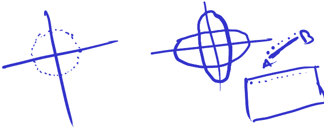

COM524T-Interactive Computer Graphics

Circle and Ellipse Drawing Algorithms

Dr. Ram Prasad Padhy

Computer Science & Engineering, IIITDM Kancheepuram

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Equation of a circle

$(x_c, y_c), r$ → x

$y = mx + c$

- General circle drawing problem: parameters are center and radius

  Center → (x_c, y_c)
- $(x - x_c)^2 + (y - y_c)^2 = r^2$ ⇒ $x^2 + y^2 = r^2$

  3 exponents
- $\Rightarrow y = y_c \pm \sqrt{r^2 - (x_c - x)^2}$
- $\boxed{\Rightarrow x = x_c \pm \sqrt{r^2 - (y_c - y)^2}}$

  $(x_c, y_c), r$

  $y = mx + c$
- Creates uneven spacing if plotted!!!

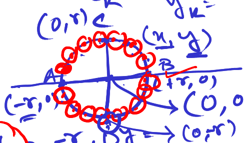
x_k, y_k
$(0, r)$
if from C
$y_{k+1} = y_k - 1$
$x_{k+1} =$
if from D
$y_{k+1} = y_k + 1$

$(y - y_c)^2 = r^2 - (x - x_c)^2$

$\Rightarrow y - y_c = \pm\sqrt{r^2 - (x - x_c)^2}$

$\Rightarrow y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$

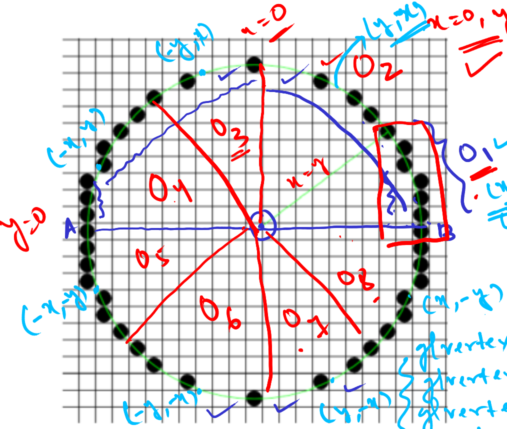
$x_k = x_k + 1, y =$



$x_k = -r, 0$
$x_k = x_k + 1$
$y =$

Drawing a circle

$x=0,\ y=0,\ x=y,\ x=-y$



glvertex(x,y)
glvertex(y,x)
glvertex(-y,x)
⋮

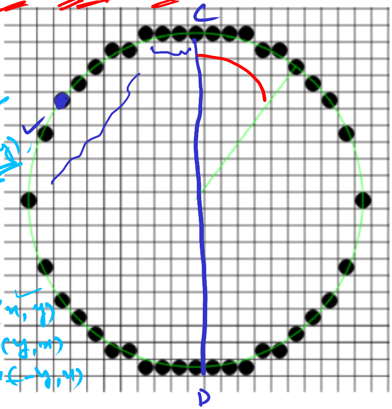


Figure : Plot with
$y = f(x) = y_c \pm \sqrt{r^2 - (x_c - x)^2}$

$x_{k+1} = x_k + 1$

Figure : Plot with
$x = f(y) = x_c \pm \sqrt{r^2 - (y_c - y)^2}$

Drawing a circle

$x^2 + y^2 = r^2$

parametric

$x = x_c + r\cos\theta$

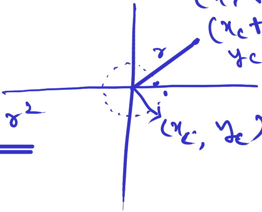
$y = y_c + r\sin\theta$

- How to eliminate uneven spacing?
- parametric polar form:
- $x = x_c + r\cos\theta$
- $y = y_c + r\sin\theta$

$x^2 + y^2 = r^2$

$(x - x_c)^2 + (y - y_c)^2 = r^2$

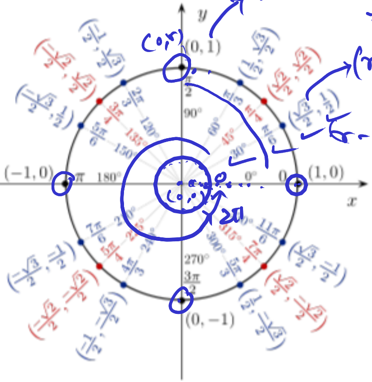
0, 1, 2, 3



Drawing a circle

(0,1) (1,0) (-1,0) (0,-1) $r=1$

- How to vary θ?
- We are aiming to plot $4r - 4$ points $\Rightarrow 4(r-1)$
- Step size for varying θ is $2\pi/4(r-1)$ $r=1$ 0
- Computation can be reduced by exploiting symmetry



