Math13 - Introduction to Statistics (code 21947), Spring 2015

Exercises 2; Probability Distributions

1) Suppose that X takes on one of the values 1, 2, 3, 4, or 5.
If P($X < 3$) = 0.4 and P($X \geq 4$) = 0.5, find:

a. P(X = 3) = 0.5 - 0.4 = 0.1
b. P(X < 4) = 1 - 0.5 = 0.5

2) Suppose that P(Z > 1.96) = 0.025. Find P($Z \leq$ 1.96)

Answer: P($Z \leq$ 1.96) = 1 - P(Z > 1.96)

3) If A and B are events such that: 0 < P(A) < 1, and 0 < P(B) < 1; select the correct answer:

1. P(A) / 2 = 0.6	always true	sometimes true	never true
2. P(A and A^c) = 0	always true	sometimes true	never true
3. P(A or A^c) = 0	always true	sometimes true	never true
4. P(A and B) $\leq$ P(A)	always true	sometimes true	never true
5. P(A | A^c) = 1	always true	sometimes true	never true
6. P(B | A) = P(B)	always true	sometimes true	never true
7. P(A or B) = P(A) + P(B)	always true	sometimes true	never true
8. P(A) = P(B)	always true	sometimes true	never true
9. P(A | B) - P(A) = 0	always true	sometimes true	never true
10. P(A | B) P(B) = P(B | A) P(A)	always true	sometimes true	never true

4) Suppose that A and B are events such that P(A) = 0.4, P(B) = 0.5, and P(A ∩ B) = 0.2.

a. Are A and B mutually exclusive? No because P(A ∩ B) is different from 0.
b. Are A and B independent? Yes because P(A ∩ B) = P(A) P(B).

5) When two balanced dice are rolled, 36 equally likely outcomes are possible. Let Z denote the sum of the dice.

a. What are the possible values of the random variable Z?
   2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
b. Find P(Z = 7)
   6 / 36
c. Find the probability distribution of Z. Leave your probabilities in fraction form.

z	P(Z = z)
2	1/ 36
3	2 / 36
4	3 / 36
5	4 / 36
6	5 / 36
7	6 / 36
8	5 / 36
9	4 / 36
10	3 / 36
11	2 / 36
12	1 / 36

d. Construct a graph of the probability distribution. We saw this in class

6) Suppose that A and B are independent events with $P(A) = 0.8$, and $P(B^c) = 0.4$

a. Find $P(A \cap B) = (0.8)(1 - 0.4) = 0.48$
b. $P(A \cup B) = (0.8) + (0.6) - (0.48) = 0.92$
c. $P(B) = 1 - 0.4 = 0.6$
d. $P(A^c \cap B) = (1 - 0.8)(0.6) = 0.12$

7) A total of 500 married working couples were polled about whether their annual salaries exceeded \$75,000. The following information was obtained:

		Husband	
		Less than \$75,000	More than \$75,000
Wife	Less than \$75,000	212	198
	More than \$75,000	36	54

a. What is the probability that the husband earns less than \$75,000?
(212 + 36) / 500
b. What is the conditional probability that the wife earns more than \$75,000 given that the husband earns more than this amount?
54 / (198 + 54)
c. What is the conditional probability that the wife earns more than \$75,000 given that the husband earns less than this amount?
36 / (212 + 36)

8) The committee on Student Life did a survey of 417 students regarding satisfaction with student government and class standing. The results follow:

	Freshman	Sophomore	Junior	Senior	***Total***
Not satisfied	17	19	23	12	***71***
Neutral	61	35	32	38	***166***
Satisfied	23	49	43	65	***180***
Total	***101***	***103***	***98***	***115***	***417***

Find the probability that a student selected at random is:

a. Satisfied = 180 / 417
b. Junior = 98 / 417
c. Satisfied, given that the student is a senior = 65 / 115
d. Neutral and freshman = 61 / 417
e. Senior, given satisfied = 65 / 180
f. Neutral or satisfied = (166 / 417) + (180 / 417)
g. At least a sophomore = 1 - (101 / 417)

9) A club has 90 members: 50 are lawyers and 50 are liars. Everyone is either a lawyer or a liar. Consider the experiment of randomly selecting a member. Let **A** be the event of selecting a lawyer. Let **B** be the event of selecting a liar.

