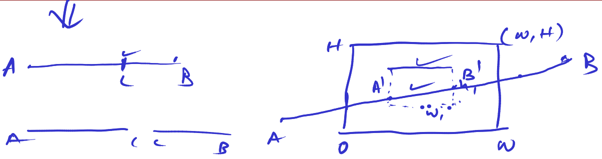
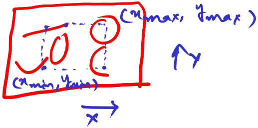
Clipping: LINES and POLYGONS



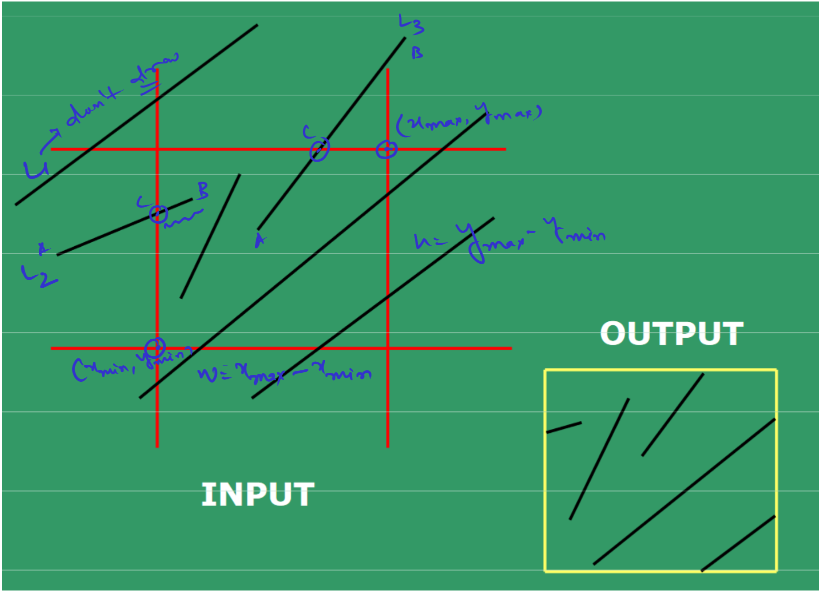
Overview

What is Clipping



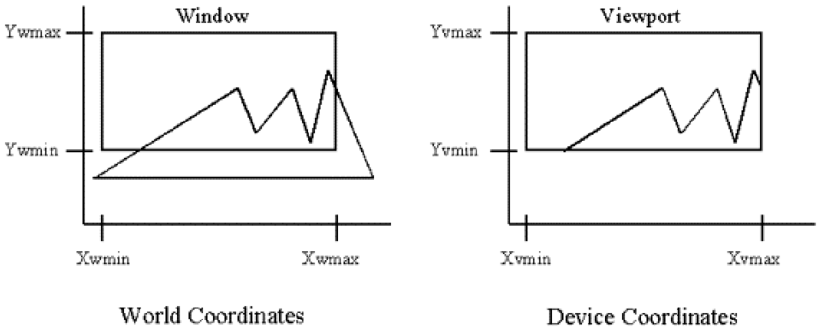
- A **Scene** is modeled as a set of objects
- **Viewing Window**: Rectangle is called as viewing window if a scene is viewed only inside the rectangle
- **Representation of Viewing Window:** (x_{min}, y_{min}) and (x_{max}, y_{max})
- **2D clipping:** Given a set of 2D objects, and a viewing window, display the portion of the objects inside the window
- **3D clipping:** Given a set of 3D $\langle x, y, z \rangle$ objects, and a 3D viewing window(Cuboid), display the portion of the objects inside the window

What is Clipping (cont.)



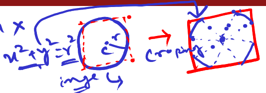
Need for Clipping

To display a portion of an object on screen



Cropping of sub image Vs Clipping of portion of Object

circle : $x^2+y^2=r^2$ $\quad$ $y^2=4ax$



2D-image → matrix → pixel → R G B

- The representation of image in memory is 2D array of numbers →
- The representation of object in memory is the values of its parameters → Line → Two points → (x_1,y_1) (x_2,y_2)
  Parabola → {Vertex, focal length}; Circle → centre, radius; Ellipse → center, major-axis length, minor-axis length
- **Input for cropping sub image:** the image(2D array) and representation of viewing rectangle
- **Input for clipping object:** The representation of objects and a viewing rectangle
  $A(x_1,y_1)$, $B(x_2,y_2)$ → crop → A', B' → image
- **Output for cropping sub image:** the portion of image(2D array) within the viewing rectangle
- **Output for clipping object:** The portion of objects within the viewing rectangle

- Advantage of representation of object with parameters:
  - Less storage requirement → parameter conversion.
    obj1 → obj2.
  - No interpolation is required when transformation is applied on the object, and hence no reduction in quality of displayed image

pixel → pixel.

B

A

all correspond pixel

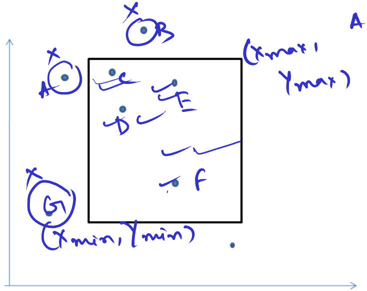
Clippings of Object of Different Dimensions

- Point Clipping
- Line Clipping
- Polygon Clipping

Point Clipping

Given the set of points, display only the points inside the rectangular window

A, B, C, D, E, F, G . . . .



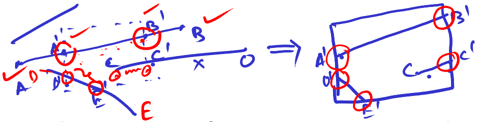
viewing window

(x_{min}, y_{min})

(x_{max}, y_{max})

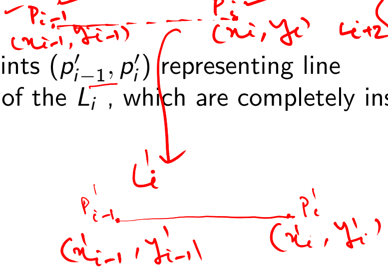
- Given a rectangle, display all the points within the rectangle

Line Clipping

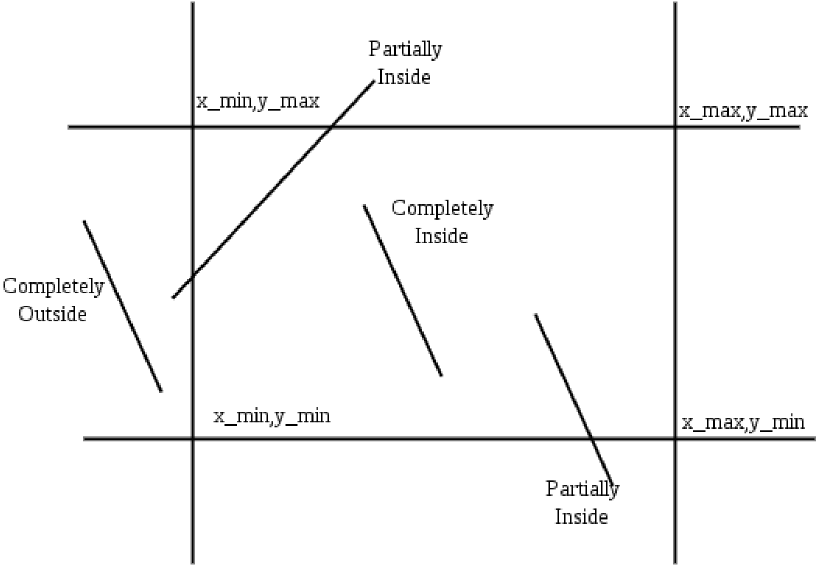


- Given a rectangle, display the portions of all line segments within the rectangle
- Input: Set of pairs of 2D points (p_{i-1}, p_i), and (x_{min}, y_{min}) and (x_{max}, y_{max}) representing viewing window
  - Each pair of pint (p_{i-1}, p_i) represents line segment L_i, where $p_i = (x_i, y_i)$
- Output: Set of pairs of 2D points (p'_{i-1}, p'_i) representing line segments L'_i that are portions of the L_i , which are completely inside the the viewing window

