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Data Structures and Algorithms

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Session: Natural Numbers

Natural Numbers

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- Natural Numbers is the most frequently occurring data type
- Used for counting
  - Money in a bank
  - Students in a course
- Used for denoting other objects
  - Telephone numbers used to identify users
  - IP address used to identify computers
  - Room numbers used to identify rooms
- Many different operations defined for numbers
  - Arithmetic, comparison

What are Natural Numbers?

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- Informally, we call *0, 1, 2, 3, …..*   as natural numbers
- To be precise, we need to define them mathematically
- Formal definitions avoid any ambiguity
- Give a clear specification to implementers
- Ensure that programs are correct
- Formally prove properties of numbers
- These can be used to write better programs

Peano Axioms

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The set N of natural numbers satisfies the following:

1. $0 \in N$
2. If $n \in N$ then there exists a number $next(n) \in N$
3. If X is any set that satisfies 1. and 2. then $N \subseteq X$

- Here *next(n)* is an operation on numbers
- Informally it denotes the number $n+1$
- All properties of numbers follow from these definitions

Meaning of Peano Axioms

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- Specify that *0* is a natural number
- If *n* is a natural number then *next(n)* is a natural number
- All natural numbers can be obtained from *0* using the *next* operation
- *0, next(0), next(next(0)),* … are the natural numbers
- Denote *next(0)* by 1, *next(next(0))* by 2, etc.

Operations on numbers

- Given only the *next* operation
- All other operations defined using it
- To define an operation
  - Define it for *0*
  - Assuming it is defined for number *n* define it for *next(n)*
  - This defines it for all natural numbers

Recursive definition

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- Numbers are the simplest example of a recursive definition
- Define an initial value
  - *0* in case of numbers
- Given a value, define operations to construct new values
  - *next* operation in the case of numbers
- Define properties that these operations must satisfy
- Almost all definitions in computer science are recursive

Mathematical Induction

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- Essentially, Peano axioms say that mathematical induction can be used to define operations and prove properties of numbers
- Do it for the base case (*0*)
- Assuming it is done for a value (*n*), do it for all values that can be constructed from it (*next(n)*)
- This general method is used for all data types

Exercise

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- Extend the definition of natural numbers to all integers
  - To do this, you need to generate new values
  - Define an operation *minus*
  - *minus(0)* must be *0*
  - There is a relation between the *next* and *minus* operations that must be specified as a property to be satisfied