CSCM 70

MATHEMATICAL SKILLS OF DATA SCIENTISTS

ASSESSED SHEET 2

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MSc DATA SCIENCE

ANSWER

UA-4 - UA4 $\frac{10}{10}$ pg-1

⇒ Exersice 1 :→ Modify the derivation of linear regression

Answer 1 →>>

Ordinary Least Squares regression operates by minimising a loss function, which is

$$\Sigma_{i=1}^{n} (\hat{Y}_i - Y_i)^2$$

✓

By changing the loss function so that outliers do not play such a big role, we can give less weight.

For Eg: Minimising

$$\Sigma_{i=1}^{n} |\hat{Y}_i - Y_i| \text{ would work}$$

↳ in genral it does not have a unique sol^n.

Other good suggestions for reducing the weight of outliers were linear regression and, if you can reduce the problem to a classification one, using logistic Regression.

Ex 2 Assume you have approximate fx^n values for a fx^n of the form $A\cos(\tau x)$ with parameters A, τ. Describe in principle how to calculate optimal guesses for A an τ

✓

Ans 2 2/ If (x_i, Y_i) are the data points, then we could use gradient descent to minimise

$$\Sigma_{i=1}^{n} (A \cos(T X_i) - Y_i)^2$$

as a function of A and T. This is a bit tricky - we're already tried using gradient descent on sine and cosine functions and if your step size is ~~too~~ large you end up hopping over the waves instead of finding the bottom of a wave.

You can deduce some good initial guesses with theoretical methods. For Eg

If X_i is a random number between 0 & 2π, then $E(Y_i) = 0$ and $Var(Y_i) = \int_0^{2\pi} (A^2 \cos^2 x)$

$$\left(\frac{1}{2\pi}\right) dx = \frac{A^2}{2}$$

So, $A = \sqrt{2}\sigma$, where σ is the sample standard deviation of your Y values, will be a great starting point.

To get a good guess for T (the wavelength) you could use mathematical techniques for studying waves. Or you could just plot it and read off a decent estimate from the plot.

Ex 3 // Comput the sample co-variance matrix of the data giving

	Prop1	Prop2	Prop3	Prop4
Trial1	2	1	3	4
Trial2	1	1	4	3
Trial3	3	1	3	3

Pg3

Answer 3 The means for each property are $(2, 1, \frac{10}{3}, \frac{10}{3})$ so we can write a deviation matrix

$$F = \begin{pmatrix} 0 & 0 & -0.3333 & 0.6667 \\ -1.0000 & 0 & 0.6667 & -0.3333 \\ 1.0000 & 0 & -0.3333 & -0.3333 \end{pmatrix}$$

and then obtain the covariance matrix as

$$W = \frac{1}{2} F^T F = \begin{pmatrix} 1.0000 & 0 & -.5000 & 0 \\ 0 & 0 & 0 & 0 \\ -.5000 & 0 & .3333 & -.1667 \\ 0 & 0 & -.1667 & .3333 \end{pmatrix}$$

$\frac{10}{10}$ UA4

Refence : SIR's Answer Booklet
Checked from canvas.

→

Ex/ Finding Eigenvector of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ to the eigenvalue 2, and one for the eigenvalue 0.

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. We are solving

$Av = 2v$. This is the same as solving

$$\left(A - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right) v = 0 \text{ (i.e.) } \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} v = 0$$

We will solve this using Gaussian elimination

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$ so if $v = \begin{pmatrix} x \\ y \end{pmatrix}$

then we have $-x + y = 0$ or $y = x$.

So $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector for the eigenvalue 2.

⇒ For the eigen value 0, do the same:

$\left(A - \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right) v = 0$ is just $Av = 0$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \mapsto \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$ so we have

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$x+y = 0$ OR

$y = x$

Then $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is an eigen vector for the eigenvalue 0.

Exercise 2

Find an eigenvector of $\begin{pmatrix} 2 & -3 \\ 2 & -5 \end{pmatrix}$ to the eigen value 1, and one for the eigenvalue -4.