L_i (A, B)
L_{i+1} (C, D)
L_{i+2} (E, F)

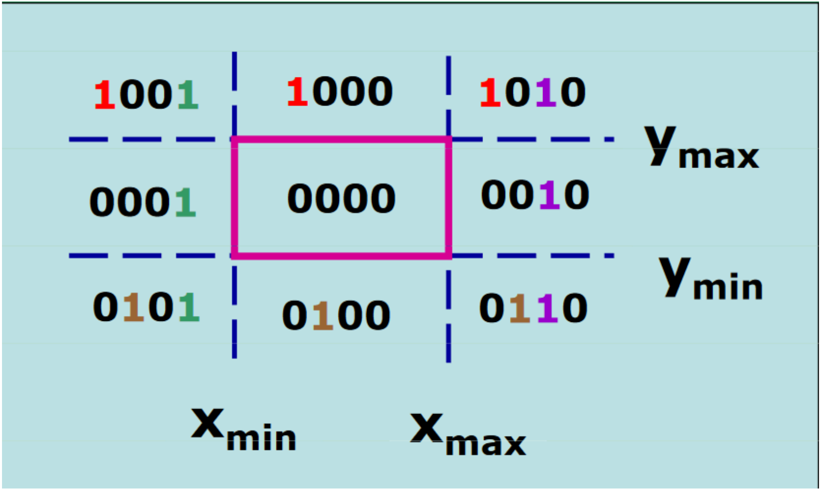


Line Clipping (cont.)



Cohen-Sutherland Line Clipping

Region Codes: Define a 4 bit code for each of the nine regions as given below



Cohen-Sutherland Line Clipping (cont.)

Bit Number	1	0
First (MSB)	Above Top Edge $Y>Y_{max}$	Below Top Edge $Y<Y_{max}$
Second	Below Bottom Edge $Y<Y_{min}$	Above Bottom Edge $Y>Y_{min}$
Third	Right of Right Edge $X>X_{max}$	Left of Right Edge $X<X_{max}$
Fourth(LSB)	Left of Left Edge $X<X_{min}$	Right of Left Edge $X>X_{min}$

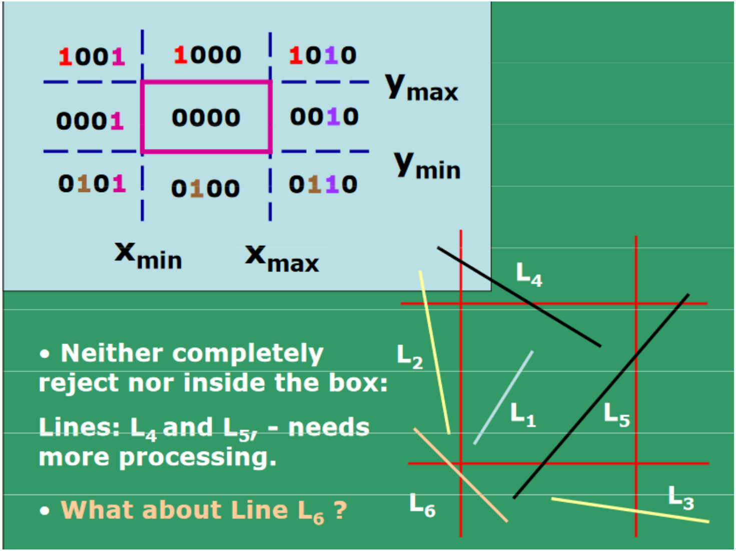
Cohen-Sutherland Line Clipping (cont.)

- To determine the region code for any point (X, Y), use:
  - $b_1 = sign(Y - Y_{max})$;
    (ie. $b_1 = 1$ iff (X, Y) is above the line $y = y_{max}$)
  - $b_2 = sign(Y_{min} - Y)$;
    (ie. $b_2 = 1$ iff (X, Y) is below the line $y = y_{min}$)
  - $b_3 = sign(X - X_{max})$;
    (ie. $b_3 = 1$ iff (X, Y) is on right of $x = x_{max}$)
  - $b_4 = sign(X_{min} - X)$;
    (ie. $b_4 = 1$ iff (X, Y) is on left of $x = x_{min}$)
  - Where $sign(a) = 1$ if $a \geq 0$; 0 otherwise;
  - sign(-4)= 0; sign(3)=1

Cohen-Sutherland Line Clipping (cont.)

- To check if the line segment is completely inside or completely outside or partially inside use region codes of endpoints of the line segment as given below
  - **Case 1: Trivial Acceptance:** If both endpoint codes are [0000], the line lies **completely inside** the box, no need to clip. This is the simplest case (e.g. L1).
  - **Case 2: Trivial Rejection:** If any line has 1 in the same bit positions of both the endpoints, then it is guaranteed to **lie outside the box completely** (e.g. L2 and L3).
  - **Case 3:** Line that does not belong to case 1 and case 2 (eg: L4, L5, L6)

Cohen-Sutherland Line Clipping (cont.)



- Clipping is required for line segment that is neither Completely IN nor completely OUT; e.g. Lines: L4, L5 and L6.
- Basic idea to clip line segment: Clip the part of the line segment which lies outside the viewing window

Algorithm Steps:

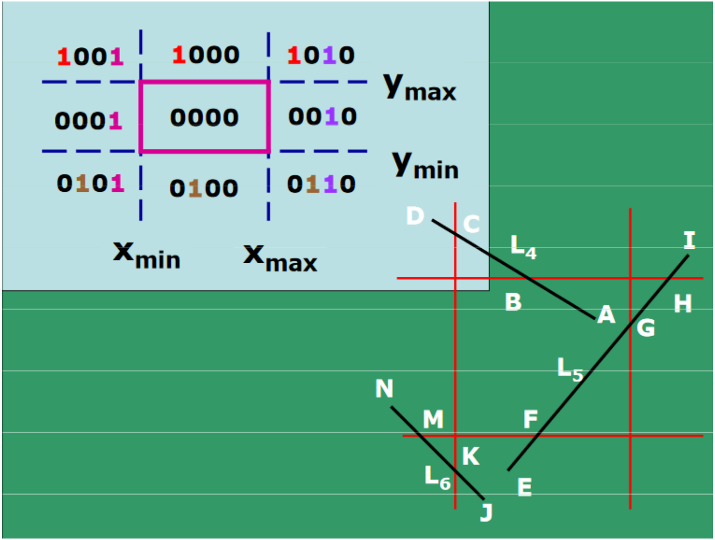
- **S1)** For end points (p_{i-1}, p_i) of each line segment L_i, Compute region codes, say c_{i-1} and c_i respectively.
- **S2)** if$((c_{i-1} == c_i) == 0000)$ accept(display) the segment L_i
  else if $((c_{i-1} \& c_i)! = 0000)$ reject(do not display) L_i
  else
  - Obtain an endpoint p that lies outside the box (at least one will ?)
  - Using the region code of that end point p, obtain the edge(the infinitely extended side of the viewing window) as follows.
    In the region code of p
    - if $b_1 = 1$, $y = y_{max}$ is to be intersected with line L_i

Cohen-Sutherland Line Clipping (cont.)

