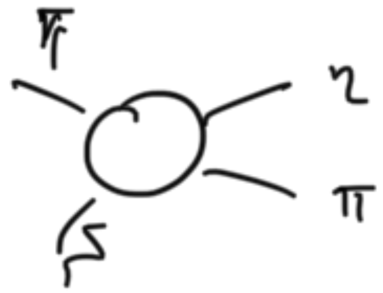
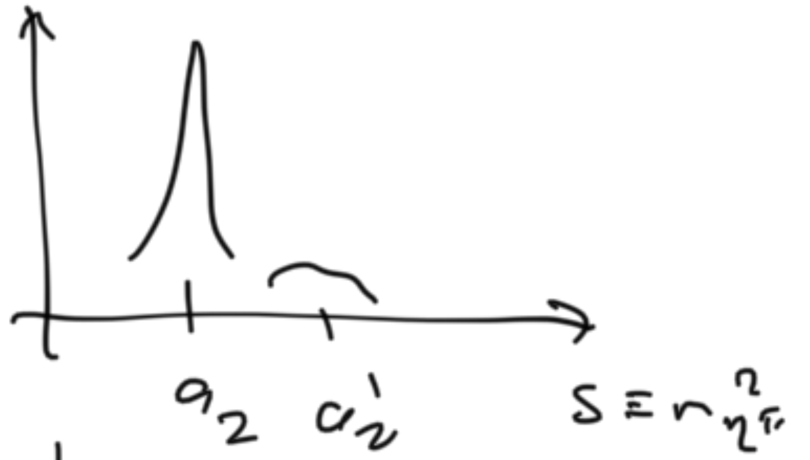
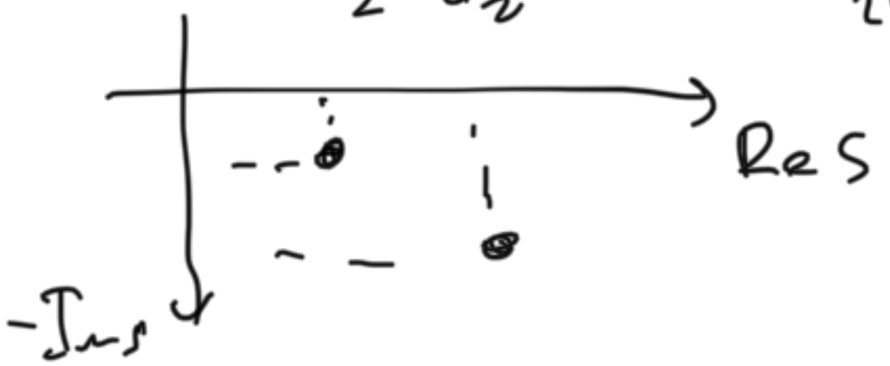
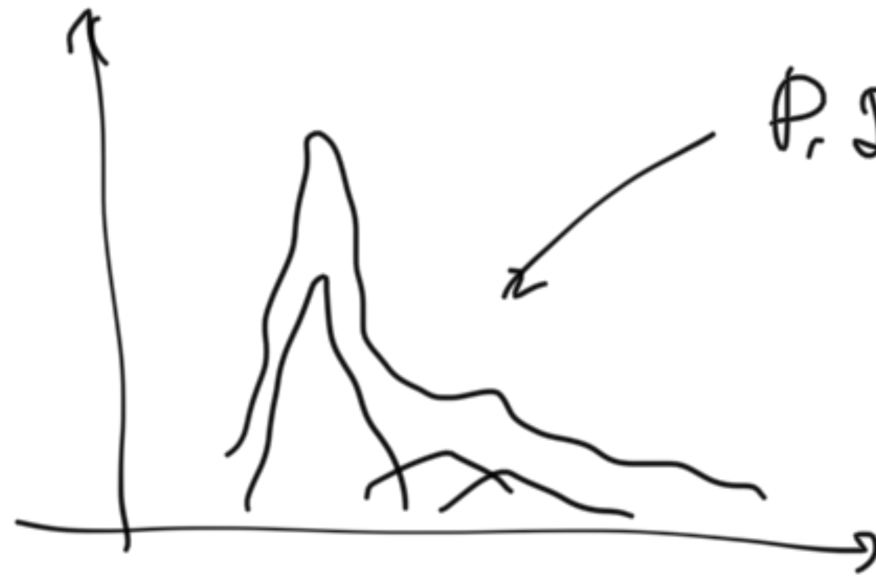
1. Study of $\eta\pi$ D-wave amplitude



$\leftarrow$ D-wave $\quad A(m^2_{\eta\pi}) \quad m^2_{\eta\pi} = (p_\eta + p_\pi)^2$