a. What is P(A), the probability that a randomly selected member is a lawyer?
   50 / 90
b. What is P(B), the probability that a randomly selected member is a liar?
   50 / 90
c. What is $P(B^c)$, the probability that a randomly selected member is not a liar?
   1 - (50 / 90)
d. What is $P(A \cap B)$, the probability that a randomly selected member is both a lawyer and a liar?
   10 / 90
e. What is P(A | B), the probability of randomly selecting a lawyer given that the member is a liar?
   10 / 50
f. What is $P(A \cap B^c)$, the probability that a randomly selected member is both a lawyer but not a liar?
   40 / 90
g. What is $P(B^c \mid A)$, the probability that a lawyer is not a liar?
   40 / 50

10) Suppose that 25% of a forest consists of trees of species A, 40% of species B, and 35% of species C.

a. What is the probability that the tree selected at random will be of species A?
   0.25
b. What is the probability that the tree selected at random will not be of species A?
   1 - 0.25 = 0.75
c. If it is known that the tree is not of species A, what is the probability that it will be of species B?
   0.40 / 0.75

11) An automobile license plate consists of three numbers followed by three letters. How many license plates are possible if:

a. repetition of numbers and letters is allowed?
   10 x 10 x 10 x 26 x 26 x 26
b. no repetition of numbers and letters is allowed? (e.g. 112ABC, 278ABA, 112ABA are not allowed)
   10 x 9 x 8 x 26 x 25 x 24
c. no repetition is allowed, the first digit must be either 1, 2 or 3, and the first letter cannot be a vowel?
   3 x 9 x 8 x 21 x 25 x 24

12) Suppose that license plates are given out in random fashion. Mr. and Mrs. Brown (a two car family) went to get automobile license plates. If repetition is allowed, what is the probability that their plates:

a. would end with the same letter? 1 / 26
b. begin with the same digit? 1 / 10
c. begin with the same digit and end with the same letter? 1 / 260

13) The following table lists the number of employees of a food chain restaurant in eight cities:

City	Employees
Minneapolis, MN	105
Newark, NJ	155
Omaha, NE	149
Portland, OR	195
San Antonio, TX	290
San Jose, CA	357
Tucson, AZ	246
Tulsa, OK	178

One of the cities is to be randomly chosen and all the employees of this city are to be interviewed. All the cities have the same probability of being chosen. Find the expected number of people who will be interviewed.
105 (1/8) + 155 (1/8) + 149 (1/8) + 195 (1/8) + 290 (1/8) + 357 (1/8) + 246 (1/8) + 178 (1/8) = 209.375 ≈ 210

14) The random variable W is the crew size of a randomly selected shuttle mission between April 1981 and July 2000. Its probability distribution is as follows:

w	2	3	4	5	6	7	8
$P(W=w)$	0.042	0.010	0.021	0.375	0.188	0.344	0.021

a. Find the mean of the random variable W = 5.777
b. Obtain the standard deviation of W = 1.263584
c. Draw the distribution of the random variable

15) A designer makes a profit of $30 on each item that is produced in perfect condition, and suffers a loss of $6 on each item that is produced in less-than-perfect condition. If each item produced is in perfect condition with probability 0.4, what is the designer's expected profit per item?
E(Profit per item) = (0.4) ($30) + (0.6) (-$6) = $12 - $3.6 = $8.4

16) Two people are randomly chosen from a group of 10 men and 20 women. Let X denote the number of men chosen, and let Y denote the number of women chosen.
a. Find $E(X)$ = 0.6597
b. Find $E(Y)$ = 1.333
c. Find $E(X + Y)$ = 2

17) A small taxi company has 4 taxis. In a month's time, each taxi will get 0 traffic tickets with probability 0.3, 1 traffic ticket with probability 0.5, or 2 traffic tickets with probability 0.2. What is the expected number of tickets per month amassed by the fleet of 4 taxis?
E(tickets per taxi) = 0 (0.3) + 1 (0.5) + 2 (0.2) = 0.9
E(tickets fleet 4 taxis) = 4 x E(tickets per taxi) = 4 (0.9) = 3.6

18) A husband's year-end bonus will be:

\$0	with probability 0.3
\$1000	with probability 0.6
\$2000	with probability 0.1

His wife's bonus will be:

\$1000	with probability 0.7
\$2000	with probability 0.3

Let S be the sum of their bonuses, and assume that the bonus of the husband is independent from the bonus of the wife. Find E(S) and Var(S).
E(Husband's bonus) = (\$0) (0.3) + (\$1000) (0.6) + (\$2000) (0.1) = \$800
Var(Husband's bonus) = \$360,000

