

### **Image Registration 3:**

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### **Handouts & Lecture Notes**

Report in Scientific American (June 2014):

"In each study, however, those who wrote out their notes by hand had a stronger conceptual understanding and were more successful in applying and integrating the material than those who used [sic] took notes with their laptops."

The Pen Is Mightier Than the Keyboard

P. A. Mueller, D. M. Oppenheimer, *Psychological Science*, Vol 25, Issue 6, pp. 1159 – 1168, April-23-2014.

- Handouts are to aid note taking, not a total replacement for note taking
- Podcasts, slides, pdfs etc on BlackBoard



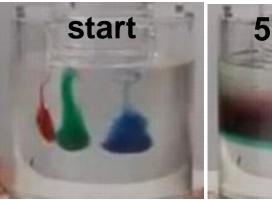
## From Elastic Solids to Fluids

## **Physics Based Models:**



#### **Elastic Registration:**

- Elasticity regularises warp
- Penalizes large deformations
- Can't deal with extreme cases





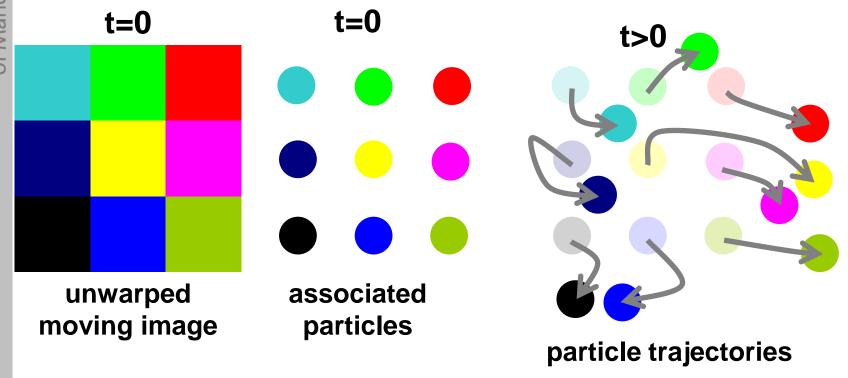




#### Fluid:

- Extremely large deformation
- Still unmixes!
- Viscosity and flow over time

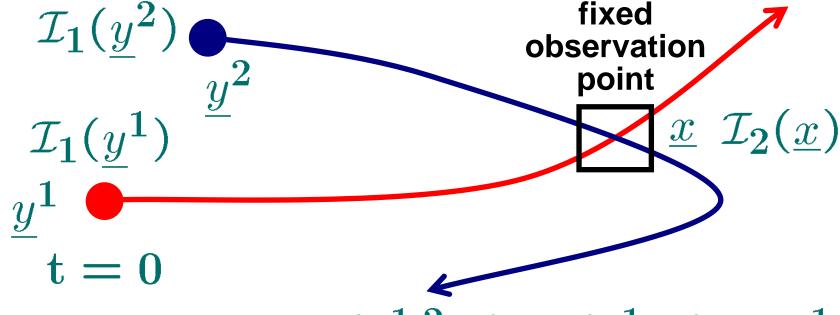
# Moving Images using Fluid Flow



warped image information

- Added time dimension allows greater flexibility
- Regulariser = fluid forces between moving particles

### Fluid Picture: Notation



Trajectories:  $\phi(\underline{y}^{1,2},\mathbf{t})$ ,  $\phi(\underline{y}^{1},\mathbf{0}) = \underline{y}^{1}$ 

- Follow just two particles
- Different particles pass observation point at different times, with different speeds, & different image values
- Trajectories can cross, just can't have two particles in the same place at the same time

### Fluid Picture: Notation

- Fluid particles that move in time,
   carry the moving image information with them
- Particle that starts at  $\underline{y}$  at t=0 carries image information  $\mathcal{I}_1(y)$
- $\boxed{\phi(\underline{y},\mathbf{0}) = \underline{y}}$
- ullet When reaches a point  $\underline{x}$  at time  $\underline{t}$
- Compare  $\mathcal{I}_1(\underline{y})$  with value  $\mathcal{I}_2(\underline{x})$
- $|\phi(\underline{y},\mathbf{t}) = \underline{x}$

- Forces acting:
- Image comparison forces  $\mathcal{I}_1(y)$  vs  $\mathcal{I}_2(\underline{x})$
- Fluid regulariser:
- Viscous forces between moving particles

## Fluids & Particle Displacements

Define (backwards) displacement field:

