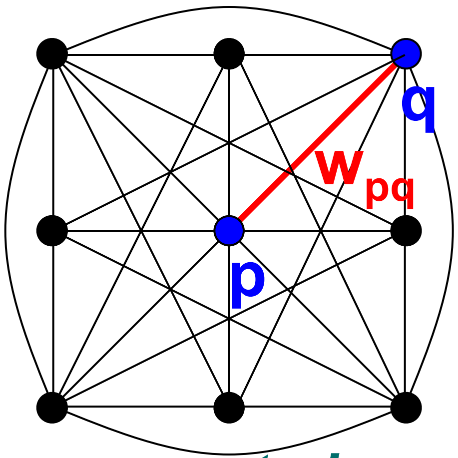
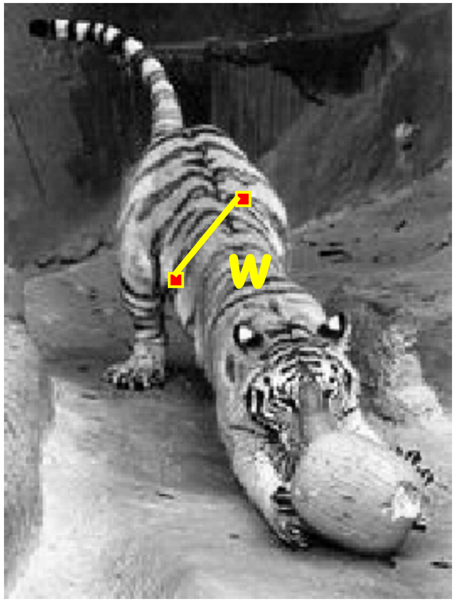
Segmentation and Clustering (part 4)

This Lecture

- **Segmentation and grouping**
  - Gestalt principles
  - Image segmentation
- **Segmentation as clustering**
  - k-Means
  - Feature spaces
- **Probabilistic clustering**
  - Mixture of Gaussians, EM
- **Model-free clustering**
  - Mean-Shift clustering
- **Graph theoretic segmentation**
  - Normalised cuts

Images as Graphs





- ***Fully-connected*** **graph**
  - Node (vertex) for every pixel
  - Link between *every* pair of pixels, **(p,q)**
  - Affinity weight (cost) w_{pq} for each link (edge)
    - w_{pq} measures similarity
    - Similarity is *inversely proportional* to difference (in color and position…)

Slide credit: Steve Seitz

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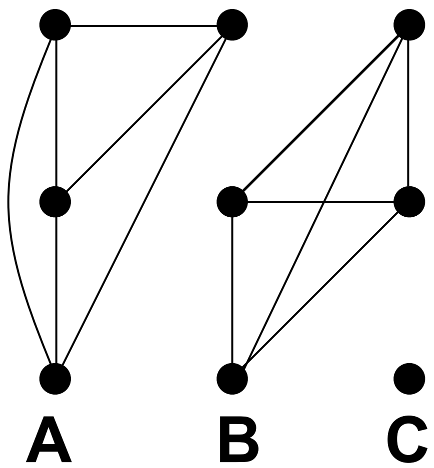
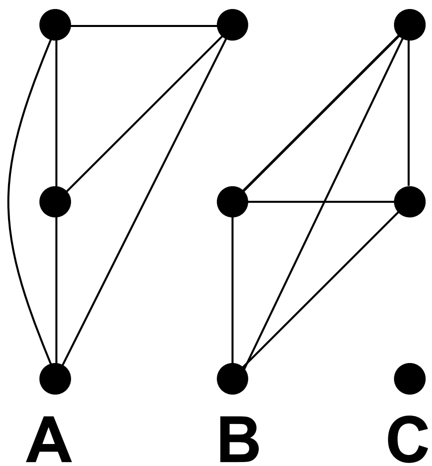
Segmentation by Graph Cuts