If we follow the question 1 steps.

By deciding A

then

Using

the

Gaussian Elimination

for the eigen value 0, 1, & -4.

By the above Steps we get :→

$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is an eigen vector for the eigen value 1.

and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is an eigen vector for the eigen value -4.

Ex 3

Proove that if v & u are eigen vectors of some matrix 'A' for some eigen value 'λ', and if $u \neq -v$, then $u+v$ is an eigenvector of A for λ too.

By defination of an eigenvector,

$Av = \lambda v$ and
$Au = \lambda u$. To see if $u+v$ is an eigen vector we need $u+v \neq 0$,

which is true since $u \neq v$, and we need to check $A(u+v)$.

By distributivity,

$$A(u+v) = Au + Av = \lambda u + \lambda v$$
$$= \lambda (u+v)$$

So $u+v$ is an eigenvector for λ as required.

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(4) Let $A \in R^{2\times 2}$ have eigenvector $(1,1)^t$ for the eigenvalue 2 and eigenvector $(-1, 1)$ for eigenvalue -1, COMPUTE A.

$\Rightarrow$ Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

We know that

$A\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix} a+b \\ c+d \end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$ and $A\begin{pmatrix}-1\\1\end{pmatrix} =$

$\begin{pmatrix} -a+b \\ -c+d \end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$.

From the first equation, we have $b = 2-a$, $d = 2-c$. Substituting into the second, we obtain $2-2a = 1$, $2-2c = -1$.

The Solutions are

$a = \frac{1}{2}$ and $c = \frac{3}{2}$ so that

$b = \frac{3}{2}$ and $d = \frac{1}{2}$ Creating

$$A = \frac{1}{2}\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}.$$

You can check by multiplying this by the vectors above to confirm ~~the~~ your answer

Other good answers include (1) observing that you can stack the two eigenvectors into a matrix to write down.

$$A \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$$

So the sol^n is $A = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^{-1}$

and (2) using the very enportant fact that

the matrix A can be written $\underline{\underline{A = S \Lambda S^{-1}}}$; where

; where; S is a matrix of eigenvectors

$$S = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

and

Λ is a matrix of eigenvalues

$$\Lambda = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

PTO

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Essentially, this means that if you use the eigen vectors as the basis vectors and write A in new co-ordinates, it turns

into the simpler matrix Λ.

This is pretty much what

eigen value & eigen vectors are for!

10/10 UAS

Reference: SIR's Answer booklet
On Canvas. Answers Checked

UA6 UA6 $\frac{10}{10}$ Pg 10

1) Exercise 1) (Drawing from urns)

compute the probility of the following outcomes:

1) Drawing (in order) red blue red from an urn containing 3 red balls and 2 blue balls.

2) Drawing (in total) 2 red and 1 blue ball from an urn containing 3 red balls and 2 blue balls.

3) Drawing (in total) 2 red and 1 blue ball from an urn containing 3 red balls and 2 blue balls, when returning the drawn balls to the urn prior to the next drawn.

Solⁿ Ex1

1) ⇒ We consider how many balls are left at each stage, both in total (denominator) and of the desired color (numerator) to get

$$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} = \frac{1}{5}$$

2) How many ways can you choose 2 red balls from 3? Or 1 blue ball from 2?

Multiply these together to get the number of ways you can choose the desired combination (the numerator). The total number of ways you can choose 3 balls is the denominator.

$$\frac{\left(\binom{3}{2}\binom{2}{1}\right)}{\binom{5}{3}} = \frac{6}{10} = \frac{3}{5}.$$

There are three orders for the 2 red balls and 1 blue ball, so this is why the answer is three times the previous answer.

