PH821: Gravitational Waves Physics and Astronomy
Department of Physics
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Course Project

Estimating the parameters of a simulated black hole binary source

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1 Introduction

Gravitational waves (GWs) are ripples in space-time that are emitted by accelerating masses. Technically speaking, every physical object that accelerates produces gravitational waves. This includes humans, cars, airplanes etc. But the masses and accelerations of objects on Earth are far too small to make gravitational waves big enough to detect with our instruments. To find big enough gravitational waves, we have to look far outside of our own solar system.

The Universe is filled with incredibly massive objects undergoing rapid accelerations that generate gravitational waves that we can now detect. Known objects are pairs of black holes or neutron stars orbiting each other, or a neutron star and black hole orbiting each other or gigantic stars blowing themselves up at the ends of their lives. Astronomers have defined four categories of gravitational waves based on what object or system generates the waves:

- Continuous
- Compact Binary Inspiral
- Stochastic
- Burst

Each category of objects generates a characteristic set of gravitational-wave signals that researchers can look for in U.S. National Science Foundation Laser Interferometer Gravitational-wave Observatory (NSF LIGO) data.

2 Problem Statement

We are given strain data (.gwf files) from three interferometers (Livingston, Hanford and Virgo). The aim of the project is to estimate the component masses and sky location of the binary source. The constraints given are as follows:

1. The component masses are between 20 and 50 solar masses
2. The spins and polarization angle are set to 0
3. The phase is set to 1.3
4. The inclination angle is set to 0.4
5. The luminosity distance is set to 900 Mpc

3 Theory

Compact binary mergers are rare events which, though powerful, generate only a tiny tidal deformation of space-time requiring extremely sensitive detectors able to probe a large volume of the Universe. Since they were built, LIGO and Virgo have been alternating observing runs and upgrades to the instruments to improve their sensitivity and stability. Binary black hole (BBH) mergers dominate the observed events, which also includes a few binary neutron star (BNS) and neutron star - black hole (NSBH) mergers. The basic features of the signal from these sources are as follows:

1. **Inspiral-merger-ringdown**: The evolution of a binary system, and the associated GW signal, during and after coalescence can be divided into three parts: the long inspiral stage, when the orbit shrinks adiabatically (slowly and smoothly), is followed by a stage where the two objects plunge toward each other and merge, then a stage where the subsequent final compact object (typically a black hole) relaxes to its quiescent state. The three stages are usually referred to as the inspiral, merger and ringdown (see figure below).

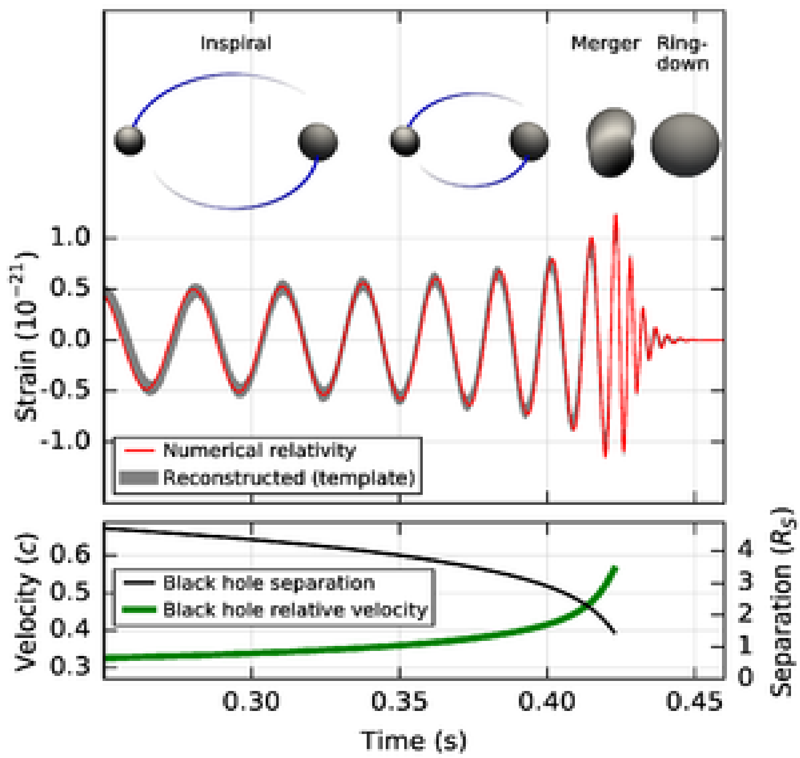


Fig 1. The various stages of a compact binary coalescence and the GW signal reconstructed from the GW150914 event observed in the LIGO detectors (ref)