P, D, F, ... waves

$\eta^0 : J^P = 0^- \; ; \; \pi^- \; J^P = 0^-$

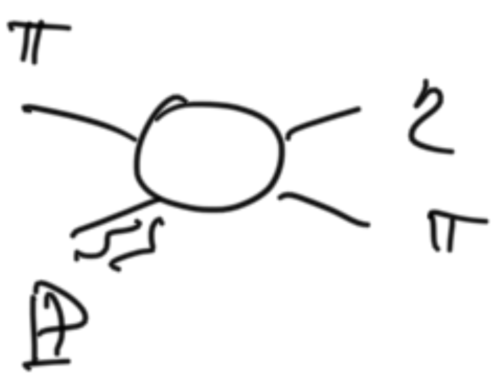
$R : (\eta\pi^-)$ charged $\; ; \; 0^- \otimes 0^- \times \underbrace{L^P}_{L \text{ wave}} \; ; \; P = (-1)^L$

$\Rightarrow$ Resonances in D-wave are 2^+ $\qquad 0^- \otimes 0^- \otimes 2^+ = 2^+$

Resonances in P-wave are 1^-

F-wave $\rightarrow 3^-$

. — —



$C(-1)^I = G \; , I = 1$ for charged particles $^{+1}_{0}$

-1

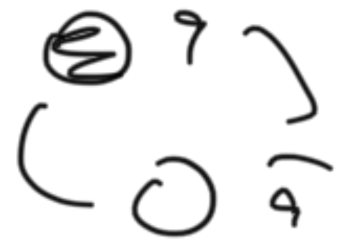
$$G(\pi P) = 1$$

$$\Rightarrow C(\text{init}) = G(-1)^I \Rightarrow -1(-1)^1 = +1 \Rightarrow C(\text{final}) = +1$$

$$P:\ 1^{-+},\quad D:\ 2^{++},\quad F:\ 3^{-+}$$

Meson:

$$\tfrac{1}{2}^+ \otimes \tfrac{1}{2}^- = 0^{-+} \oplus 1^{--} \leftarrow \text{s-wave}$$



