
Photon  
Emission  
Computed  
Tomography

Radioisotope



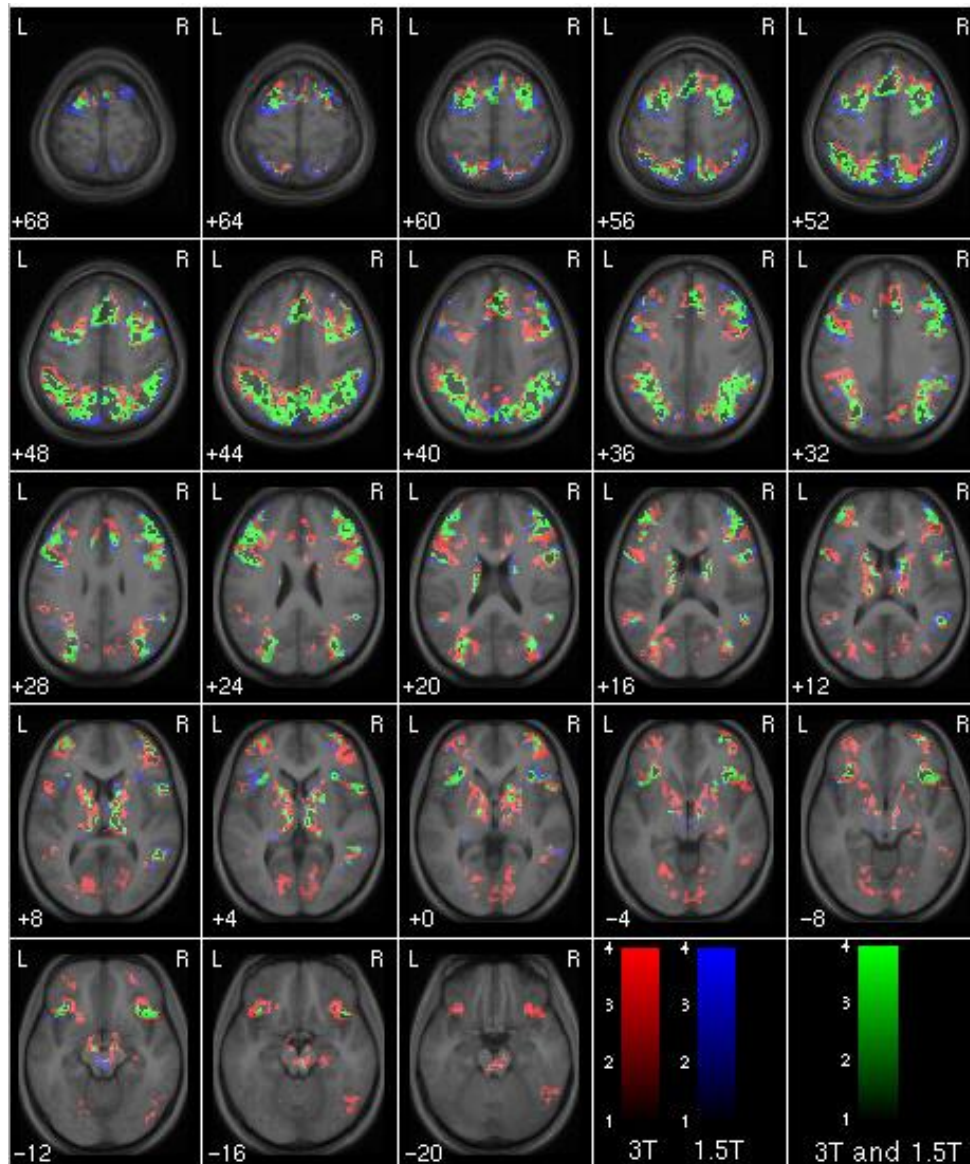
**PET:**  
Positron  
Emission  
Tomography

Radioisotope



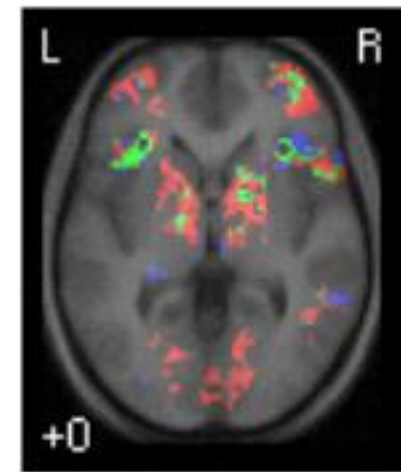


# Data Fusion:



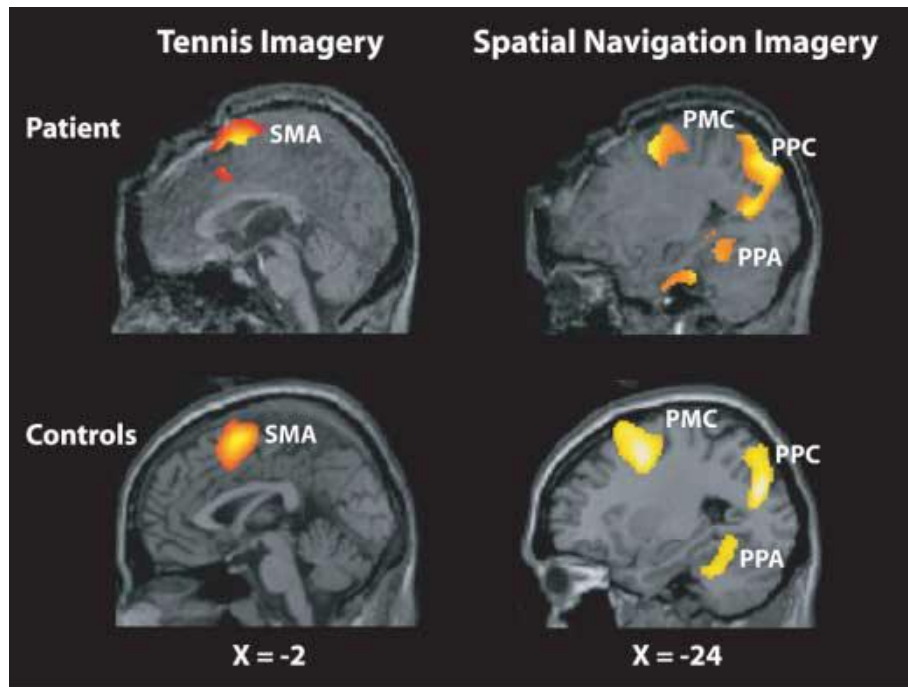
**Functional (fMRI)  
overlaid on  
Anatomical MRI:**