- **Break Graph into Segments**
  - Delete links that cross between segments

Slide credit: Steve Seitz

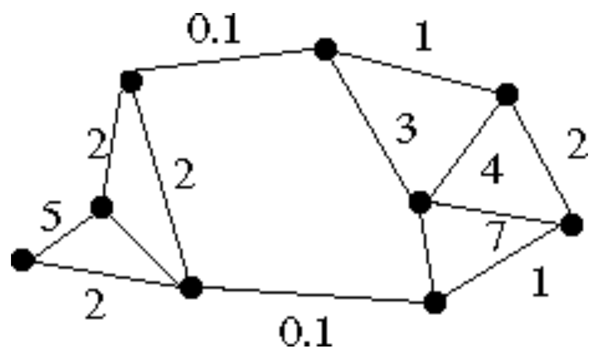
Segmentation by Graph Cuts

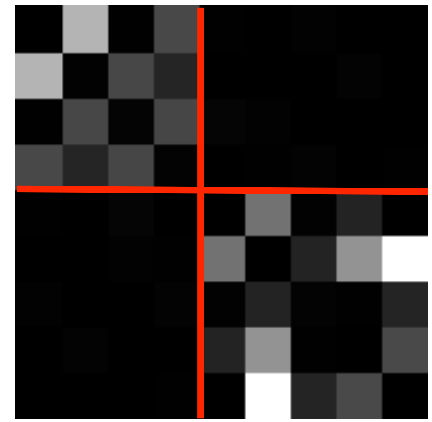


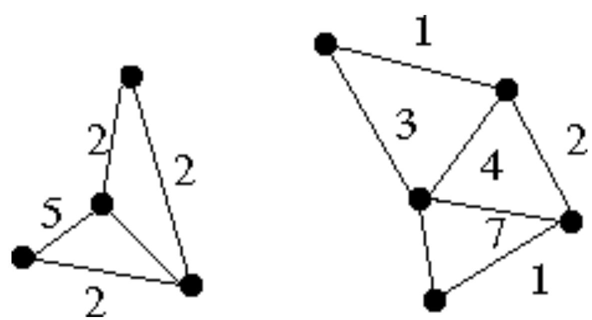
- **Break Graph into Segments**
  - Delete links that cross between segments
  - Easiest to break links that have low similarity (low weight)
    - Similar pixels should be in the same segments
    - Dissimilar pixels should be in different segments

Slide credit: Steve Seitz

Graph Cut







Here, the cut is nicely defined by the block-diagonal structure of the affinity matrix.

