++++++++++++++++++ Lab 006 ++++++++++++++++++

++++++++++++++++++ 2154638 ++++++++++++++++++

Mathematical Skills for Data Scientists Lab Exercises 6

CSCM70

Lab 6 Solutions

Mathematical Skills for Data Scientists Lab Exercises 6 – 4 Marks

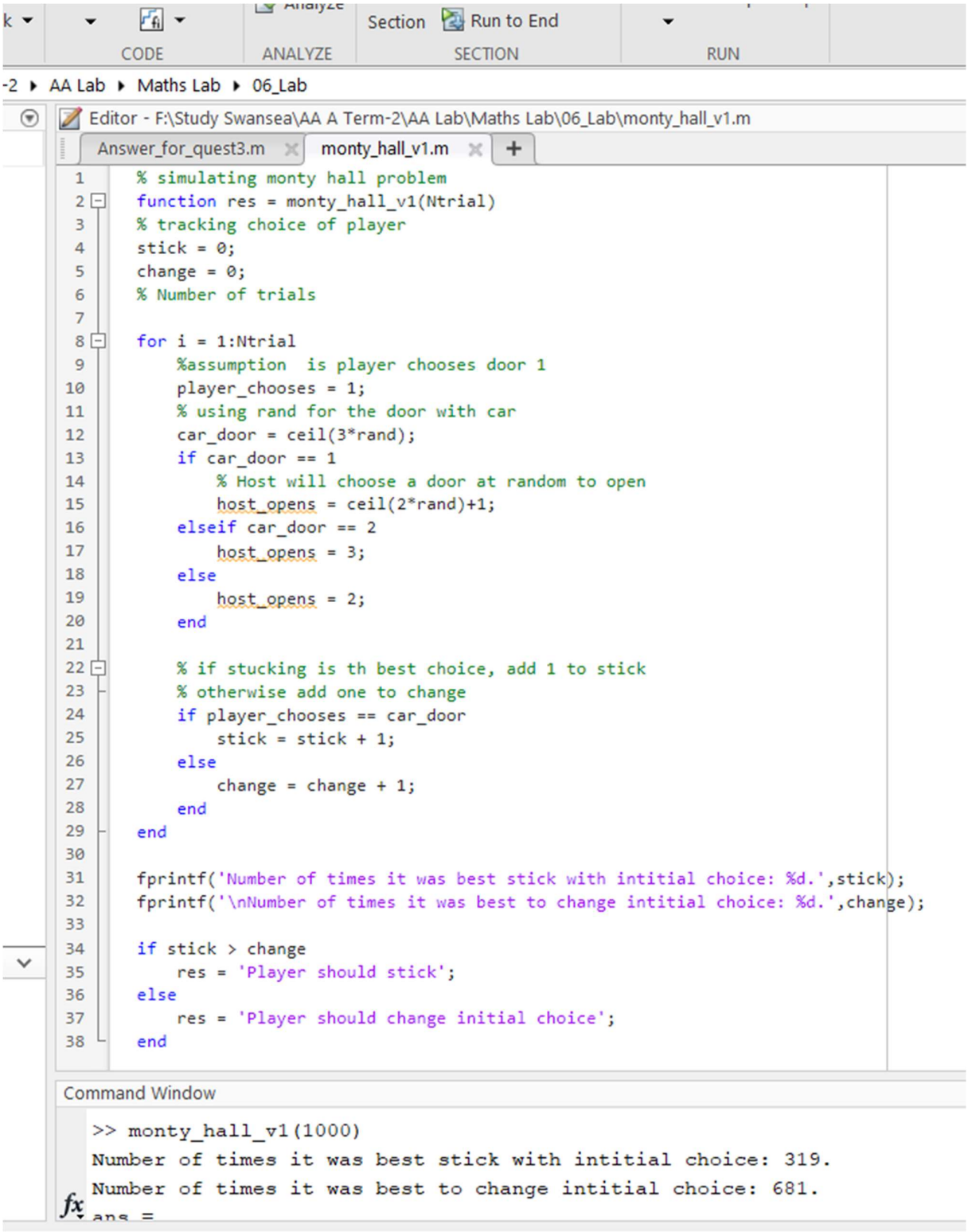
Question 1

The Monty Hall game: There are three doors. Two doors have a goat behind them. The other door has a new car behind it. You choose a door. The game host, Monty, who knows what is behind each door, opens one of the two doors which you did not choose, revealing a goat. If both doors had goats behind them, Monty chooses between the two with equal likelihood. Monty invites you to change your initial decision. What should you do? Switch, or stick with your first choice? (**Important**: Your goal is to win a car, not a goat.)