3) Sol^n

The chances of drawing (in order) red red blue are

$$\frac{3}{5}\;\frac{3}{5}\;\frac{2}{5} = \frac{18}{125}$$

But just like in the previous question, it might happen in three different orders, so the probability is

$$\frac{3 \cdot 18}{125} = \frac{54}{125} \cong 0.432.$$

DESC :-

The Monty Hall game :

There are 3 ~~doctors~~ doors. Two doors have a goat behind them. The other door has a new car behind it.
You choose a door. The game host, Monty, who knows what is behind each door, opens one of the two doors which you did not choose, revealing a goat.

If both doors had goats behind them, Monty chooses between the two with equal likelihood.

Monty invites you to change your initial decision. What should you do? Switch, or stick with your first choice?
(Important: Your goal is to win a car not a goat.).

Ex2 What is the best choice in the Monty Hall game? Justify your answer mathematically.

Answer 2) Call the door opened "Door 1". Call the door Monty opens in response "Door 2". Call the remaining door "Door 3".
The question is what the probability is of the car behind Door 1, before —

PTO

- before Monty acts, is $\frac{1}{3}$.

We want the posterior probability, given that Monty open's Door 2.

$P($ Car in Door 1 | Monty Opens Door 2 $) =$

$$= P(\text{Car in Door 1}) \frac{P(\text{Monty opens Door 2} \mid \text{Car in Door 1})}{P(\text{Monty opens Door 2})}$$

The probability that Monty opens Door 2 is $\frac{1}{2}$, because he has two doors to choose from.

If there is a car behind Door 1 this changes nothing - he still chooses freely between the two doors.

So $P($ Monty opens Door 2 | Car in Door 1 $) = \frac{1}{2}$ also

Then

$P($ Car in Door 1 | Monty opens Door 2 $) =$

$$= \frac{1}{3} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}.$$

So it is wiser to switch to Door 3

You can also check

$$P\left(\begin{array}{c|c}\text{Car in} & \text{Monty opens}\\ \text{Door 3} & \text{Door 2}\end{array}\right) = P\left(\begin{array}{c}\text{Car in}\\ \text{Door 3}\end{array}\right) \frac{P\left(\begin{array}{c|c}\text{Monty opens} & \text{Car in}\\ \text{Door 2} & \text{Door 3}\end{array}\right)}{P\left(\begin{array}{c}\text{Monty opens}\\ \text{Door 2}\end{array}\right)}$$

The prior probability, P (Car in Door 3), is also $\frac{1}{3}$. P(Monty opens Door 2) is still $\frac{1}{2}$

What changes is the conditional probability. If you have opened Door 1, and the Car is behind Door 3, Monty is required to open Door 2, making the

conditional probability 1.

$$P(\text{Car in Door 3} \mid \text{Monty opens Door 2}) = \frac{1}{3} \cdot \frac{1}{1/2} = \frac{2}{3}$$

Ex 3 A medical test has a chance of 1% of giving a wrong answer. Only 0.1% of the population have the illness tested for. If a person is randomly selected for testing and the test comes up positive, what are chances for the person to have the illness?

Sol^n

$$P(\text{Ill} \mid \text{Positive}) = P(\text{Ill}) \frac{P(\text{Positive} \mid \text{Ill})}{P(\text{Positive})}$$

The denominator is always ~~a~~ the tricky bit, the probability of a positive test. It comes from two possibilities.

P(Positive) = P(Positive and Ill) +

+ P(Positive and Not Ill) $= 0.99 \times 0.001 + 0.01 \times 0.999 = 0.01098$

So,

$$P(Ill \mid Positive) = 0.001 \; \frac{0.99}{0.01098} \simeq 0.090.$$

In simple words,

the chance is still only 9%.

Ex 4 You are betting on coin tosses, and at the start belive that there is a 1 in 10 chance of your opponent using a coin with two heads. Calculate how likely you should consider your opponent cheating after seeing 3 subsequent heads.

Ans 4

$$P(\text{Cheating} \mid 3\text{ Heads}) = P(\text{Cheating}) \frac{P(3\text{Heads} \mid \text{Cheating})}{P(3\text{ Heads})}$$

The probability of 3 Heads coming up is $\frac{1}{8}$ with a fair coin, but 1 if your opponent is cheating.