**Link activation to  
specific anatomical  
structures**



# ASIDE: fMRI & Mind-Reading!

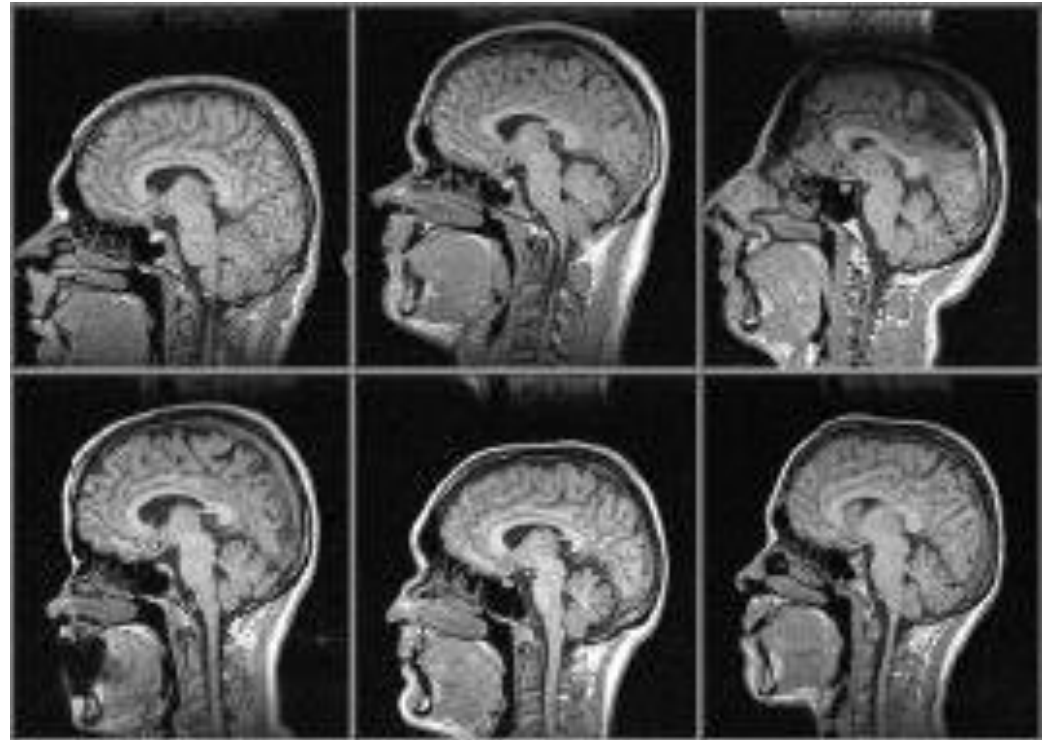
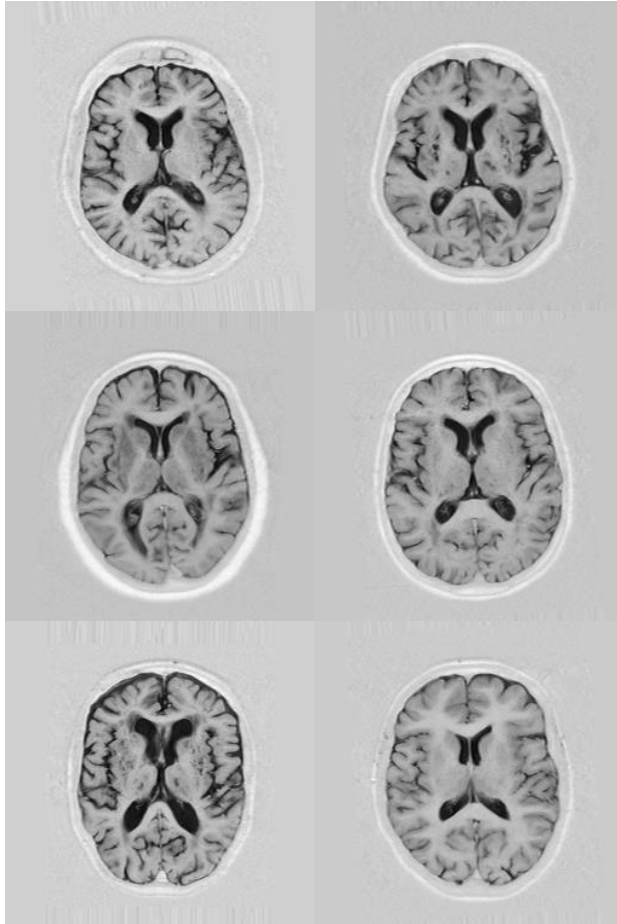
- Patients in persistent vegetative states (PVS)
- Show brain activity & can answer yes/no questions



Owen et al. ,  
*Detecting Awareness in  
the Vegetative State*  
Science 8 September  
2006

- Scott Routley able to answer that he was not in pain
- Raises important ethical issues.....