Image Source: Forsyth & Ponce

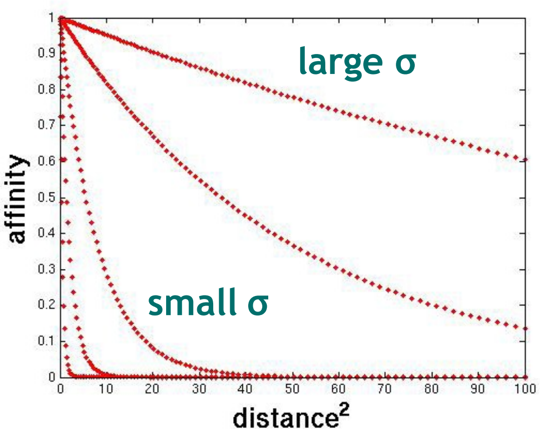
Measuring Affinity

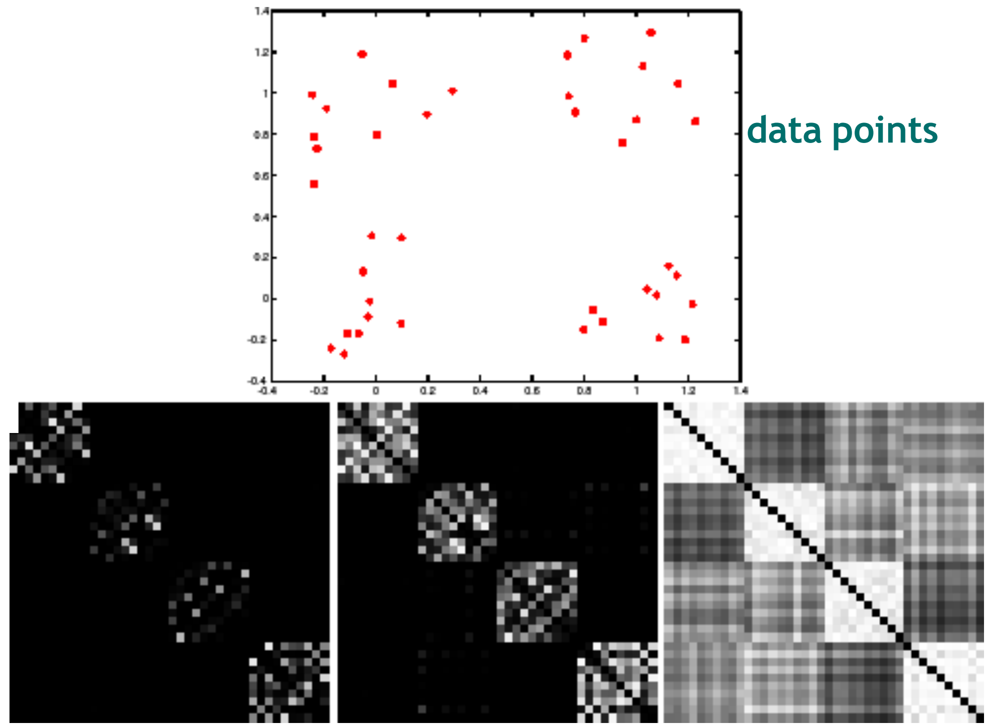
- Distance $$aff(x,y) = \exp\left\{-\frac{1}{2\sigma_d^2}\left\|x-y\right\|^2\right\}$$
- Intensity $$aff(x,y) = \exp\left\{-\frac{1}{2\sigma_d^2}\left\|I(x)-I(y)\right\|^2\right\}$$
- Color $$aff(x,y) = \exp\left\{-\frac{1}{2\sigma_d^2}\underbrace{dist\left(c(x),c(y)\right)}_{\text{(some suitable color space distance)}}{}^2\right\}$$

Source: Forsyth & Ponce

Scale Affects Affinity

- Small σ: group only nearby points
- Large σ: group far-away points

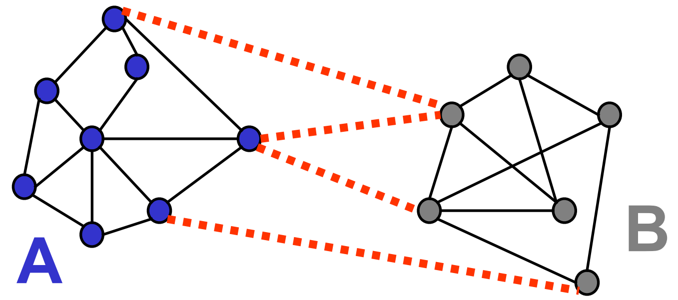




Small σ   Medium σ   Large σ

Image Source: Forsyth & Ponce

Cuts in a graph



- **Graph Cut**
  - Set of edges whose removal makes a graph disconnected
  - Cost of a cut
    - Sum of weights of cut edges: $cut(A,B) = \sum_{p \in A, q \in B} w_{p,q}$