$$P:\ 1^{+-} \oplus 0^{++}\ 1^{++}\ 2^{++}$$

$$D:\ 2^{-+} \oplus 1^{--}\ 2^{--}\ 3^{--}$$

does not have 0^{+-} 2^{+-}, 1^{-+}, 3^{-+}

J^{PC} exotic states

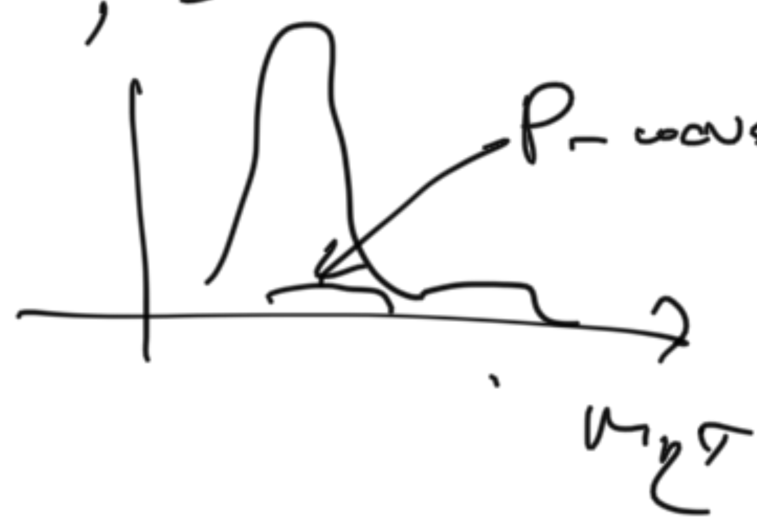
$$C(q\bar{q}) = (-1)^{L+S}$$

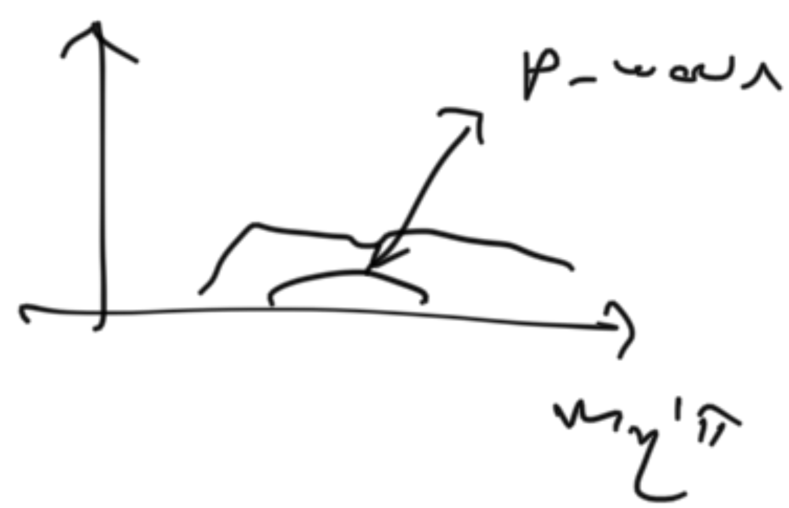
contribute to the quantum numbers

$$\Rightarrow 1^{-+},\ 3^{-+}$$

P wave of $\eta\pi$, $\eta'\pi$

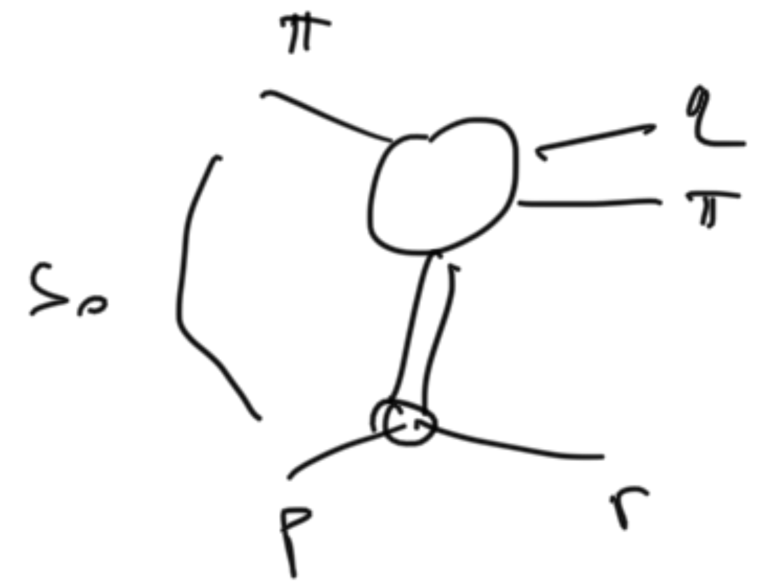
$\Rightarrow$ exotic state





$\pi_1(1600)$

Separation of the partial waves



$E_b = 190$ GeV

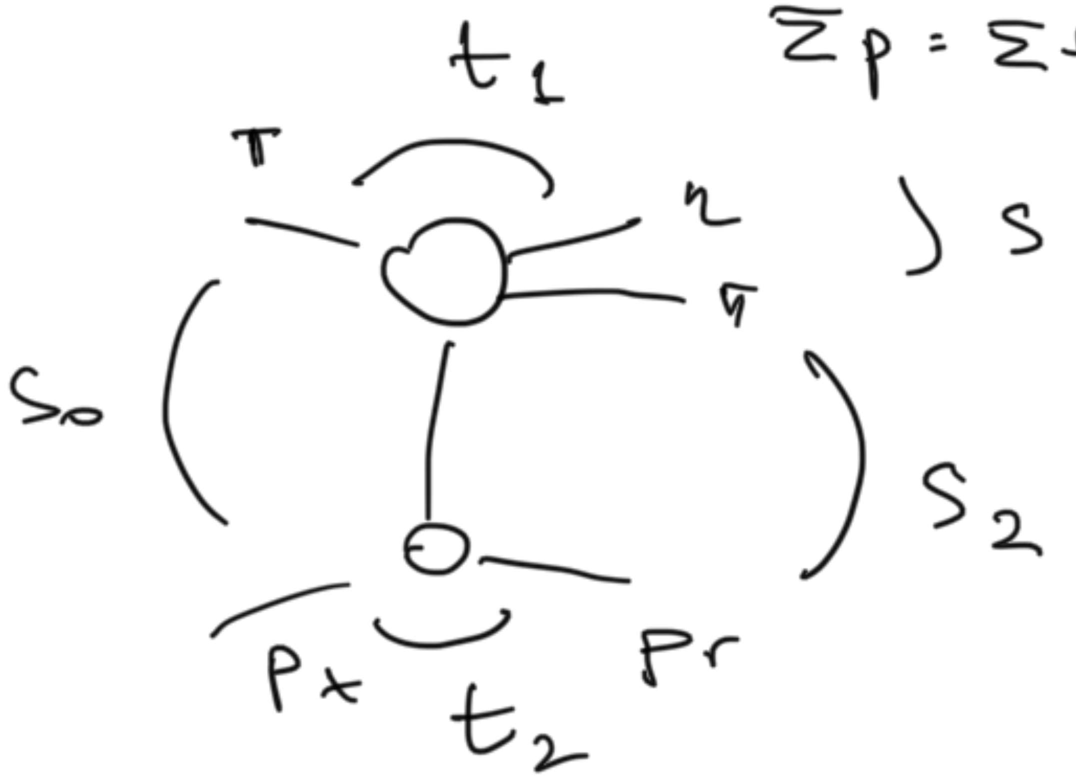
$s_0 \simeq (19\ \text{GeV})^2 = (p_b + p_t)^2 \longrightarrow$