# Population Studies

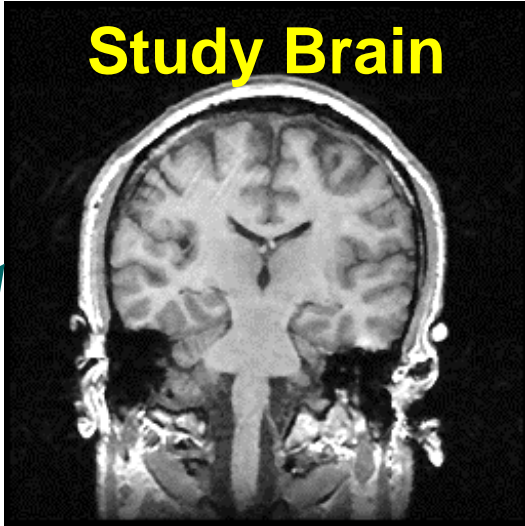


- Normal and abnormal anatomical variation

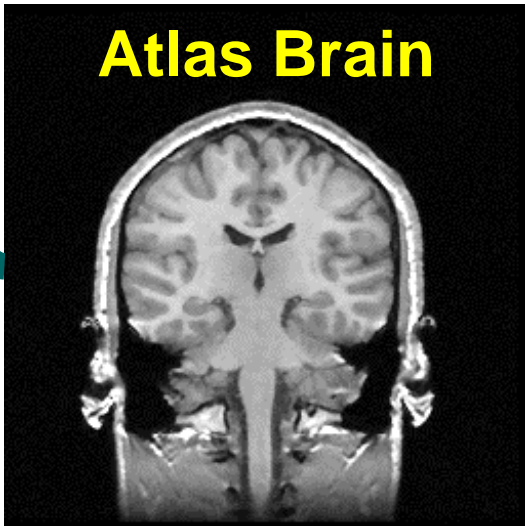


# Individual to Atlas

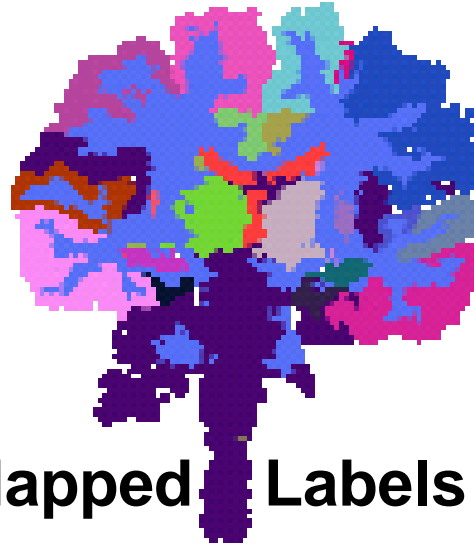
**Study Brain**



**Atlas Brain**



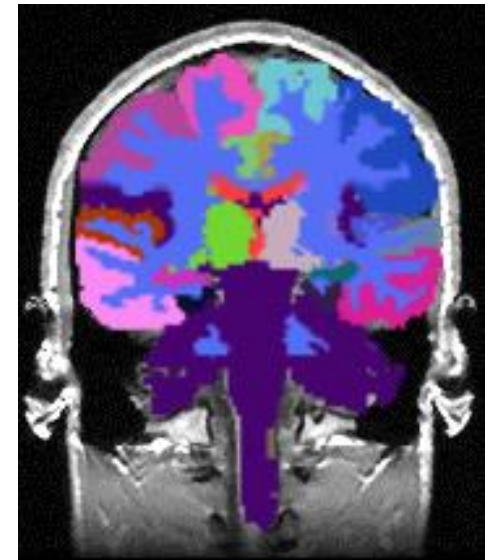
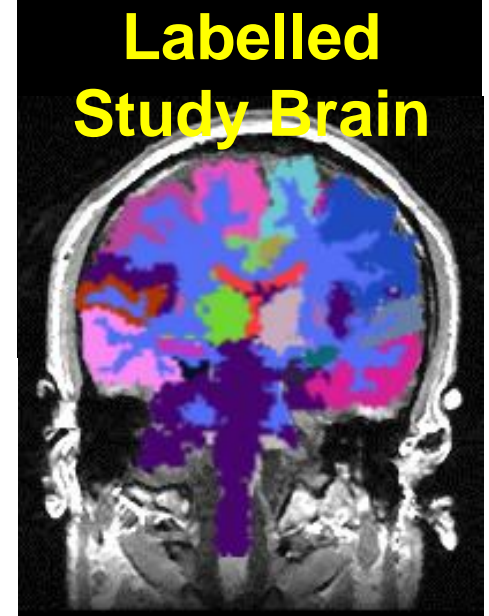
**Mapped Labels**