- if $b_2 = 1$, $y = y_{min}$ is to be intersected with line L_i
- if $b_3 = 1$, $x = x_{max}$ is to be intersected with line L_i
- if $b_4 = 1$, $x = x_{min}$ is to be intersected with line L_i

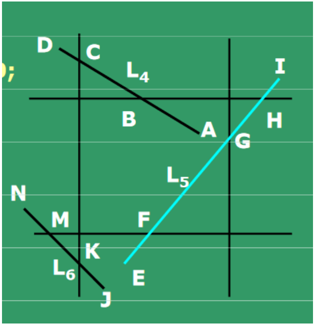
- **Question:** If more than one bit are 1s, which one is to be used?
  -Ans: use any bit
- Let the intersection point be I, replace the end point p with I, and compute the region code for I
  (ie. the line segment (p, q) has become (I, q), and this process is called as clipping)
- Repeat S2 if Codes for I and q are not 0000

Cohen-Sutherland Line Clipping (cont.)



Cohen-Sutherland Line Clipping (cont.)

e.g. Take Line L_5(endpoints - E and I): E has region code 0100 (to be clipped w.r.t. bottome dge);



- So EI is clipped to FI;
- Region code of F is 0000; But Region code of I is 1010; Clip(w.r.t.top edge) to get FH.
- Region code of H is 0010; Clip(w.r.t.right edge) to get FG.

Cohen-Sutherland Line Clipping (cont.)

- Since region code of G and also R are 0000, display the segment FG.
- Alternative process of clipping to get FG
  - Intersect line EI with $x = x_{max}$ to get G
  - Intersect line EG with $y = y_{min}$ to get F
  - Display the segment FG as codes of both F and G are 0000

Cohen-Sutherland Line Clipping (cont.)

How to find intersection of line passing through the end points of a segment with an edge of the viewing window

- When the end points of the segment are (x_1, y_1) and (x_2, y_2), and edge is $y = y_{max}$(Top Edge)
- slope of the line: $m = (y_2 - y_1)/(x_2 - x_1)$
- Line equation: $y - y_1 = m(x - x_1)$
- sub $y = y_{max}$ in line eqn. we get, $y_{max} - y_1 = m(x - x_1)$
- Therefor
  $x = x_1 + (1/m)(y_{max} - y_1) = x_1 + (x_2 - x_1)/(y_2 - y_1) * (y_{max} - y_1)$
- The Intersection point is (x, y),
  where $x = x_1 + (x_2 - x_1)(y_{max} - y_1)/(y_2 - y_1)$,
  $y = y_{max}$

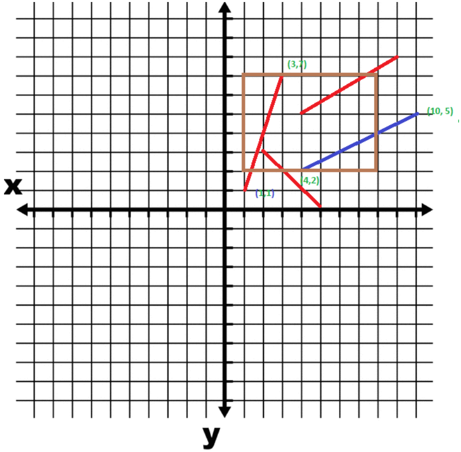
Cohen-Sutherland Line Clipping (cont.)

Similarly the intersection with other edges, namely Bottom, Right and left can be derived

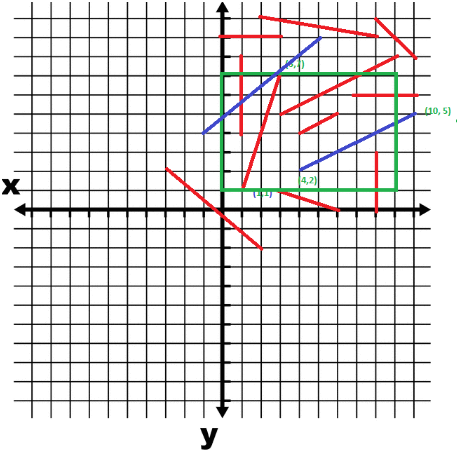
The Intersection points (x, y) in each of the types is given below

- Bottom Edge:
  $x = x_1 + (x_2 - x_1) * \frac{y_{min} - y_1}{y_2 - y_1}$
  $y = y_{min}$
- Right Edge:
  $x = x_{max}$
  $y = y_1 + (y_2 - y_1) * \frac{x_{max} - x_1}{x_2 - x_1}$
- Left edge:
  $x = x_{min}$
  $y = y_1 + (y_2 - y_1) * \frac{x_{min} - x_1}{x_2 - x_1}$

HW: Trace Cohen-Sutherland algorithm on the following inputs

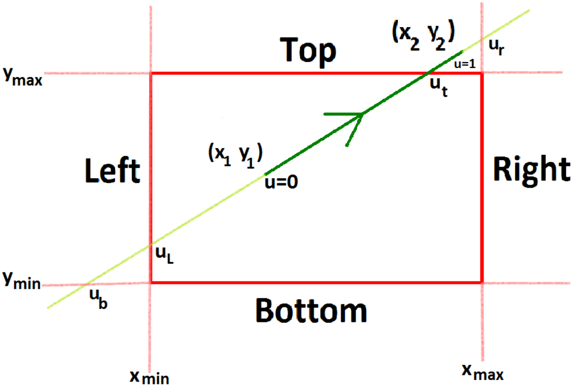


Cohen-Sutherland Line Clipping (cont.)



Note:
Viewing window is in brown in the first, and green in the second. A few endpoints are explicitly given other end points can be calculated from the graph

Basic Idea: 1) Extend the line segment and the window boundaries; 2) Find the visible portion using intersection points of the line with each of the window boundaries



Liang-Barsky Line Clipping (cont.)

- Parametric equation of the **line segment** with end points (x_1, y_1) and (x_2, y_2):
  $x = x_1 + u\Delta x;$
  $y = y_1 + u\Delta y, 0 \leq u \leq 1$
  Where, $\Delta x = x_2 - x_1; \Delta y = y_2 - y_1$
- Parametric equation of the **line** passing through points (x_1, y_1) and (x_2, y_2):
  $x = x_1 + u\Delta x;$
  $y = y_1 + u\Delta y, -\infty \leq u \leq \infty$
  Where, $\Delta x = x_2 - x_1; \Delta y = y_2 - y_1$
- The point (x, y) is considered to be within a rectangle with parameters (x_{min}, y_{min}) and (x_{max}, y_{max})
  iff
  - $x_{min} \leq x_1 + u\Delta x \leq x_{max}$
  - $y_{min} \leq y_1 + u\Delta y \leq y_{max}$
- Equivalently

Liang-Barsky Line Clipping (cont.)