So $P(3\text{ Heads}) = 0.9 \times \frac{1}{8} + 0.1 \times 1$

$P(3\text{ Heads}) = 0.2125$

$$P(\text{Cheating} \mid 3\text{ Heads}) = 0.1 \frac{1}{0.2125} \simeq 0.47$$

10/10

Reference: Sir's answer booklet on Canvas Checked.

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UA - 7

10/10

Ex 1)) Does picking $n \in N$ uniformly at random make sense? If Not, can you think of a way to assign non-zero probabilities to all $n \in N$?

Sol^n 1) No, because if we give probability p to any given number, then the total probability over all of N will be unbounded.
On the other hand, if we give probability 0 to each number, then the total probability will be zero.
(This differs from the situation for continuous variables, where we can integrate them up.

To give non-zero probabilites to all $n \in N$, we need to assign a positive number to every n so that they add up to 1

An example of how to do this is with a geometric series, so that n is picked with probability 2^{-n-1}.

Starting from $n=0$, we have probability 1/2, 1/4, 1/8, so on. You can model this as follows. Toss a coin until a head comes up. n is the number of tails you saw along the way.

Ex 2 A computer genrates a random number 'X' between '0' and ~~100~~ '10'. The likelihood of any particular number appearing is proportional to the square of that number.

Write down a probability density function for X. (If you need to compute an integral you can simply ask a computer - there is no need to demonstrate the steps of the integration)

Ans 2 The density function must take the form

$$f(x) = k x^2 \quad \text{to be}$$

proportional to the square. But to have total probability 1 we need

$$\int_0^{10} f(x)\,dx = 1,$$

(i.e.) $\frac{1}{3} x^3 \Big|_0^{10} = \frac{1000}{3} k = 1$ so $k = \frac{3}{1000} = 0.003$

Therefore $f(x) = 0.003\, x^3$ for

$$0 \leq x \leq 10 \quad \text{and}$$

$$f(x) = 0 \quad \text{otherwise.}$$

Ex3 If $x \in [0, 1]$ is picked uniformly at random, what is the probability of 'x' being:

1. → less or equal than $\frac{1}{3}$

2. → Strictly less than $\frac{1}{3}$

3. → between $\frac{1}{4}$ and $\frac{1}{3}$

4. → exactly $\frac{1}{2}$

Sol^n 3 The probability density function is

$$f(x) = 1 \text{ for } 0 \leq x \leq 1 \text{ and '0' otherwise}$$

These probabilities can be given as integrals of 1, or just as areas under the graph of 1, i.e. areas of rectangles with height 1.

The area of a rectangle of hight 1 is exactly it's width.

1. The rectangle has width $\frac{1}{3}$ so, the probability is $\frac{1}{3}$

2. It makes no difference is the end point is included - including or

excluding it makes no difference to the width of the rectangle.
This is still $\frac{1}{3}$.

3. The width of this rectangle is $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$, so $\frac{1}{12}$ is the probability.

4. This is a rectangle of width 0, so the probability is 0,

Of course, $\frac{1}{2}$ is a perfectly "possible" choice of number in $[0,1]$,

but there are uncountably many numbers in $[0,1]$ so there is no way to assign a positive probability to each of them.

Even the trick ~~form~~ from Q 1 won't work here

Reference: SIR's Answer booklet on Canvas. Checked.

10/10

UA-4 - 10 / 10
UA-5 - 10 / 10
UA-6 - 10 / 10
UA-7 - 10 / 10

$\frac{10}{10}$

Q 1 $\frac{10}{10}$

AA-2

Qust 1 Do all UA's given after AA-1 ?

Answer 1 -) UA 4, UA 5, UA 6, UA 7.
with Answers..

