+++++++++++++++++ Lab 004 +++++++++++++++++

+++++++++++++++++ 2154638 +++++++++++++++++

Mathematical Skills for Data Scientists
Lab Exercises 4

CSCM70

Code:

Newton.m GradientDescent.m

1.) Newton.m

```
% CSCM 70 ----------- LAB 04 -----------
% CSCM 70 ----------- 2154638 -----------
% =================================================================

% =========================================================
% PALLAV SHUKLA

% ------------ Mentioned by Sir to Mention ---------------------

filename = which(mfilename('fullpath'))
fileInfo = dir(filename)

% --------------------------------------------------------------



function res =Newton(f,initial_est)
syms x
fprime = diff(f,x);
root_expression = x - (f/fprime);
root_func = matlabFunction(root_expression);

root = initial_est;
currentError = 1;
roots = zeros(20,1);

iterCount = 0;
```

```
while currentError > 0.0001
    if iterCount == 20
        disp(roots)
        error('20 iteration comlete with results shown as shown.')
    end
    oldroot = root;
    root = feval(root_func,oldroot);
    currentError = abs(root-oldroot);
    iterCount = iterCount + 1;
    roots(iterCount) = root;
end

res = root;

X = sprintf('The root is estimated to be %f. The value of the function is
%f.',res,subs(f,x,res));
disp(X)

% ==================================================================
%  ----------- END Newton -----------
% ==========================================================
```

2.) GradientDescent.m

```
% CSCM 70 ----------- LAB 04 -----------
% CSCM 70 ----------- 2154638 -----------
% ==================================================================

% ==========================================================
% PALLAV SHUKLA

% ------------ Mentioned by Sir to Mention ---------------------

filename = which(mfilename('fullpath'))
fileInfo = dir(filename)

% ---------------------------------------------------------------

function res=GradientDescent(f,initial_est,step_size)

syms x y

format shortG

minimizer = initial_est;
value = subs(f,[x; y],initial_est);
grad = [diff(f,x); diff(f,y)];
h = step_size;

next_step_expression = [x;y] - h*grad;
next_step_func = matlabFunction(next_step_expression);

currentNorm = 1;
estimates = zeros(20,3);

iterCount = 0;

while currentNorm > 0.001
    if iterCount == 20
        disp(estimates)
```

```
        error('20 iterations have been carried out without convergence. Results as shown (x, y, value).')
    end
    oldvalue = value;
    minimizer = feval(next_step_func, minimizer(1), minimizer(2));
    value = subs(f,[x;y], minimizer);
    currentNorm = norm(subs(grad,[x; y], minimizer));
    iterCount = iterCount + 1;
    estimate = [minimizer(1) minimizer(2) value];
    estimates(iterCount,:) = estimate;
end

res = minimizer;

X = sprintf('The minimizer is estimated to be (%f,%f). The value of the function there is %f.',res(1),res(2),value);
disp(X)

% ==================================================================
%  ----------- END GradientDescent -----------
% ==========================================================
```

Lab Sheet Starting

```
OVERALL x & y defined
```

```
syms x y
```

Exercise 1: Newton’s Method in Excel/Matlab

Question : $x \mapsto x^3 - 5\ (f(x) = x^3 - 5)$

f=x^3-5;

Newton(f,1)

OUTPUT EXCEL:

	A	B	C	D	E	F	G	H	I	J	K
1	a_n	f(a_n)	f'(a_n)	a_n+1	n	a_n+1 = a_n - f(a_n)/f'(a_n)			f(x)=x^3 - 5	n= 1,2,3,4,5.....	
2	1	-4	3	2.333333	1				f'(x)=3x^2	derivative	
3	2.333333333	7.703703704	16.33333	1.861678	2						
4	1.861678005	1.45228739	10.39753	1.722002	3						
5	1.72200188	0.106235773	8.895871	1.71006	4						
6	1.710059737	0.000735046	8.772913	1.709976	5						
7	1.709975951	3.60136E-08	8.772053	1.709976	6						
8	1.709975947	0	8.772053	1.709976	7						
9	1.709975947	0	8.772053	1.709976	8						
10	1.709975947	0	8.772053	1.709976	9						
11	1.709975947	0	8.772053	1.709976	10						

