Local Features (part 2)

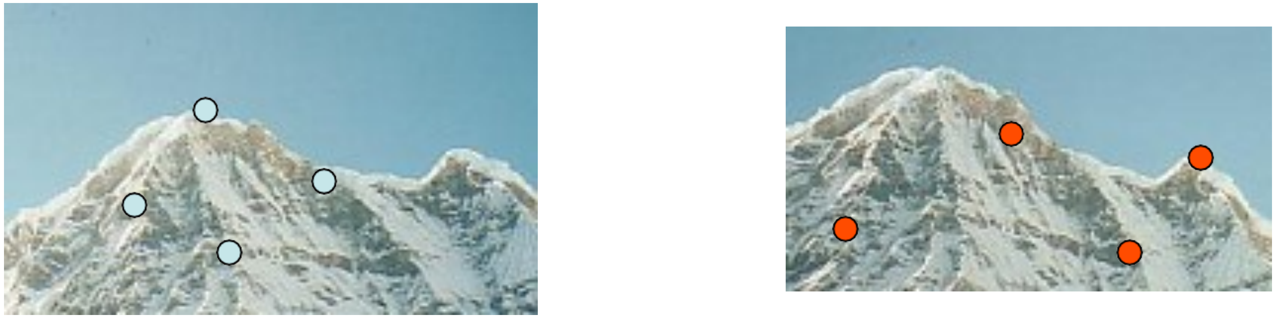
Topics of This Lecture

- **Local Invariant Descriptors (= Features) (part 1)**
  - Motivation
  - Requirements, Invariances
- **Local Interest Point Detection (Keypoint Localization)**
  - Harris detector **(part 2)**
- **Scale Invariant Region Selection (part 3)**
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector
- **Local Descriptors (part 4)**
  - Orientation normalization
  - SIFT

Common Requirements

- **Problem 1:**

  Detect the same point *independently* in both images



No chance to match!

We need a repeatable detector!

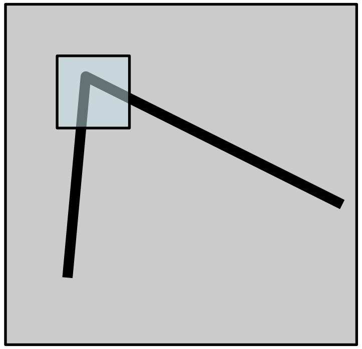
Motivation for Corners

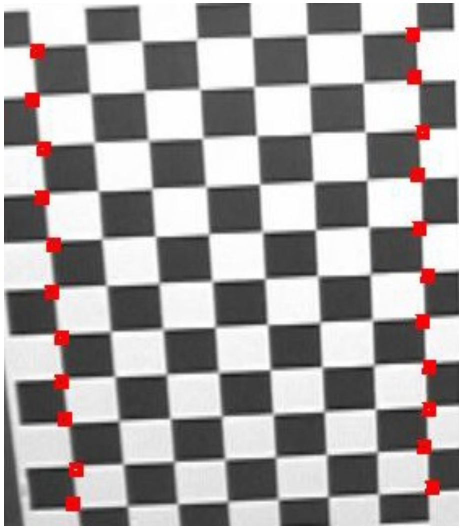
Edges only localise in one direction

Corners provide repeatable points for matching, so are worth detecting.

Idea:

In the region around a corner, image gradient has two or more dominant directions

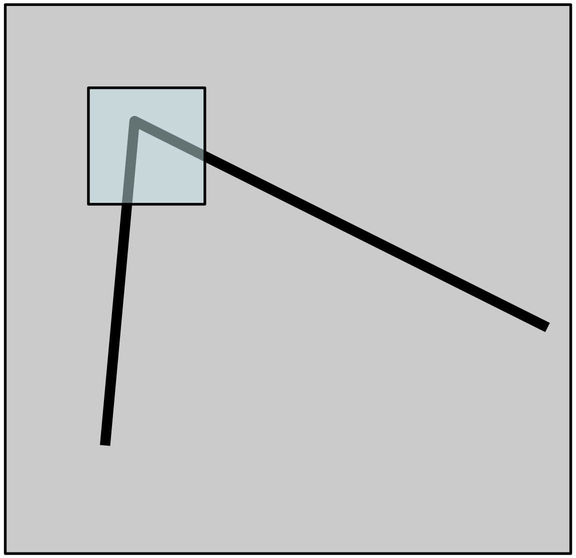




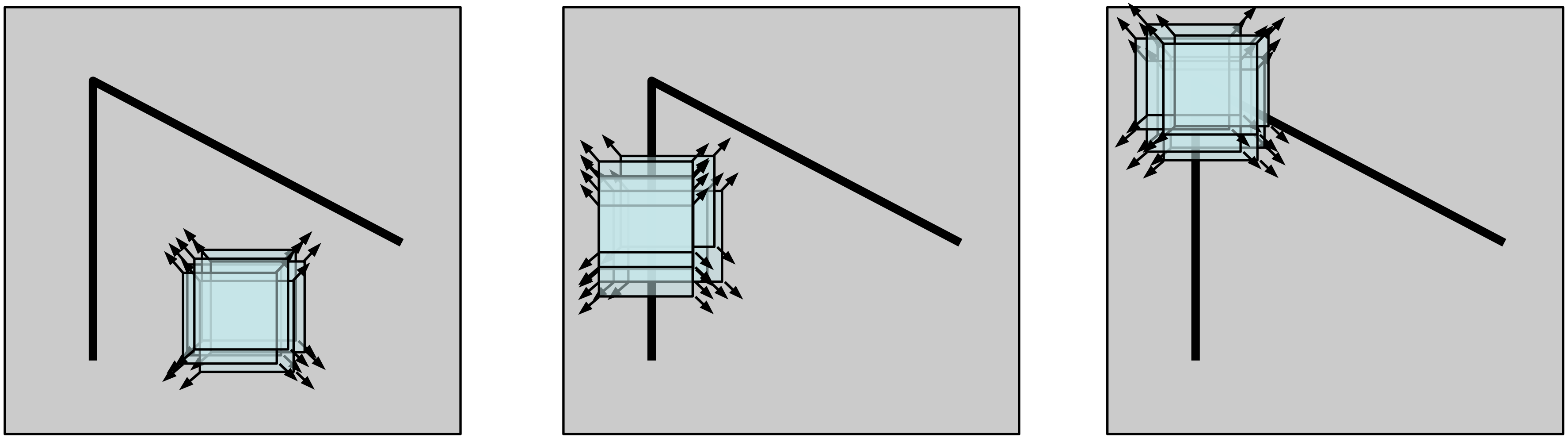
Harris Corner detector

Design criteria

- Shifting a window in *any direction* should give *a large change* in intensity



Corners as Distinctive Interest Points



"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

Harris Detector Formulation

- **Change of intensity for the shift [u,v]:**

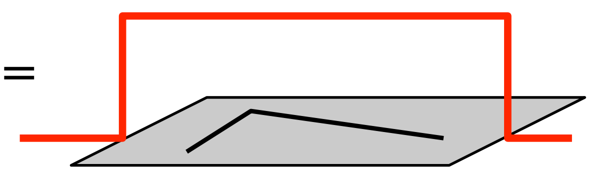
$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