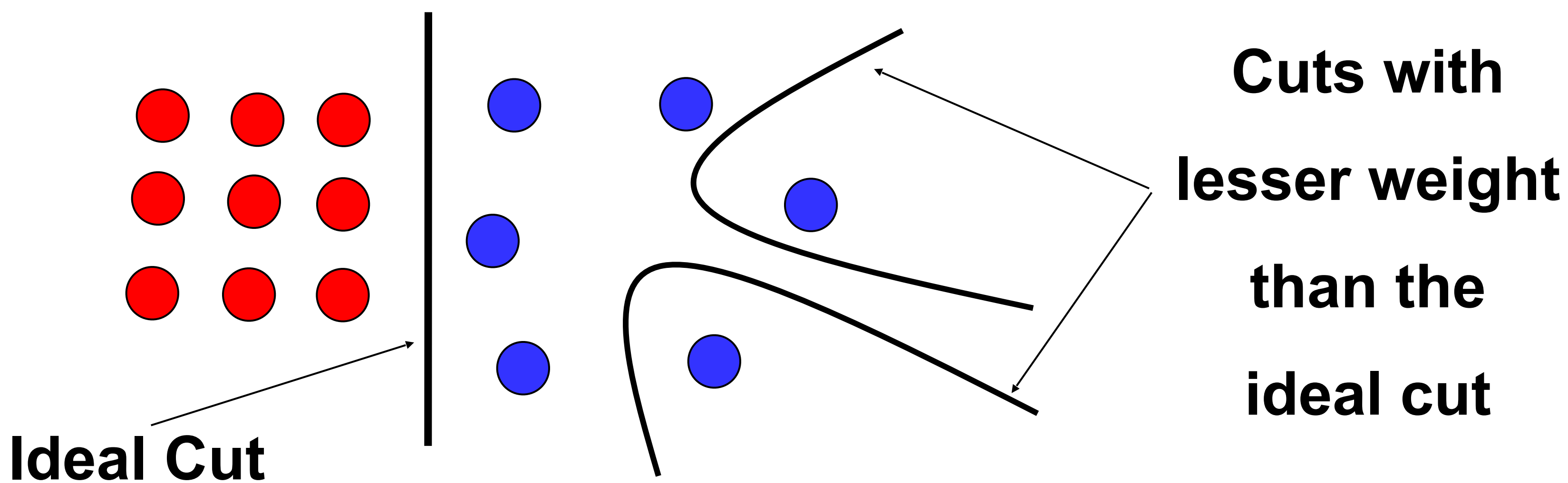
- **A graph cut gives us a segmentation**
  - What is a “good” graph cut and how do we find one?

Source: Steve Seitz

Minimum Cut

Minimum Cut

- **We can do segmentation by finding the *minimum cut* in a graph**
  - Efficient algorithms exist for doing this
- **But minimum cut is not always the best cut …**
  - Weight of cut proportional to number of edges in the cut
  - Minimum cut tends to cut off very small, isolated components



Normalized Cut (NCut)

- A minimum cut penalizes large segments
- This can be fixed by normalizing for size of segments
- The **normalised cut** cost is:

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

Where:

$\boldsymbol{assoc}(\boldsymbol{A},\boldsymbol{A}) = \sum_{p \in A, q \in A} \boldsymbol{w}_{p,q}$ is the *association* (sum of all weights) within a cluster

$\boldsymbol{assoc}(\boldsymbol{A},\boldsymbol{V}) = \boldsymbol{assoc}(\boldsymbol{A},\boldsymbol{A}) + \boldsymbol{cut}(\boldsymbol{A},\boldsymbol{B})$ is the sum of all the weights associated with nodes in *A*