$2 \to 3$

$$A(p_b, p_t; p_\eta, p_\pi, p_r) \qquad 4 \cdot 5 = 20 \text{ variables}$$

$$\begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 4 & 4 & 4 & 4 \end{array}$$

$$20 \underbrace{-\,5}_{\text{masses are fixed}} \underbrace{-\,4}_{\substack{\Sigma E = \Sigma E \\ \Sigma p = \Sigma p}} \underbrace{-\,3}_{\text{Rotat.}} \underbrace{-\,3}_{\text{boost}} = 5 \text{ variables}$$



$$s = (p_\pi + p_\eta)^2$$

$$s_0 = (p_b + p_t)^2$$

$$s_2 = (p_r + p_\pi)^2$$

$$t_1 = (p_\pi - p_\eta)^2$$

$$t_2 = (p_t - p_r)^2$$

$A(s_0, s_1, s_2, t_1, t_2)$ ← the most general amplitude

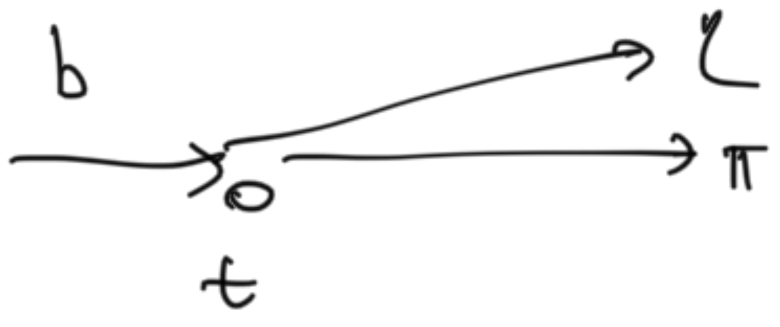
$s_0 \gg m_i^2$ $\qquad$ s_0 — fixed $\qquad$ $t_1 \to \cos\theta$

$t_2 \ll m_i$ $\qquad$ $s_1 = m_{\eta\pi}^2$ $\qquad$ $s_2 \to \varphi$

$t_2 \leftarrow$ keep

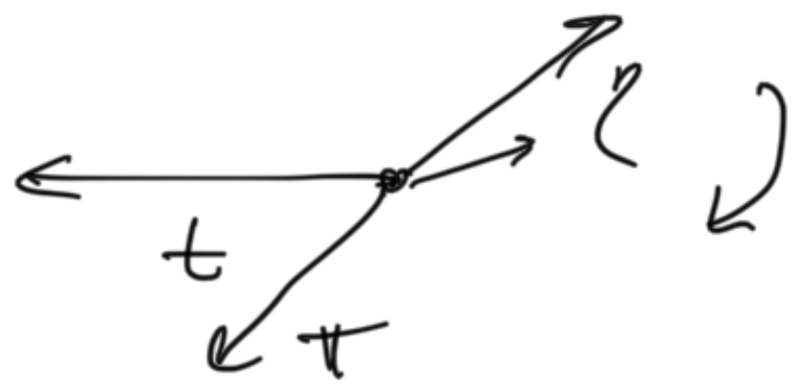
$$A \sim e^{-b|t_2|}\,\hat{A}(s_1, s_2, t_2)$$