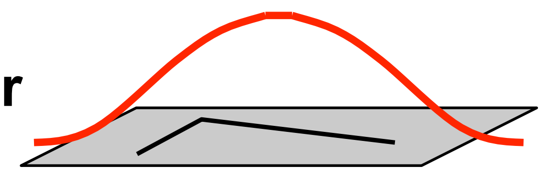
Shifted intensity

Intensity

Window function $w(x,y) =$



or



1 in window, 0 outside

Gaussian

Harris Detector Formulation

- For small shift, the measure of change can be approximated:

$$E(u,v) \cong [u,v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y \in w} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region - the area we are checking for corner

Harris Detector Formulation

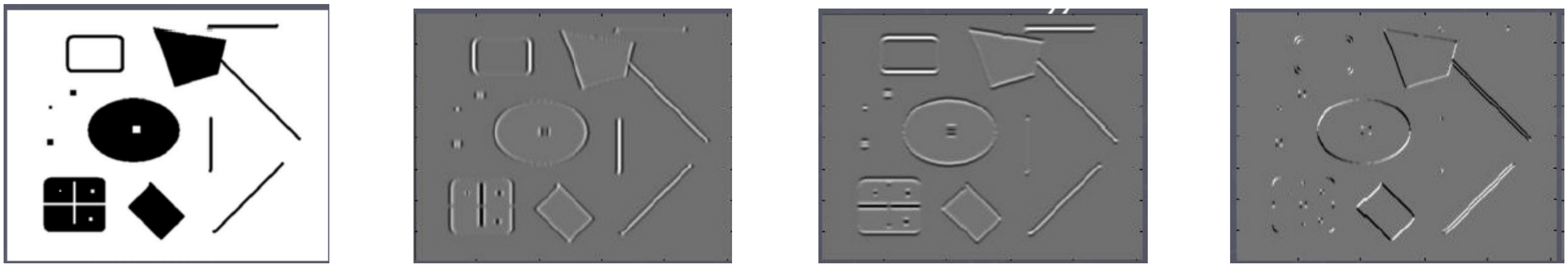


Image I | I_x | I_y | I_xI_y

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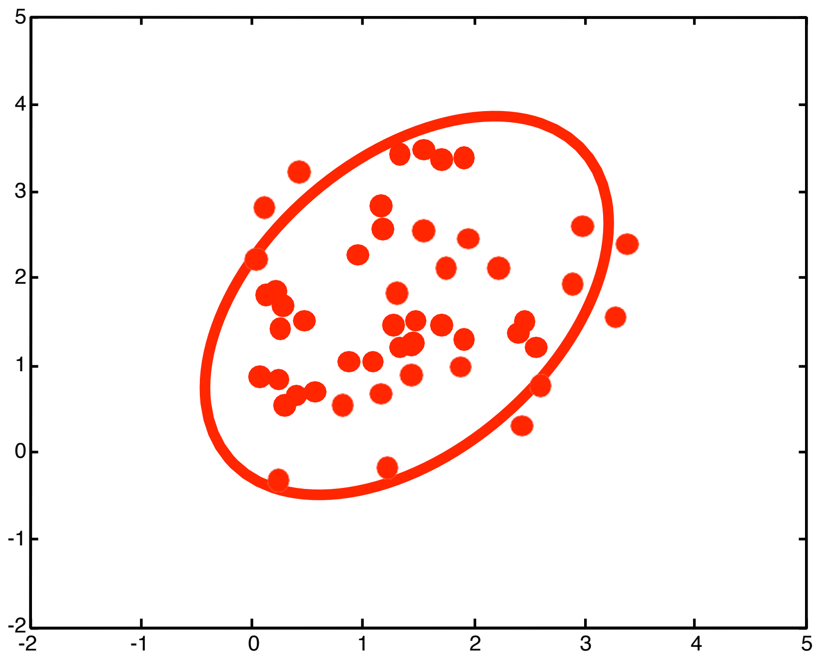
Gradient with respect to x, times gradient with respect to y

Sum over image region - the area we are checking for corner

Multivariate Gaussian model

$$\mathcal{N}(\underline{x}\ ;\ \underline{\mu}, \Sigma) = \frac{1}{(2\pi)^{d/2}} |\Sigma|^{-1/2} \exp\left\{ -\frac{1}{2} (\underline{x} - \underline{\mu})^T \Sigma^{-1} (\underline{x} - \underline{\mu}) \right\}$$

a symmetric “positive definite” covariance matrix Σ



Maximum Likelihood estimates

$$\hat{\mu} = \frac{1}{m} \sum_i x^{(i)}$$

1st moment of the data

$$\hat{\Sigma} = \frac{1}{m} \sum_i (x^{(i)} - \hat{\mu})^T (x^{(i)} - \hat{\mu})$$

Mean centered 2nd moment of the data

Harris Detector Formulation

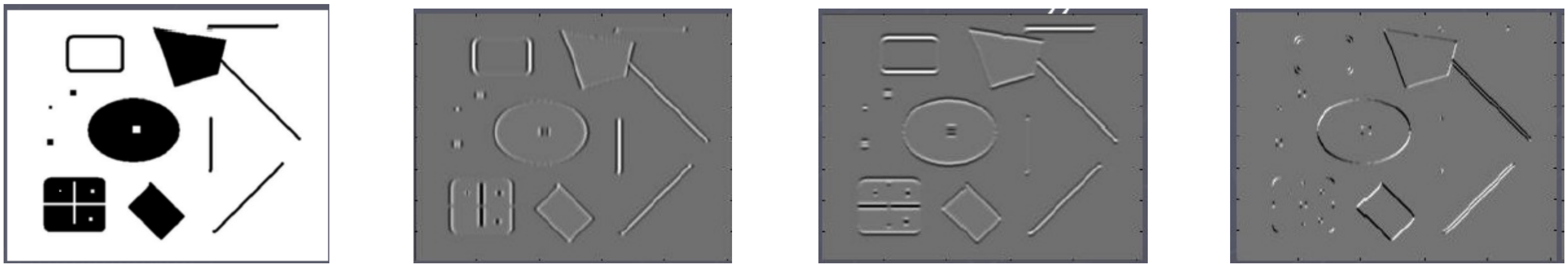


Image I | I_x | I_y | I_xI_y

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$$M = \sum_{x,y \in w} w(x,y) \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