Answer 1

monty_hall_v1.m

CODE



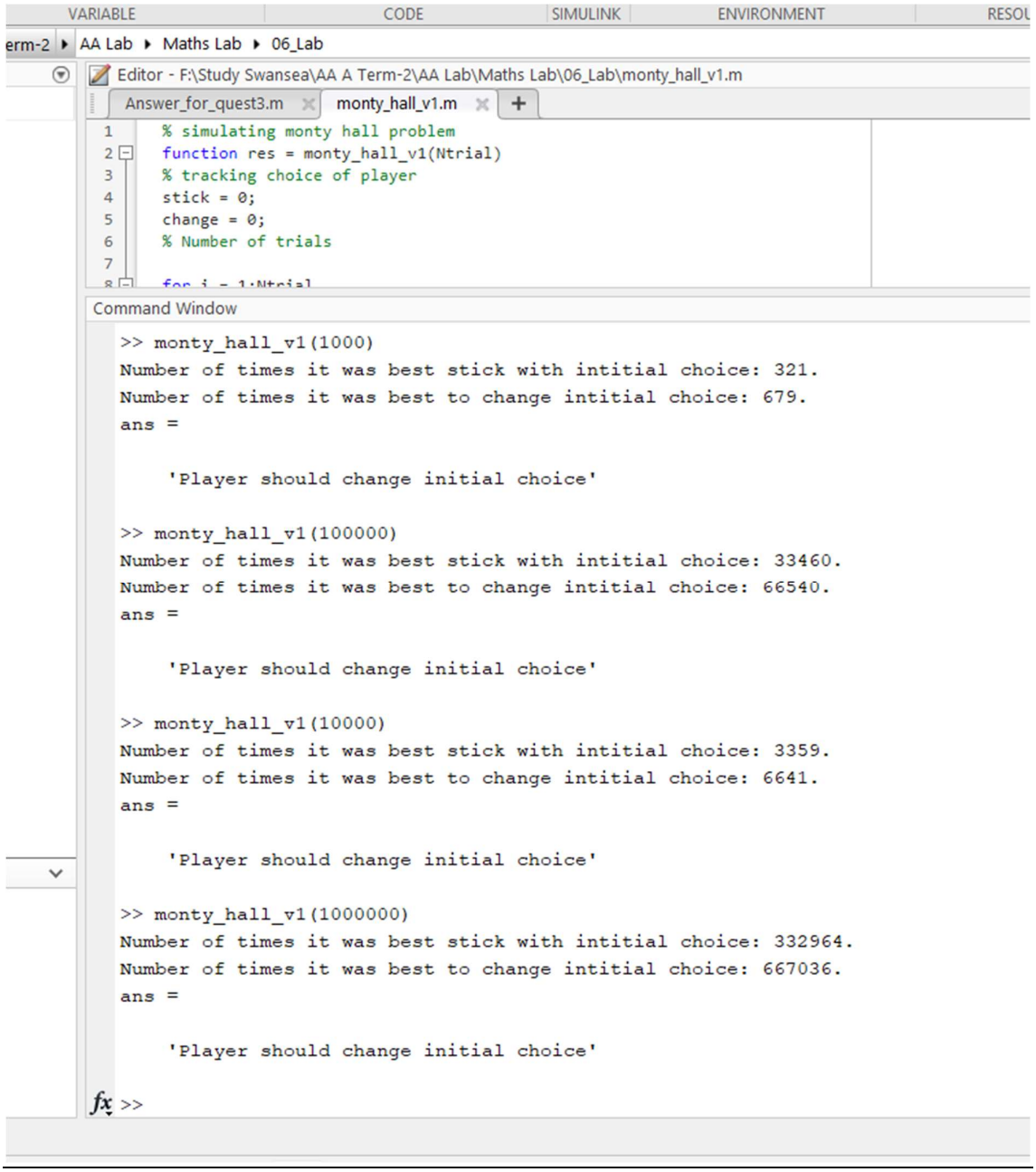
Code In easier form to read.

```
% simulating monty hall problem
function res = monty_hall_v1(Ntrial)
% tracking choice of player
stick = 0;
change = 0;
% Number of trials

for i = 1:Ntrial
    %assumption  is player chooses door 1
    player_chooses = 1;
    % using rand for the door with car
    car_door = ceil(3*rand);
    if car_door == 1
        % Host will choose a door at random to open
        host_opens = ceil(2*rand)+1;
    elseif car_door == 2
        host_opens = 3;
    else
        host_opens = 2;
    end

    % if stucking is th best choice, add 1 to stick
    % otherwise add one to change
    if player_chooses == car_door
        stick = stick + 1;
    else
        change = change + 1;
    end
end

fprintf('Number of times it was best stick with intitial choice: %d.',stick);
fprintf('\nNumber of times it was best to change intitial choice: %d.',change);

if stick > change
    res = 'Player should stick';
else
    res = 'Player should change initial choice';
end
```

