**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:  **B**

*μ* = 45 minutes; *σ* = 8 minutes

Service manager committed to the customer to finish work in one hour (60 min). And started working after 10 min. So, time left for him to fulfill his commitment is (60-10) = 50 min.

Probability of service time exceeding 50 min = 0.26598552904870054 ≈ **0.26.**

(Used stats.norm.cdf)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer:  **False.**

Because in a normal distribution, 50% of the observations fall above the mean and 50% of observations fall above the mean. And 34.1% of all observations are present below one standard deviation distance on each side from mean. In the above case, 44 is the value which is exactly one standard deviation away from mean value 38. So, 34.1% of the observations above the mean are present between the values 38 and 44. And rest 15.9% of observations lie above 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:  **True.**

Because *μ* = 38; *σ* = 6 and

proportion of the employees under the age of 30 ≈ 9.1% 0f 400 ≈ **36.**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2), we can say that *X1, X*2 are following independent and identical distributions. Which mean both variables have same type of probability distributions but outcome of one variable doesn’t affect the distribution of another variable.

**So, they both are identical and at the same time independent too.**

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:  **D**

For a normal distribution symmetric around the mean of 100 with a standard deviation of 20, 99% confidence interval for a random variable is 48.5 to 151.5.

**(Used stats. norm. interval)**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer:

Total mean of annual profits = mean of profits of division 1 +mean of profits of division 2

=5 +7 =12(million dollars) =

Mean profits in million rupees = 12\*45 = 540 million rupees

Standard deviation of annual profits = square root of variance of annual profits

Variance of annual profits = variance of division 1+ variance of division 2 = 9+16 =25

SD = square root of (25) = 5 (in million$) = 225 million rupees.

95% Confidence interval of annual profits is = 99.0081034 to 980.9918965 million rupees. (Used stats. norm. interval)

1. Specify the 5th percentile of profit (in Rupees) for the company.

Answer:

5th percentile of annual profits of company = **169.9079339 million rupees**

(Used stats.norm.cdf)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

Probability of division 1 making loss = 0.0477903

Probability of division 2 making loss = 0.040059

**Division 1 have more probability of making loss.**