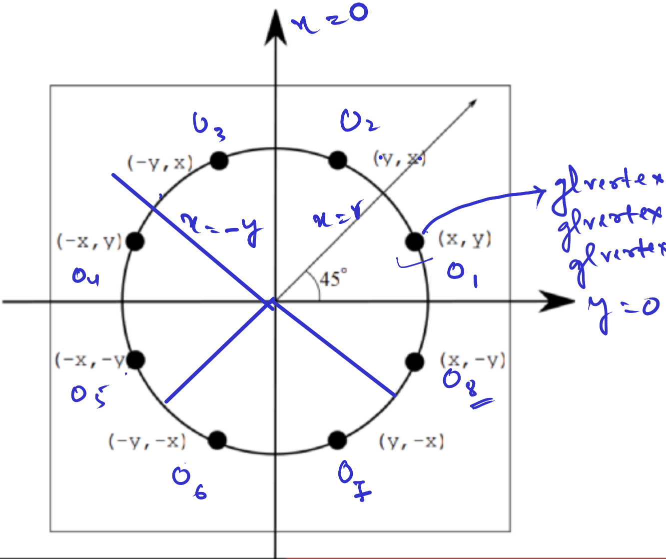
$(x_c, y_c) = (0,0)$

$(r\cos\theta, r\sin\theta)$

$(r-1)$ points in one Quadrant

Figure : Parametric circle drawing

Drawing a circle



glvertex2i(x,y)
glvertex2i(y,x)
glvertex2i(-y,x)
⋮

Drawing a circle

General line drawing

(x, y)

- $\Rightarrow y = y_c \pm \sqrt{r^2 - (x_c - x)^2}$
- $\Rightarrow x = x_c \pm \sqrt{r^2 - (y_c - y)^2}$
- Uneven plotting won't be there if we plot 1/8th of a circle! one octant
- Each pixel computation would need 3 addition (or subtraction) and 3 exponentiation (or root) operation

complexity is high

Midpoint Circle Drawing

MCD

$x^2+y^2=r^2$ $c=(0,0)$, radius $=r$.

$(x-x_c)^2+(y-y_c)^2=r^2$

- $f_{circle} = x^2 + y^2 - r^2$
- Without loss of generalization, we consider center at $(0, 0)$
-

$x^2+y^2=r^2$

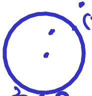
$\Rightarrow x^2+y^2-r^2=0$

$f(x,y)$

radius $=r$

$f_{circle}(x,y)$

$$f_{circle}(x, y) = \begin{cases} < 0; & \text{if} \quad (x, y) \text{ is inside the boundary} \\ = 0; & \text{if} \quad (x, y) \text{ is on the boundary} \\ > 0; & \text{if} \quad (x, y) \text{ is outiside the boundary} \end{cases}$$



(x,y)

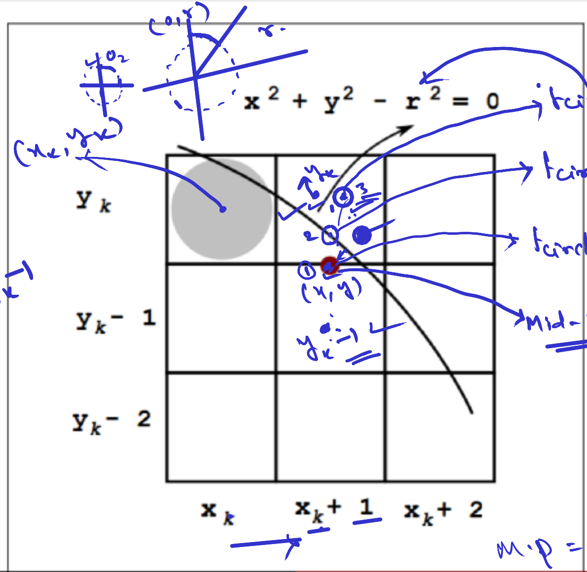
$x^2+y^2=r^2$

$\Rightarrow f_{circle}=0$

$x^2+y^2-r^2<0$

$x^2+y^2-r^2>0$

Finding Decision Parameter



$(0, r)$

x_k, y_k

x_{k+1}, y_{k+1}

$x_k + 1$ $y_k / y_k - 1$

$y_k - \frac{1}{2}$

$\frac{y_k + y_k - 1}{2}$

$(x_k + 1, y_k)$

$(x_k + 1, y_k - 1)$

$M.P = (x_k + 1, y_k - \frac{1}{2})$

Finding Decision Parameter

- Assuming (x_k, y_k) is plotted, we want to plot (x_{k+1}, y_{k+1})
- Without loss of generalization, we consider $x_{k+1} = x_k + 1$ and we check if $y_{k+1} = y_k$ or $y_{k+1} = y_k - 1$
- p_k

$$= f_{circle}(x_k + 1, \boxed{y_k - \tfrac{1}{2}})$$
$$= (x_k + 1)^2 + (y_k - \tfrac{1}{2})^2 - r^2$$