Ref: From Sir's Lecture - Link attached in bottom

OUTPUT MATLAB: COMMAND WINDOW

Output



Question 2

Exercise 2 (Excel). **[Example]You are betting on coin tosses, and at the start believe that there is a 1 in 10 chance of your opponent using a coin with two heads. Calculate how likely you should consider your opponent cheating after seeing 3 subsequent heads. (no marks)**

Answer 2

Attaching the excel:

Answer2_Coin_Toss.xlsx Excel Sheet above.

OUTPUT EXCEL:

OUTPUT

No. of Toss	Prior P(Cheating)	Update Factor P(Head|Cheating) / P(Head)	Posterior P(Cheating|Head)		P(Head|Cheating) = 1	P(Head) = P(Cheating).P(Head/Cheating) + P'(Cheating).P(H/no Cheating)	
1	0.1	1.818181818	0.181818182				
2	0.18181818	1.692307692	0.307692308				
3	0.30769231	1.529411765	0.470588235				
4	0.47058824	1.36	0.64				
5	0.64	1.219512195	0.780487805				
6	0.7804878	1.123287671	0.876712329				
7	0.87671233	1.065693431	0.934306569				
8	0.93430657	1.033962264	0.966037736				
9	0.96603774	1.017274472	0.982725528				
10	0.98272553	1.008712488	0.991287512				
11	0.99128751	1.004375304	0.995624696				
12	0.9956247	1.002192448	0.997807552				
13	0.99780755	1.001097427	0.998902573				
14	0.99890257	1.000549015	0.999450985				
15	0.99945099	1.000274583	0.999725417				
16	0.99972542	1.00013731	0.99986269				
17	0.99986269	1.00006866	0.99993134				
18	0.99993134	1.000034331	0.999965669				
19	0.99996567	1.000017166	0.999982834				
20	0.99998283	1.000008583	0.999991417				
21	0.99999142	1.000004292	0.999995708				
22	0.99999571	1.000002146	0.999997854				
23	0.99999785	1.000001073	0.999998927				
24	0.99999893	1.000000536	0.999999464				
25	0.99999946	1.000000268	0.999999732				
26	0.99999973	1.000000134	0.999999866				
27	0.99999987	1.000000067	0.999999933				
28	0.99999993	1.000000034	0.999999966				
29	0.99999997	1.000000017	0.999999983				
30	0.99999998	1.000000008	0.999999992				

Question 3

Exercise 3 (Excel). Consider the setting of Exercise 2, but start with an initial probability of cheating of $\frac{1}{1.000.00}$. Show how the probability assigned to cheating vary as the number of subsequent heads increases. When is the threshold of 50% exceeded? When the threshold of 99.9%? (marks 2)

Answer 3

Excel Sheet above.