$\boxed{\eta\pi \text{ rest frame}}$

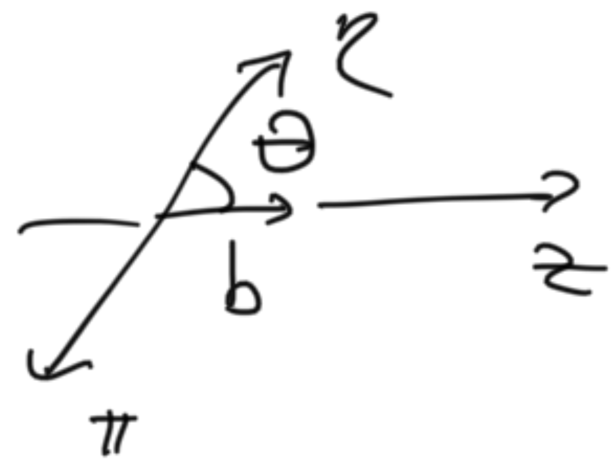


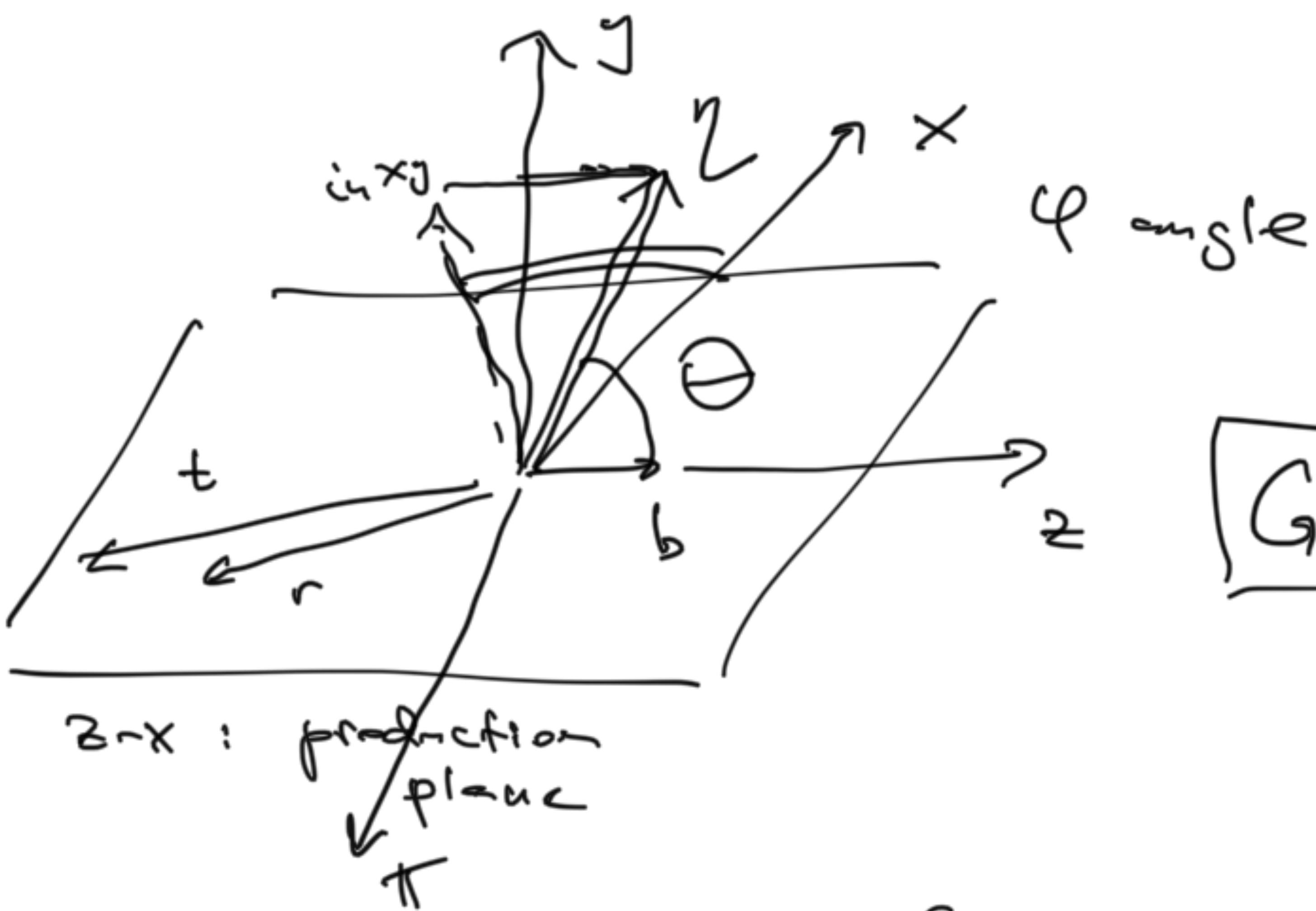
$\boxed{\text{lab frame}}$

boost $\Longleftarrow$



$\boxed{\eta\pi \text{ rest frame}}$





(θ, φ) are sperical angles of p_η

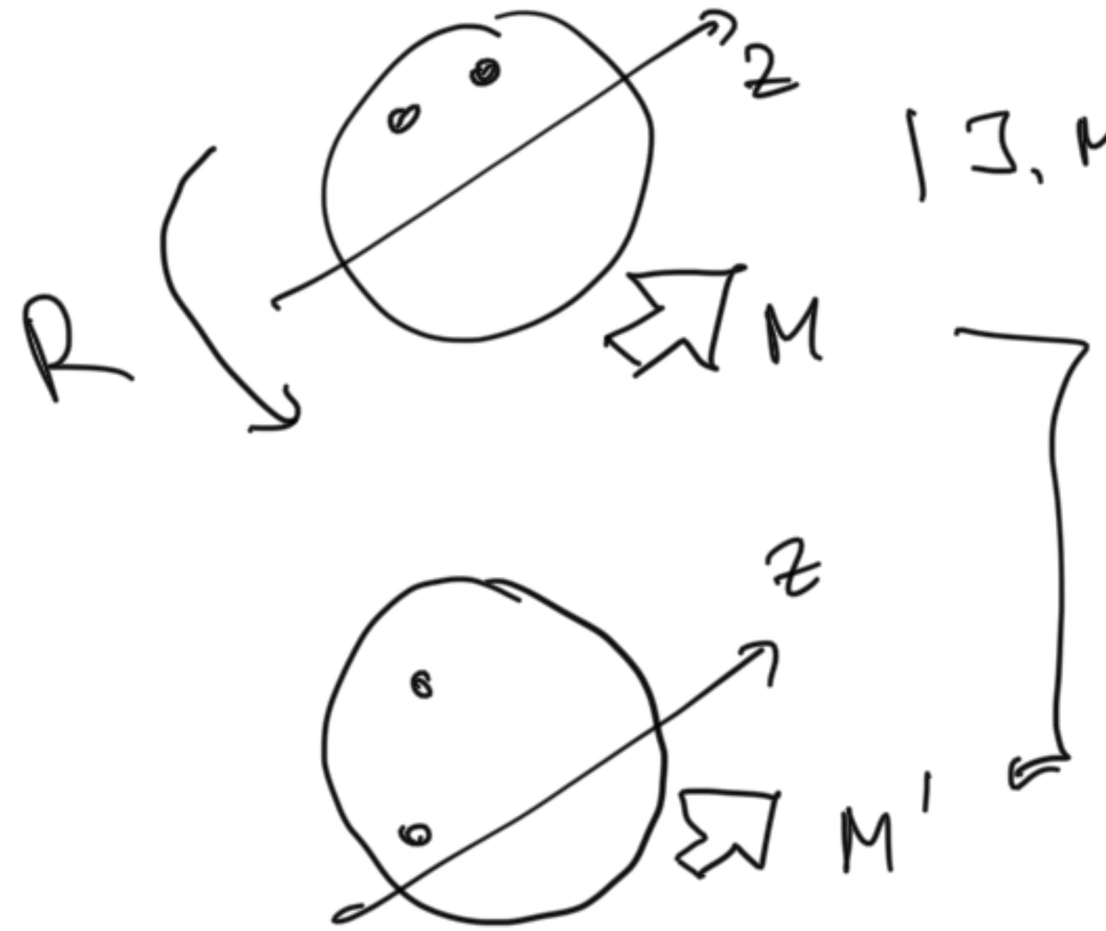
$\boxed{\text{GJ frame}}$

$$A(m_{\eta\pi}, \cos\Theta, \varphi) = \sum_{LM} b_{LM}(m_{\eta\pi})\, \Psi_{LM}(\cos\Theta, \varphi)$$

we are going to truncate the sum at L_{max} M_{max}

$$\Psi_{LM}(\cos\Theta, \varphi) = \sqrt{\frac{2L+1}{4\pi}}\; \underbrace{d^{L}_{M,0}(\Theta)}_{\text{wigner functions}}\; \sin(M\varphi)\cdot\sqrt{2}$$

has spin J



all possible values of M'

$|J, M\rangle$ state

$$R\,|JM\rangle = \sum_{M'} \underline{\underline{D^{J}_{M'M}(R)}}\,|JM'\rangle$$