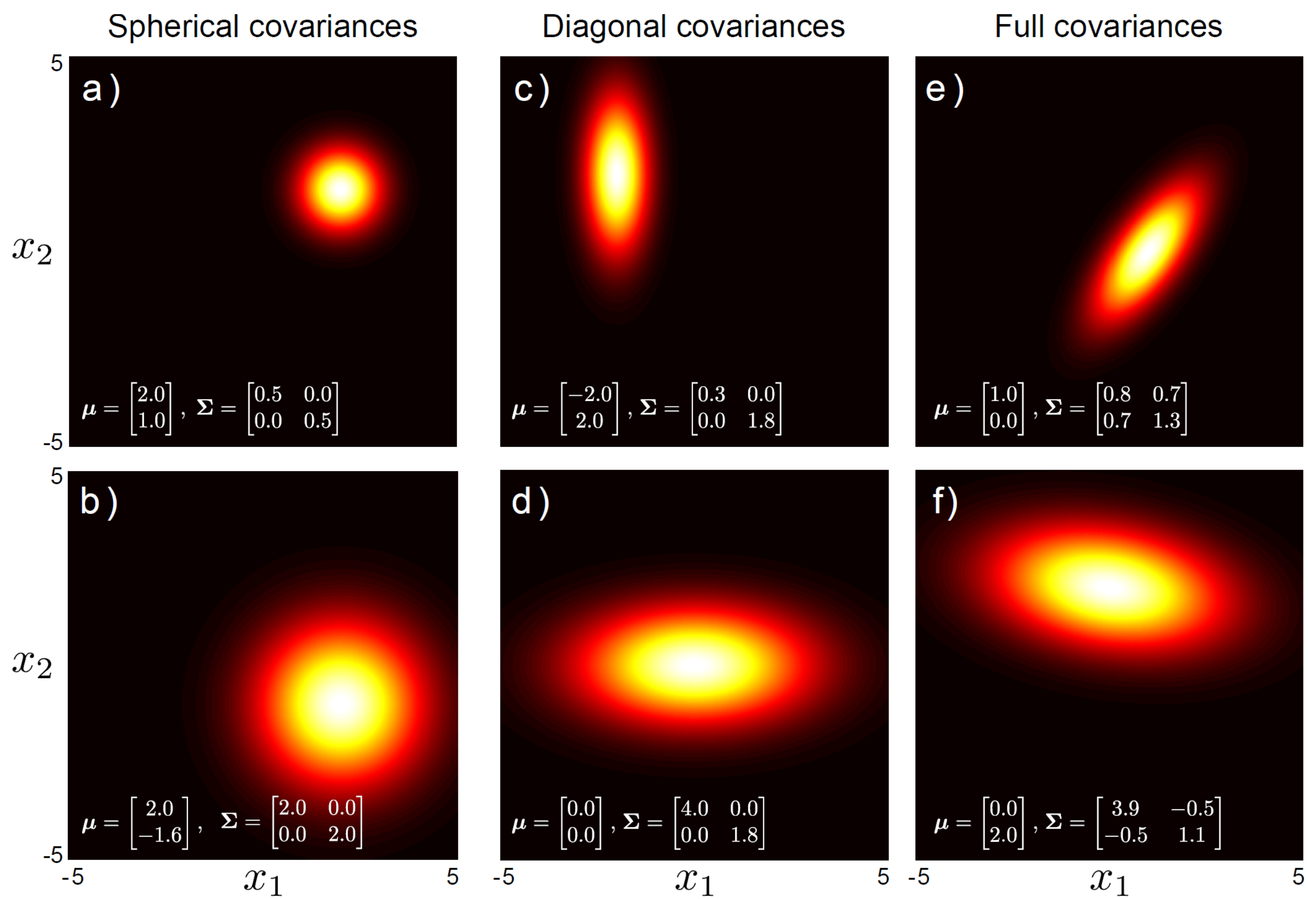
Sum over image region - the area we are checking for corner

Second Moment matrix

Types of covariance

Covariance matrix has three forms, termed spherical, diagonal and full

$$\boldsymbol{\Sigma}_{spher} = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \quad \boldsymbol{\Sigma}_{diag} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \quad \boldsymbol{\Sigma}_{full} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$



slide credit: Simon J.D. Prince

Statistical Interpretation of SVD

- Singular value decomposition

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \boldsymbol{u_1} & \boldsymbol{u_2} \end{array} \\ \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \end{array} \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}^T$$

$$= U \cdot D \cdot V^T$$

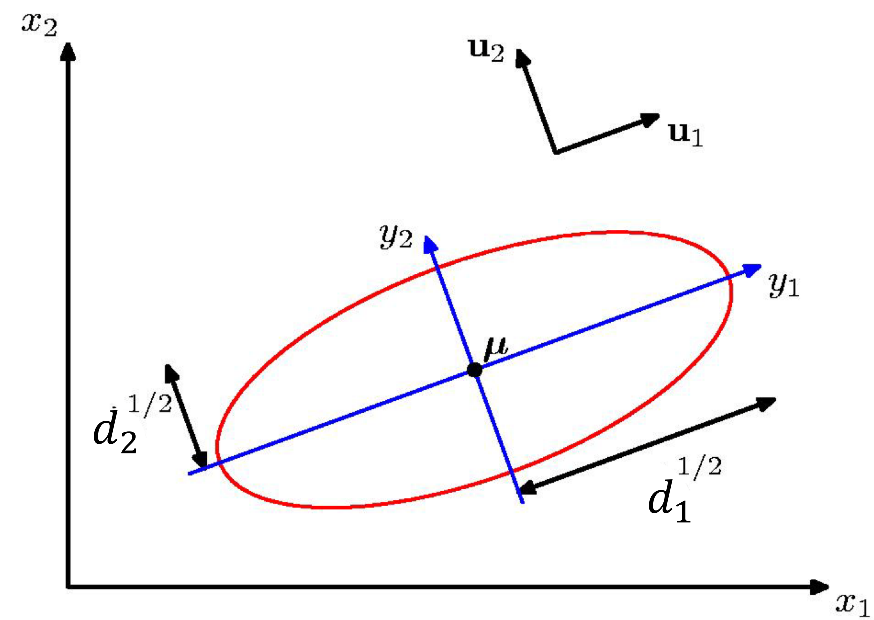
$$U \cdot U^T = V \cdot V^T = I$$

Here:

$A = U \cdot D \cdot U^T$

$$\begin{array}{lcc} & \boldsymbol{u_1} & \boldsymbol{u_2} \\ \textbf{eigenvectors} & \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} & \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \end{array}$$

eigenvalues $d_1, d_2 \geq 0$



A square 2×2 Matrix :

$tr(A) = \alpha_{11} + \alpha_{22}$

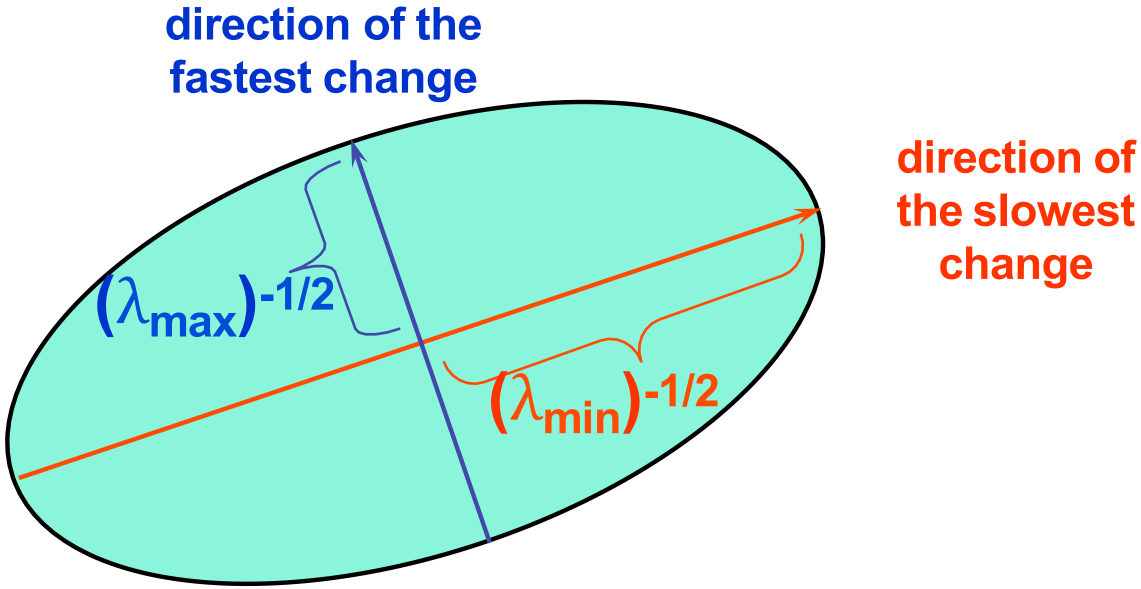
$\det(A) = \alpha_{11}\,\alpha_{22} - \alpha_{12}\,\alpha_{21}$

Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

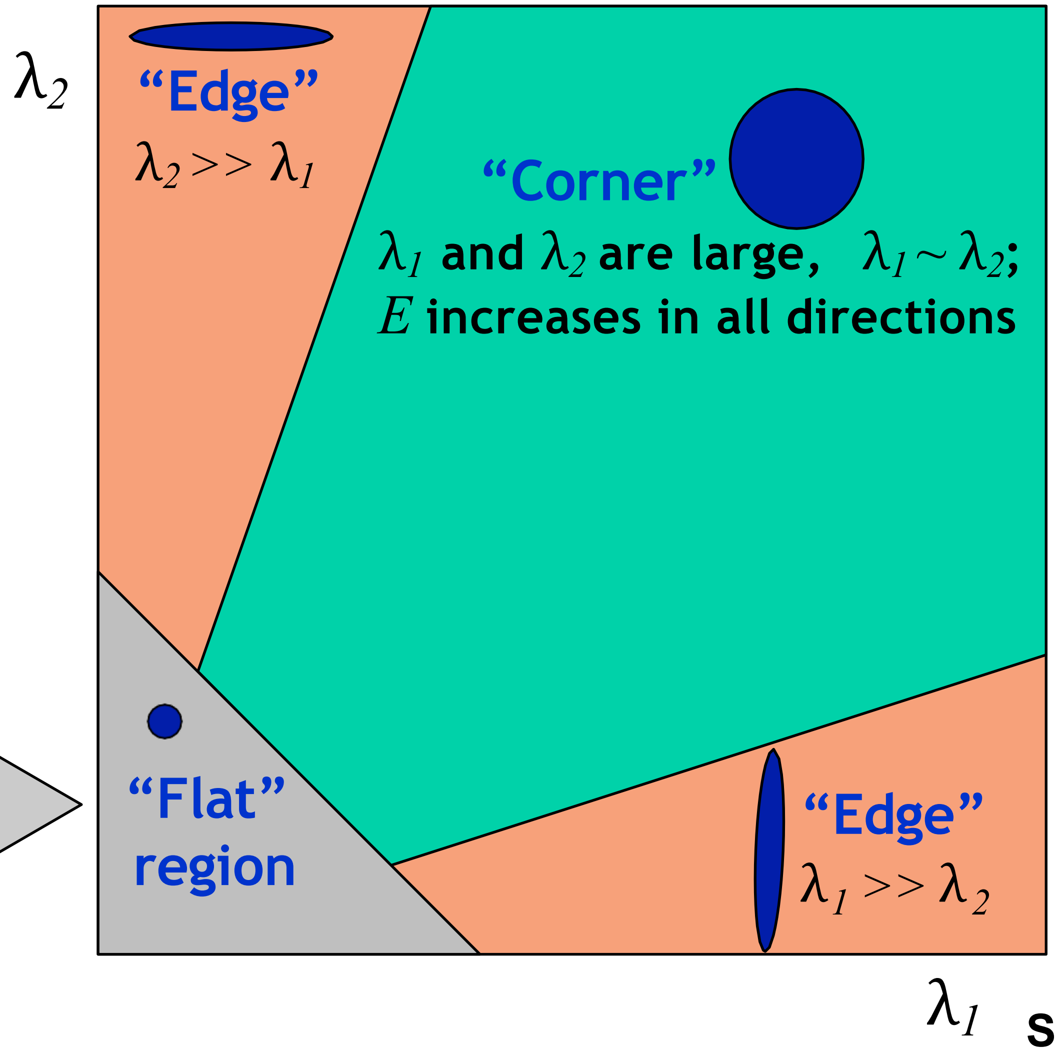
λ_1, λ_2 – eigenvalues of M



Interpreting the Eigenvalues

- **Classification of image points using eigenvalues of M:**

λ_1 and λ_2 are small;
E is almost constant
in all directions



Harris Detector: Mathematics

Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

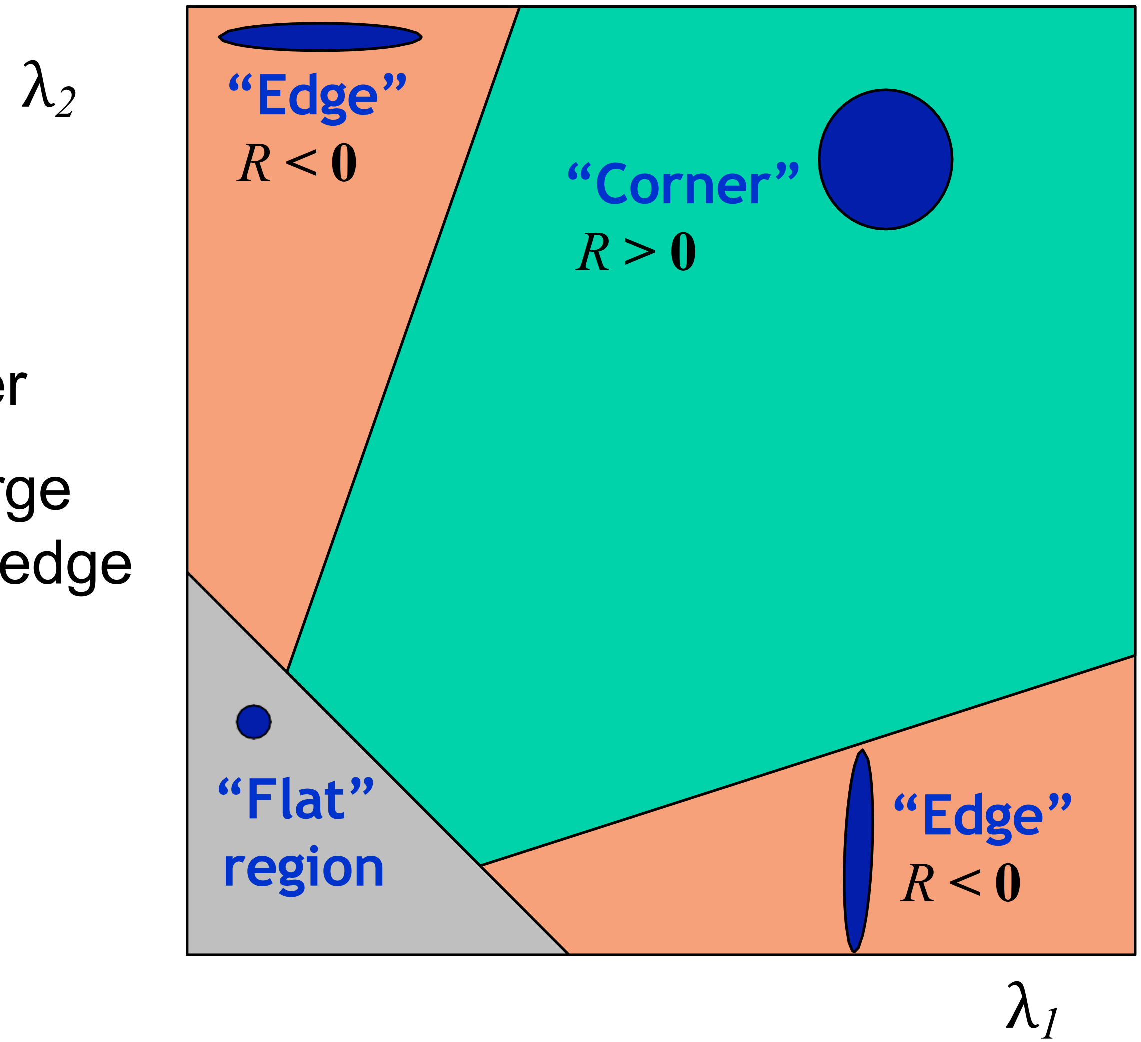
$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