So, We have tried it for two instances
1.) When P(cheating) = 1/1000
and
2.) When P(cheating) = 1/1,00,000

➔ FIrst

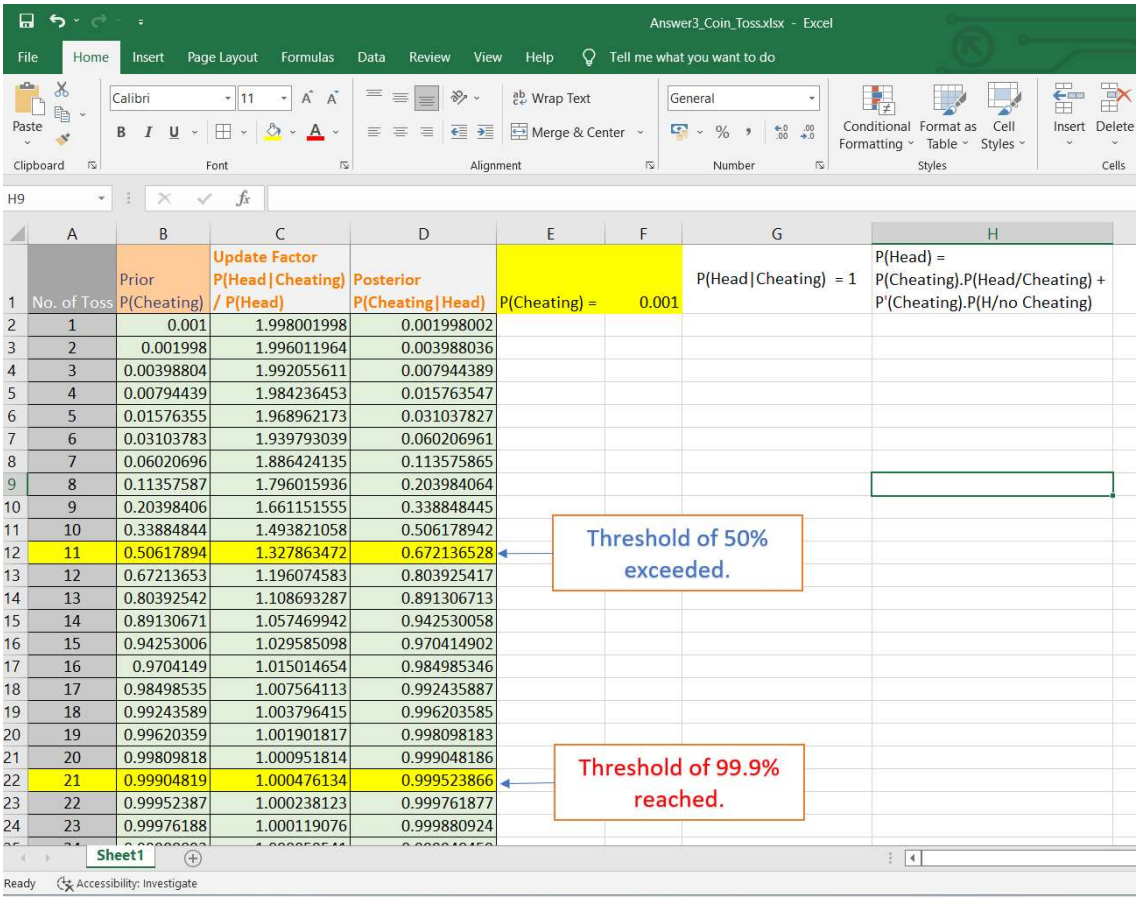
0In column F we choose the initial probability of **cheating as 1/1000** and the result as shown below: The thresholds are highlighted in yellow.

That is 50% threshold is exceeded at 11^{th} subsequent head appearing .

Also, 99.9% threshold is reached at about 21^{st} subsequent head appearing with initial probability of 1/1000.

OUTPUT EXCEL:

OUTPUT 1



	A	B	C	D	E	F	G	H
1	No. of Toss	Prior P(Cheating)	Update Factor P(Head|Cheating) / P(Head)	Posterior P(Cheating|Head)	P(Cheating) =	0.001	P(Head|Cheating) = 1	P(Head) = P(Cheating).P(Head/Cheating) + P'(Cheating).P(H/no Cheating)
2	1	0.001	1.998001998	0.001998002				
3	2	0.001998	1.996011964	0.003988036				
4	3	0.00398804	1.992055611	0.007944389				
5	4	0.00794439	1.984236453	0.015763547				
6	5	0.01576355	1.968962173	0.031037827				
7	6	0.03103783	1.939793039	0.060206961				
8	7	0.06020696	1.886424135	0.113575865				
9	8	0.11357587	1.796015936	0.203984064				
10	9	0.20398406	1.661151555	0.338848445				
11	10	0.33884844	1.493821058	0.506178942				
12	11	0.50617894	1.327863472	0.672136528				
13	12	0.67213653	1.196074583	0.803925417				
14	13	0.80392542	1.108693287	0.891306713				
15	14	0.89130671	1.057469942	0.942530058				
16	15	0.94253006	1.029585098	0.970414902				
17	16	0.9704149	1.015014654	0.984985346				
18	17	0.98498535	1.007564113	0.992435887				
19	18	0.99243589	1.003796415	0.996203585				
20	19	0.99620359	1.001901817	0.998098183				
21	20	0.99809818	1.000951814	0.999048186				
22	21	0.99904819	1.000476134	0.999523866				
23	22	0.99952387	1.000238123	0.999761877				
24	23	0.99976188	1.000119076	0.999880924				

24	23	0.99976188	1.000119076	0.999880924
25	24	0.99988092	1.000059541	0.999940459
26	25	0.99994046	1.000029772	0.999970228
27	26	0.99997023	1.000014886	0.999985114
28	27	0.99998511	1.000007443	0.999992557
29	28	0.99999256	1.000003722	0.999996278
30	29	0.99999628	1.000001861	0.999998139
31	30	0.99999814	1.00000093	0.99999907
32	31	0.99999907	1.000000465	0.999999535
33	32	0.99999953	1.000000233	0.999999767
34	33	0.99999977	1.000000116	0.999999884
35	34	0.99999988	1.000000058	0.999999942
36	35	0.99999994	1.000000029	0.999999971
37	36	0.99999997	1.000000015	0.999999985
38	37	0.99999999	1.000000007	0.999999993
39	38	0.99999999	1.000000004	0.999999996
40	39	1	1.000000002	0.999999998
41	40	1	1.000000001	0.999999999
42	41	1	1	1
43	42	1	1	1
44	43	1	1	1
45	44	1	1	1
46	45	1	1	1

➔ Second

Output 2

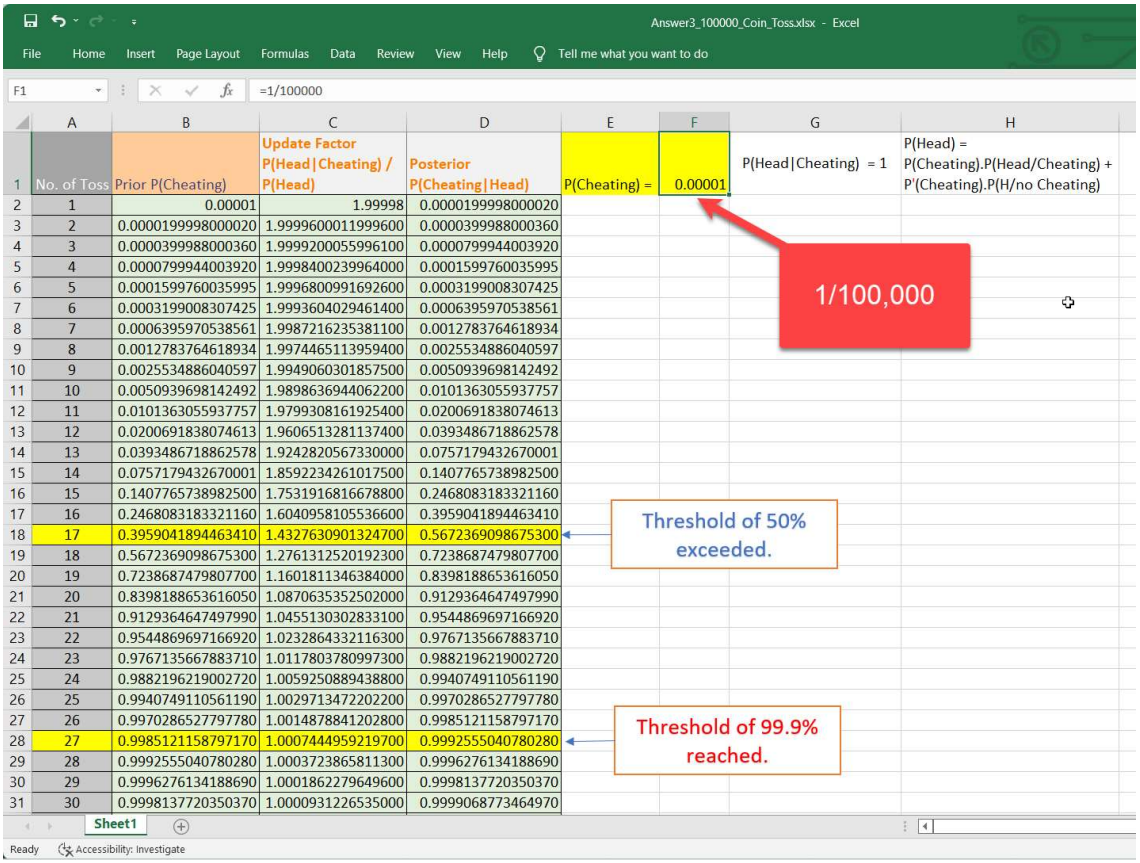
Answer3_100000_Coin_Toss.xlsx Excel Sheet above.