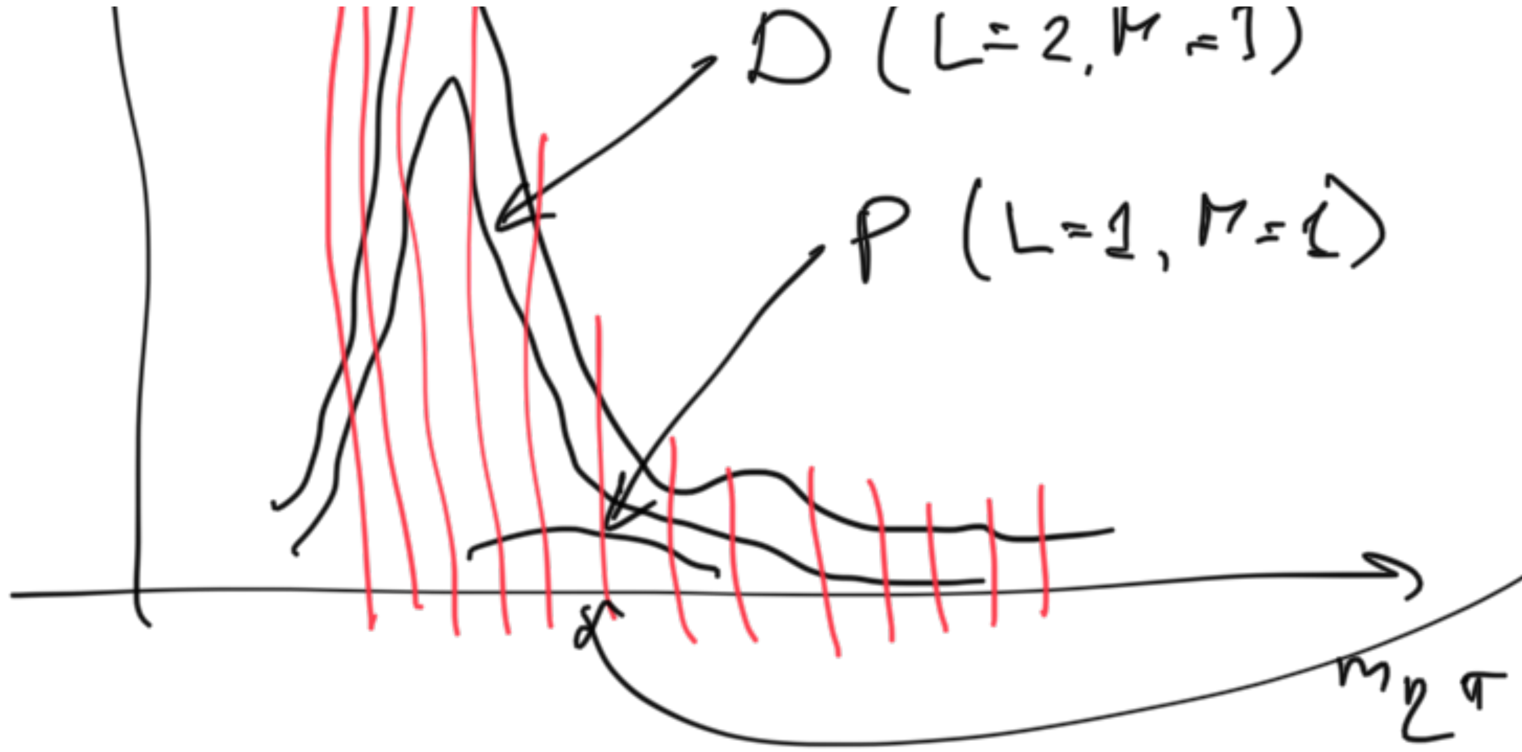
$$R = R_z(\varphi)\, R_y(\Theta)$$

$$D_{M'M}(R) = \underbrace{e^{-iM'\varphi}}_{\uparrow\ \text{complex}}\, d^{J}_{M'M}(\Theta)$$