Quest 2 :-) Compute the gradients of the following function of X and Y and find extrema

$$f(x,y) = x^2 + xy + y^2 + 5x - 5y + 3.$$

Changed Later by SIR

Answer 2

$$f(x,y) = x^2 + xy + y^2 + 5x - 5y + 3$$

$$\nabla f = \begin{bmatrix} \frac{\delta f}{\delta x} \\ \\ \frac{\delta f}{\delta y} \end{bmatrix} = \begin{bmatrix} 2x + y + 5 \\ \\ x + 2y - 5 \end{bmatrix}$$

i.e. $\nabla f = (2x + y + 5)\,\hat{i} + (x + 2y - 5)\,\hat{j}$

$$\nabla_x f = 2x + y + 5$$

$$\nabla_y f = x + 2y - 5$$

For extrema

$2x + y + 5 = 0$ $\qquad$ $x + 2y - 5 = 0$

$x = 5 - 2y$

$10 - 4y + y + 5 = 0$

$10 + 5 = 3y$

$y = 5$

$\therefore x = 5 - 10 = -5$

☐→ So $(-5, 5)$ is a extrema point.

VERIFICATION - Second dererative test.

$$D = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - (f_{xy}(x_0, y_0))^2$$

$f_{xx}(x, y) = 2$ $\qquad$ $f_{x,y}(x, y) = 0$

$f_{yy}(x, y) = 2$

$D = 2 \times 2 = 4$

That is $D > 0$

Since $D > 0$ & $f_{xx}(x_0, y_0) > 0$

$\therefore$ we have a local minimum at $(-5, 5)$

Q3 //

The equation

$$2x^3 - 7x^2 - x + 12 = 0$$

has a root near $x = 1.5$. Do the first three steps of Newton's Method by hand, to find this root.

Sol3.

$$2x^3 - 7x^2 - x + 12 = 0$$

$f(x) = 2x^3 - 7x^2 - x + 12$ ~~#@~~

has a root near $= 1.5$

$$f'(x) = 6x^2 - 14x - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} =$$

$$= x_n - \frac{2x_n^3 - 7x_n^2 - x_n + 12}{6x_n^2 - 14x_n - 1}$$

Taking $x_0 = 1.5$ *

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} =$$

$$= 1.5 - \frac{(2(1.5)^3 - 7(1.5)^2 - 1.5 + 12)}{6(1.5)^2 - 14(1.5) - 1}$$

$$x_1 = 1.5 + \frac{\cancel{1.5}\,3}{\cancel{8.5}\,17} \Rightarrow 1.5 + \frac{3}{17} = 1.6764705$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 + \frac{3}{17} - \frac{0.073275}{-7.60927}$$

$$= 1.686103$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$f(x_2) = 2(1.686103)^3 - 7(1.686103)^2 - (1.686103) + 12$$

$$= 0.000286$$

$$f'(x_2) = -7.54778$$

$$\boxed{\therefore \quad x_3 = 1.686141}$$ <u>Answer</u>

Q4 Find an eigenvector of $\begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{pmatrix}$ ~~to~~ the eigenvalue 2, and one for eigenvalue 3.

Sol^n

$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{pmatrix}$$

$\Rightarrow$ $A \rightarrow$ square matrix.

V is a non-zero vector is eigenvector of A if

$Av = \lambda v$, for some number λ

$|A - \lambda I| = 0$

$\Rightarrow$ Given eigen values $\lambda_1 = 2$ & $\lambda_2 = 3$

Case 1: $\rightarrow$ When $\lambda_1 = 2$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 4z = 2x$ || $2y = 2y$ || $3x + y - 3z = 2z$

$4z = x$ || || $12z + y = 5z$

$y = -7z$

$$\begin{bmatrix} 4z \\ -7z \\ z \end{bmatrix}$$

$\therefore \begin{bmatrix} 4c \\ -7c \\ c \end{bmatrix}$ for any $c \in \mathbb{R}$ (field)

$ex = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix}$ ... etc

CASE 2 :→

when $\lambda_2 = 3$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow x + 4z = 3x \;||\; 2y = 3y \;||\; 3x + y - 3z = 3z$

$2x = 4z$ & $\therefore y = 0$ $\quad 6z + y - 3z = 3z$

$x = 2z$ $\quad y = 0$

$$\begin{bmatrix} 2z \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore \begin{bmatrix} 2c \\ 0 \\ c \end{bmatrix}$ for any $c \in \mathbb{R}$ (field)

$ex = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix}$ ... etc.