Newton(f,1)

OUTPUT MATLAB: COMMAND WINDOW

```
>> f = x^3-5;
>> Newton(f,1)
      1.5714
      1.7881
      1.6486
      1.7497
      1.6802
      1.7302
      1.6952
      1.7202
      1.7026
      1.7152
      1.7063
      1.7126
      1.7081
      1.7113
       1.709
      1.7106
      1.7095
      1.7103
      1.7097
      1.7101

Error using Newton
20 iteration comlete with results shown as shown.
```

Whereas Newton(f,1.7) gives

```
>> Newton(f,1.7)
The root is estimated to be 1.709935. The value of the function is -0.000362.

ans =

    1.7099

>> |
```

Exercise 2: (Newton's Method in Excel)

Question: $x \mapsto x^3 - 2 \; (f(x) = x^3 - 2)$

f(x)=x^3 – 2
f'(x)=3x^2   `% this is by hand`

a_n+1 = a_n - f(a_n)/f'(a_n)

OUTPUT EXCEL:

C5 =3*(A5)^2

	A	B	C	D	E	F	G	H	I	J	K
1	a_n	f(a_n)	f'(a_n)	a_n+1	n	a_n+1 = a_n - f(a_n)/f'(a_n)			f(x)=x^3 - 2	n= 1,2,3,4,5.....	
2	1	-1	3	1.333333333	1				f'(x)=3x^2	derivative	
3	1.333333333	0.37037037	5.333333333	1.263888889	2						
4	1.263888889	0.018955225	4.79224537	1.259933493	3						
5	1.259933493	5.92593E-05	4.762297224	1.25992105	4						
6	1.25992105	5.85259E-10	4.762203157	1.25992105	5						
7	1.25992105	0	4.762203156	1.25992105	6						
8	1.25992105	0	4.762203156	1.25992105	7						
9	1.25992105	0	4.762203156	1.25992105	8						
10	1.25992105	0	4.762203156	1.25992105	9						
11	1.25992105	0	4.762203156	1.25992105	10						
12	1.25992105	0	4.762203156	1.25992105	11						
13	1.25992105	0	4.762203156	1.25992105	12						
14	1.25992105	0	4.762203156	1.25992105	13						

Exercise 3: (Newton's Method in MatLab)

1.) $x \mapsto x^2$

f = x^2;
Newton(f,1)

OUTPUT MATLAB: COMMAND WINDOW

Command Window

```
>>
>> syms x
>> f = x^2

f =

x^2

>> Newton(f,1)
The root is estimated to be 0.000061. The value of the function is 0.000000.
```

2.) $x \mapsto \sin(\cos x)$

f=sin(cos(x))
Newton(f,1)

OUTPUT MATLAB: COMMAND WINDOW

```
>> f=sin(cos(x));
Newton(f,1)
The root is estimated to be 1.570763. The value of the function is 0.000034.

ans =

    1.5708

>>
```

3.) $x \mapsto x^7 - 5$

f=x^7 - 5;
Newton(f,1)

OUTPUT MATLAB: COMMAND WINDOW

```
>> f=x^7 - 5;
Newton(f,1)

The root is estimated to be 1.258519. The value of the function is 0.000555.

ans =

      1.2585
```

3.4) $x \mapsto \sqrt{(}x)$

f=x^(0.5);
Newton(f,1)

OUTPUT MATLAB: COMMAND WINDOW

```
>> f=x^(0.5);
Newton(f,1)

The root is estimated to be Inf. The value of the function is Inf.

ans =

   Inf
```

& with f = sqrt(x) the output is as below :

```
Command Window

     1.2585

>> f = sqrt(x)

f =

x^(1/2)
```

```
Editor - F:\Study Swansea\AA A Term-2\AA Lab\Maths Lab\04_Lab\Newton.m
Newton.m  GradientDescent.m  untitled *  +
31        iterCount = iterCount + 1;
```

```
Command Window

x^(1/2)

>> Newton(f,1)
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1
    -1
     1

Error using Newton
20 iteration comlete with results shown as shown.
```

Basically we will not be able to get a proper root for this one.