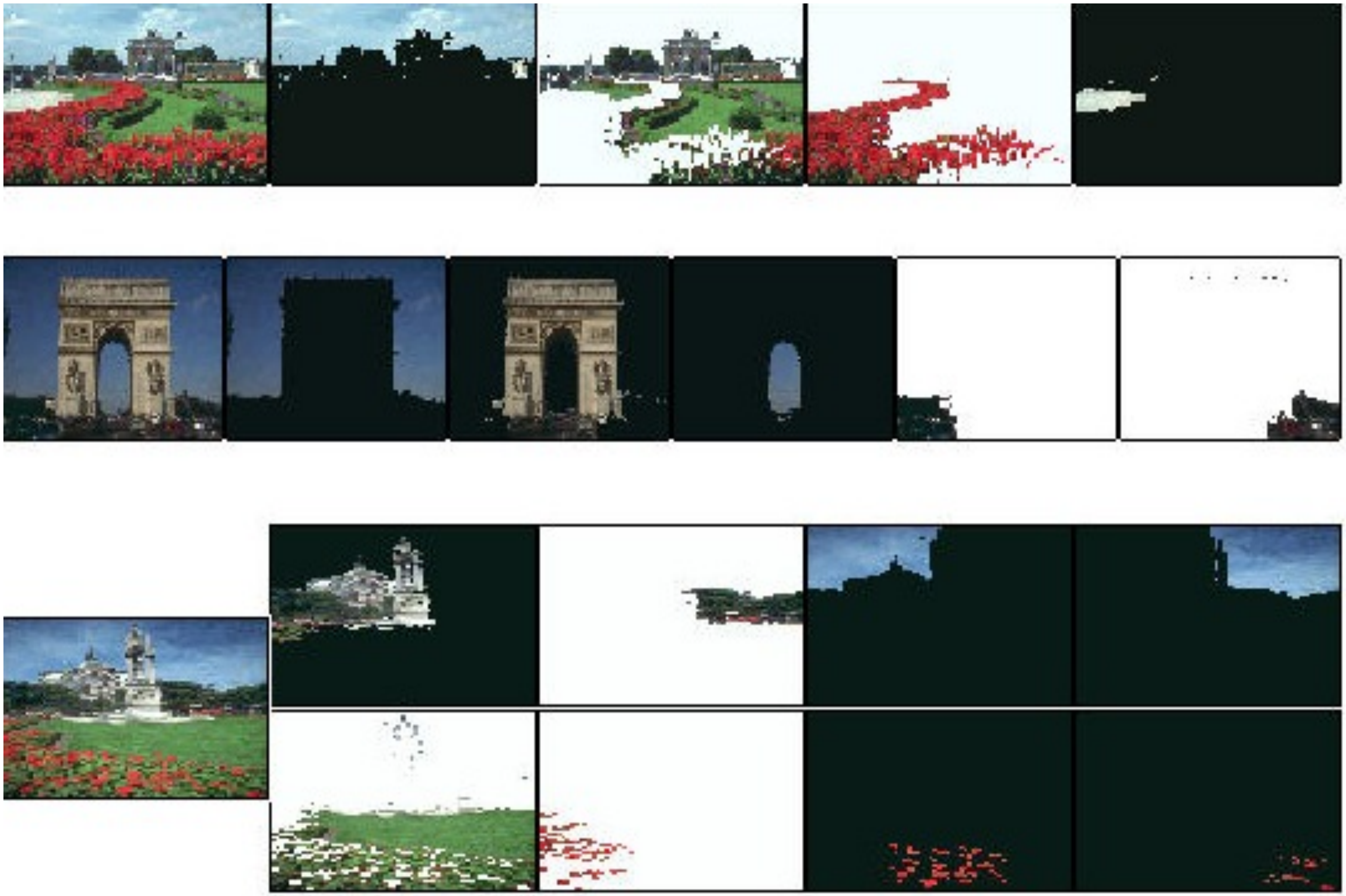
Intuition: Big segments will have a large *assoc(A,V)*, thus decreasing *Ncut(A, B)*

- Finding the globally optimal cut is NP-complete,
  but a relaxed version can be solved using *a generalized eigenvalue* problem

For details, see

J. Shi and J. Malik. Normalized cuts and image segmentation. PAMI 2000

Color Image Segmentation with NCuts



NCuts Matlab code available at http://www.cis.upenn.edu/~jshi/software/

Image Source: Shi & Malik

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Summary: Normalized Cuts

- **Pros:**
  - Generic framework, flexible to choice of function that computes weights ("affinities") between nodes
  - Does not require any model of the data distribution

- **Cons:**

Summary: Normalized Cuts

- **Pros:**
  - Generic framework, flexible to choice of function that computes weights (“affinities”) between nodes
  - Does not require any model of the data distribution

- **Cons:**
  - Time and memory complexity can be high
    - Dense, highly connected graphs $\Rightarrow$ many affinity computations
    - Solving eigenvalue problem

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Graph-cuts and GrabCut

GrabCut

Only user input is the box!