**Atlas Labels**

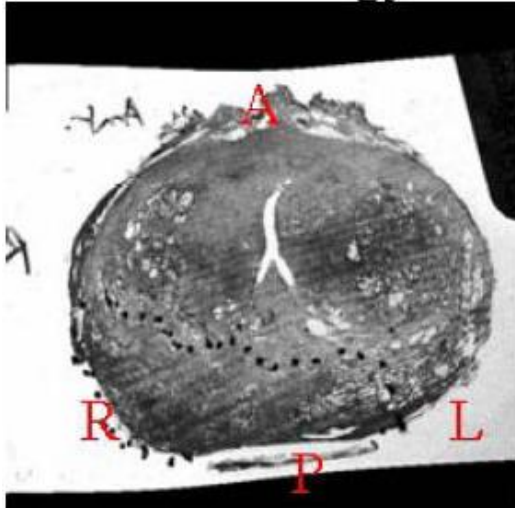


**Labelled Study Brain**

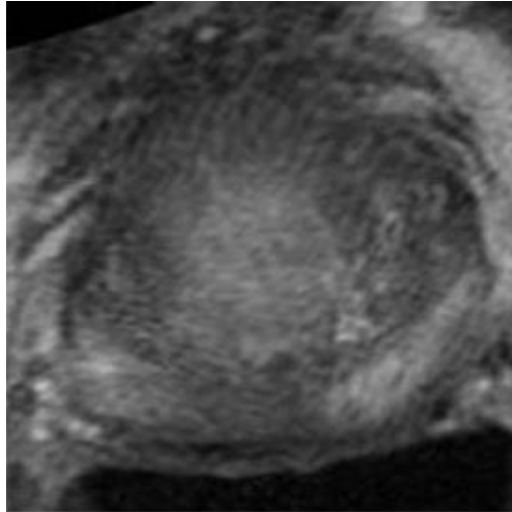


Courtesy of J-P. Thirion, INRIA

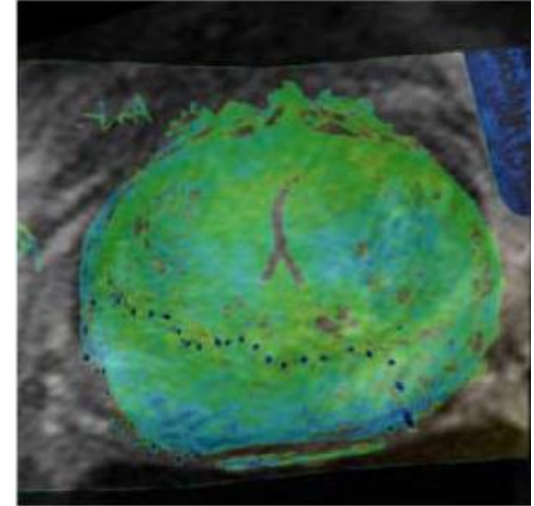
# Histological Sections: Removed prostates



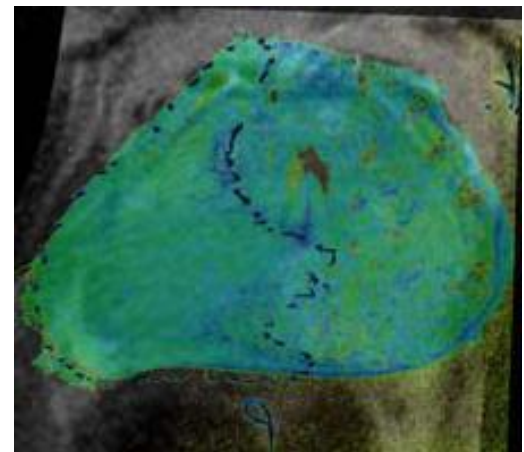
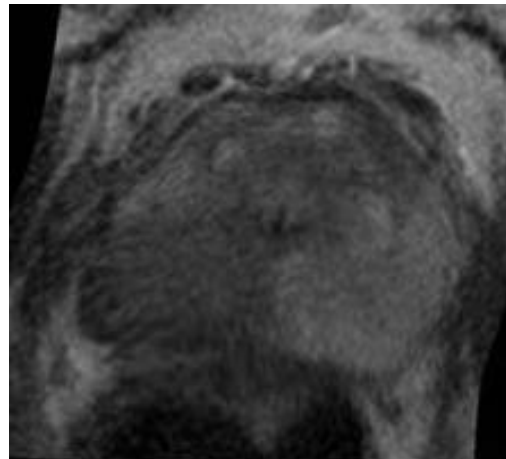
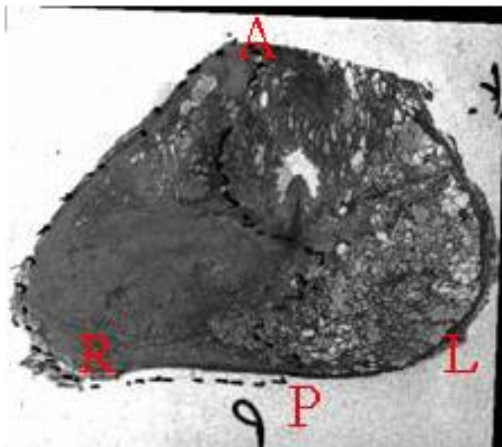
**Histological  
section of prostate**