- $u(-\Delta x) \leq x_1 - x_{min}$
- $u\Delta x \leq x_{max} - x_1$
- $u(-\Delta y) \leq y_1 - y_{min}$
- $u\Delta y \leq y_{max} - y_1$

Each of these four inequalities, can be expressed as:
$u.p_k \leq q_k; k = 1, 2, 3, 4$

Where the parameters are defined as:

- $p_1 = -\Delta x, q_1 = x_1 - x_{min}$
- $p_2 = \Delta x, q_2 = x_{max} - x_1$
- $p_3 = -\Delta y, q_1 = y_1 - y_{min}$
- $p_4 = \Delta y, q_1 = y_{max} - y_1$

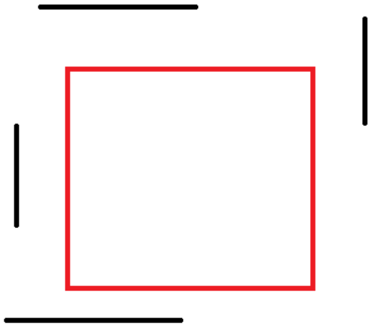
Liang-Barsky Line Clipping (cont.)

Based on these four inequalities, we can find the following conditions for line clipping:

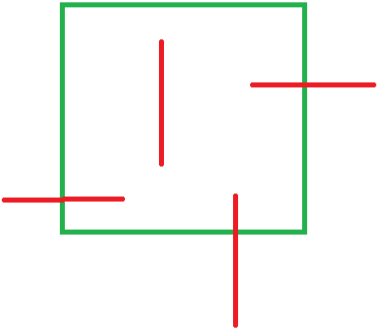
- If $p_k = 0$, the line is parallel to X or Y axes:
  - $k = 1 \implies \Delta x = 0 \implies x = x_1$ which is a line parallel to Y axis
  - $k = 2 \implies \Delta X = 0 \implies x = x_1$ which is a line parallel to Y axis
  - $k = 3 \implies \Delta y = 0 \implies y = y_1$ which is a line parallel to X axis
  - $k = 4 \implies \Delta y = 0 \implies y = y_1$ which is a line parallel to X axis

  Observations:
- **Case 1:** for any k, for which $p_k = 0$:
  - if $q_k < 0$, the line is completely outside the boundary (When $k = 1$, $q_1 < 0 \implies \mathbf{x_1} < \mathbf{x_{min}}$, and $p_1 = 0 \implies \mathbf{x} = \mathbf{x_1}$. Similarly for other K, the observation can be proved)
  - $q_k \geq 0$, the line is inside the boundary, and hence requires clipping

Liang-Barsky Line Clipping (cont.)

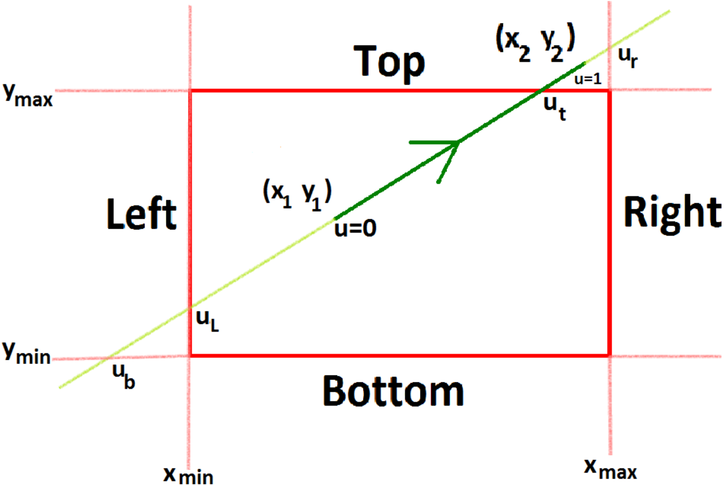


When pk=0 and qk<0



When pk=0 and qk>0

Liang-Barsky Line Clipping (cont.)



Line when $\Delta x > 0$ and $\Delta y > 0$;

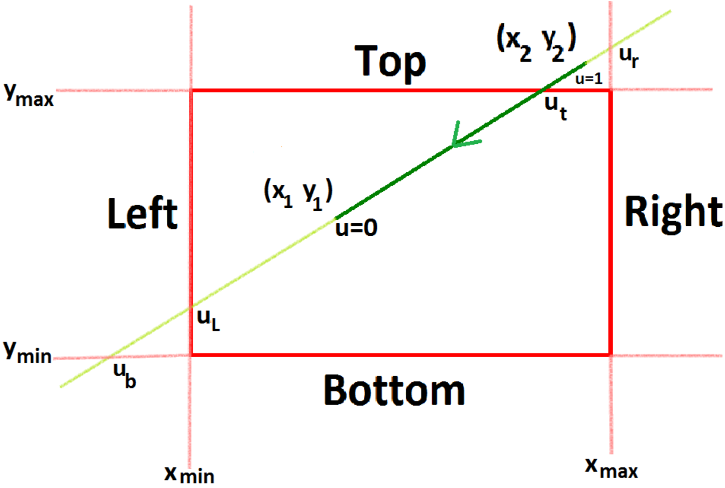
When $\Delta x > 0$ and $\Delta y > 0$

- (x, y) will increase in both x and y direction as u increases as $x = x_1 + u\Delta x$; $y = y_1 + u\Delta y$
- $p_1 = -\Delta x < 0$; $p_3 = -\Delta y < 0$
- But $p_2 = \Delta x > 0$; $p_4 = \Delta y > 0$
- Notice that the line moves from out side to inside of left edge(due to $p_1 < 0$) and also the line moves from outside to inside of the bottom edge(due to $p_3 < 0$)
- The line moves from inside to outside of top edge(due to $p_4 > 0$) and also the line moves from inside to outside of the right edge(due to $p_2 > 0$)
- Hence the clipping boundary corresponding to k:
  - When k=1, Left edge
  - When k=2, Right edge
  - When k=3, Bottom edge

- When k=4, Top edge

- **Conclusion when $\Delta x > 0$ and $\Delta y > 0$:**
  - If $p_k < 0$, the line proceeds from the outside to the inside of the corresponding clipping boundary line (consider infinite extensions in both line and window boundary).
  - If $p_k > 0$, the line proceeds from the inside to the outside of the corresponding clipping boundary line (consider infinite extensions in both line and window boundary).

Liang-Barsky Line Clipping (cont.)



Line when $\Delta x < 0$ and $\Delta y < 0$;

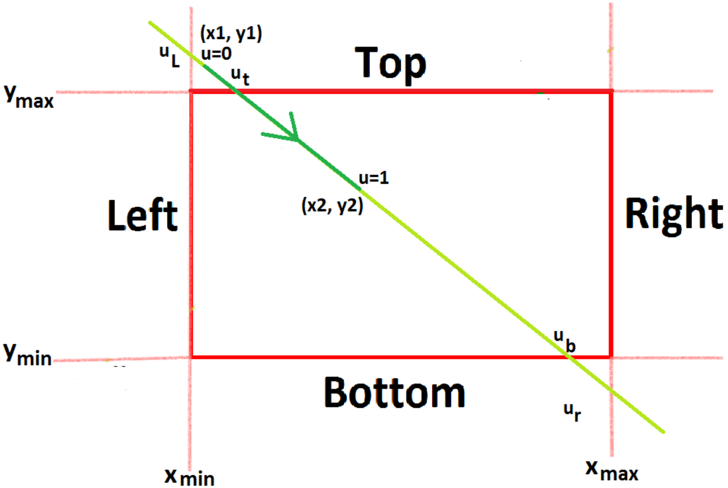
When $\Delta x < 0$ and $\Delta y < 0$

Liang-Barsky Line Clipping (cont.)