$f_{circle}(M)$

$f_{circle} = x^2 + y^2 - r^2$

$p_k < 0,\ y_{k+1} = y_k$

$p_k > 0,\ y_{k+1} = y_k - 1$

P_0 : integer

$P_1 = f(P_0)$

$\Rightarrow (x_k+1)^2 = P_k - (y_k - 1/2)^2 + r^2$

$$P_{k+1} = f(P_k) = f_{circle}(x_{k+1}+1,\ y_{k+1}-1/2)$$

$$= \left[(x_{k+1}+1)^2 + (y_{k+1}-1/2)^2 - r^2\right]$$

$$= \left[\{(x_k+1)+1\}^2 + (y_{k+1}-1/2)^2 - r^2\right]$$

$(a+b)^2 = a^2+b^2+2ab$

$$= \left[(x_k+1)^2 + 1 + 2(x_k+1) + (y_{k+1}-1/2)^2 - r^2\right]$$

$(a-b)^2 \to a^2+b^2-2ab$

$$= P_k + r^2 - (y_k-1/2)^2 + 1 + 2(x_k+1) + y_{k+1}^2 + 1/4 - y_{k+1} - r^2$$

$(a-b)^2 \to a^2+b^2-2ab$

circle

* parabola, hyperbola

$$= P_k - y_k^2 - \cancel{1/4} + y_k + 1 + 2(x_k+1) + y_{k+1}^2 + \cancel{1/4} - y_{k+1}$$

V.P

$$= P_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$$= f(P_k)$$

$$P_{k+1} = f(P_k) = f_{circle}(x_{k+1}+1,\ y_{k+1}-1/2)$$

$$= \left[(x_{k+1}+1)^2 + (y_{k+1}-1/2)^2 - r^2\right]$$

$(a+b)^2$

$= a^2 + b^2 + 2ab$

$$= \left[\{(x_k+1)+1\}^2 + (y_{k+1}-1/2)^2 - r^2\right]$$

$a^2 \quad a \quad b$

$$= \left[(x_k+1)^2 + 1 + 2(x_k+1) + (y_{k+1}-1/2)^2 - r^2\right]$$

$(a-b)^2 \rightarrow a^2+b^2-2ab$

circle

* parabola, hyperbola

$$= P_k + r^2 - (y_k - 1/2)^2 + 1 + 2(x_k+1) + y_{k+1}^2 + 1/4 - y_{k+1} - r^2$$

$(a-b)^2 \rightarrow a^2+b^2-2ab$

$$= P_k - y_k^2 - \cancel{1/4} + y_k + 1 + 2(x_k+1) + y_{k+1}^2 + \cancel{1/4} - y_{k+1}$$

V.P

$$= P_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$= f(P_k)$

Finding Decision Parameter

x_{k+1} 0 0

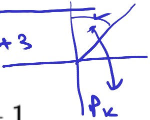
$x_k + 1$

Mid point lies inside the circle

- p_{k+1}
  $= f_{circle}(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$
  $= [(x_k + 1) + 1]^2 + (y_{k+1} - \frac{1}{2})^2 - r^2$

  $y_k - 1$

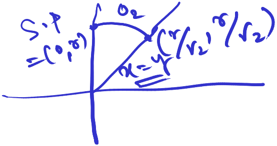
  $P_k + 2x_k + 3$



- $p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$

$r/\sqrt{2}, r/\sqrt{2} =$ $\left\{ x = y \right.$

$x^2+y^2=r^2$



P_k, P_{k+1}

P_0

$p_k > 0, \quad y_{k+1} = y_k - 1$

$$p_{k+1} = p_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$$= p_k + 2(x_k+1) + \left[(y_k-1)^2 - y_k^2\right] - (y_{k+1} - y_k) + 1$$

$$= p_k + 2(x_k+1) + \left[\cancel{y_k^2} + 1 - 2y_k - \cancel{y_k^2}\right] - (y_{k+1} - y_k) + 1$$

$$= p_k + 2(x_k+1) + 1 - 2y_k - y_{k+1} + y_k + 1$$

$$= p_k + 2(x_k+1) + 1 - 2y_k - \cancel{y_k} + 1 + \cancel{y_k} + 1$$

$$\boxed{\begin{cases} = p_k + 2x_{k+1} + 2 - 2y_k + 2 \\ = p_k + 2x_{k+1} + 1 - 2(y_k - 1) \\ = p_k + 2x_{k+1} + 1 - 2y_{k+1} \end{cases}}$$

$$p_{k+1} = p_k + 2x_k + 3 - 2y_k + 2$$

$$= p_k + 2x_k + 5 - 2y_k$$

(x_k, y_k)