Harris Detector: Mathematics

$$R = \det(M) - \alpha\, \mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

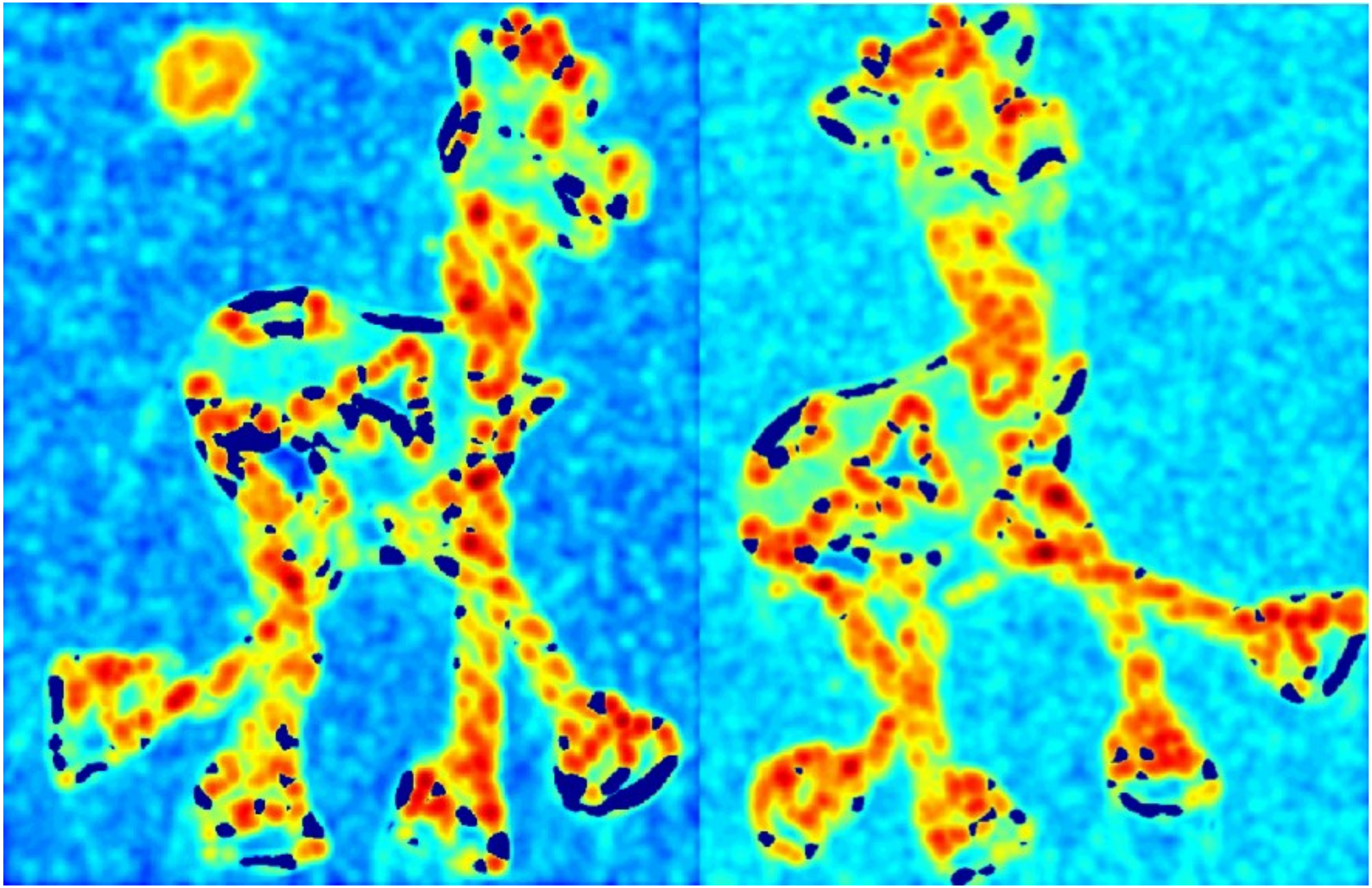
- *R* depends only on eigenvalues of M
- *R* is large for a corner
- *R* is negative with large magnitude for an edge
- |*R*| is small for a flat region

α: constant (0.04 to 0.06)