⇒ Q5 Compute the sample covariance matrix of the following data (by hand with details):

	Prop 1	Prop 2	Prop 3	Prop 4
Trial 1	2	2	5	96
Trial 2	4	3	1	105
Trial 3	1	3	2	98

Answers Covariance matrix

	Prop 1	Prop 2	Prop 3	Prop 4
TRIAL 1	2	2	5	96
TRIAL 2	4	3	1	105
TRIAL 3	1	3	2	98

$$X = \begin{bmatrix} 2 & 2 & 5 & 96 \\ 4 & 3 & 1 & 105 \\ 1 & 3 & 2 & 98 \end{bmatrix}$$

$\Rightarrow$ Then we find the mean

$$\bar{X} = \begin{bmatrix} \frac{7}{3} & \frac{8}{3} & \frac{8}{3} & \frac{299}{3} \end{bmatrix}$$

variance - co variance matrix S is calculated by

$$\Rightarrow \quad S = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})'$$

here n = no. of trials = 3

$$\text{Covariance Matrix} = \begin{bmatrix} Var(Prop1) & cov(P_2, P_1) & cov(P_3 P_1) & cov(P_4, P_1) \\ cov(P_1, P_2) & var(Prop2) & cov(P_3, P_2) & cov(P_4, P_2) \\ cov(P_1, P_3) & cov(P_2, P_3) & var(Prop3) & cov(P_4, P_3) \\ cov(P_1, P_4) & cov(P_2, P_4) & cov(P_3 P_4) & var(Prop4) \end{bmatrix}$$

$$X_i - \bar{X} = \begin{bmatrix} 2 - \frac{7}{3} & 2 - \frac{8}{3} & 5 - \frac{8}{3} & 96 - \frac{299}{3} \\ 4 - \frac{7}{3} & 3 - \frac{8}{3} & 1 - \frac{8}{3} & 105 - \frac{299}{3} \\ 1 - \frac{7}{3} & 3 - \frac{8}{3} & 2 - \frac{8}{3} & 98 - \frac{299}{3} \end{bmatrix}$$

$$X_1 - \bar{X} = \begin{array}{cccc} \text{Prop 1} & \text{Prop 2} & \text{Prop 3} & \text{Prop 4} \end{array}$$
$$\begin{bmatrix} -0.3333 & -0.6667 & 2.3333 & -3.6667 \\ 1.6667 & 0.333 & -1.6667 & 5.3333 \\ -1.3333 & 0.3333 & -0.6667 & -1.6667 \end{bmatrix}$$

$\rightarrow$ (1)

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \end{bmatrix}$$

[There can be another way (*) follow star]

Variance $\Rightarrow$

$$var(x) = \frac{\sum_1^n (x_i - \bar{x})^2}{n-1}$$

Covariance: $$cov(x,y) = \frac{\sum_1^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\therefore var(Prop1) = \frac{((-0.333)^2 + (1.6667)^2 + (1.333)^2)}{3-1}$$

$\rightarrow n = 3$ (as there are 3 trials)

