HW1

0.1f. The set of all integers equal to and added to n

0.1e. The set of all strings made up of 0s and 1s and each of the strings backwords/reversed

0.6d. Range = $\{6, 7, 8, 9, 10\}$

Domain = $\{(1,6)(1,7)(1,8)(1,9)(1,10)$
$(2,6)(2,7)(2,8)(2,9)(2,10)$
$(3,6)(3,7)(3,8)(3,9)(3,10)$
$(4,6)(4,7)(4,8)(4,9)(4,10)$
$(5,6)(5,7)(5,8)(5,9)(5,10)\}$

0.6e. $f(4) = 7$, so $g(4, f(4)) = g(4,7) = 8$

1. Base: $n=1$

$$LHS = \sum_{i=1}^{1} \frac{1}{1(1+1)} = \frac{1}{2}$$

$$RHS = \frac{1}{1+1} = \frac{1}{2}$$

LHS = RHS ✓

Induction Hypothesis: $n = k$

$$\sum_{i=1}^{k} \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Inductive Step: $n = k+1$

$$LHS: \sum_{i=1}^{k+1} \frac{1}{(k+1)((k+1)+1)} = \sum_{i=1}^{k+1} \frac{1}{(k+1)(k+2)} =$$

$$\sum_{i=1}^{k} \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} =$$

$$\frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} =$$

$$\frac{k^2+k+k+1}{(k+1)(k+2)} = \frac{k(k+1)+(k+1)}{(k+1)(k+2)}$$

$$\frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$$

RHS: $\frac{k+1}{(k+1)+1} = \frac{k+1}{k+2}$

LHS = RHS ✓

2. Base: $n=1$

LHS: $1^3 = 1$

RHS: $\left[\frac{1(1+1)}{2}\right]^2 = \left[\frac{2}{2}\right]^2 = 1^2 = 1$

LHS = RHS ✓

Induction Hypothesis: $n = k$

$$1^3+2^3+3^3+\ldots+k^3 = \left[\frac{k(k+1)}{2}\right]^2$$

Inductive Step: $n = k+1$

LHS: $1^3+2^3+3^3+\ldots+(k+1)^3 =$

$1^3+2^3+3^3+\ldots+k^3+(k+1)^3 =$

$\left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 =$

$\frac{k^2(k+1)^2}{4} + (k+1)^3 =$

$(k+1)^2\left(\frac{k^2+4(k+1)}{4}\right) =$

$\left[\frac{(k+1)(k+2)}{2}\right]^2$

$$(k+1)^2\left(\frac{k^2+2k+2k+4}{4}\right) =$$

$$(k+1)^2\left(\frac{k(k+2)+2(k+2)}{4}\right) =$$

$$(k+1)^2\left(\frac{(k+2)(k+2)}{4}\right) (k+1)^2\left(\frac{k+2}{2}\right)^2 =$$

$$\left[\frac{(k+1)(k+2)}{2}\right]^2$$

RHS: $\left[\frac{(k+1)((k+1)+1)}{2}\right]^2 = \left[\frac{(k+1)(k+2)}{2}\right]^2$

LHS = RHS ✓