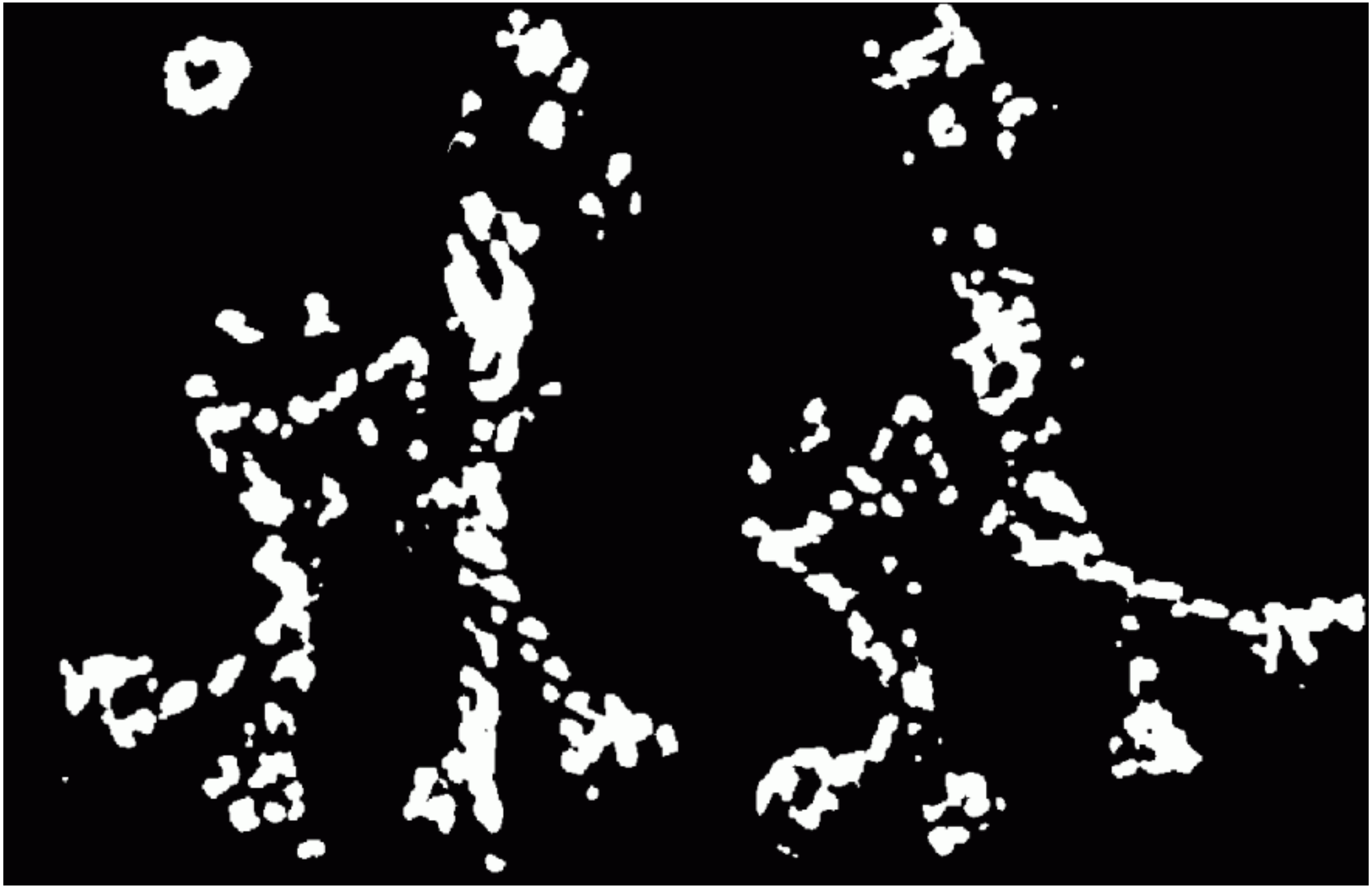
Harris Detector: Workflow

Harris Detector: Workflow



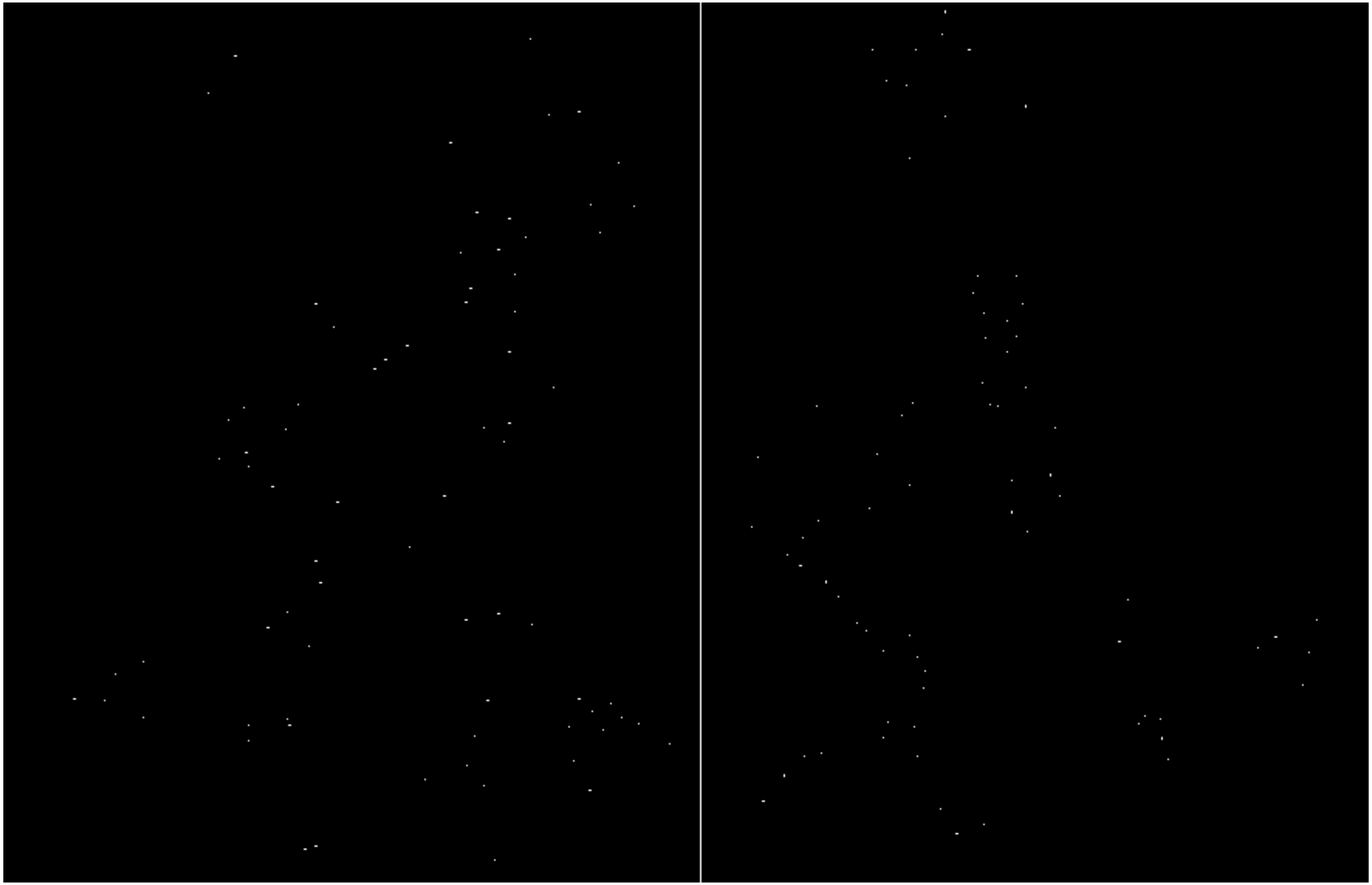
- **Compute corner responses *R***

Harris Detector: Workflow



- **Find points with large corner response: *R* > threshold**

Harris Detector: Workflow



- **Take only the local maxima of *R***

Harris Detector: Workflow

- **Resulting Harris points**

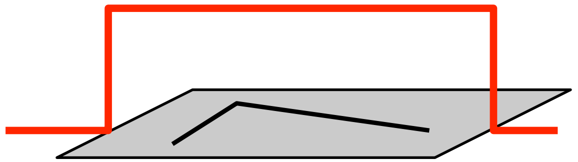
Window Function $w(x,y)$

$$M = \sum_{x,y \in w} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

- Sum over square window

$$M = \sum_{x,y \in w} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



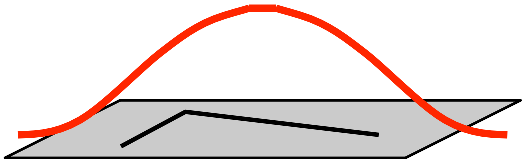
1 in window, 0 outside

- Problem: not rotation invariant

Option 2: Smooth with Gaussian

- Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

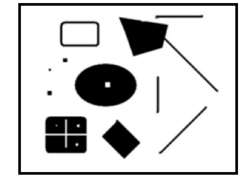


Gaussian

- Result is rotation invariant

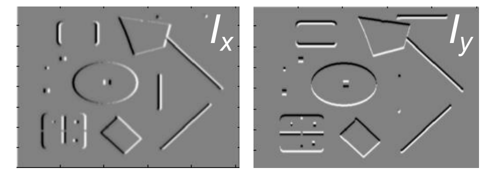
Harris Detector (Fast approximation)