- (x, y) will decrease in both x and y direction as u increases as $x = x_1 + u\Delta x$; $y = y_1 + u\Delta y$
- $p_1 = -\Delta x > 0$; $p_3 = -\Delta y > 0$
- But $p_2 = \Delta x < 0$; $p_4 = \Delta y < 0$
- Notice that the line moves from out side to inside of right edge(due to $p_2 < 0$) and also the line moves from outside to inside of the top edge(due to $p_4 < 0$) (Justification when $k = 1$:
  $p_1 > 0 \implies \Delta x < 0 \implies x = x_1 + u\Delta x$ will decreases as u increases. As u moves from $-\infty$ *to* ∞, the x coordinate of the line ($x = x_1 + u\Delta x$) moves inside to outside of left clipping boundary)
- Notice that the line moves from inside to outside of bottom edge(due to $p_3 > 0$) and also the line moves from inside to outside of the left edge(due to $p_1 > 0$) (Justification when $k = 1$:
  $p_1 > 0 \implies \Delta x < 0 \implies x = x_1 + u\Delta x$ will decreases as u increases. As u moves from $-\infty$ *to* ∞, the x coordinate of the line ($x = x_1 + u\Delta x$) moves inside to outside of left clipping boundary. Similarly, for other k, boundary crossing can be proved)

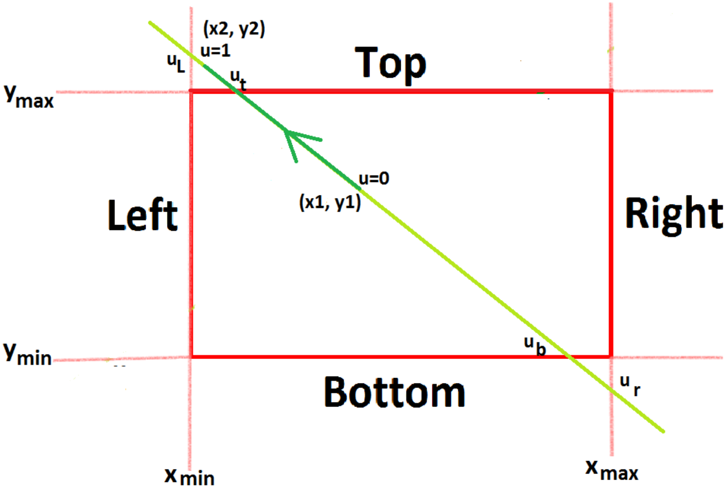
Liang-Barsky Line Clipping (cont.)

- Hence the clipping boundary corresponding to k:
  - When k=1, Left edge
  - When k=2, Right edge
  - When k=3, Bottom edge
  - When k=4, Top edge
- **Conclusion when $\Delta x < 0$ and $\Delta y < 0$:**
  - If $p_k < 0$, the line proceeds from the outside to the inside of the corresponding clipping boundary line (consider infinite extensions in both line and window boundary).
  - If $p_k > 0$, the line proceeds from the inside to the outside of the corresponding clipping boundary line (consider infinite extensions in both line and window boundary).



Line when $\Delta x > 0$ and $\Delta y < 0$; Hw: prove the same conclusion

Liang-Barsky Line Clipping (cont.)



Line when $\Delta x < 0$ and $\Delta y > 0$; HW: Prove the same conclusion

Liang-Barsky Line Clipping (cont.)

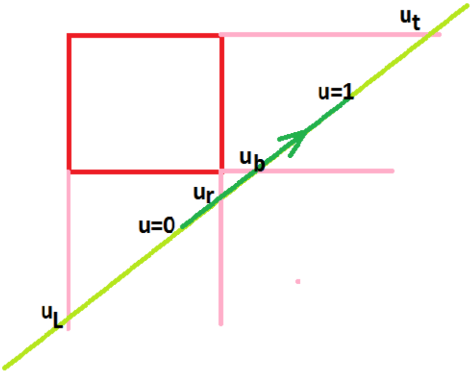
- Conclusion for arbitrary line:
  - If $p_k < 0$, the line proceeds from the outside to the inside of the corresponding clipping boundary line
    called as case 2
  - If $p_k > 0$, the line proceeds from the inside to the outside of the particular clipping boundary
    called as case 3
- In both case 2 and 3, the intersection parameter is calculated as follows:
  - Consider $q_1/p_1 = (x_1 - x_{min})/(-\Delta x)) = (x_{min} - x_1)/\Delta x = u$ (WKT $x = x_1 + u\Delta x$ for any x. put $x = x_{min}$)
  - Similarly, we can prove $u = \frac{q_k}{p_k}$ for other k
  - Hence $u = \frac{q_k}{p_k}$ is the parameter for intersecting point of line and corresponding window boundary(in case of $k = 1$, the boundary is $x = x_{min}$)

Liang-Barsky Line Clipping (cont.)

- Recall that the parametric equation of line passing through end points (x_1, y_1) $and(x_2, y_2)$ of the corresponding line segment is $x = x_1 + u\Delta x$; $y = y_1 + u\Delta y$ $-\infty < u < \infty$.
  - When $u = 0$, the point on the line is (x_1, y_1); and
  - When $u = 1$, the point on the line is (x_2, y_2)

Liang-Barsky Line Clipping (cont.)

- **Clipping:**
  - Let u_1 and u_2 be the values of u such that u_1 and u_2 correspond to the beginning and ending of portion of the line segment inside the window
  - Then $u_1 = max(\ \{q_k/p_k \mid p_k < 0\} \bigcup \{0\}\ )$,
  - Let $u_2 = min(\ \{q_k/p_k \quad p_k > 0\} \bigcup \{1\}\ )$
  - if$(u_1 > u_2)$ then reject the line as the line lies completely outside the window
else
Obtain (x, y) and (x', y') by substituting u_1 and u_2 for u respectively in the parametric line equation



Let u_l, u_b, u_r, u_t be the parameter values corresponding to the intersection of line and left, bottom, right and top boundaries of the window respectively

$$u_1 = max(0, u_l, u_b) = u_b; \;\; u_2 = min(1, u_r, u_t) = u_r; \;\; u_1 > u_2$$

Liang-Barsky Line Clipping (cont.)

Show that if($u_1 > u_2$) then the line lies completely outside the window
else
the line segment with endpoints (x, y) and (x', y') obtained when u_1 and u_2 are substituted for u in the parametric line equation lies completely inside the window

The proof can be given through geometry by considering each of the possible cases on Δx and Δy for given end points of a line segment

There are four cases:

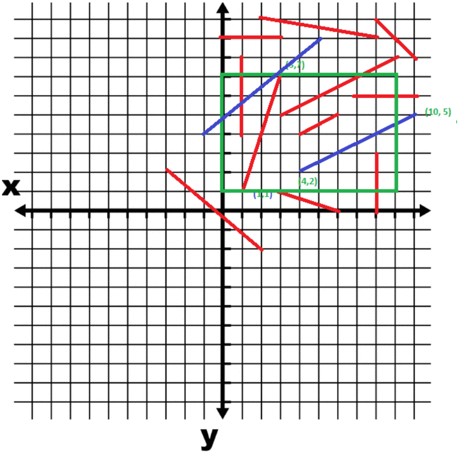
1. $\Delta x > 0$ and $\Delta y > 0$ (already done)
2. $\Delta x > 0$ and $\Delta y < 0$
3. $\Delta x < 0$ and $\Delta y > 0$
4. $\Delta x < 0$ and $\Delta y < 0$

Liang-Barsky Line Clipping (cont.)