Exercise 4: Gradient Descent in Excel/Matlab

Question : $x^2 + (y-2)^2$

f(x,y) = x^2+(y-2)^2

OUTPUT MATLAB: COMMAND WINDOW

```
f =

(y - 2)^2 + x^2

>> GradientDescent(f,[1;1],0.1)
          0.8           1.2          1.28
         0.64          1.36        0.8192
        0.512         1.488       0.52429
       0.4096        1.5904       0.33554
      0.32768        1.6723       0.21475
      0.26214        1.7379       0.13744
      0.20972        1.7903      0.087961
      0.16777        1.8322      0.056295
      0.13422        1.8658      0.036029
      0.10737        1.8926      0.023058
     0.085899        1.9141      0.014757
     0.068719        1.9313     0.0094447
     0.054976         1.945     0.0060446
      0.04398         1.956     0.0038686
     0.035184        1.9648     0.0024759
     0.028147        1.9719     0.0015846
     0.022518        1.9775     0.0010141
     0.018014         1.982    0.00064904
     0.014412        1.9856    0.00041538
     0.011529        1.9885    0.00026585

Error using GradientDescent
20 iterations have been carried out without convergence. Results as shown (x, y,
value).
```

OUTPUT EXCEL:

	A	B	C	D	E	F	G	H
1	Exercise 4: x^2 + (y-2)^2							
2	x	y	f(x,y)	df/dx	df/dy			h
3	1	1	2	2	-2			0.1
4	0.8	1.2	1.28	1.6	-1.6			
5	0.64	1.36	0.8192	1.28	-1.28			
6	0.512	1.488	0.524288	1.024	-1.024			
7	0.4096	1.5904	0.33554432	0.8192	-0.8192			
8	0.32768	1.67232	0.214748365	0.65536	-0.65536			
9	0.262144	1.737856	0.137438953	0.524288	-0.524288			
10	0.2097152	1.7902848	0.08796093	0.4194304	-0.4194304			
11	0.16777216	1.83222784	0.056294995	0.33554432	-0.33554432			
12	0.134217728	1.865782272	0.036028797	0.268435456	-0.268435456			
13	0.107374182	1.892625818	0.02305843	0.214748365	-0.214748365			
14	0.085899346	1.914100654	0.014757395	0.171798692	-0.171798692			
15	0.068719477	1.931280523	0.009444733	0.137438953	-0.137438953			
16	0.054975581	1.945024419	0.006044629	0.109951163	-0.109951163			
17	0.043980465	1.956019535	0.003868563	0.08796093	-0.08796093			
18	0.035184372	1.964815628	0.00247588	0.070368744	-0.070368744			
19	0.028147498	1.971852502	0.001584563	0.056294995	-0.056294995			
20	0.022517998	1.977482002	0.00101412	0.045035996	-0.045035996		Answer	
21	0.018014399	1.981985601	0.000649037	0.036028797	-0.036028797			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			

Exercise 5: Gradient descent in Excel

$$(1-x)^2 + 100(y-x^2)^2$$

f = x^2+(y-2)^2; close to 11
GradientDescent(f,[1;1],0.1)

OUTPUT EXCEL:

	A	B	C	D	E	F	G	H	I	J	K
1	Exercise 5: (1-x)^2 +(100* (y-x^2)^2)										
2	x	y	f(x,y)	df/dx	df/dy			h	df/dx		
3	1	1	0	0	0			0.01	2*(x-1)+400*x*(x^2-y)		
4	1	1	0	0	0				df/dy		
5	1	1	0	0	0				200*(y-x^2)		
6	1	1	0	0	0						
7	1	1	0	0	0						
8	1	1	0	0	0						
9	1	1	0	0	0						
10	1	1	0	0	0						
11	1	1	0	0	0						
12	1	1	0	0	0		The value of function is 0 at the minimum 1,1 itself.				
13											