- Compute Second moment matrix

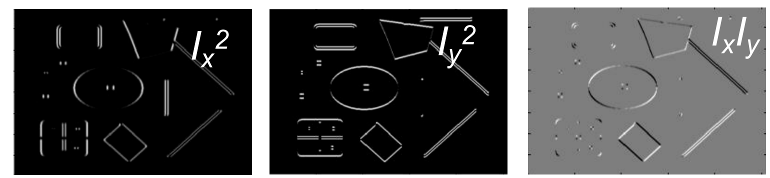


$$M(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives
(optionally, blur first)



2. Square of derivatives



3. Gaussian filter *g(s$_I$)*

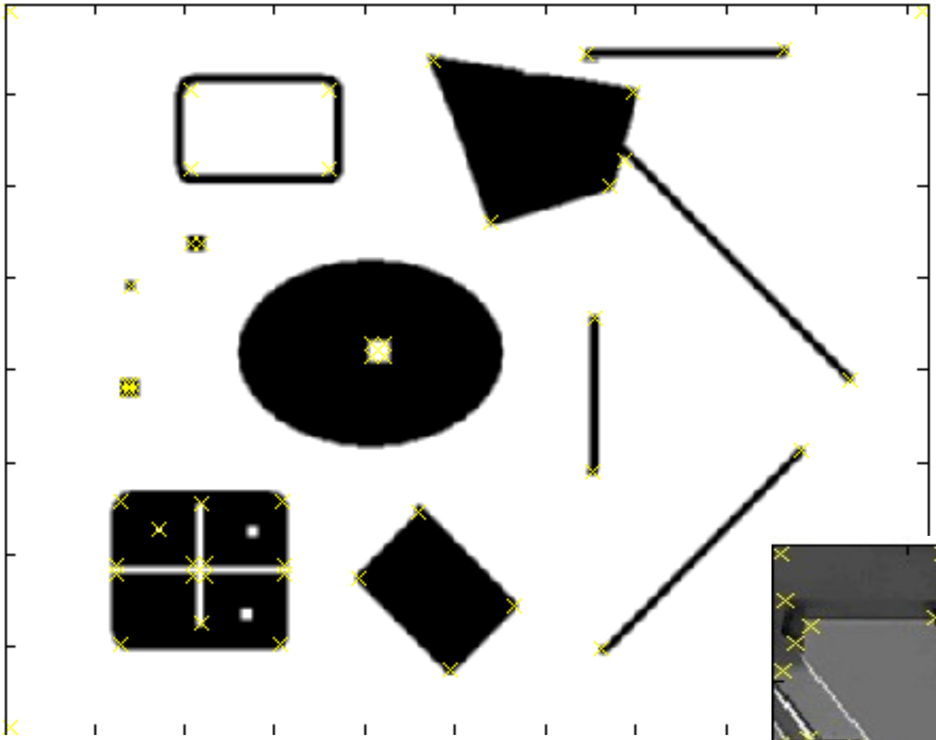
4. Cornerness function – both eigenvalues are strong

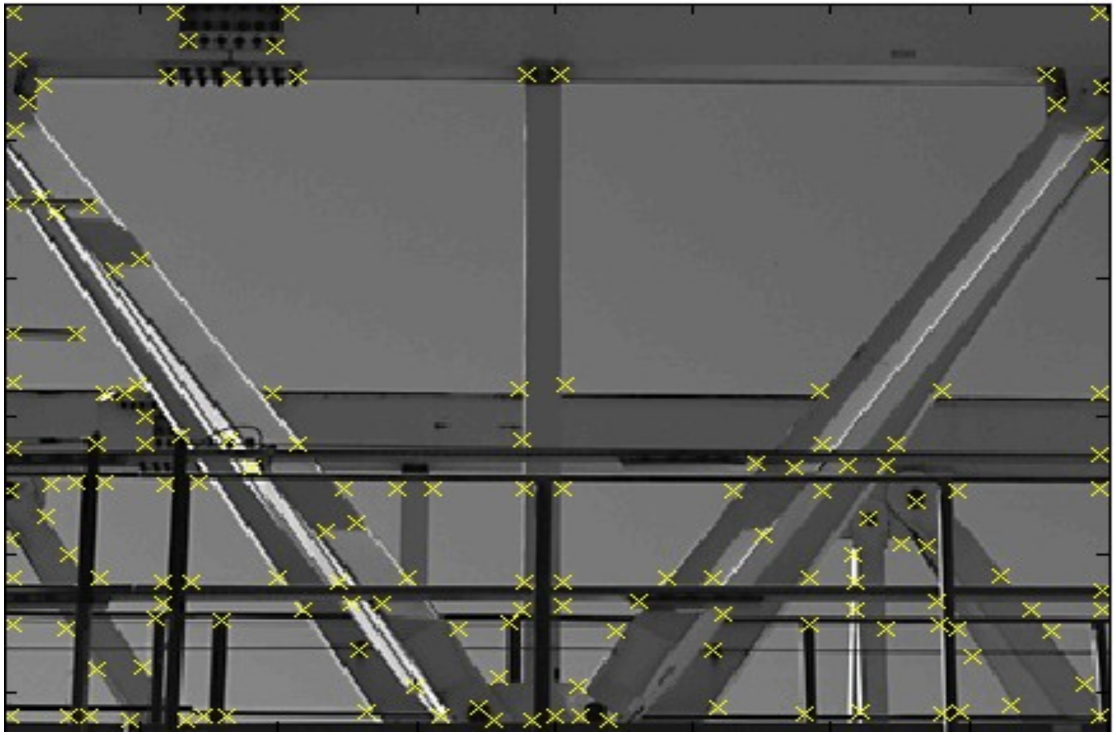
$$R = \det[M(\sigma_I,\sigma_D)] - \alpha[\mathrm{trace}(M(\sigma_I,\sigma_D))]$$
$$= g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2)+g(I_y^2)]^2$$

5. Non-maxima suppression

Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]