Finding Decision Parameter

$p_k = f_{circle}(M, P)$

- $p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$
- When p_k was negetive, i.e. $y_{k+1} = y_k$, $p_{k+1} = p_k + 2x_{k+1} + 1$
- When p_k was positive, i.e. $y_{k+1} = y_k - 1$
  $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$
- Evaluation of x_{k+1} and y_{k+1} can be done as follows:
- $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$ when we start from $(0, r)$ and move towards $(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}})$

Midpoint lies inside the circle

$p_k + 2x_k + 3$

$P_0 = 1 - r$

$P_0 < 0$

$P_k > 0$, illuminate (x_k+1, y_k-1)

$x = y$

$P_k > 0, \quad y_{k+1} = y_k - 1$

$$P_{k+1} = P_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$$= P_k + 2(x_k+1) + \left[(y_k-1)^2 - y_k^2\right] - (y_{k+1} - y_k) + 1$$

$$= P_k + 2(x_k+1) + \left[\cancel{y_k^2} + 1 - 2y_k - \cancel{y_k^2}\right] - (y_{k+1} - y_k) + 1$$

$$= P_k + 2(x_k+1) + 1 - 2y_k - y_{k+1} + y_k + 1 \qquad y_{k+1} \to y_k - 1$$

$$= P_k + 2(x_k+1) + 1 - 2y_k - \cancel{y_k} + 1 + \cancel{y_k} + 1$$

$$\boxed{\begin{aligned} &= P_k + 2x_{k+1} + 1 - 2y_k + 2 \\ &= P_k + 2x_{k+1} + 1 - 2(y_k - 1) \\ &= P_k + 2x_{k+1} + 1 - 2y_{k+1} \end{aligned}}$$

$$P_{k+1} = P_k + 2x_k + 3 - 2y_k + 2 \qquad (x_k, y_k)$$

$$= P_k + 2x_k + 5 - 2y_k$$

Finding p_0 and halting the algorithm

- p_0
  $= f_{circle}(1, r - \frac{1}{2})$
  $= 1^2 + (r - \frac{1}{2})^2 - r^2$
  $= \frac{5}{4} - r$
  $= 1 - r$ (rounding p_0 for an integer value of r)
- Starting from $(0, r)$, stop when $x > y$

$p_k = f_{circle}(M, P)$

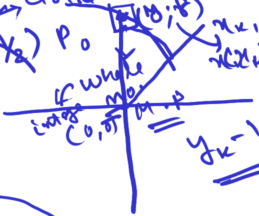
$p_k = 0$

$f_{circle}(x_k + 1, y_k - 1/2)$ p_0

x_k, y_k

$y_k - 1/2$

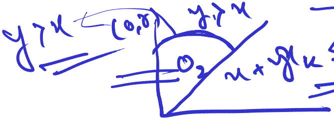
$x^2 + y^2 = r^2$



Restrict points in all octants

$p_0 < 0$

$p_1 = 2$.
illuminate (x_k+1, y_k)



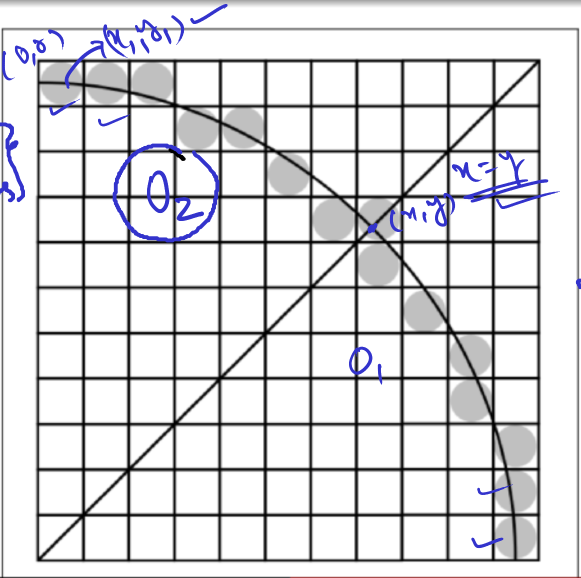
$x_k \le y_k$

$p_k > 0$,
$p_1 = 2$.
illuminate (x_k+1, y_k-1)

Circle drawn on a quadrant

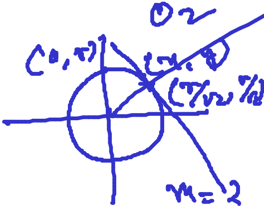
Decision parameters?

why



x = 0
y = 0
x = y
x = -y

$x^2+y^2=r^2$ circle



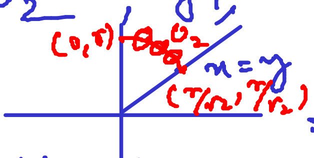
$\Rightarrow 2x+2y\cdot\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$ $\quad m = \frac{-x}{y}$