OUTPUT MATLAB: COMMAND WINDOW

- MATLAB verification:

```
f =

(x - 1)^2 + 100*(- x^2 + y)^2

>> GradientDescent(f,[1;1],0.1)
The minimizer is estimated to be (1.000000,1.000000). The value of the function there is 0.000000.

filename =

    '/MATLAB Drive/GradientDescent.m'


fileInfo =

  struct with fields:

       name: 'GradientDescent.m'
     folder: '/MATLAB Drive'
       date: '06-Nov-2022 20:45:12'
      bytes: 1100
      isdir: 0
    datenum: 7.3883e+05


ans =

     1
     1
```

Exercise 6: Gradient descent in Matlab

6.1 $x^2 \cos y$

f = (x^2)*cos(y);
GradientDescent(f,[1;1],0.1)

OUTPUT MATLAB: COMMAND WINDOW

```
>> f = (x^2)*cos(y)

f =

x^2*cos(y)

>> GradientDescent(f,[1;1],0.1)
     0.89194      1.0841      0.37206
     0.80851      1.1545      0.26436
     0.74312      1.2143      0.19275
     0.69124       1.266      0.14339
     0.64976      1.3116      0.10821
     0.61645      1.3524     0.082338
     0.58973      1.3895     0.062712
     0.56847      1.4237     0.047365
      0.5518      1.4557     0.034979
     0.53912      1.4859     0.024643
     0.52998      1.5149     0.015701
     0.52406      1.5429    0.0076565
     0.52113      1.5704   0.00011671
     0.52109      1.5975   -0.0072568
     0.52387      1.6247    -0.014778
     0.52952      1.6521    -0.022764
     0.53811        1.68    -0.031565
     0.54985      1.7088    -0.041591
     0.56497      1.7387    -0.053358
     0.58386      1.7702    -0.067533

Error using GradientDescent
20 iterations have been carried out without convergence. Results as shown (x, y, value).
```

- Changing step size:

```
>> f = (x^2)*cos(y);
>> GradientDescent(f,[1;1],0.0001)
     0.99989      1.0001      0.54011
     0.99978      1.0002      0.53993
     0.99968      1.0003      0.53974
     0.99957      1.0003      0.53955
     0.99946      1.0004      0.53937
     0.99935      1.0005      0.53918
     0.99924      1.0006      0.53899
     0.99914      1.0007       0.5388
     0.99903      1.0008      0.53862
     0.99892      1.0008      0.53843
     0.99881      1.0009      0.53824
      0.9987       1.001      0.53806
      0.9986      1.0011      0.53787
     0.99849      1.0012      0.53768
     0.99838      1.0013       0.5375
     0.99827      1.0013      0.53731
     0.99817      1.0014      0.53712
     0.99806      1.0015      0.53694
     0.99795      1.0016      0.53675
     0.99784      1.0017      0.53657

Error using GradientDescent
20 iterations have been carried out without convergence. Results as shown (x, y, value).
```

- Changed the step size as 1

Then :

```
>> GradientDescent(f,[1;1],1)
The minimizer is estimated to be (0.000772,-107.964538). The value of the function there is 0.000000.

ans =

   0.00077225
      -107.96
```

```
GradientDescent(f,[1;1],1)
The minimizer is estimated to be (0.000772,-107.964538). The value of the function there is
0.000000.

ans =

   0.00077225
      -107.96
```

6.2 $x + y$

f = x+y;
GradientDescent(f,[1;1],0.1)