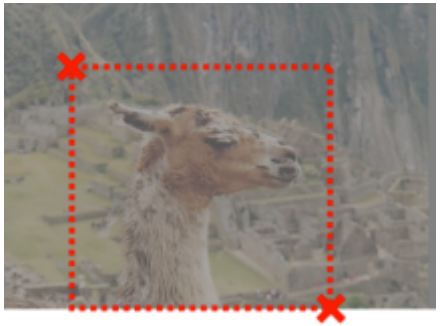
grab **cut paste**

Rother et al., "Interactive Foreground Extraction with Iterated Graph Cuts," SIGGRAPH 2004

Combining region and boundary information

GrabCut

user input

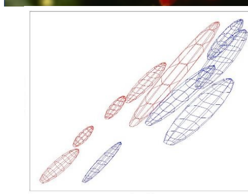


result

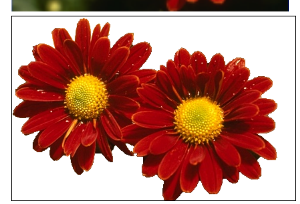
regions &
boundary

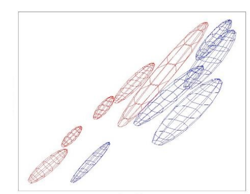
GrabCut : iterate between two steps

1. **Segmentation using graph cuts**
   - Requires having foreground model



2. **Foreground-background modelling using unsupervised clustering**
   - Requires having segmentation





GrabCut: Example Results

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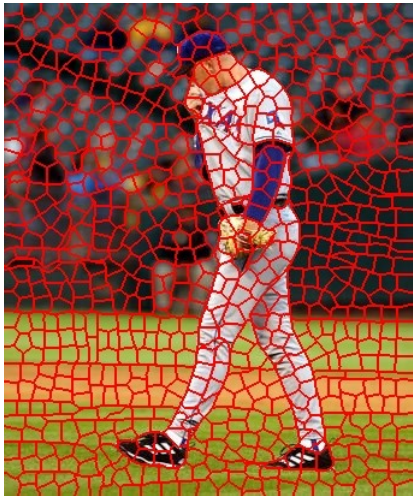
Improving Efficiency of Segmentation

- **Problem: Images contain many pixels**
  - Even with efficient graph cuts
- **Efficiency trick: Superpixels**
  - Group together similar-looking pixels for efficiency of further processing.
  - Cheap, local over-segmentation
- **Several different approaches possible but Important to ensure that superpixels**
  - Do not cross boundaries
  - Have similar size
  - Have regular shape
  - Algorithm has low complexity
- **Superpixel code available here**

  http://www.cs.sfu.ca/~mori/research/superpixels/



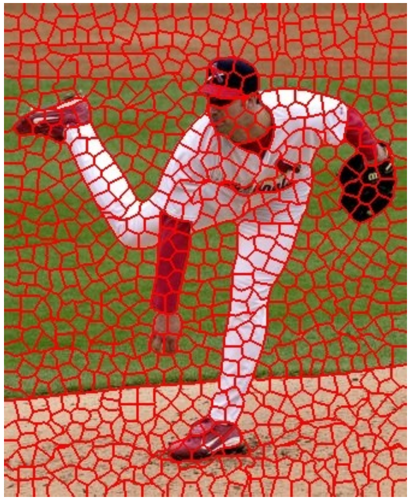


Image source: Greg Mori

Last but not least …

- **How to evaluate segmentation?**

$$F = 2PR/(P+R)$$

- <u>Precision ***P***</u>: the percentage of the marked boundary points that are real ones
- <u>Recall ***R***</u>: the percentage of real boundary points that were marked

Reading

- ***Forsyth, Ponce:*** **Computer Vision: A Modern Approach: A Modern Approach.**
- ***J. Shi and J. Malik*****,** **Normalized cuts and image segmentation. PAMI 2000**
- ***Y. Boykov and M. Jolly*****, Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D Images, ICCV 2001.**
- ***Rother et al.*****, "Interactive Foreground Extraction with Iterated Graph Cuts," SIGGRAPH 2004.**