$\Rightarrow \underline{|m|} = \frac{x}{y}$ $\quad \underline{f(x,y)}$

$\underline{|m|<1} \rightleftharpoons f'(x,y)$

in O_2 $\quad y>x$



$\left|\frac{dy}{dx}\right|<1$

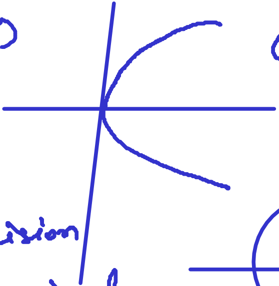
$\Rightarrow |dx|>|dy|$

$\underline{\text{S.P}}: (r/\sqrt{2}, r/\sqrt{2})$

$x_{k+1}=x_k-1$

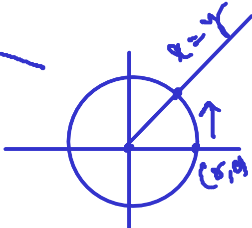
$y_{k+1} = y_k / y_k+1 \to$ depending on D.P

$y^2 = 4ax$ M.C.D Quadrant-1



Home-work

* How to find the Decision parameters in O, of a circle



Slope of tangent $|m| > 1$

$\left|\frac{dy}{dx}\right| > 1$

$|dy| > |dx|$

radius = r

center = (x_c, y_c)

$y_{k+1} = y_k + 1$

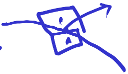
$x_{k+1} = x_k \mid x_k - 1$

P_0, P_k, P_{k+1}

Midpoint circle drawing algorithm

1. Input: r
2. $(x_0, y_0) \leftarrow (0, r)$
3. Load (x_0, y_0)
4. $p_0 \leftarrow (1 - r)$
5. $k \leftarrow 0$
6. if $(p_k < 0)$
   $x_{k+1} \leftarrow x_k + 1$, $y_{k+1} \leftarrow y_k$, Load $(x_k + 1, y_k)$
   $p_{k+1} \leftarrow p_k + 2x_{k+1} + 1$ OR $p_{k+1} \leftarrow p_k + 2x_k + 3$
7. Otherwise
   $x_{k+1} \leftarrow x_k + 1$, $y_{k+1} \leftarrow y_k - 1$, Load $(x_k + 1, y_k - 1)$
   $p_{k+1} \leftarrow p_k + 2x_{k+1} + 1 - 2y_{k+1}$ OR $p_{k+1} \leftarrow p_k + 2x_k + 5 - 2y_k$
8. $k \leftarrow (k + 1)$
9. Repeat steps 6,7,8 while $x_k \leq y_k$

$P_k = 0$



M.P

x_k, y_k

$x_k, y_k - 1$

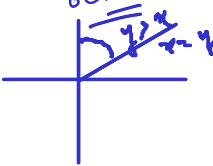
if whole integer no. = M.P

$y_k - 1/2$

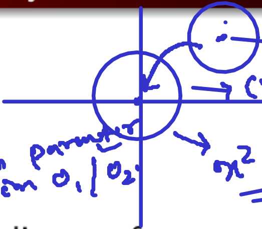
of vertex

Reflect points in all octants

$x_k \leq y_k$



A simple example by MCD



(3,4) $(x-3)^2+(y-4)^2 = r^2$

$(x_k, y_k) \rightarrow (x_k+3, y_k+4)$

$x^2+y^2=r^2 \rightarrow$ D.P

① find out the decision parameter in O_1 | O_2

② use these D.P

Find the coordinate of raster points on the circle with $r = 10$ having center at $(3, 4)$ $\rightarrow O_2$

MCD $x^2+y^2=r^2 \longrightarrow (x-3)^2+(y-4)^2=r^2$

center $\rightarrow (0,0)$

$x^2+y^2=10^2 \xrightarrow{+3,\ +4}$ (x_k, y_k)
+3   +4

Center → (3,4), radius = 10 $\quad O_2$

$P_0 = 1 - r = 1 - 10 = -9$

$(x_0, y_0) =$

$P_k < 0$, $x_{k+1} = x_k + 1$, $y_{k+1} = y_k$

$P_{k+1} = P_k + 2x_{k+1} + 1$

$P_k \geq 0$, $x_{k+1} = x_k + 1$, $y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$

K	P_k	x_{k+1}	y_{k+1}	P_{k+1}
-1		(0	10)	-9
0	-9	(1	10)	-6
1	-6	(2	10)	-1
2	-1	(3	10)	6
3	6	(4	9)	-3
4	-3	(5	9)	8
5	8	(6	8)	5
6	5	(7	7)	6

in octant 2

Center (0,0) ← $x^2 + y^2 = r^2$

Center (3,4) → $(x_k, y_k) \rightarrow (x_k + 3, y_k + 4)$

A simple example by MCD

$p_0 = 1 - 10 = -9$
$(x_0, y_0) \leftarrow (0, 10)$
$2x_0 = 0$
$2y_0 = 20$

k	p_k	x_{k+1}	y_{k+1}	$2x_{k+1}$	$2y_{k+1}$
-	-	0	10	0	20
0	-9	1	10	2	20
1	-6	2	10	4	20
2	-1	3	10	6	20
3	6	4	9	8	18
4	-3	5	9	10	18
5	8	6	8	12	16
6	5	7	7	14	14

