Computational physics
Homework 7
8 Nov 2023

Name: *P. Demidov*
GitHub: PalPaulD

Problem 1

You can find the code in the file **hw7, problem 1.py**. I'll go through the code. What do functions do?

- **f(x)**. Some say that it calculates the values of $f(x) = (x - 0.3)^2 e^x$ and the name hints... But this is just a rumor.
- **parabolic_step**. Takes the point where the interval starts (a), some point in the center (c) and the end of the interval (b). Then it calculates the minimum of a parabola that can be fitted based on these three points using

$$x = b - \frac{1}{2}\frac{(b-a)^2(f(b)-f(c)) - (b-c)^2(f(b)-f(a))}{(b-a)(f(b)-f(c)) - (b-c)(f(b)-f(a))}$$

  Then it chooses the subinterval which contains the minimum and returns it back.
- **golden_section_step**. Eats the boundaries of the interval and makes one step with the golden section method. How? Define

$$\omega = \frac{-1+\sqrt{5}}{2},$$

  then

$$x_1 = \omega a + (1-\omega)b, \qquad x_2 = (1-\omega)a + \omega b.$$

  If $f(x_1) < f(x_2)$, return the interval (a, x_2), otherwise return (x_1, b).

Now the code does the *real* job. Firstly we start with declaring the boundaries of the initial interval and taking it's middle as c. Also one can set the tolerance of minimization by changing ε. The first loop is responsible for using the parabolic steps, it breaks right after one of the conditions stated in the lecture notes is violated. Also, if it happens, the code remembers it, so it can run another loop using golden section steps.

At the very end I also print the result of scipy Brent's method to compare. 0.3 vs 0.3, looks pretty good.

Problem 2

The code can be found in the file **hw7, problem 2.py**. Firstly, the code uploads the data from the file. Then it calculates the likelihood function for the data provided by the standard formula

$$\mathcal{L} = \log f(\mathbf{x}|\theta) = \prod_{i=1}^{n} \rho(x_i|\theta),$$

where $\rho(x|\theta)$ is a probability density distribution for i.i.d. variables of the sample set. Finally, the minimization is performed using scipy method "minimize". The code prints out the fitted values with the covariance matrix. Also it ouputs the plot of the obtained distribution with the parameters on it:

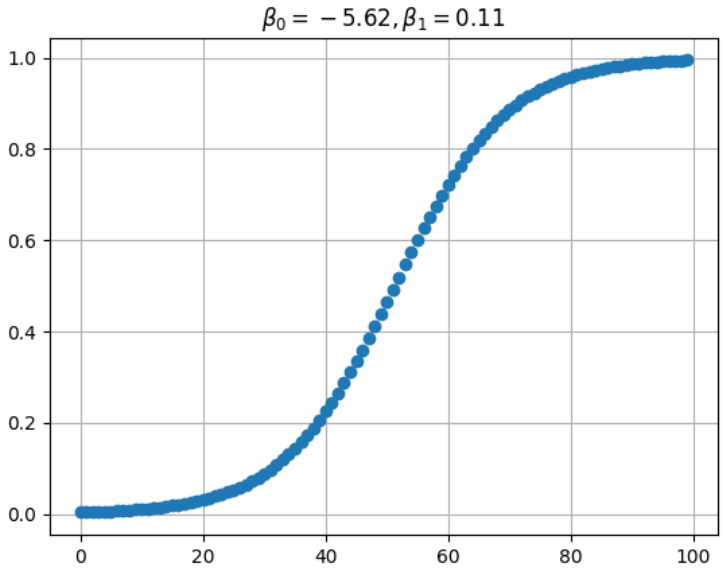

Figure 1: The fitted plot and the values

Observe that $p = 0.5$ corresponds to the age around 50. Makes sense! Though, I guess, the movie was released in 2008, so maybe 30-40 would make more sense.

Anyway, the reasonable answer convinces us that the minimization of $\mathcal{L}$ was done well.