2. **Polarization**: GWs are transverse waves and have two tensor, independent modes of polarization in GR, referred to as + (plus) and × (cross). For a given polarization mode, space contracts or expands along orthogonal axes, with the axes of the + mode being rotated by 45 degrees with respect to the axes of the × mode. The GW signal emitted by the binary is circularly polarized (and strongest) for directions perpendicular to the orbital plane (i.e. for a system viewed face-on, from the point of view of the observer), linearly polarized (and weakest) for directions in the orbital plane (i.e. for a system viewed edge-on), and elliptically polarized in other directions.

3. **Chirp pattern and signal-to-noise ratio**: The dominant frequency of the emitted GWs corresponds to twice the orbital frequency of the binary. As the orbit gradually shrinks, the frequency, frequency derivative and amplitude of the GW signal increase, shaping it into the famous chirp pattern (see figure below). Although the radiated GW strain is largest at the end of the inspiral process, the signal spends more cycles at lower frequencies than at higher frequencies, so that its spectral amplitude decreases with frequency, following a f7/6 dependency; see Figure 4. The ratio of the signal power to the detector noise power spectrum determines the density of the squared signal-to-noise ratio (SNR), whose integral over the detector bandwidth gives the squared SNR.

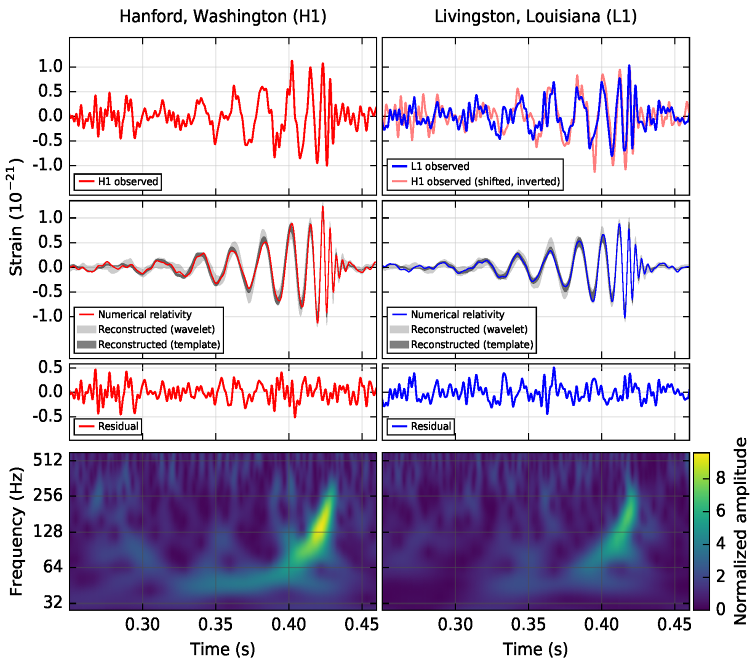


Fig 2. The event GW150914 as it appeared in the LIGO Hanford and LIGO Livingston detectors, in the time domain and in the time-frequency domain. The chirp pattern is visible. (ref)

4. **Source parameters**: The detected waveform depends on the parameters of the source, which can therefore be inferred from the signal measured in one or several detectors. Parameter estimation is performed via Bayesian inference, exploring a multi-dimensional space (15 parameters) to identify the set of parameters that best matches the data, and plausible ranges for the various parameters. The parameters are further in detail.

4a. **Intrinsic parameters**: Intrinsic parameters affect the dynamics of the system and drive the amplitude and phase evolution of the signal. They include the masses of the two orbiting objects, m1 (lighter object) and m2 (heavier object) and their spins.

4b. **Extrinsic parameters**: The source's extrinsic parameters include the time and phase at coalescence. The others are related to the location of the source with respect to the detector (luminosity distance and two angles for direction) and the orientation of the binary with respect to the detector (inclination angle and polarization angle). The luminosity distance and the inclination affect the strength of the GW signal received.