The Algorithm

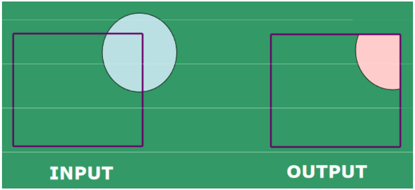
- if ($p_k = 0$ and $q_k < 0$) for any k , eliminate the line and stop
  else
  proceed to the next step
- Find $u_1 = max(\ \{q_k/p_k \mid p_k < 0\} \bigcup \{0\}\ )$
- Find $u_2 = min(\ \{q_k/p_k \quad p_k > 0\} \bigcup \{1\}\ )$
- if($u_1 > u_2$) eliminate the line as it completely lies outside the window
  else
  - Find $x_1' = x_1 + u_1 \Delta x$
  - Find $y_1' = y1 + u_1 \Delta y$
  - Find $x_2' = x_1 + u_2 \Delta x$
  - Find $y_2' = y_1 + u_2 \Delta y$
  - Output (x_1', y_1') and (x_2', y_2')

Liang-Barsky Line Clipping (cont.)



- Show the trace of Liang-Barsky algorithm for the above input

What about Circle/Ellipse clipping or for curves ??

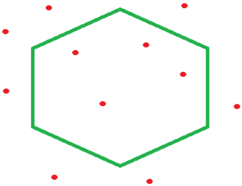


Home Work:

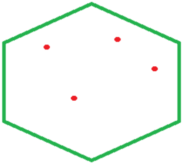
- Show that if($u_1 > u_2$) then the line lies completely outside the window; else the line segment with endpoints (x, y) and (x', y') obtained when u_1 and u_2 are substituted for u in the parametric line equation lies completely inside the window when
  - $\Delta x > 0$ and $\Delta y < 0$
  - $\Delta x < 0$ and $\Delta y > 0$
  - $\Delta x < 0$ and $\Delta y < 0$

Keep viewing window as polygon

- Clip Points
- Clip Lines
- Clip Polygons

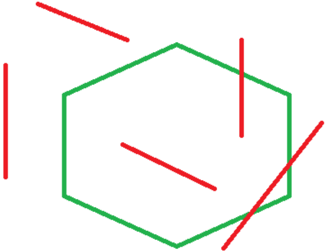


Input for point clipping with a polygon window

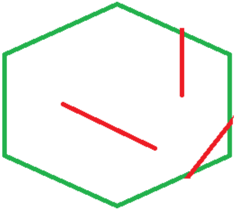


Output of point clipping with the polygon window

Clipping Lines using Polygon Window

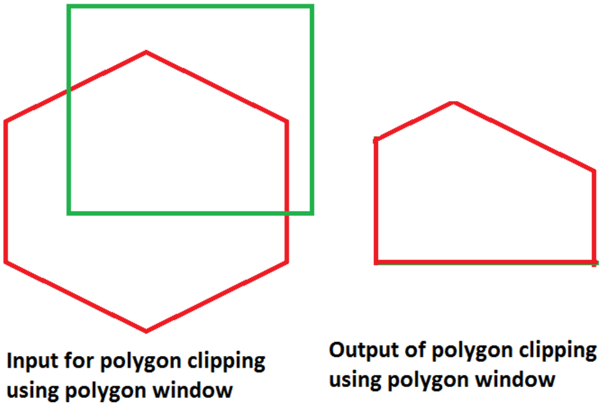


Input for line clipping using polygon window



Output of line clipping using polygon window

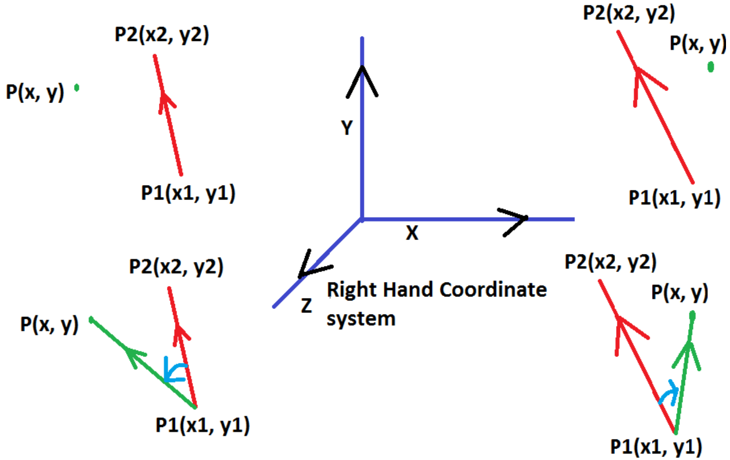
Clipping Polygon using Polygon Window



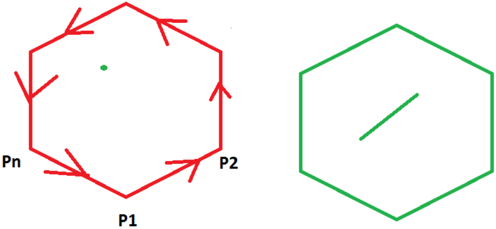
Clipping with Polygon Window (cont.)

- **Basic Problem:** Given a point p and a directed line l, check if p lies on left or right of the directed line
- **Soln:**
  - Consider the vectors $\vec{P_1P_2}$ and $\vec{P_1P}$
  - Find the cross product $\vec{P_1P_2} \times \vec{P_1P}$
  - if sign $(\vec{P_1P_2} \times \vec{P_1P})$ is positive then p lies on left; else on right
  - How to find $(\vec{P_1P_2} \times \vec{P_1P})$
    - $\vec{P_1P_2} = (x_2 - x_1, y_2 - y_1)$
    - $\vec{P_1P} = (x - x_1, y - y_1)$
    - $\vec{P_1P_2} \times \vec{P_1P} = ((x_2 - x_1)(y - y_1) - (y_2 - y_1)(x - x_1))\vec{k}$
    - if $((x_2 - x_1)(y - y_1) - (y_2 - y_1)(x - x_1)) > 0$ then p lies on left; else on right

Left-Right test:

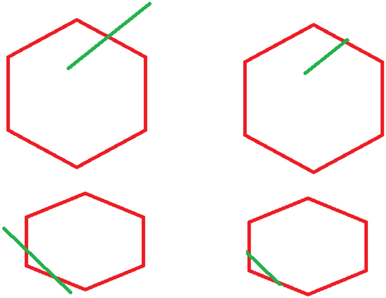


Inside-Outside test: Given a point P and a polygon window W, check if P lies inside W

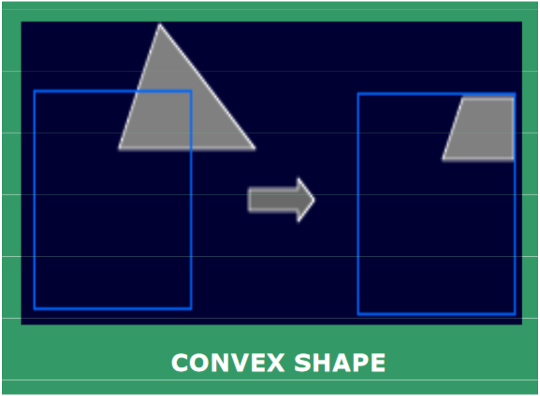


How to use

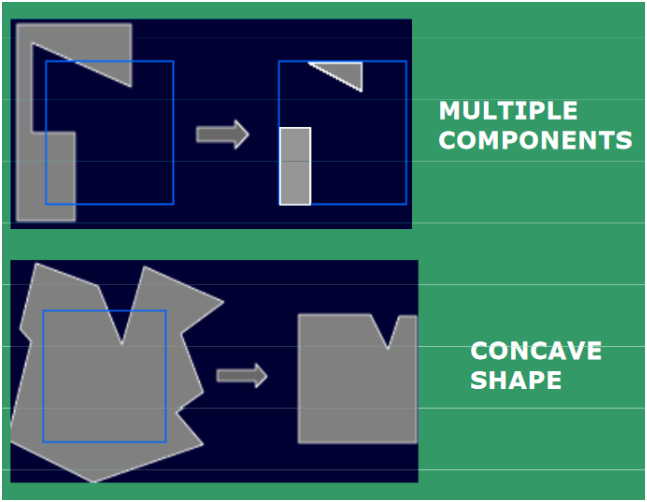
inside-outside test to clip line with polygon window



Examples of Clipping Polygon using Polygon Window



Clipping with Polygon Window (cont.)