A simple example by MCD

Shifting center at $(3, 4)$

x	y	$x + x_c$	$y + y_c$
0	10	3	14
1	10	4	14
2	10	5	14
3	10	6	14
4	9	7	13
5	9	8	13
6	8	9	12
7	7	10	11

in octant 2

Midpoint Circle Drawing

$O_2 \rightarrow |m| < 1$

$O_1 \rightarrow |m| > 1$

$x = y$

- What would happen if we do not stop at $x_{k+1} = y_{k+1}$?
- If we dont consider the computation, would the answer be correct? NO
- WHY? because our decision parameter p_k is designed to check inclusion or exclusion of the midpoint along y axis and not along x axis, which role is reversed once we cross $x = y$

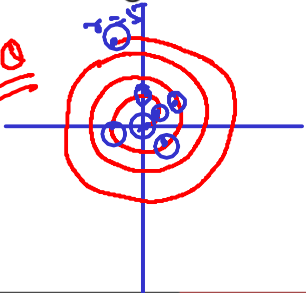
$\{ y_k, \; y_k - 1 \}$

$y_k - 1/2$

Spiral Drawing

Can a spiral be drawn based on any circle drawing algorithm?

$r = k\theta$



① General Formula

② polar co-ordinates

③ MCD

Spiral Drawing

(x_c, y_c)

circle $\begin{cases} x = r\cos\theta + x_c \\ y = r\sin\theta + y_c \end{cases}$

$r_o = 5, \; r_i = 0$

- $x = x_c + r_\theta \cos\theta$ ($r_\theta \to 0$)
- $y = y_c + r_\theta \sin\theta$
- $r_\theta = \frac{(r_{outer} - r_{inner}) \times \theta}{2\pi}$ keeps on recomputed as intermediate value of r

  $r_\theta = \frac{(5-0)\times 0}{2\pi} = 0 = r_\theta$

  $\theta \to [0, 2\pi]$
- How to vary θ?
- We aim to plot $4r - 4$ points for a circle with radius r
- Step size for varying θ is $2\pi/4(r_\theta - 1)$ ($\to r_{outer}$)

At end $r_\theta = \frac{(5-0)2\pi}{2\pi} = 5$ r_{outer}

$r_\theta = 0$ at begin

$\frac{2\pi}{4(r_{outer} - 1)}$

Equation of an ellipse

- General ellipse drawing problem: parameters are center and radius along x and y axes
- $(\frac{x-x_c}{r_x})^2 + (\frac{y-y_c}{r_y})^2 = 1$
- $\Rightarrow y = y_c \pm \frac{r_y}{r_x}\sqrt{r_x^2 - (x_c - x)^2}$
- $\Rightarrow x = x_c \pm \frac{r_x}{r_y}\sqrt{r_y^2 - (y_c - y)^2}$
- Creates uneven spacing if plotted!!!

Drawing an ellipse

- How to eliminate uneven spacing?
- parametric polar form:
- $x = x_c + r_x \cos\theta$
- $y = y_c + r_y \sin\theta$

Drawing an ellipse

- How to vary θ?
- We are aiming to plot $p = max(4r_x - 4, 4r_x - 4)$ points
- Step size for varying θ is $2\pi/p$
- Computation can be reduced by exploiting symmetry

Rationally choosing the independent axes

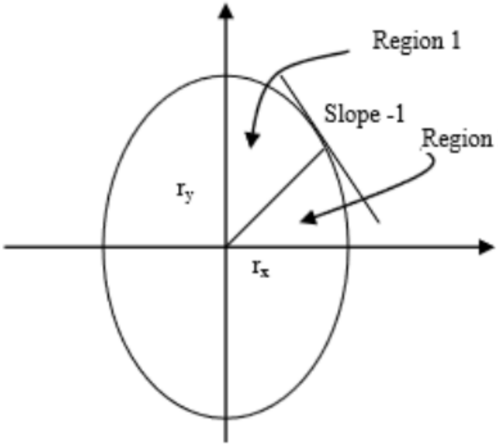


Figure : Rationally choosing the independent axes

Midpoint Ellipse Drawing

- $f_{ellipse} = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$
- Without loss of generalization, we consider center at $(0,0)$
- $$f_{ellipse}(x,y) = \begin{cases} <0; & \text{if} \quad (x,y) \text{ is inside the boundary} \\ =0; & \text{if} \quad (x,y) \text{ is on the boundary} \\ >0; & \text{if} \quad (x,y) \text{ is outiside the boundary} \end{cases}$$
- Slope of ellipse at $(x,y) = \frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y}$