$= 2.333$

Similarly $Var(Prop\,2) = 0.3333$

and $Var(Prop\,3) = 4.3333$

and $Var(Prop\,4) = 22.3333$

$$\Rightarrow \text{Now } cov(P_1, P_2) = \frac{(A_{11} \times A_{12} + A_{21} \times A_{22} + A_{31} \times A_{32})}{3-1}$$

$$= \frac{((-0.3333 \times -0.6667) + (1.6667 \times 0.3333) + (-1.3333 \times 0.3333)}{2}$$

$$= \frac{(0.2222 + 0.5555 - 0.4444)}{2}$$

$$= 0.16667$$

$$\Rightarrow cov(P_1, P_3) = \frac{((-0.3333 \times 2.3333) + (1.6667 \times (-1.6667)) + (-1.3333 \times (-0.667)))}{2}$$

$$= -1.3333$$

$$cov(P_1, P_4) = 6.1667 \times$$

$$\left(\frac{(-0.3333 \times -3.6667) + (1.6667 \times 5.3333) + (-1.3333 \times -1.6667)}{2} \right)$$

$\therefore$ Covariance Matrix =

$$\begin{bmatrix} 2.3333 & 0.1667 & -1.3333 & 6.1667 \\ 0.1667 & 0.3333 & -1.1667 & 1.8333 \\ -1.3333 & -1.1667 & 4.3333 & -8.1667 \\ 6.1667 & 1.8333 & -8.1667 & 22.3333 \end{bmatrix}$$

↳ ANSWERS.

PTO

You can also process the above question by solving it in this format (Shown below

Q6

(*) Continued.

- After calculating $X - \bar{X}$ we get a Matrix we take a transpose of it then.

$$\Rightarrow N = \frac{1}{2} F^T F =$$

$$= \frac{1}{2} \begin{bmatrix} -0.33 & 1.66 & -1.33 \\ -0.66 & 0.34 & 0.34 \\ 2.34 & -1.66 & -0.66 \\ -3.66 & 5.34 & -1.66 \end{bmatrix}_{4\times3} \begin{bmatrix} -0.33 & -0.66 & 2.34 & -3.66 \\ 1.67 & 0.34 & -1.66 & 5.34 \\ -1.33 & 0.34 & -0.66 & 1.66 \end{bmatrix}_{3\times4}$$

$$= \frac{1}{2} \begin{bmatrix} 4.667 & 0.333 & -2.65 & 12.33 \\ 0.333 & 0.333 & -2.33 & 3.66 \\ -2.65 & -2.33 & 8.67 & -16.33 \\ 12.33 & 8.67 & -16.33 & 44.66 \end{bmatrix}$$

A6

$$= \begin{bmatrix} 2.333 & 0.166 & -1.333 & 6.166 \\ 0.166 & 0.333 & -1.166 & 1.833 \\ -1.333 & -1.166 & 4.333 & -8.166 \\ 6.166 & 1.833 & -8.166 & 22.333 \end{bmatrix}$$

ANSWER

Another way to approach the same problem

(*) Please do not reduce Marks
I have talked with Prof. for this thing (*)

Q6. There are 5 balls in an urn, of which 4 balls are red and 1 ball is blue.
To do the following:

- draw a ball from the urn at random, note its color, do not return the ball to the urn;
- draw a second ball, note its color, do not return the ball to the urn;
- finally draw a third ball and note its color.

(i) Give the sample space Ω for this experiment.

Calculate probability for each outcome

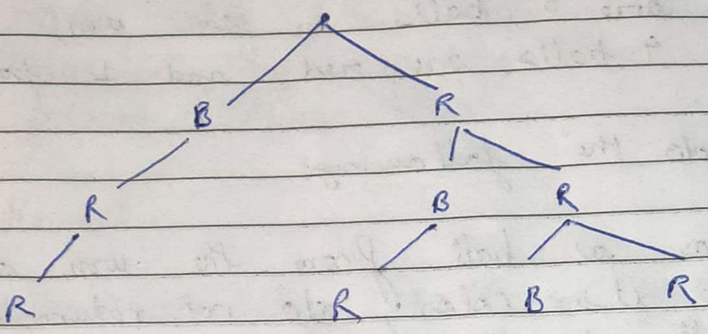
(ii) Consider the following event,
— (One of the firts 2 Ball is Blue)

(too much text) Sorry.