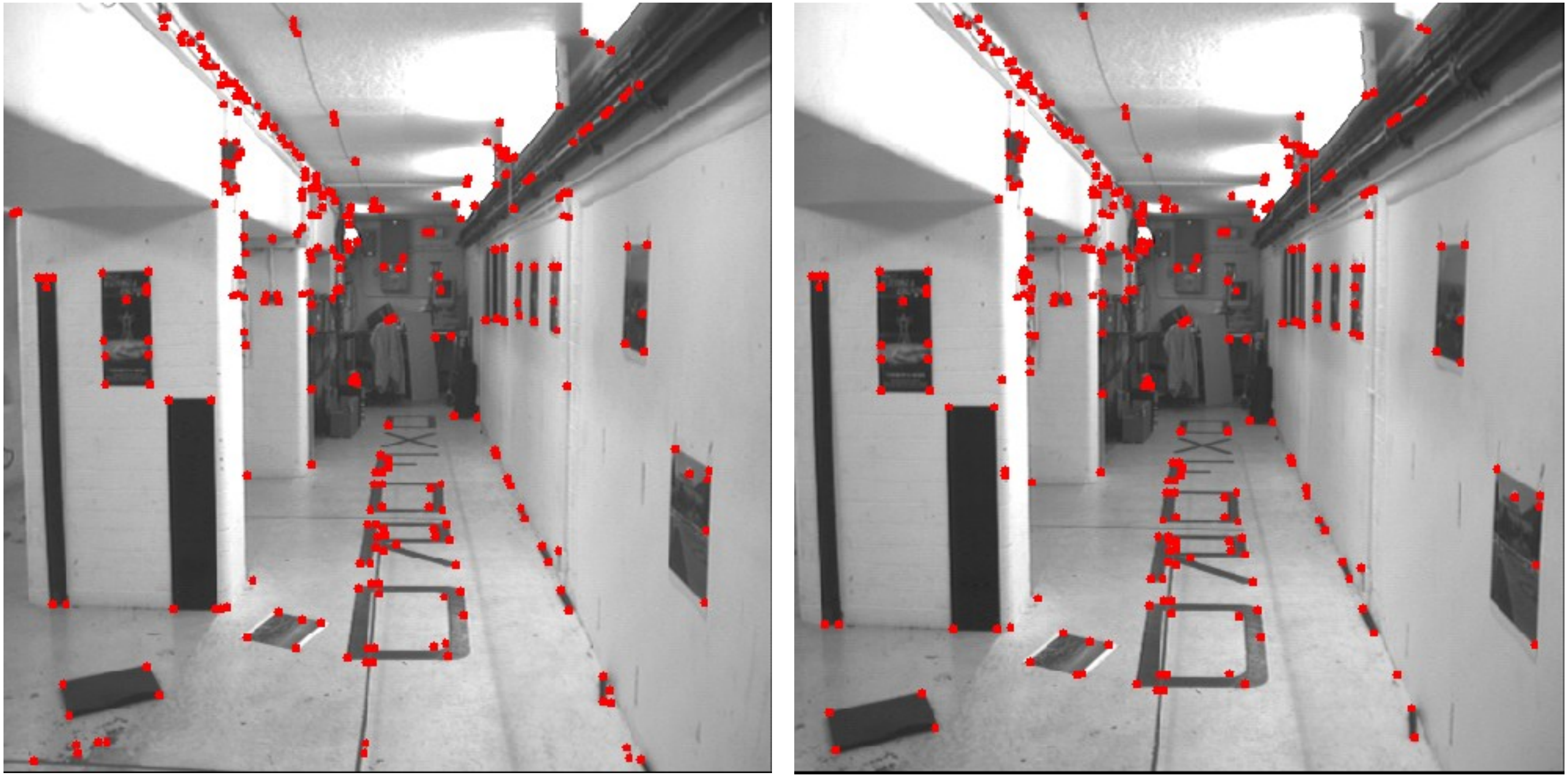




Effect: A very precise corner detector.

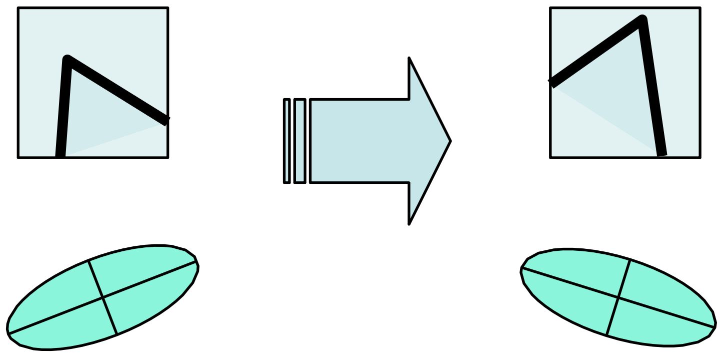
image credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]



Harris Detector: Properties

- **Rotation invariance?**

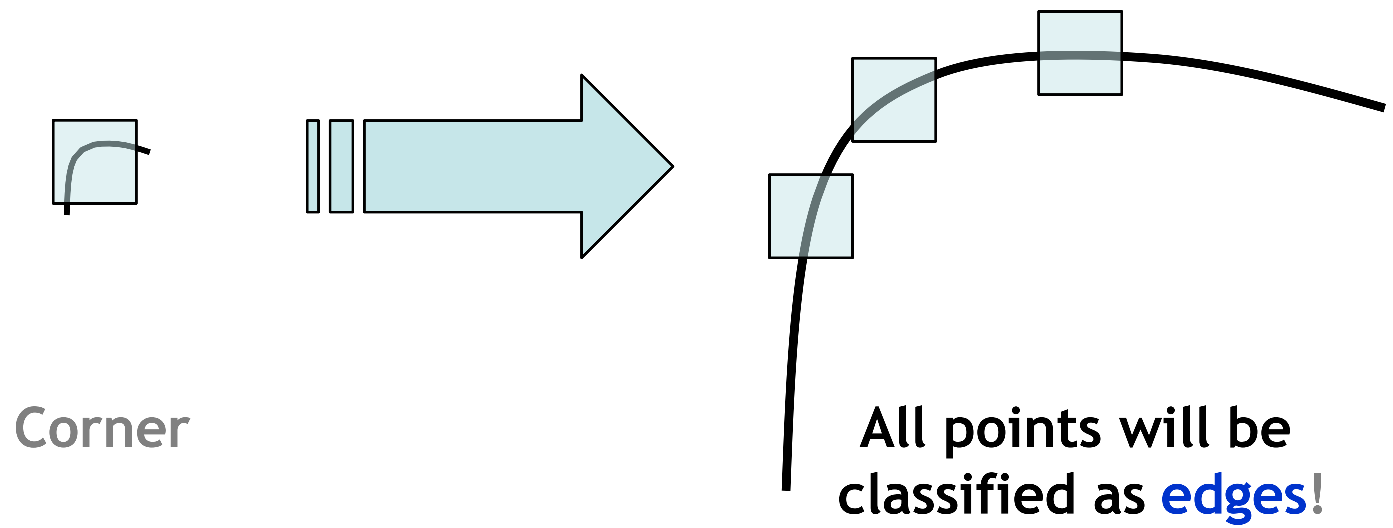


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Harris Detector: Properties

- Rotation invariance
- Scale invariance?



Not invariant to image scale!

Next Lecture

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