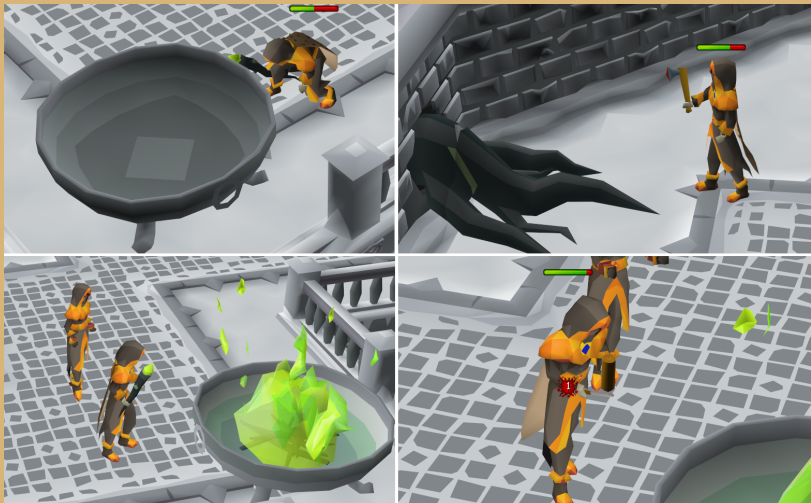# Optimizing Runescape
Wintertodt

Actions

A player can perform several actions during a game. Some of them give firemaking experience. We can model this skilling boss to gain insight, make predictions, and guide players.

Policies

$$c_n \equiv \delta_{\text{root}}^n M_{\text{root}} + \delta_{\text{kindling}}^n M_{\text{kindling}}$$

$$c_n^{\text{OBR}} \equiv \begin{cases} M_{\text{root}} & \text{if } n \leq T/P_{\text{root}} \\ 0 & \text{otherwise.} \end{cases}$$

$$c_n^{\text{KTB}} \equiv \begin{cases} M_{\text{kindling}} & \text{if } n \leq 500/P_{\text{kindling}} \\ M_{\text{root}} & \text{if } n \leq \frac{T}{P_{\text{root}}} - \frac{P_{\text{kindling}}}{P_{\text{root}}} \lceil 500/P_{\text{kindling}} \rceil \\ 0 & \text{otherwise.} \end{cases}$$

The amount and order that the player performs these actions changes the experience and points earned. We can define *policies* that describe a sequence of player actions, eg:
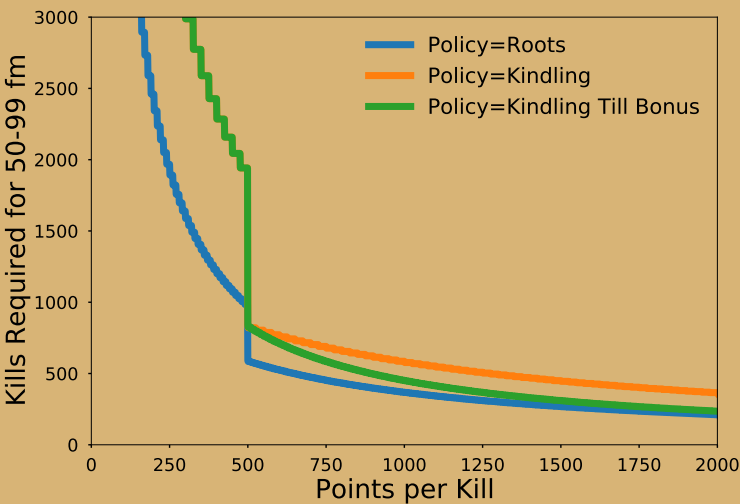
1. Only Burn Roots (OBR)

2. Kindle Till Bonus (KTB)

c_n^{OBR} says that Bruma roots are burned until reaching a target of T points. c_n^{KTB} says that Bruma kindling is burned for the first 500 points followed by roots until reaching T points.

Experience & Kills

Within a Fight: $E_{n+1} = E_n + c_n \mathcal{L}(E_n)$

Between Fights: $\mathcal{E}^{k+1} = \mathcal{E}^k + E_{N(T;\text{policy})}^{\text{policy}} + \theta(T - 500) M_{\text{bonus}} \mathcal{L}(\mathcal{E}^k)$


The experience gained from an action depends on the previous actions since the player may level up during a fight. After each fight, we also add in the bonus experience. So, the experience gained can be modeled as a set of two *recursive* equations. The Only Burn Kindling, and Only Burn Roots policies act as upper and lower bounds for the kills required to reach a given level (for any playstyle/policy).

Rewards

$$\text{Rolls} = \begin{cases} \min(1 + p/500, 28) & \text{if } p \geq 500 \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{Total Value} = \text{Kills} \times \text{Rolls} \times \text{Roll Value}$$
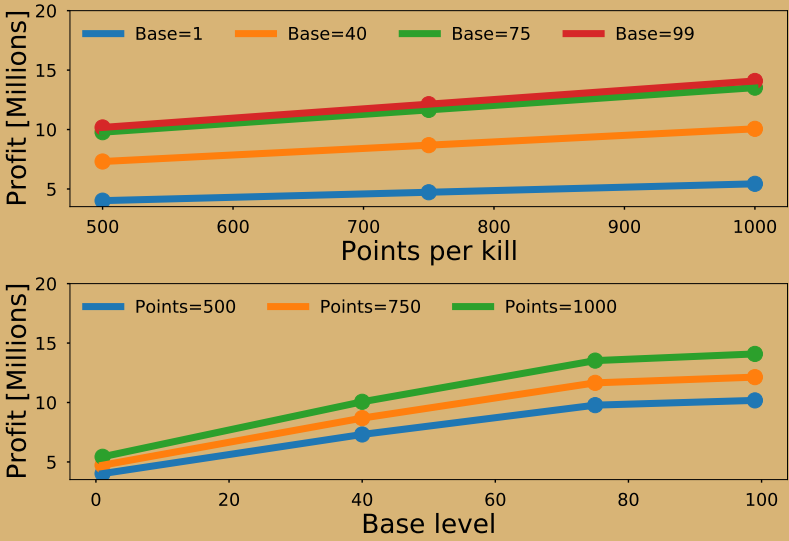
The value of a crate comes from some of the player's other skill levels. We can use the Wiki calculator to determine the value of a crate for a player with a base skill level. Combining this with the number of kills required gives an estimate for the total value expected over the course of 99 firemaking.

Takeaways
(Under the assumptions of this model!)

1. 50-99 takes a *maximum* of 839 kills (70h).

2. 50-99 at base 40s yields about 7-8 million gp.

3. Value doesn't increase much past base 70s

4. Value increases linearly as points earned increases.

5. For 500 point games, 591-839 kills are required (49-70h) (for any policy!)

6. For 750 point games, 455-690 kills are required (38-58h) (for any policy!)