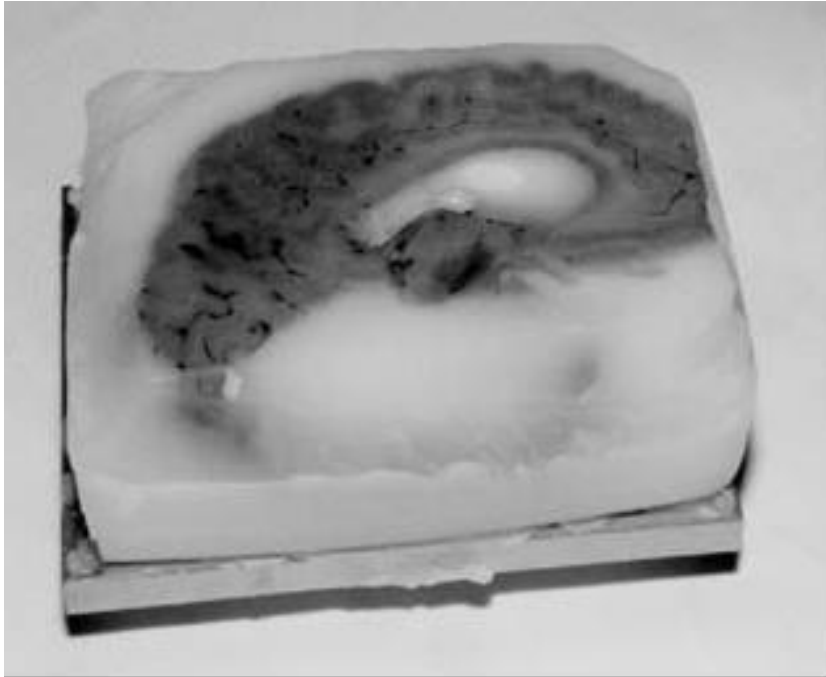
**Registered MR  
image**



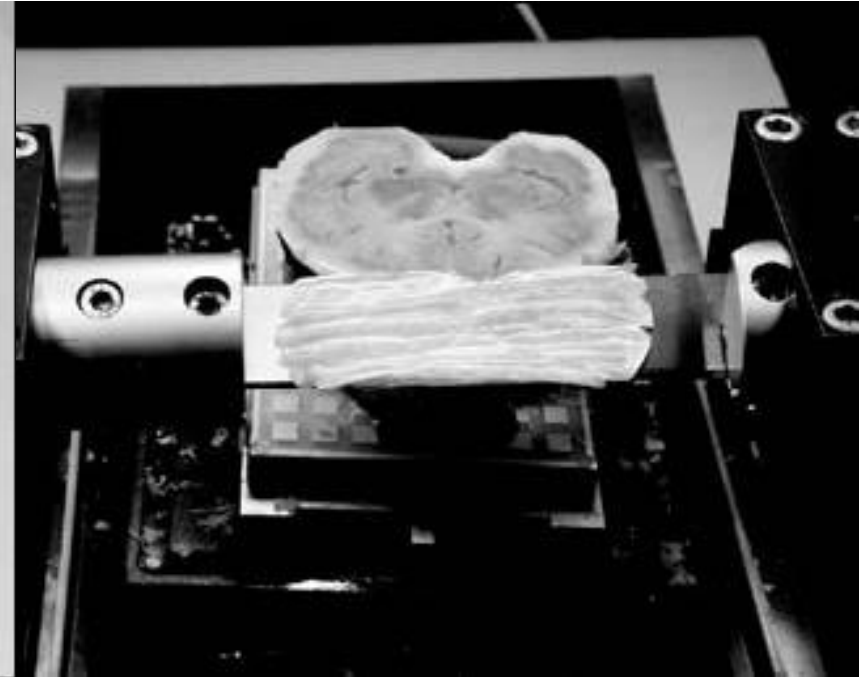
**Superimposed**



# Example: Histological sections



**Whole human brain  
embedded in paraffin wax**

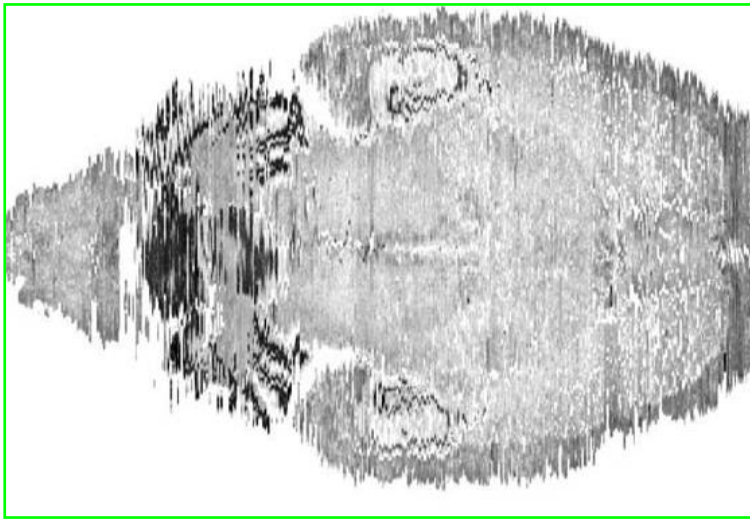


**Microtome**

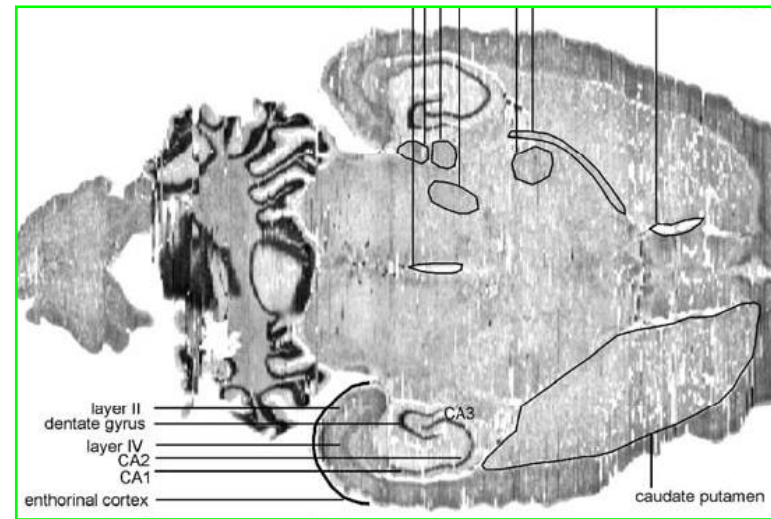
- Stretched, dried, wax removed, stained, mounted
- Distortion occurs during this process

# Histological Sections: Rat Brain

Modersitzki-, Schmitt and Wirtz,  
*Registration of Histological Serial Sectionings,*  
Mathematical Models for Registration and Applications to Medical Imaging



Before registration



After **volume preserving** registration

- Artificial slice of reconstructed 3D rat brain from histological sections



# Basic Structure of Pairwise Registration Algorithms

# Pairwise Registration Algorithm

- Two images, source and target (also called moving & reference, moving & stationary, template & reference etc etc etc!)
- Warp one image into the frame of the other
- Assess image matching
- Optimize the match

## Three Main Questions:

- **How** do we measure image match/image difference?
- **How** do we represent image warping?
- **How** do we find the optimum match?