Particle passing point  $\underline{x}$  at time t started from  $\underline{x} - \underline{u}(\underline{x}, \mathbf{t})$  at  $\mathbf{t} = \mathbf{0}$ 

Particle velocity as it passes this point [Reg2, slide 19]:

$$\underline{v}(\underline{x}, \mathbf{t}) = \frac{\partial \underline{u}(\underline{x}, \mathbf{t})}{\partial \mathbf{t}} + (\underline{v}(\underline{x}, \mathbf{t}) \cdot \overrightarrow{\nabla}) \underline{u}(\underline{x}, \mathbf{t})$$

Regulariser forces: in elastic case (no flow)

$$\underline{F}_{\text{elas}}(\underline{x}) = \mu \overrightarrow{\nabla}^2 \underline{u}(\underline{x}) + (\lambda + \mu) \overrightarrow{\nabla} (\overrightarrow{\nabla} \cdot \underline{u}(\underline{x}))$$

Fluids: similar, but Eulerian velocity rather than displacement:

$$F_{ ext{visc}}(\underline{x}, ext{t}) = \mu \overrightarrow{\nabla}^2 \underline{v}(\underline{x}, ext{t}) + (\lambda + \mu) \overrightarrow{\nabla} (\overrightarrow{\nabla} \cdot \underline{v}(\underline{x}, ext{t}))$$
massless & inertialess visco-elastic fluid

## Fluid Registration:

- Driving force:
  - **Body forces = Image difference forces**
- Fluid massless and inertialess:
  - Flows so that viscous forces counteract applied forces
- Penalises flow not displacement (which can be large)
- When reaches match, body forces vanish, flow stops

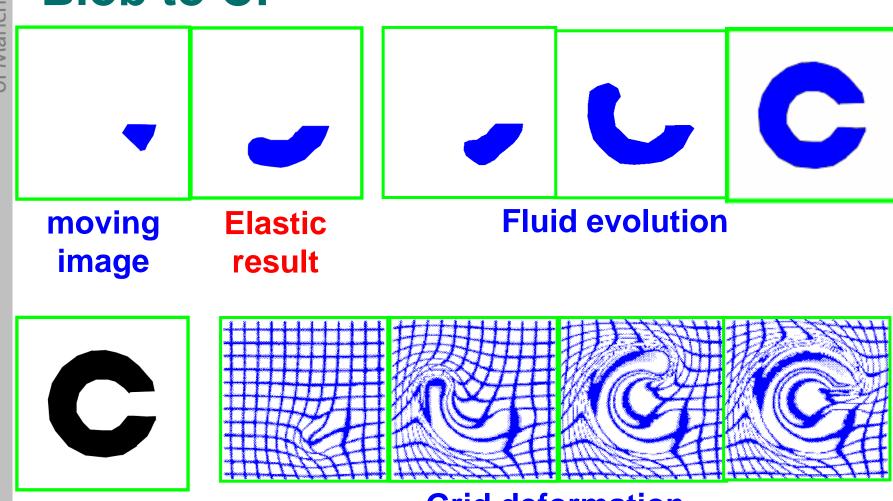
#### **Elastic Solution:**

Body forces balance displacement force

#### Fluid Solution:

Flows until body forces vanish

## Blob to C:



target

**Grid deformation Boundary is fixed** 

### Fluid: Solution

Christensen, Rabbitt, and Miller Deformable Templates Using Large Deformation Kinematics, IEEE Transactions on Image Processing, 5(10), pages 1435—1447, (1996).

- Start from zero displacement
- Evaluate image-match body forces
- ullet Viscous forces balance hence find flow fields  $\underline{v}(\underline{x},\mathbf{t})$
- Update displacements:  $\underline{u}(\underline{x}, \mathbf{t}) \mapsto \underline{u}(\underline{x}, \mathbf{t} + \Delta \mathbf{t})$
- Compute new body forces
- Repeat until body forces vanish

# Fluid Registration: Summary

- Like elastic, greater flexibility than parametric warps
- Like elastic, simplified and viscosity fixed across image
- Unlike elastic, can generate very large displacements
- As toy example shows, does so without tears or folds
- Computationally intensive, but suitable where considerable fine-scale matching, as in brains

### **Other Flow-Based Methods:**

 Large Deformation Diffeomorphic Metric Mapping (LDDMM)
 limited time flow

Trajectory:  $\phi(\underline{y}, \mathbf{t})$ , from  $\mathbf{t} = \mathbf{0}$  to  $\mathbf{t} = \mathbf{1}$ 