Midpoint Ellipse Drawing

- The algorithm starts from $(0, r_y)$ (slope $= 0$)
- We move towards $(r_x, 0)$ (slope $= -\infty$)
- We move from Region 1 to 2 when $\frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y} \geq -1$, i.e. $r_y^2 x \geq r_x^2 y$
- Midpoint parameters are formed in different way in two regions
- Region 1: Change in $x >$ Change in y
- Region 2: Change in $y >$ Change in x

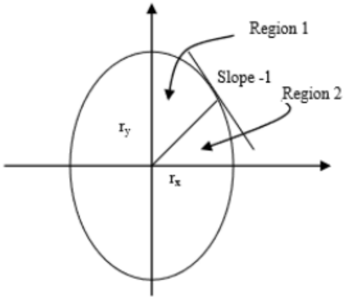


Figure : Rationally choosing the independent axes

Finding Decision Parameter in Region 1

- Assuming (x_k, y_k) is plotted, we want to plot (x_{k+1}, y_{k+1})
- We consider $x_{k+1} = x_k + 1$ and we check if $y_{k+1} = y_k$ or $y_{k+1} = y_k - 1$
- $p1_k$
  $= f_{ellipse}(x_k + 1, y_k - \frac{1}{2})$
  $= r_y^2(x_k + 1)^2 + r_x^2(y_k - \frac{1}{2})^2 - r_x^2 r_y^2$

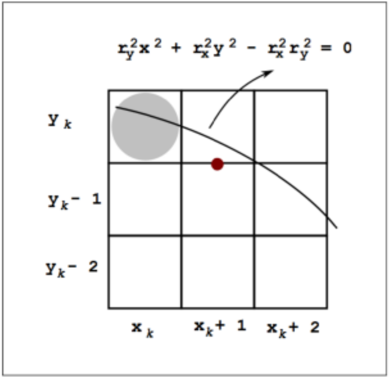


Figure : Decision parameter in Region 1

Finding Decision Parameter in Region 1

- $p1_k$
  $= f_{ellipse}(x_k + 1, y_k - \frac{1}{2})$
  $= r_y^2(x_k + 1)^2 + r_x^2(y_k - \frac{1}{2})^2 - r_x^2 r_y^2$
- $p1_{k+1}$
  $= f_{ellipse}(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$
  $=$
  $r_y^2[(x_k + 1) + 1]^2 + r_x^2(y_{k+1} - \frac{1}{2})^2 - r_x^2 r_y^2$
- $p1_{k+1}$
  $= p1_k + 2r_y^2(x_k + 1) + r_y^2 +$
  $r_x^2[(y_{k+1} - \frac{1}{2})^2 - (y_k - \frac{1}{2})^2]$
- IF $p1_k < 0$
  $y_{k+1} = y_k$
  $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$
- OTHERWISE
  $y_{k+1} = y_k - 1$
  $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}$

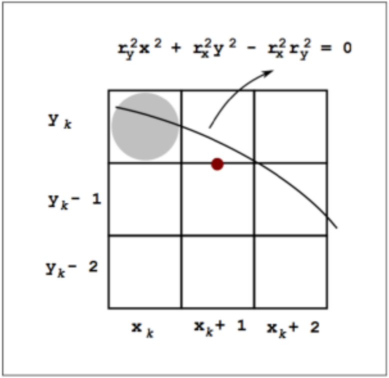


Figure : Decision parameter in Region 1

Finding $p1_0$

- $p1_0$
  $= f_{ellipse}(1, r_y - \frac{1}{2})$
  $= r_y^2 + r_x^2(r_y - \frac{1}{2})^2 - r_x^2 r_y^2$
  $= r_y^2 - r_x^2 r_y + \frac{r_x^2}{4}$

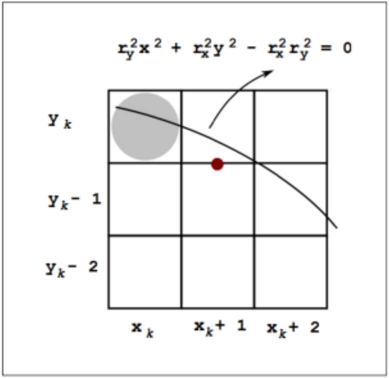


Figure : Decision parameter in Region 1

Finding Decision Parameter in Region 2

- Assuming (x_k, y_k) is plotted, we want to plot (x_{k+1}, y_{k+1})
- We consider $y_{k+1} = y_k - 1$ and we check if $x_{k+1} = x_k$ or $x_{k+1} = x_k + 1$
- $p2_k$
  $= f_{ellipse}(x_k + \frac{1}{2}, y_k - 1)$
  $= r_y^2(x_k + \frac{1}{2})^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2$