Now we are trying with P(Cheating) = 1 / 100,000.

The thresholds are highlighted in yellow.
Here, 50% threshold is exceeded at 17th subsequent head appearing .

Also, 99.9% threshold is reached at about 27th subsequent head appearing with initial probability of 1/100,000.

OUTPUT EXCEL:



No. of Toss	Prior P(Cheating)	Update Factor P(Head|Cheating) / P(Head)	Posterior P(Cheating|Head)
1	0.00001	1.99998	0.0000199998000020
2	0.0000199998000020	1.9999600011999600	0.0000399988000360
3	0.0000399988000360	1.9999200055996100	0.0000799944003920
4	0.0000799944003920	1.9998400239964000	0.0001599760035995
5	0.0001599760035995	1.9996800991692600	0.0003199008307425
6	0.0003199008307425	1.9993604029461400	0.0006395970538561
7	0.0006395970538561	1.9987216235381100	0.0012783764618934
8	0.0012783764618934	1.9974465113959400	0.0025534886040597
9	0.0025534886040597	1.9949060301857500	0.0050939698142492
10	0.0050939698142492	1.9898636944062200	0.0101363055937757
11	0.0101363055937757	1.9799308161925400	0.0200691838074613
12	0.0200691838074613	1.9606513281137400	0.0393486718862578
13	0.0393486718862578	1.9242820567330000	0.0757179432670001
14	0.0757179432670001	1.8592234261017500	0.1407765738982500
15	0.1407765738982500	1.7531916816778800	0.2468083183321160
16	0.2468083183321160	1.6040958105536600	0.3959041894463410
17	0.3959041894463410	1.4327630901324700	0.5672369098675300
18	0.5672369098675300	1.2761312520192300	0.7238687479807700
19	0.7238687479807700	1.1601811346384000	0.8398188653616050
20	0.8398188653616050	1.0870635352502000	0.9129364647497990
21	0.9129364647497990	1.0455130302833100	0.9544869697166920
22	0.9544869697166920	1.0232864332116300	0.9767135667883710
23	0.9767135667883710	1.0117803780997300	0.9882196219002720
24	0.9882196219002720	1.0059250889438800	0.9940749110561190
25	0.9940749110561190	1.0029713472202200	0.9970286527797780
26	0.9970286527797780	1.0014878841202800	0.9985121158797170
27	0.9985121158797170	1.0007444959219700	0.9992555040780280
28	0.9992555040780280	1.0003723865811300	0.9996276134188690
29	0.9996276134188690	1.0001862279649600	0.9998137720350370
30	0.9998137720350370	1.0000931226535000	0.9999068773464970