Velocity field:  $\underline{v}(\underline{x}, \mathbf{t})$  as before

Objective function:

$$\mathcal{E} = \underbrace{\mathcal{D}(\mathcal{I}_1, \mathcal{I}_2; \phi(\cdot, 1))}_{\text{image match at t=1}} + \underbrace{\lambda \mathcal{R}(\phi)}_{\text{trade-off warp term}}$$
 
$$\mathcal{R}(\phi) = \int\limits_0^1 ||\underline{v}(\cdot, t)||^2 \mathrm{d}t \quad \text{depends on whole of flow}$$
 
$$||\underline{v}(\cdot, t)||^2 = \int \mathrm{d}\underline{x} (\mathrm{L}\underline{v}(\underline{x}, t))^2 \text{norm on velocity fields}$$

Differential operator: 
$$\mathbf{L} = \left(-\alpha \overrightarrow{\nabla}^2 + \gamma\right)^{\beta}$$

## **Pairwise Registration:**

- Large number of methods of pairwise registration:
   Image similarity term
   Warp representation (parametric, non-parametric)
   Exact regulariser
   2D, 3D or time series
- Different methods for different tasks

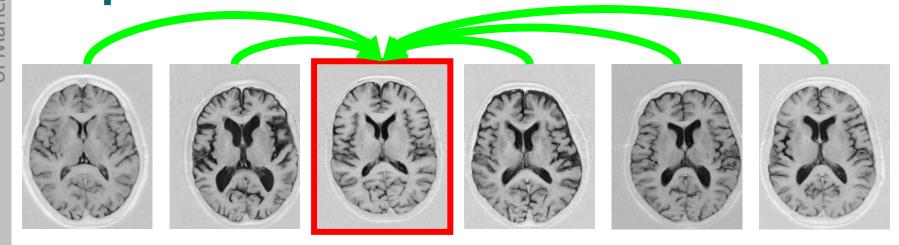
#### **BUT:**

- What if we want to register a whole population?
- Groupwise image registration?



# **Groupwise Image Registration**

# **Repeated Pairwise**



Large set of images

### **Repeated Pairwise:**

Chose a reference image, register all to this, 1-by-1

- Depends which reference you choose
- Doesn't use all available image information
- Group can help resolve ambiguities

## **Groupwise Registration:**

- Suppose we are warping whole set to common frame
- Image Matching:
- Pairwise:

Difference of two aligned images, pixel-by-pixel

- Groupwise:
  - Whole distribution of differences, pixel-by-pixel Some regions may be harder to match than others
- Use statistics of that distribution to evaluate groupwise image matching term
- Shows improvements over repeated pairwise

### Link between Registration & Modelling

### Image & Shape Models (AAM, ASM etc):

Dense groupwise correspondence for training set

### **Registration:**

Output is such a correspondence

### **Groupwise Registration:**

- Use quality of model to evaluate quality of registration
   Model built using bad registration generates images unlike actual training images
- Information-theoretic approach:
   Minimum Description Length (MDL): Rissanen

# Minimum Description Length (MDL):

### **Objective function for registration:**

- Build appearance model from correspondence
- Use model to encode all training images (i.e., generate them from the model)
- Transmit model as binary message more modes = longer message
- Transmit training set encoded using the model better the model fit to data, shorter the encoding
- Total length of message is objective function
- Simple to describe, harder to make work!

## MDL for Shapes & Images:

MDL Shape Models:

Davies, Twining & Taylor

Statistical Models of Shape: Optimisation & Evaluation

Springer, 2008 (electronic access via library)

MDL Appearance models & Groupwise registration:

Stephen Marsland, Carole J. Twining and Chris J. Taylor

A minimum description length objective function for groupwise non-rigid image registration

Image and Vision Computing, Volume 26(3), Pages 333-346, 2008

### **Recent Research Issue:**

- Lots of different methods of registration
- Many in routine use

#### Issue:

- Proper evaluation & comparison
   Ground-truth annotation difficult if not impossible
- Goal: evaluation without ground-truth
- Paper in progress at the moment.....

### In Conclusion:

Image Registration:

Intellectually-challenging area

Requires mathematical and computational sophistication

Uses:

Making sense of vast quantity of medical image data

Practical applications: image-guided surgery

**Research applications:** 

Many interesting questions that you couldn't attempt to answer without image registration