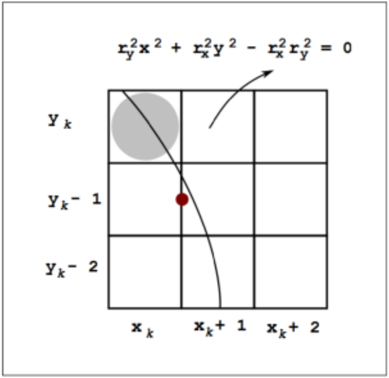


Figure : Decision parameter in Region 2

Finding Decision Parameter in Region 2

- $p2_k$
  $= f_{ellipse}(x_k + \frac{1}{2}, y_k - 1)$
  $= r_y^2(x_k + \frac{1}{2})^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2$
- $p2_{k+1}$
  $= f_{ellipse}(x_{k+1} + \frac{1}{2}, y_{k+1} - 1)$
  $=$
  $r_y^2(x_{k+1} + \frac{1}{2})^2 + r_x^2[(y_k - 1) - 1]^2 - r_x^2 r_y^2$
- $p2_{k+1}$
  $= p2_k - 2r_x^2(y_k - 1) + r_x^2 +$
  $r_y^2[(x_{k+1} + \frac{1}{2})^2 - (x_k + \frac{1}{2})^2]$
- IF $p2_k > 0$
  $x_{k+1} = x_k$
  $p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$
- OTHERWISE
  $x_{k+1} = x_k + 1$
  $p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2 + 2r_y^2 x_{k+1}$

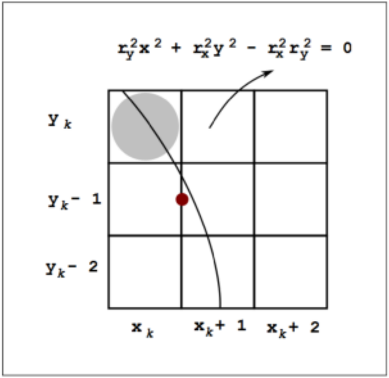


Figure : Decision parameter in Region 2

Finding $p2_0$

- $p2_0$
  $= f_{ellipse}(x_0 + \frac{1}{2}, y_0 - 1)$
  $= r_y^2(x_0 + \frac{1}{2})^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$
- (x_0, y_0) is the last point of Region 1.
- $(x_0, y_0) = (\left\lceil \sqrt{\frac{r_x^4}{r_x^2 + r_y^2}} \right\rceil, \left\lfloor \sqrt{\frac{r_y^4}{r_x^2 + r_y^2}} \right\rfloor)$

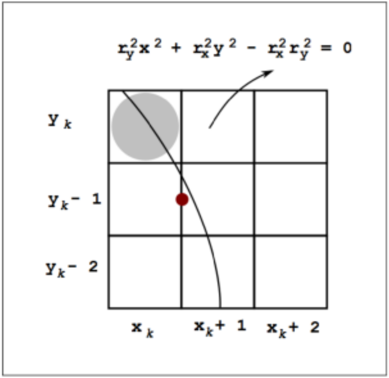


Figure : Decision parameter in Region 2

Midpoint ellipse drawing algorithm

1. Input: r_x, r_y
2. $(x_0, y_0) \leftarrow (0, r_y)$
3. Load (x_0, y_0)
4. $p1_0 \leftarrow r_y^2 - r_x^2 r_y + r_x^2/4$
5. $k \leftarrow 0$
6. if $(p1_k < 0)$
   $x_{k+1} \leftarrow x_k + 1,\ y_{k+1} \leftarrow y_k,$
   Load $(x_k + 1, y_k)$
   $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$
7. Otherwise
   $x_{k+1} \leftarrow x_k + 1,\ y_{k+1} \leftarrow y_k - 1,$
   Load $(x_k + 1, y_k - 1)$
   $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}$
8. $k \leftarrow (k + 1)$
9. Repeat steps 6,7,8 while $r_y^2 x_k \not> r_x^2 y_k$
10. $(x_0, y_0) \leftarrow (x_k, y_k)$
11. Load (x_0, y_0)
12. $p2_0 \leftarrow$
    $r_y^2(x_0 + 1/2)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$
13. $k \leftarrow 0$
14. if $(p2_k > 0)$
    $x_{k+1} \leftarrow x_k,\ y_{k+1} \leftarrow y_k - 1,$
    Load $(x_k, y_k - 1)$
    $p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$
15. Otherwise
    $x_{k+1} \leftarrow x_k + 1,\ y_{k+1} \leftarrow y_k - 1,$
    Load $(x_k + 1, y_k - 1)$
    $p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2 + 2r_y^2 x_{k+1}$
16. $k \leftarrow (k + 1)$
17. Repeat steps 14,15,16 while $y_k > 0$