# Image Matching

## Two main strands:

- **Geometric Matching:**

**Extract sparse features (manually or automatically) :**  
points, lines, edges, ridge lines, surfaces etc

**Register extracted features**

**Problems:** manual annotation, matching features,  
interpolating/extrapolating the paired features

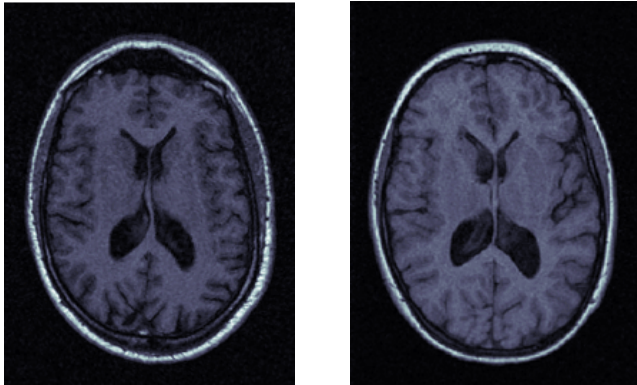
- **Intensity-Based Registration:**

**pixel-by-pixel comparison warped & stationary images**

**Agrees with our intuition about matching**

**Makes maximal use of available image information**

# Image & Warped Image: Matching



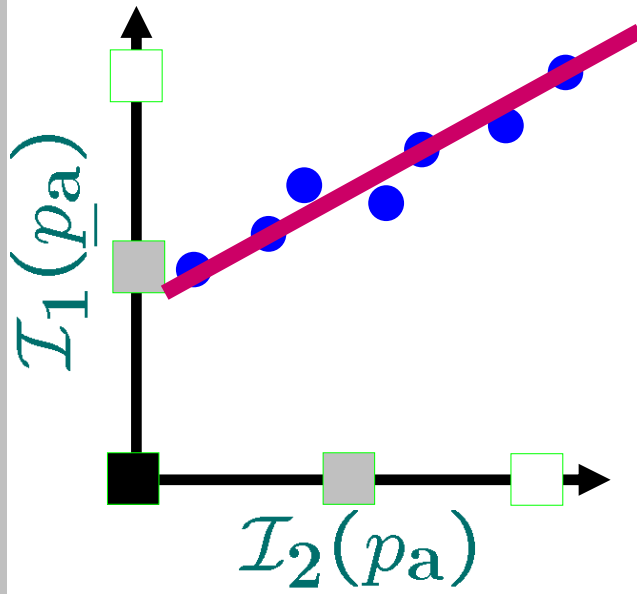
$\mathcal{I}_1(\underline{X})$        $\mathcal{I}_2(\underline{X})$

- Both defined on same pixellated grid:  $\underline{X} = \{\underline{p}_a\}$
- Pixel-by-pixel comparison:  $\mathcal{I}_1(\underline{p}_a)$  versus  $\mathcal{I}_2(\underline{p}_a)$

- Sum of Absolute Differences (SAD):  $\sum_a |\mathcal{I}_1(\underline{p}_a) - \mathcal{I}_2(\underline{p}_a)|$
- Sum of Squared Differences (SSD):  $\sum_a (\mathcal{I}_1(\underline{p}_a) - \mathcal{I}_2(\underline{p}_a))^2$
- Both assume best match = same values
- If this is **not** the case, **correlation** or **mutual information**
- Resampling the images onto common pixel grid?



# Correlation & Mutual Information



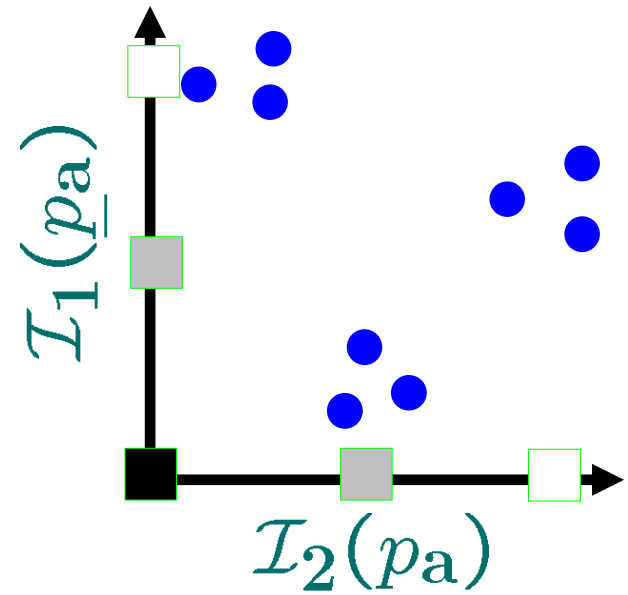
Subtract mean of each image

$$\hat{I}(\underline{p}_a) = I(\underline{p}_a) - \frac{1}{n_p} \sum_b I(\underline{p}_b)$$