A6

PTO

A6



SAMPLE SPACE

- BRR
- RBR
- RRB
- RRR

NOTE

It is not Mentioned that R is Seperately

$\therefore \Omega = \{BRR, RBR, RRB, RRR\}$.

(i) Case: 1 BRR → 1st Ball drawn is blue

$\Rightarrow \frac{1}{5} \times 1 \times 1 = \frac{1}{5}$

Case: 2 RBR

$\Rightarrow \frac{4}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{1}{5}$

Case: 3 RRB

$\Rightarrow \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{5}$

Case: 4 = RRR

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

(ii)

A: One of first 2 balls is blue

Event A can be written as = {BRR, RBR}

Clearly $\underline{A \subset \Omega}$.

where $A = \{BRR, RBR\}$
& from (i) $\Omega = \{BRR, RBR, RRB, RRR\}$.

Probability

$$P(A) = \underline{P(BRR) + P(RBR)}$$

$$= \frac{1}{5} \times 1 \times 1 + \frac{4}{5} \times \frac{1}{4} \times 1$$

$$= \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

Therefore $P(A) = \boxed{\frac{2}{5}}$

~~Quest 6~~ In $\boxed{\text{small view}}$ →

4 Red Ball = r

1 blue Ball = b

(i) Sample space : $brr \rightarrow 1/5 \quad 4/4 \quad 3/3 = 1/5$

$rbr \rightarrow 4/5 \quad 1/4 \quad 3/3 = 1/5$

$rrb \rightarrow \frac{4}{5} \quad \frac{3}{4} \quad \frac{1}{3} = 1/5$

$rrr \rightarrow \frac{4}{5} \quad \frac{3}{4} \quad \frac{2}{3} = 2/5$

(ii) Sample Space = brr

rbr

EXTRA

EXTRA — If R is seprate

If R is seprate then it can be a case like as follows.

$4R \Rightarrow R_1 \quad R_2 \quad R_3 \quad R_4$

$1B \rightarrow B_1$

$S \Rightarrow \{R_1R_2R_3, R_1R_2R_4, R_2R_3R_4, R_1R_2B_1, R_1R_3B_1, R_1R_4B_1, R_2R_3B_1, R_3R_4B_1, R_1R_3R_4, R_2R_4B_1\}$

They are 10 // [List of all possible outcom.

$$C(5\ 3) = \frac{5!}{3! \times 2!} = 10$$

$${}^5C_3 = 10$$

Each outcome has $\left(\frac{1}{10}\right) = 0.1$ proba.

Q7 Additi

A7

(i)

(ii)

(iii)

Then

Q7 Additional Question. *

A7

dice 1	die 2 or 3	dice 4, or 5 or 6
URN I	URN II	URN III
2B	1B	1B
3W	3W	2W
= 5	= 4	= 3

(i) Probability, a ball is drawn from URN I $= \frac{1}{6} = P(E_1)$ ← By dice Roll

(ii) Probability, a ball is drawn from URN II $= \frac{2}{6} = \frac{1}{3} = P(E_2)$

(iii) Probability, a ball is drawn from URN III $= \frac{3}{6} = \frac{1}{2} = P(E_3)$

$P(A)$ = Probability of drawing black ball

Then, Probability that ball drawn is black

$= P(A|E_1) + P(A|E_2) + P(A|E_3)$

$= \frac{2}{5} + \frac{1}{4} + \frac{1}{3} = \frac{12+15+20}{60} = \boxed{\frac{47}{60}}$ Ans 7(i)

*

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(7.ii) Probability that black ball is drawn from URN 'I', (i.e) $P(E_1|A)$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{6}\times\frac{2}{5}}{\frac{1}{6}\times\frac{2}{5}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{3}}$$

$$= \frac{\frac{1}{15}}{\frac{1}{15}+\frac{1}{12}+\frac{1}{6}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{\frac{1}{5}}{\frac{4+5+10}{20}}$$

$$= \frac{4}{19} = \boxed{\frac{4}{19}}$$ Ans