F1 =1/100000

	A	B	C	D
31	30	0.9998137720350370	1.0000931226535000	0.9999068773464970
32	31	0.9999068773464970	1.0000465634948100	0.9999534365051910
33	32	0.9999534365051910	1.0000232822894600	0.9999767177105430
34	33	0.9999767177105430	1.0000116412802500	0.9999883587197540
35	34	0.9999883587197540	1.0000058206740000	0.9999941793259970
36	35	0.9999941793259970	1.0000029103454700	0.9999970896545280
37	36	0.9999970896545280	1.0000014551748500	0.9999985448251470
38	37	0.9999985448251470	1.0000007275879600	0.9999992724120440
39	38	0.9999992724120440	1.0000003637941100	0.9999996362058900
40	39	0.9999996362058900	1.0000001818970900	0.9999998181029120
41	40	0.9999998181029120	1.0000000909485500	0.9999999090514480
42	41	0.9999999090514480	1.0000000454742800	0.9999999545257220
43	42	0.9999999545257220	1.0000000227371400	0.9999999772628600
44	43	0.9999999772628600	1.0000000113685700	0.9999999886314300
45	44	0.9999999886314300	1.0000000056842900	0.9999999943157150
46	45	0.9999999943157150	1.0000000028421400	0.9999999971578580
47	46	0.9999999971578580	1.0000000014210700	0.9999999985789290
48	47	0.9999999985789290	1.0000000007105400	0.9999999992894640
49	48	0.9999999992894640	1.0000000003552700	0.9999999996447320
50	49	0.9999999996447320	1.0000000001776300	0.9999999998223660
51	50	0.9999999998223660	1.0000000000888200	0.9999999999111830
52	51	0.9999999999111830	1.0000000000444100	0.9999999999555920
53	52	0.9999999999555920	1.0000000000222000	0.9999999999777960
54	53	0.9999999999777960	1.0000000000111000	0.9999999999888980
55	54	0.9999999999888980	1.0000000000055500	0.9999999999944490
56	55	0.9999999999944490	1.0000000000027800	0.9999999999972250
57	56	0.9999999999972250	1.0000000000013900	0.9999999999986120
58	57	0.9999999999986120	1.0000000000006900	0.9999999999993060
59	58	0.9999999999993060	1.0000000000003500	0.9999999999996530
60	59	0.9999999999996530	1.0000000000001700	0.9999999999998260
61	60	0.9999999999998260	1.0000000000000900	0.9999999999999130
62	61	0.9999999999999130	1.0000000000000400	0.9999999999999570
63	62	0.9999999999999570	1.0000000000000200	0.9999999999999780

63	62	0.9999999999999570	1.0000000000000200	0.9999999999999780
64	63	0.9999999999999780	1.0000000000000100	0.9999999999999890
65	64	0.9999999999999890	1.0000000000000100	0.9999999999999950
66	65	0.9999999999999950	1.0000000000000000	0.9999999999999970
67	66	0.9999999999999970	1.0000000000000000	0.9999999999999990
68	67	0.9999999999999990	1.0000000000000000	0.9999999999999990
69	68	0.9999999999999990	1.0000000000000000	1.0000000000000000
70	69	1.0000000000000000	1.0000000000000000	1.0000000000000000
71	70	1.0000000000000000	1.0000000000000000	1.0000000000000000
72	71	1.0000000000000000	1.0000000000000000	1.0000000000000000
73	72	1.0000000000000000	1.0000000000000000	1.0000000000000000
74	73	1.0000000000000000	1.0000000000000000	1.0000000000000000
75	74	1.0000000000000000	1.0000000000000000	1.0000000000000000
76	75	1.0000000000000000	1.0000000000000000	1.0000000000000000
77	76	1.0000000000000000	1.0000000000000000	1.0000000000000000
78	77	1.0000000000000000	1.0000000000000000	1.0000000000000000
79	78	1.0000000000000000	1.0000000000000000	1.0000000000000000
80	79	1.0000000000000000	1.0000000000000000	1.0000000000000000
81	80	1.0000000000000000	1.0000000000000000	1.0000000000000000
82				

References:

1.) Sir's Lecture.
- *Sign in to Panopto Panopto*. Available at: https://swanseauniversity.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=3f0aa22c-548e-4b87-a73c-af5400c6ceda (Accessed: November 28, 2022).

2.) Sir's Lecture Class
- *Sign in to Panopto Panopto*. Available at: https://swanseauniversity.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=40baf037-d115-4806-9204-af5500a5904b (Accessed: November 28, 2022).

End

++++++++++++++++++ Lab 006 ++++++++++++++++++

++++++++++++++++++ 2154638 ++++++++++++++++++