E(Wife's bonus) = (\$1000) (0.7) + (\$2000) (0.3) = \$1300
Var(Wife's bonus) = \$210,000

E(S) = E(Husband's bonus + Wife's bonus) = \$800 + \$1300 = \$2100
Var(S) = Var(Husband's bonus) + Var(Wife's bonus) = \$360,00 + \$210,000 = \$570,000

19a) An engineering firm is deciding whether to prepare a bid for a construction project. If the probability of getting the contract is 0.4, and the probability that the weather will be bad is 0.6, find the following probabilities: we saw this in class

Outcome	**Probability**
Not getting the contract	
Getting the contract and weather is bad	
Getting the contract and weather is good	

19b) Consider the following three possible outcomes for the gross profit: we saw this in class

1) If it does not get the contract, then the firm will make a gross profit of \$0.
2) If it gets the contract and the weather is bad, the firm will make a gross profit of \$3,000.
3) If it gets the contract and the weather is good, the firm will make a gross profit of \$6,000.

What is the company's expected gross profit?

What is the standard deviation of the company's gross profit?

19c) Knowing that the cost for preparing the bid is \$800, What is the company's **expected net profit**? *Hint: The net profit is the gross profit minus the preparation cost.*

Use Binomial Probability Formula for problems 20, 21, 22

20) A fair coin is tossed independently five times. P(heads) = P(tails) = 0.5. Compute:

a. probability of all heads = 0.03125
b. probability of no heads = 0.03125
c. probability of at least one heads = 0.96875
d. probability of more heads than tails = 0.5
e. probability of less than three heads = 0.5

21) A certain couple is equally likely to have either a boy or a girl. If the family has four children, let Y be the number of boys.

a. Identify the possible values of the random variable Y = 0, 1, 2, 3, 4
b. Determine the probability distribution of Y

k	P(Y = k)
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

c. Find the probability that the couple has exactly two boys = 0.375
d. Find the probability that the couple has at least two boys = 0.6875
e. Find the probability that the couple has at most two boys = 0.6875
f. Find the probability that the couple has between one and three two boys, inclusive 0.875
g. Find the probability that the couple has children all of the same gender = 0.125

22) A large shipment of headphones ordered by a retail store contains 5% of defective devices. The retail store performs an inspection of the shipment by randomly selecting 20 headphones.

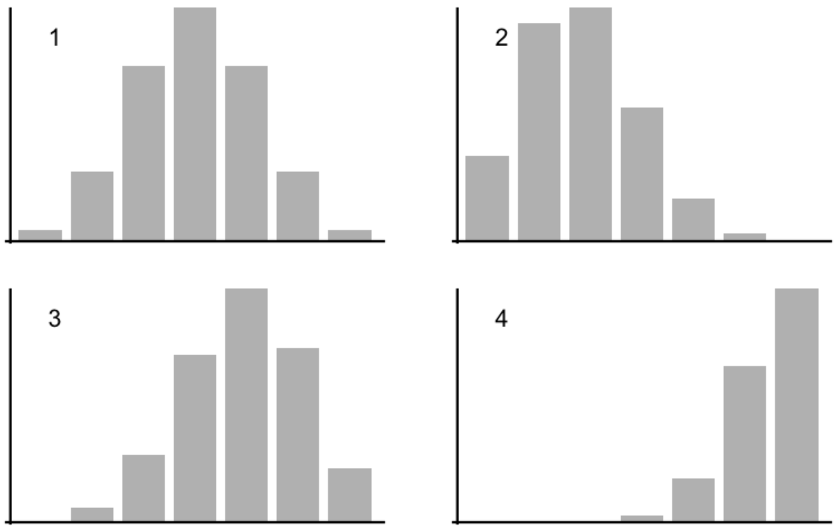
a. Find the probabilities that the inspection finds 0, 1, 2, 3 defective headphones. (*Hint: treat the random variable as a binomial random variable*):

x	**P(X = x)**
0	0.3584
1	0.3773
2	0.1886

3	0.0595

b. The retail store has decided to accept the shipment if the inspection yields ***no more than two*** defective headphones. What is the probability of finding no more than two defective headphones? 0.3584 + 0.3773 = 0.7357

23) The following figure shows four binomial distributions with n = 6 trials. Match the given probability of success with the corresponding graph.



a) $p = 0.30$ goes with graph 2
b) $p = 0.50$ goes with graph 1
c) $p = 0.65$ goes with graph 3
d) $p = 0.90$ goes with graph 4