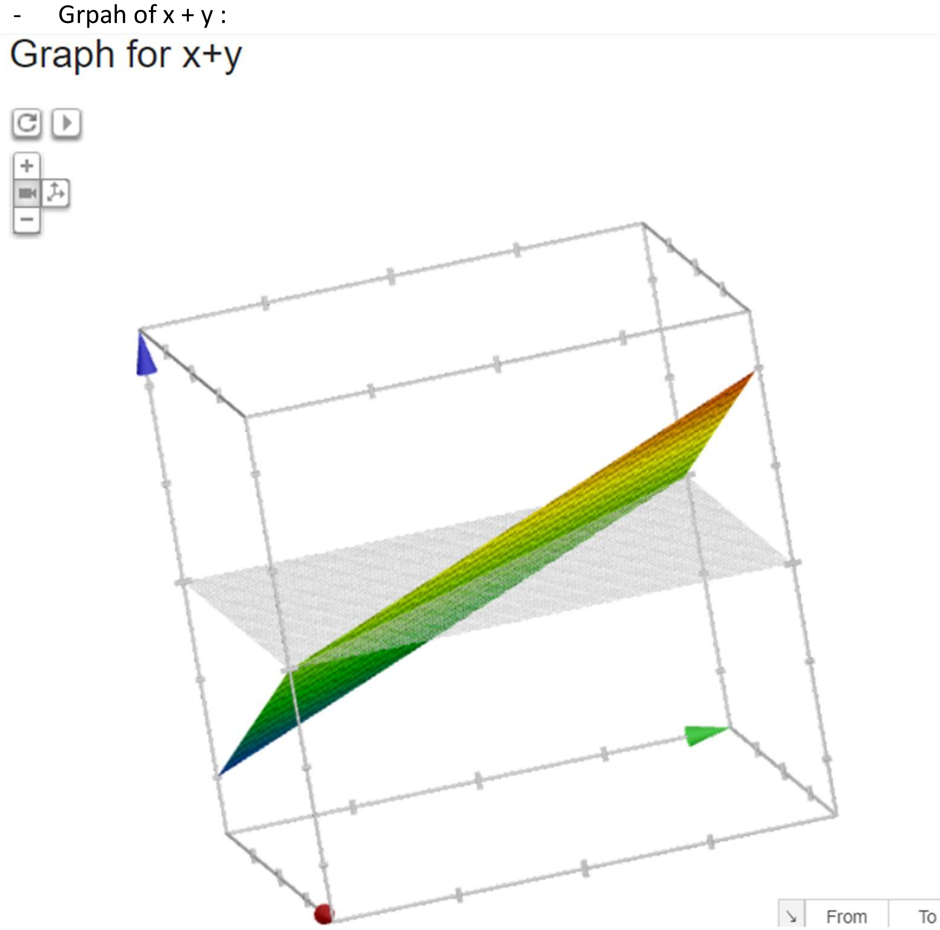
OUTPUT MATLAB: COMMAND WINDOW

```
>> f = x+y;
>> GradientDescent(f,[1;1],0.1)
          0.9          0.9          1.8
          0.8          0.8          1.6
          0.7          0.7          1.4
          0.6          0.6          1.2
          0.5          0.5            1
          0.4          0.4          0.8
          0.3          0.3          0.6
          0.2          0.2          0.4
          0.1          0.1          0.2
   1.3878e-16   1.3878e-16   2.7756e-16
         -0.1         -0.1         -0.2
         -0.2         -0.2         -0.4
         -0.3         -0.3         -0.6
         -0.4         -0.4         -0.8
         -0.5         -0.5           -1
         -0.6         -0.6         -1.2
         -0.7         -0.7         -1.4
         -0.8         -0.8         -1.6
         -0.9         -0.9         -1.8
           -1           -1           -2

Error using GradientDescent
20 iterations have been carried out without convergence. Results as shown (x, y, value).
```

- Changing h size:

```
>> GradientDescent(f,[1;1],0.0001)
      0.9999      0.9999      1.9998
      0.9998      0.9998      1.9996
      0.9997      0.9997      1.9994
      0.9996      0.9996      1.9992
      0.9995      0.9995       1.999
      0.9994      0.9994      1.9988
      0.9993      0.9993      1.9986
      0.9992      0.9992      1.9984
      0.9991      0.9991      1.9982
       0.999       0.999       1.998
      0.9989      0.9989      1.9978
      0.9988      0.9988      1.9976
      0.9987      0.9987      1.9974
      0.9986      0.9986      1.9972
      0.9985      0.9985       1.997
      0.9984      0.9984      1.9968
      0.9983      0.9983      1.9966
      0.9982      0.9982      1.9964
      0.9981      0.9981      1.9962
       0.998       0.998       1.996

Error using GradientDescent
20 iterations have been carried out without convergence. Results as shown (x, y, value).
```