4 Data Analysis and Results

We use gwpy, bilby, lalsuite and pycbc - to analyze the data and infer parameters of the source.

To identify the event time from the strain data, we use **matched filtering** This.

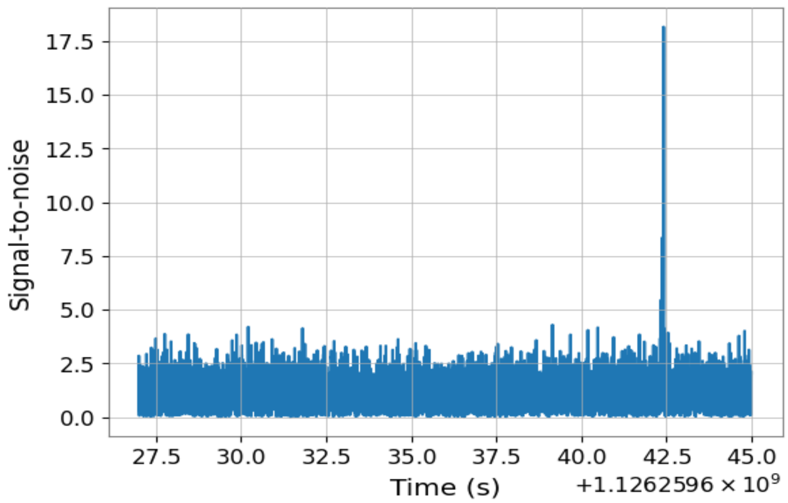


Fig 3. SNR plot from matched filtering

As shown in the plot, the event time is found to be around **1126259642.5 secs** with an SNR of ~ 18.

Once we have the event time, we use an interval without the signal to estimate the PSD of the interferometers. For this, we divide the 30 seconds time interval into two equal parts, the first one without the signal and the second one with the signal. We then plot strain data with the Amplitude Spectrum Density (ASD): this is just the square root of the PSD and has the right units to be comparable to the frequency-domain strain data.

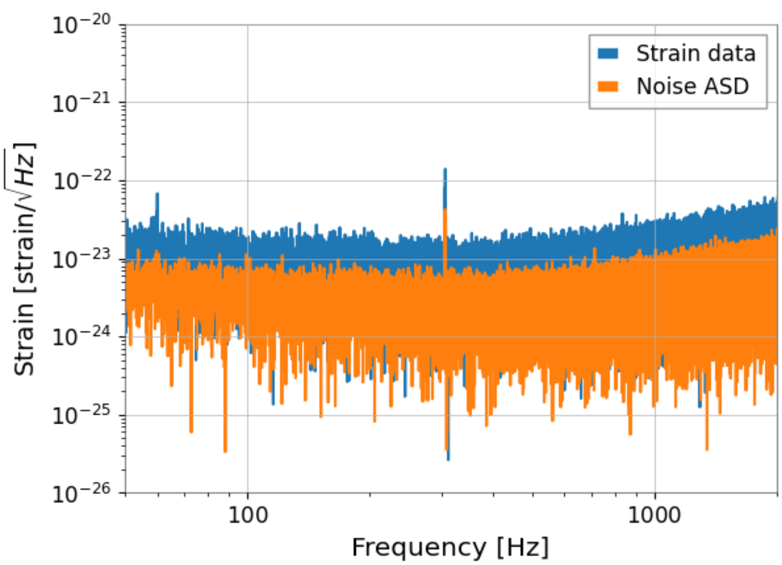


Fig 4. Noise ASD and strain data

We now use the **bilby Bayesian inference library** to estimate the parameters based on a suitable prior.

The prior distribution is chosen as follows:

S.No.	Parameter	Prior
1.	mass_1	Uniform(minimum=20, maximum=50)
2.	mass_2	Uniform(minimum=20, maximum=50)
3.	phase	1.3
4.	geocent_time	Uniform(minimum=event_time-0.1, maximum=event_time+0.1)
5.	a_1	0.0
6.	a_2	0.0
7.	tilt_1	0.0
8.	tilt_2	0.0
9.	phi_12	0.0
10.	phi_jl	0.0
11.	dec	Cosine()
12.	ra	Uniform(minimum=0.0, maximum=2π)
13.	theta_jn	0.4
14.	psi	0.0
15.	luminosity_distance	900