Normalization $\int d\cos\Theta\, d\varphi\, |\Psi_{LM}(\varphi, \Theta)|^2 = 1$

$A \to I \sim |A|^2$ $\qquad I \cong \left|\sum_{LM} b_{LM}\Psi_{LM}\right|^2$

$b_{LM}(m_{\eta\pi})$



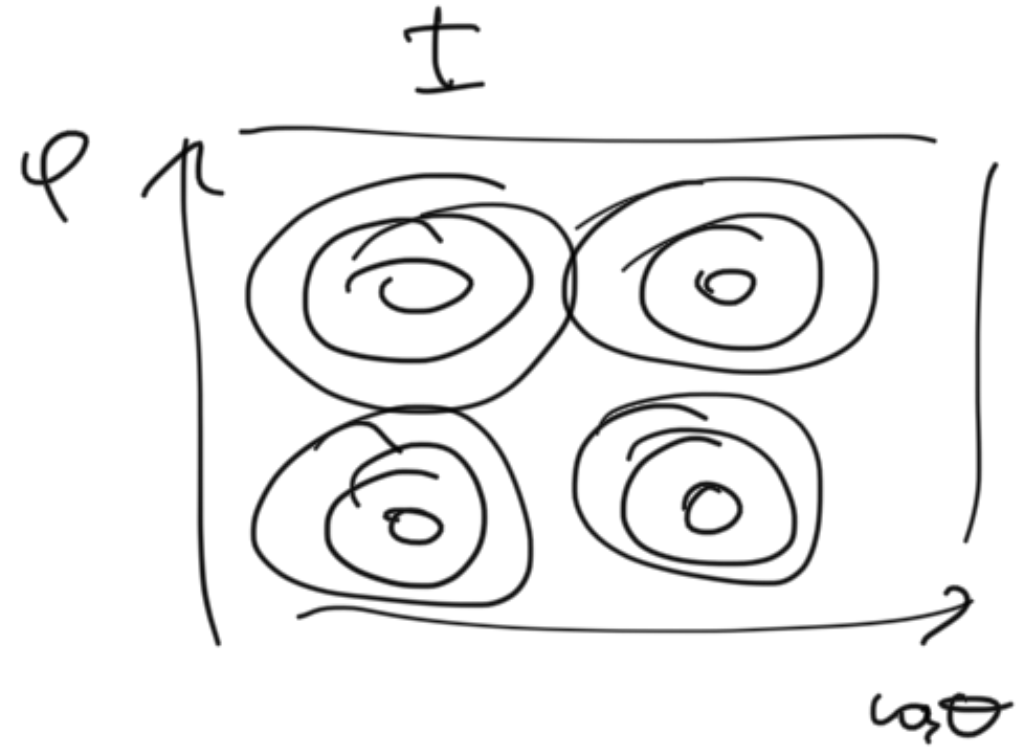
bin # 34 M_{MAX}

$$I = \left| \sum^{L_{MAX}} b^{(34)}_{LM} \Psi_{LM}(\theta, \varphi) \right|$$

constants

$$I^{bin\#34}(\cos\theta, \varphi) \Leftarrow \left| \sum b_{LM} \Psi(\theta, \varphi) \right|^2 = I_{PWA}(\theta, \varphi)$$

fit parameters



L=2 M=1

Log Likelihood method

$$P(\theta, \varphi) = \frac{I_{PWA}(\theta, \varphi)}{\int I_{PWA}(\theta, \varphi)\, d\Omega}$$

$$L(\text{Data}) = \prod_{e=1}^{N_{Data}} P(\theta_e, \varphi_e)$$; Likelihood is a high as closer model I_{PWA} to the data distribution

maximized

N_D

$$LLH = \log L = \sum_{e=1} \log P(\theta_e, \varphi_e) \longleftarrow \text{maximized}$$

$$NLLH = -\log L \longleftarrow \text{minimized}$$

$$P = \frac{I_{PWA}(\theta, \varphi)}{\int I_{PWA}\, d\Omega}, \qquad d\Omega = d\cos\theta\, d\varphi$$

$$b_{LM} \to \alpha b_{LM} \Rightarrow I_{PWA} \to \alpha^2 I \Rightarrow P \to P$$

$\Rightarrow$ Such fit will only be sensitive to b_{LM}/b_{21}

$$\int I_{PWA}(\Omega)\, d\Omega = \underbrace{\sum_{L,M} |b_{LM}|^2 \longrightarrow N_D}_{\text{normalization constraint}}$$

$$L_{Ext} = Pois(\mu) \cdot \prod_{e=1}^{N_D} P(\Omega_e)$$

$$\int I_{PWA}\, d\Omega \Rightarrow N_D \; : \; Pois = \frac{\mu^{N_D} e^{-\mu}}{N_D!}$$

$$\mu = \int I_{PWA}\, d\Omega$$

dL_{ext} … $\Rightarrow \mu = N$

$$\frac{\quad}{d\mu} = 0 \Rightarrow \mu = N_D$$

$$\Rightarrow \quad NLLH_{exp} = -\log L_{ext} =$$

$$= -\log\left[\frac{\cancel{\mu^N}\, e^{-\mu}}{N!}\prod_{e=1}^{N}\frac{I(\Omega_e)}{\cancel{\mu}}\right] =$$

$$= -\left[\sum_{i=1}^{N}\log I_{PWA}(\Omega_i) - \mu - \log N!\right] =$$

$$= -\sum_{e=1}^{N}\log I_{PWA}(\Omega_e) + \int I_{PWA}\, d\Omega =$$

$$= -\sum_{e=1}^{N_D}\log I(\Omega_e|\{b_{LM}\}) + \sum_{LM}|b_{LM}|^2$$

Truncation in LM : $L \leq 6$, $M \leq 2$

$$\hookrightarrow (L,M) \in \{(2,3), (1,1), (4,1)\}$$