- Grpah of x + y :



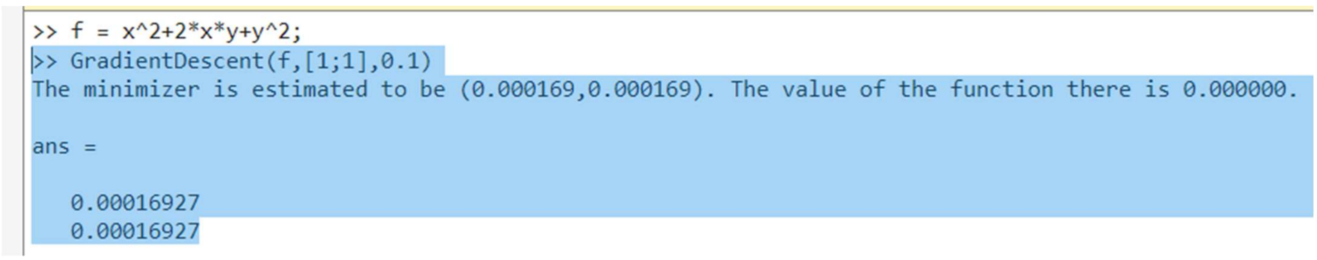
That is it’s a plane . Hence finding a minimum is not possible as every time we’ll find a lower value always.

Ideally as it is a plane, So it does make sense that we will be getting a lower value again and again

6.3 $x^2 + 2xy + y^2$

f = x^2+2*x*y+y^2;
GradientDescent(f,[1;1],0.1)

OUTPUT MATLAB: COMMAND WINDOW



```
>> f = x^2+2*x*y+y^2;
>> GradientDescent(f,[1;1],0.1)
The minimizer is estimated to be (0.000169,0.000169). The value of the function there is 0.000000.

ans =

   0.00016927
   0.00016927
```

```
GradientDescent(f,[1;1],0.1)
The minimizer is estimated to be (0.000169,0.000169). The value of the function there is
0.000000.

ans =

   0.00016927
   0.00016927
```

We are getting a min value at the first go itself, So no need for iteration. On this one.

Exercise 7

Notes

[Gradient descent - Wikipedia]

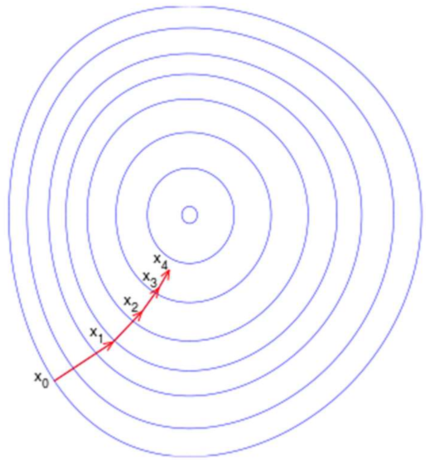


Figure 1 : Illustration of gradient descent on a series of level sets

using a step size Γ that is too small would slow convergence, and a Γ too large would lead to divergence,

Γ → gamma

Gradient descent can be used to solve a system of linear equations Ax -b = 0

reformulated as a quadratic minimization problem.

References:

1.) Sir's Lecture.
 - Sign in to your account. Login.microsoftonline.com. Retrieved November 7, 2022, from https://canvas.swansea.ac.uk/courses/36431/external_tools/95
2.) Canvas Material Provided.
 - Sign in to your account. Login.microsoftonline.com. Retrieved November 7, 2022, from https://canvas.swansea.ac.uk/courses/36431/assignments/249906
3.) Wikipedia link Given in sheet.
 - Wikipedia Contributors. (2019, April 19). Gradient descent. Wikipedia; Wikimedia Foundation. https://en.wikipedia.org/wiki/Gradient_descent

End

++++++++++++++++++ Lab 004 ++++++++++++++++++

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