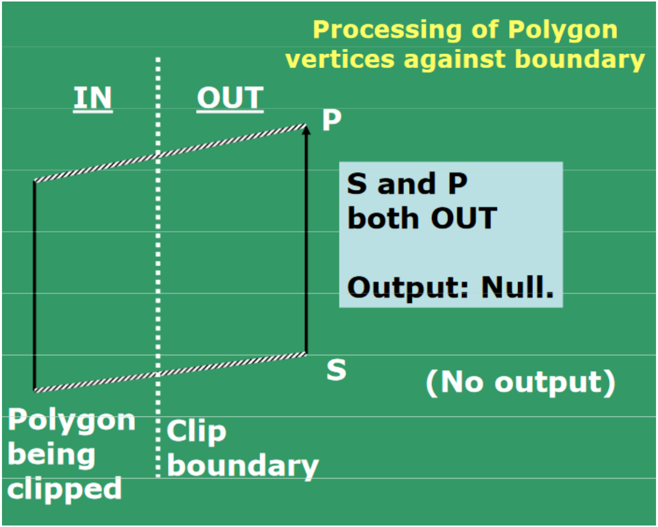
Sutherland-Hodgman Algorithm to clip a polygon with polygon window

- Input: Vertex sequence $P_1, ...P_N$ representing polygon to be clipped, and another vertex sequence $Q_1, ...Q_M$ representing convex polygon window(clipping window)
- Output: The sequence of vertices of clipped polygon
- For each edge $E = (A, B)$ in clipping polygon window
  for each edge $\overrightarrow{P_{i-1}P_i}$ in the polygon to be clipped
  - If both P_{i-1} and P_i lie on the left of the edge E then output the vertex P_i
  - If both P_{i-1} and P_i lie on the right of the edge E then output nothing
  - If P_{i-1} is on the left and P_i is on the right of the edge E, then find the intersection point I between the extended edge E and the line segment $\overrightarrow{P_{i-1}P_i}$, and output I

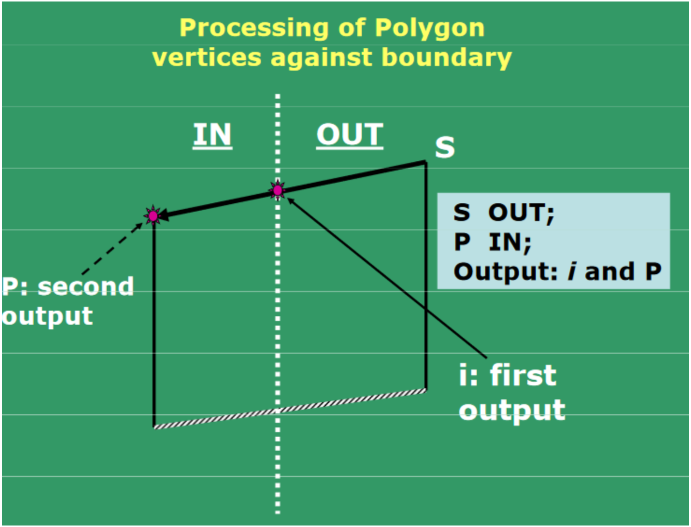
Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)

- If P_{i-1} is on the right and P_i is on the left of the edge E, then find the intersection point I between the extended edge E and the line segment $\overrightarrow{P_{i-1}P_i}$, and output I and P_i

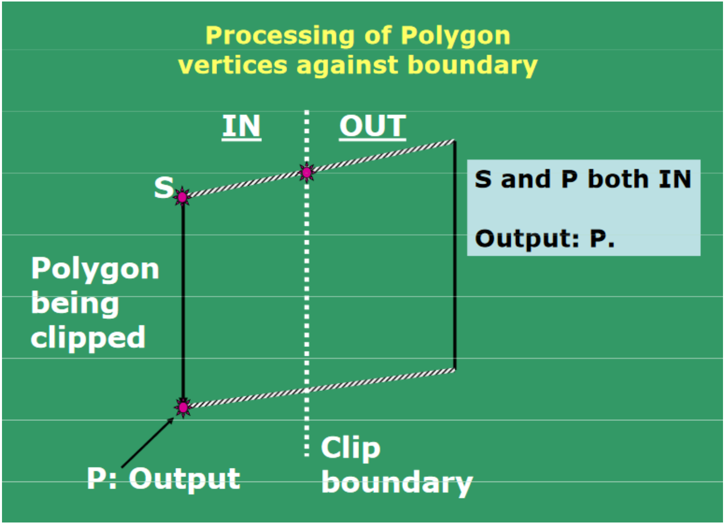
Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)



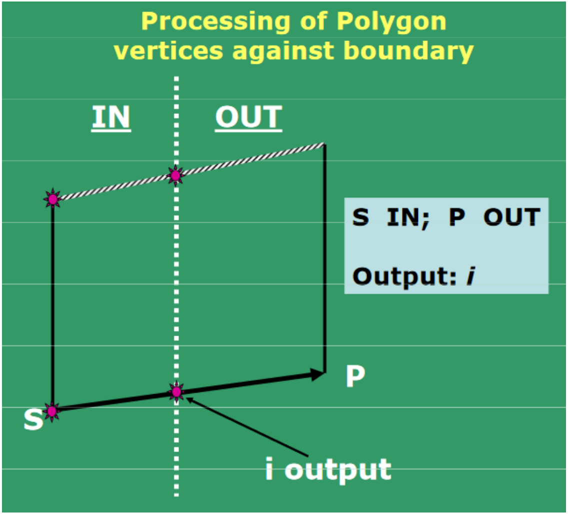
Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)



Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)

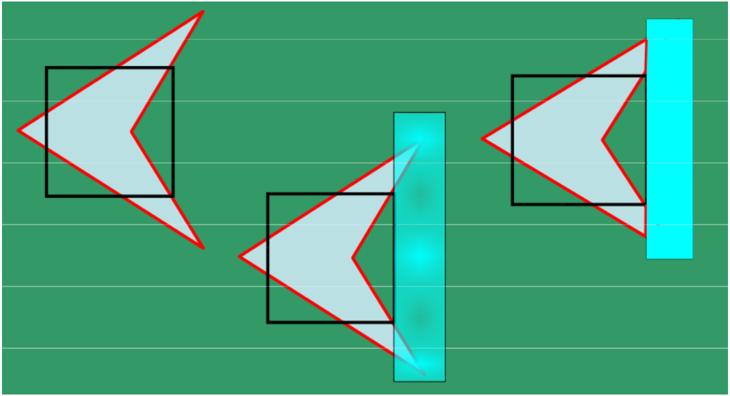


Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)