Table 1: Prior distribution

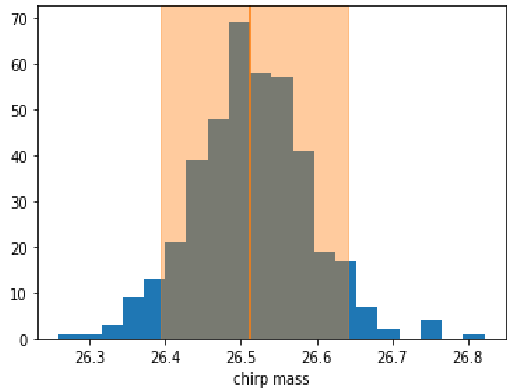
This prior is used to calculate the likelihood of getting the observed signal and therefore, is used to estimate the optimal source parameters.

The results obtained are summarized below (all masses are in solar mass):

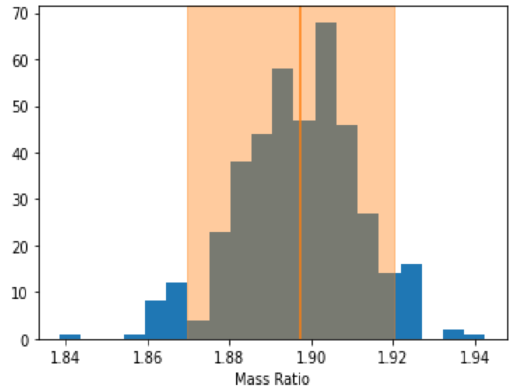
S.No.	Parameter	Value	90% Confidence Interval	
			Lower bound	Upper bound
1.	Chirp Mass	26.53	26.39	26.64
2.	Mass Ratio	1.90	1.87	1.92
3.	mass_1	22.33	22.12	22.58
4.	mass_2	42.38	42.21	42.52
5.	ra	1.38	1.36	1.40
6.	dec	-1.21	-1.23	-1.19

Table 2: Estimated parameters

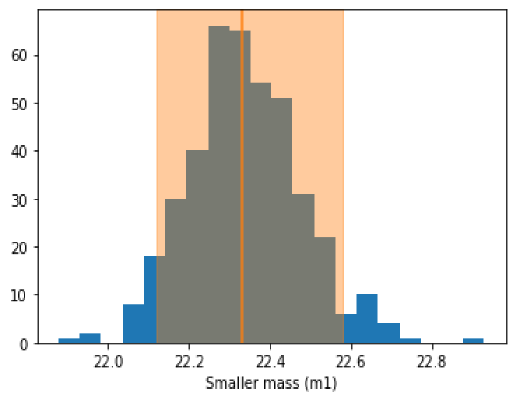
For better visualization, histograms are plotted for all the estimated parameters to indicate 90% confidence intervals, which are shown below:



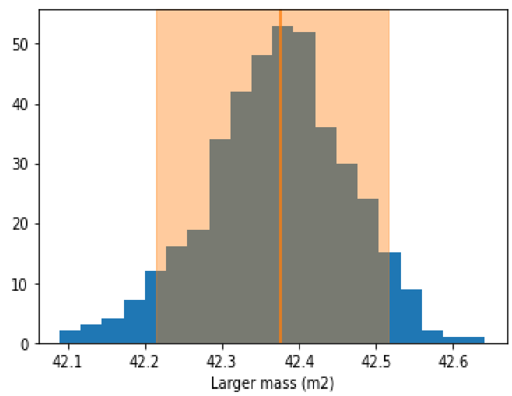
(a) Chirp Mass Distribution



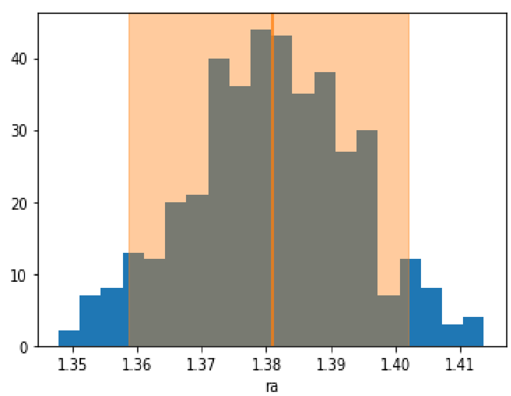
(b) Mass Ratio Distribution



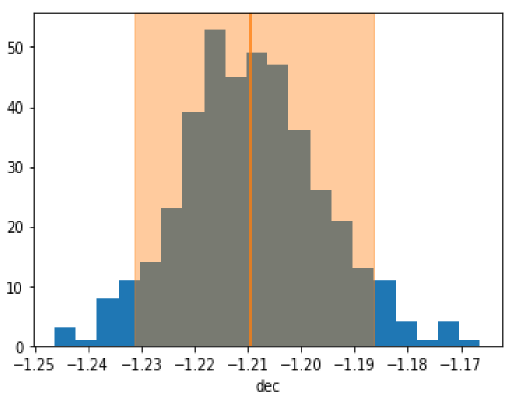
(a) Smaller Mass (m1) Distribution



(b) Larger Mass (m2) Distribution



(a) RA Distribution



(b) Dec Distribution

Fig 5. Histograms for estimated parameters

The corner plots for the estimated parameters are also shown below:

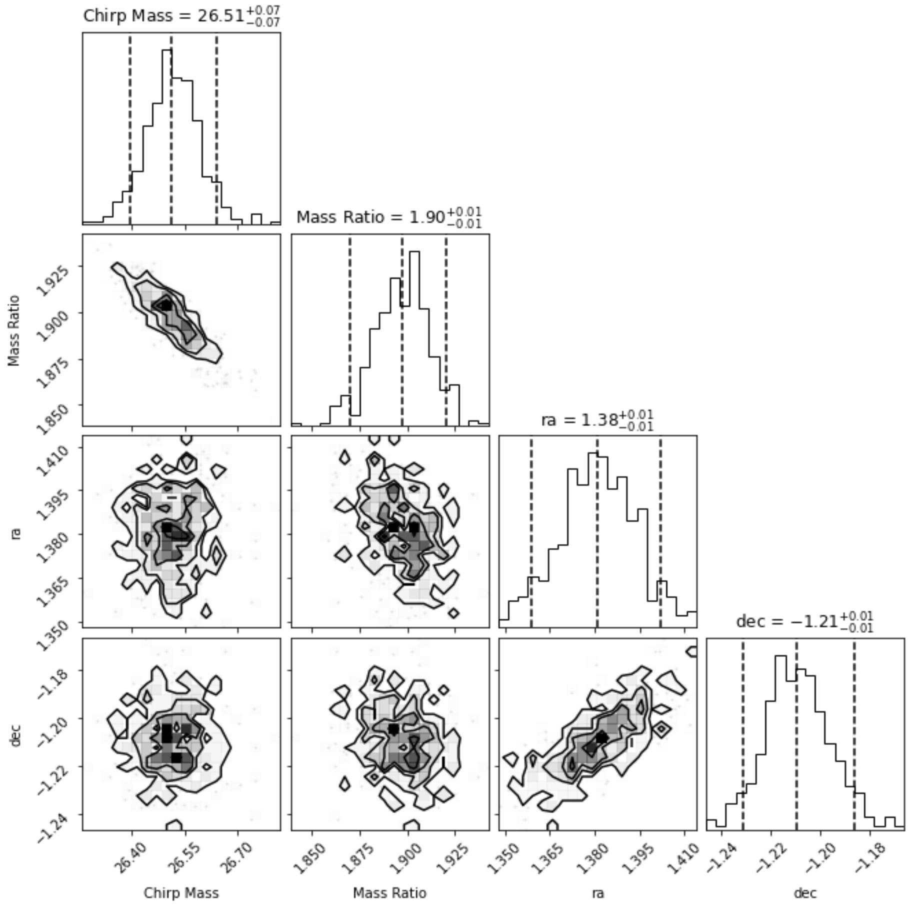


Fig 6. Corner plots for estimated parameters

5 Conclusion

Through this project, we learnt how Bayesian inference can be used to infer properties (here, component masses and sky location) of a binary GW source by maximizing log likelihood. It is also important to note that the convergence of this method relies on the prior distribution chosen.

6 Appendix

Please refer to the github repository for codes.

7 References

1.GW Parameter Estimation Workshop 2020 Tutorial notebooks
2.Basic Physics of GW150914
3.An introduction to Bayesian inference in gravitational-wave astronomy