Normalised cross-correlation

$$\frac{\sum_a \hat{I}_1(\underline{p}_a) \hat{I}_2(\underline{p}_a)}{\sqrt{\sum_b (\hat{I}_1(\underline{p}_b))^2 \sum_c (\hat{I}_2(\underline{p}_c))^2}}$$

**Tries to make joint distribution  
a straight line**



Mutual Information (Viola, 1995):

Useful when no simple ordering  
relation between intensities:

**multi-modality registration**

**Tries to make joint distribution as  
peaky as possible**

# Mutual Information & Entropy

- Histogram, bins:  $\{i\}$  probabilities:  $\{P_i\}$ ,  $\sum_i P_i = 1$
- Entropy:  $E = -\sum_i P_i \log P_i$

Single bin:  $E = 0$   
Else  $E > 0$

## Mutual Information:

- Entropy of distribution of pixel values for image 1
- Entropy of distribution of pixel values for image 2
- **minus** entropy of joint 2D histogram

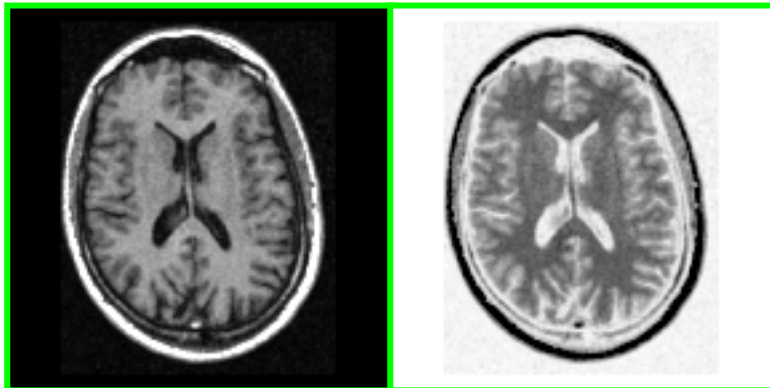
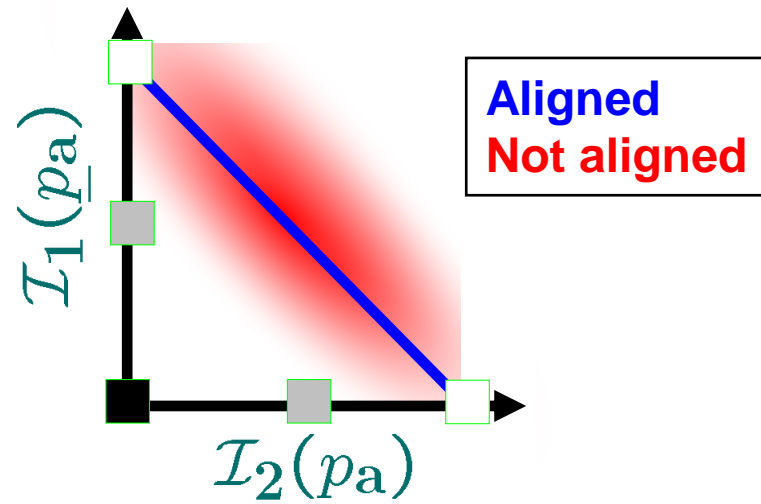
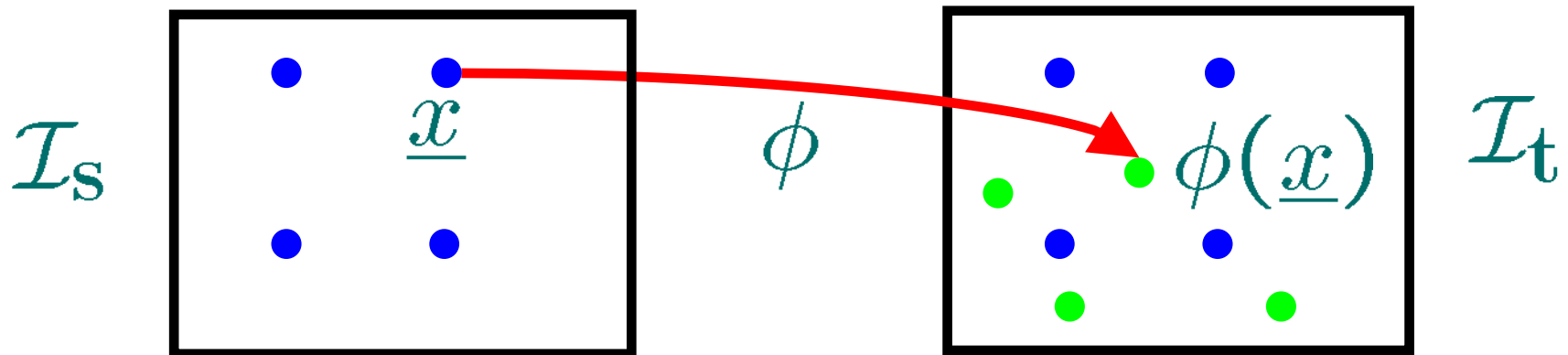