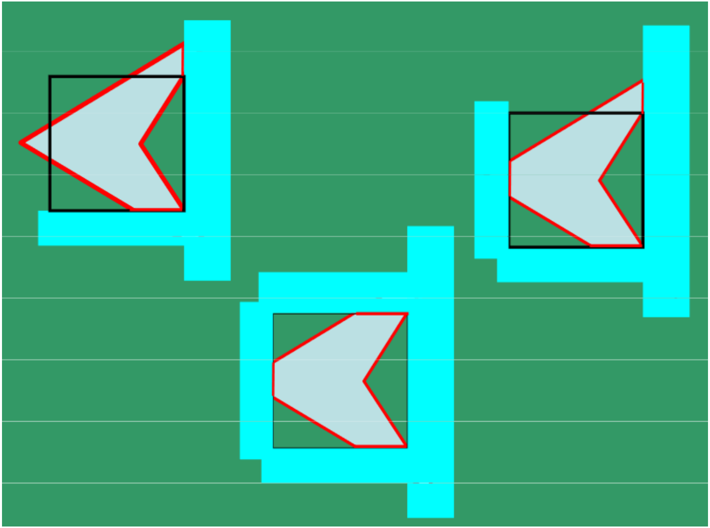


Trace of the algorithm

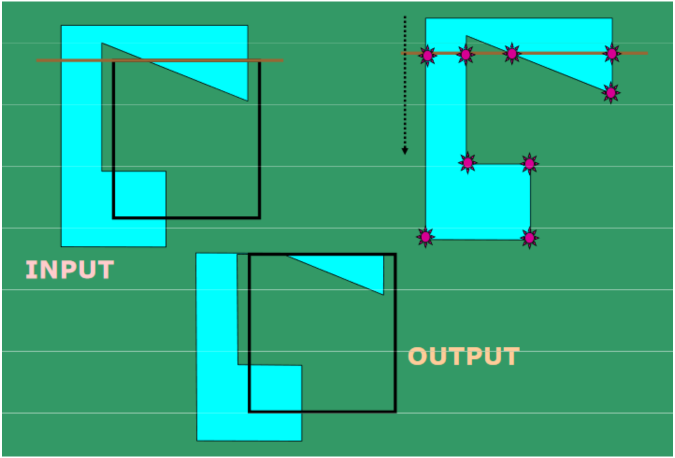
May have to add new vertices to the list.



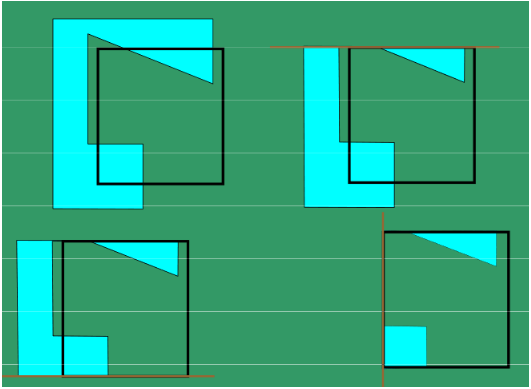
Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)



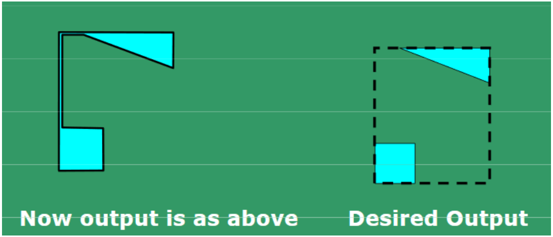
Problems with multiple components



Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)

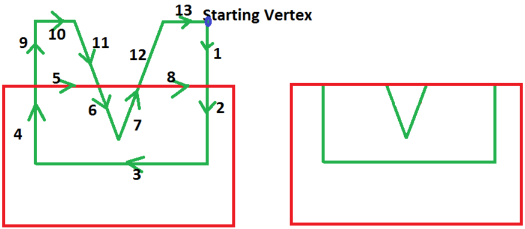


Sutherland-Hodgman Algorithm to clip a polygon with polygon window (cont.)



- Any Idea??
- Weiler-Atherton algorithm

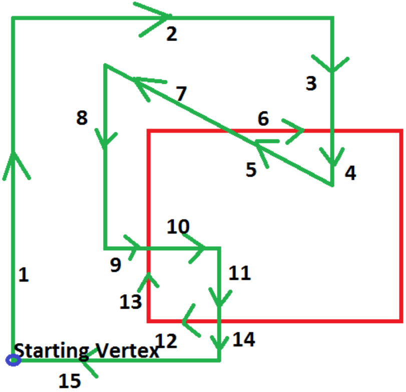
Weiler-Atherton algorithm to clip a polygon with polygon window

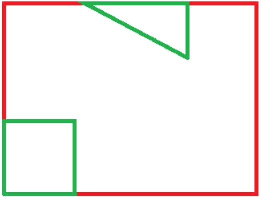


- For say, clockwise processing of polygons, follow:
  - For OUT $->$ IN pair, follow the polygon boundary
  - For IN $->$ OUT pair, follow Window boundary

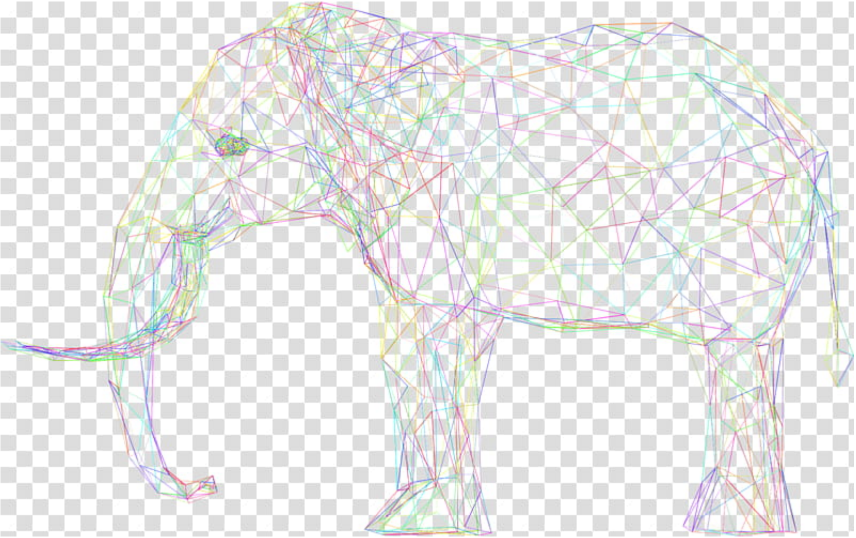
Weiler-Atherton algorithm to clip a polygon with polygon window

- For say, clockwise processing of polygons, follow:
  - For OUT $->$ IN pair, follow the polygon boundary
  - For IN $->$ OUT pair, follow Window boundary



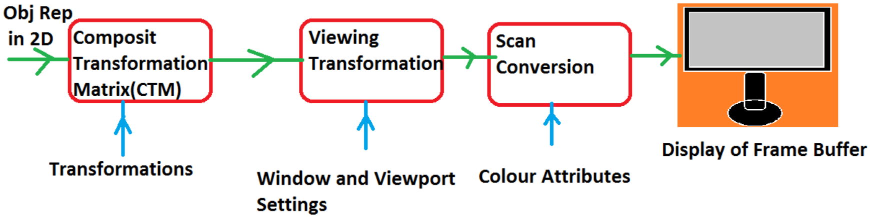


2D Graphics Pipeline



2D Graphics Pipeline (cont.)

2D Graphics Pipeline (cont.)



Acknowledgements

- Some of the slides have been adopted from NPTEL and different internet sources. The due credits are acknowledged.

Thank You! :)