Image & Inverse image



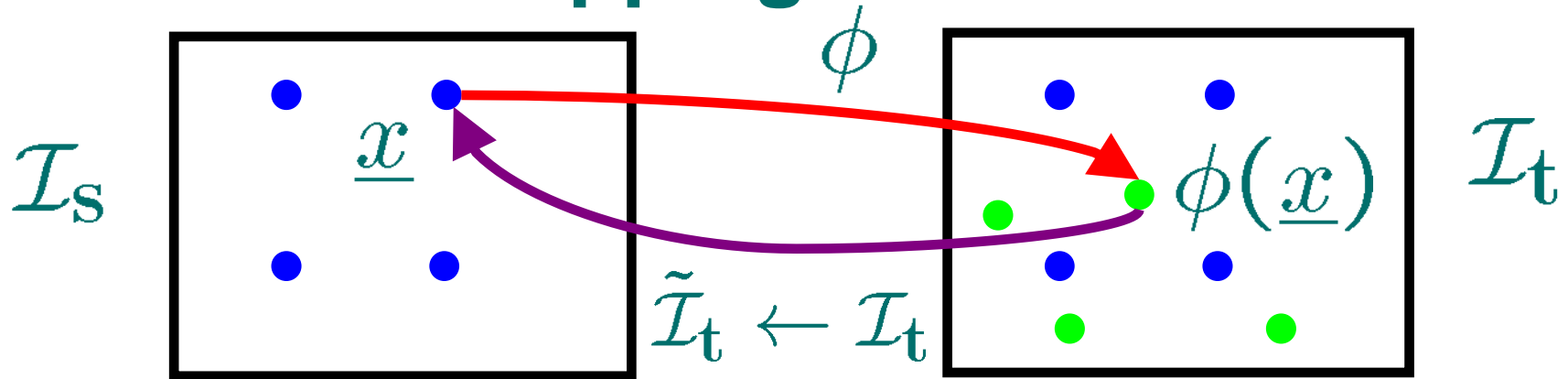
# Sampling the Warped Image

# Push-Forward Mapping



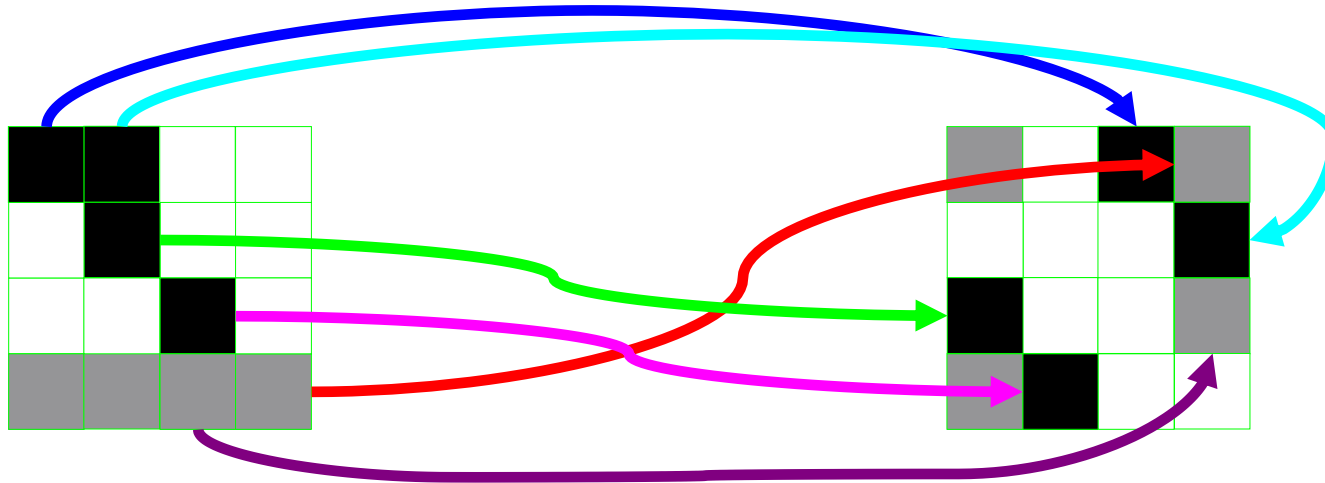
- $\mathcal{I}_s$  &  $\mathcal{I}_t$  both defined on regular pixel grid  $\underline{X}$
- Mapping:  $\phi : \underline{x} \mapsto \phi(\underline{x})$
- Warped source:  $\tilde{\mathcal{I}}_s : \tilde{\mathcal{I}}_s(\phi(\underline{X})) \doteq \mathcal{I}_s(\underline{X})$
- Pixel-by-pixel comparison in frame of target image
- Comparing  $\tilde{\mathcal{I}}_s(\phi(\underline{X}))$  with  $\mathcal{I}_t(\underline{X})$  :  
resample source from **irregular grid** to **regular grid**
- Computationally expensive

# Pull-Back Mapping



- $\mathcal{I}_s$  &  $\mathcal{I}_t$  both defined on regular pixel grid  $X$
- Mapping:  $\phi : \underline{x} \mapsto \phi(\underline{x})$
- Pixel-by-pixel comparison in frame of **source** image
- Warped target:  $\tilde{\mathcal{I}}_t : \tilde{\mathcal{I}}_t(\underline{X}) \doteq \mathcal{I}_t(\phi(\underline{X}))$
- Compare  $\mathcal{I}_s$  with  $\tilde{\mathcal{I}}_t$
- Computing  $\tilde{\mathcal{I}}_t : \mathcal{I}_t(\underline{X}) \rightarrow \mathcal{I}_t(\phi(\underline{X}))$   
resample target from **regular grid** to **irregular grid**
- Computationally easier

# Image matching alone not enough



- **If we:** Shuffle pixels in any fashion
- Add in intensity changes due to interpolation
- **Result:** Match almost anything to almost anything!

## Ideal Solution:

- Retain proximity:  
close in one image = close in the other
- Deformation field  $\phi(\underline{x})$  continuous and smooth
- No folds or tears

# Summary:

- Why registration & example image data

## Pairwise Registration:

- Image matching, single or multi modal cases
- Creating warped image
- Why image matching alone isn't enough!

## Next Lectures:

## Warp Regularisation:

- Parametric Warps:

Meshes, barycentric coordinates & splines

- Non-Parametric Warps:

Dense deformation fields, elastic solids & fluids