CSCM 70

MATHEMATICAL SKILLS OF DATA SCIENTISTS

ASSESSED SHEET - 1

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MSc DATA SCIENCE

(7)

Quest :> 1

Ans

$$2x_1 - 2x_2 + 2x_3 = 1$$
$$-3x_1 - 6x_2 = -1$$
$$x_1 - 7x_2 + 10x_3 = 2$$

(i) Write this in matrix form $AX = b$
Identify A, X & b.

(ii) Find x_1, x_2, x_3 from echelon form

Sol^n (2)(i)

$$2x_1 - 2x_2 + 2x_3 = 1$$
$$-3x_1 - 6x_2 = -1$$
$$x_1 - 7x_2 + 10x_3 = 2$$

So,

$$\underset{\text{constant}}{A} \quad \underset{\text{variable}}{X} = \underset{\text{Result}}{b}$$

$$\begin{bmatrix} 2 & -2 & 2 \\ -3 & -6 & 0 \\ 1 & -7 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$A \Rightarrow \begin{bmatrix} 2 & -2 & 2 \\ -3 & -6 & 0 \\ 1 & -7 & 10 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

2(ii) Augmented Matrix :-

$$\left[\begin{array}{ccc|c} 2 & -2 & 2 & 1 \\ -3 & -6 & 0 & -1 \\ 1 & -7 & 10 & 2 \end{array}\right] R_1 \to R_1/2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1/2 \\ -3 & -6 & 0 & -1 \\ 1 & -7 & 10 & 2 \end{array}\right] R_2 \to R_2 + 3R_1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1/2 \\ 0 & -9 & 3 & 1/2 \\ 1 & -7 & 10 & 2 \end{array}\right] R_3 \to R_3 - R_1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1/2 \\ 0 & -9 & 3 & 1/2 \\ 0 & -6 & 9 & 3/2 \end{array}\right] R_2 \to -R_2/9$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1/2 \\ 0 & -9 & 3 & 1/2 \\ 0 & -6 & 9 & 3/2 \end{array}\right] \quad R_2 \rightarrow -R_2/9$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1/2 \\ 0 & 1 & -1/3 & -1/18 \\ 0 & -6 & 9 & 3/2 \end{array}\right] \quad R_1 \rightarrow R_1 + R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 4/9 \\ 0 & 1 & -1/3 & -1/18 \\ 0 & -6 & 9 & 3/2 \end{array}\right] \quad R_3 = R_3 + 6R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 4/9 \\ 0 & 1 & -1/3 & -1/18 \\ 0 & 0 & 7 & 7/6 \end{array}\right] \quad R_3 = R_3/7$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 4/9 \\ 0 & 1 & -1/3 & -1/18 \\ 0 & 0 & 1 & 6 \end{array}\right] \quad R_1 \rightarrow R_1 - 2R_3/3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & -1/3 & -1/18 \\ 0 & 0 & 1 & 1/6 \end{array}\right] \quad R_2 \rightarrow R_2 + R_3/3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/6 \end{array}\right] \quad \boxed{\because \text{IDENTITY MATRIX FORM REACHED}}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 0 \\ 1/6 \end{bmatrix}$$

↳ Values.

Therefore, at $x = 3$ is the local Minimum

and

$x = 1$ is the local Maximum.

Question 3: Check wether vectors are linearly dependent or independent.

(i) $(1, 1, 1), (1, 2, 1), (2, 3, 4)$ in R^3 over R.

Soln: (i) $(1, 1, 1), (1, 2, 1), (2, 3, 4)$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 4 \end{vmatrix} \quad R_2 \to R_2 - R_1$$

So, basically we are performing row transmission.

Again $\Rightarrow$ $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 4 \end{vmatrix}$ $\quad R_2 \to R_2 - R_1$

$\Rightarrow$ $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 4 \end{vmatrix}$ $\quad R_3 \to R_3 - 2R_1$

$\Rightarrow$ $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix}$ $\quad R_3 \to R_3 - R_2$

$\Rightarrow$ $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$ $\quad R_3 \to \dfrac{R_3}{2}$

$\Rightarrow$ $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $\quad R_1 \to R_1 - R_3$

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $\quad R_1 \to R_1 - R_2$

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ Hence Linearly Independent

We got 1 in all diagonal ~~Mat~~ element position and zero in below & above hence it leads to Linear Independence.

3 (ii) $(1,-1,1), (1,-2,2)(2,-1,-1)$ in R^3 on R

Solⁿ 3 (ii)

↳ $(1,-1,1), (1,-2,2), (2,-1,-1)$

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & -1 & -1 \end{vmatrix} \quad R_2 \to R_2 - R_1$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ 2 & -1 & -1 \end{vmatrix} \quad R_3 \to R_3 - 2R_1$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -2 \end{vmatrix} \quad R_2 \to -R_2$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{vmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{vmatrix} \quad R_3 \to -R_3$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} \quad R_2 \to R_2 + R_3$$

Again
$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} \quad R_2 \rightarrow R_2 + R_3$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad R_1 \rightarrow R_1 - R_3$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad R_1 \rightarrow R_1 + R_2$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Hence Linearly Independent.

AN 1

uestion 4)

(i) $\lim_{x \to 2} (x^2 - 4x)$

Answer => (i) So here limit tends to '2' so first we will write the normalise form of the above question that is :→

$$\lim_{x \to 2} (x^2 - 4x)$$

$$\Rightarrow \lim_{x \to 2} x(x-4)$$

$$\Rightarrow \lim_{x \to 2} 2(2-4)$$

$$\Rightarrow 2 - 2$$

$$\boxed{= -4}$$ Answer..

uestion 4

(ii) $\lim_{x \to 0} \frac{|x|}{x}$

Answer => * • Key point when limit tends to zero : It can go either to "0^+" or "0^-". So we need to calculate for both of them.

• These are called RHL Right Hand Limit & LHL - Left hand limit Respectively.

$$\lim_{x \to 0} \frac{|x|}{x}$$

$\Rightarrow \lim_{x \to 0^+} \frac{x}{x} = 1$, Right hand limit
$(x > 0)$
x just greater than 0

$\Rightarrow \lim_{x \to 0^-} \frac{-x}{x} = -1$ left hand limit
$(x < 0)$
x just less than 0

For existence of limit of a real valued function at a certain point the value of left hand limit and right hand limit should exist and have same value

Since here LHL $\neq$ RHL $\therefore$ Limit Doesn't exist.

Question 5//

Given: $f(x) = x^2 - 3x$

Find? $\Rightarrow \frac{df(x)}{dx}$ using def of derivative

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sol^n

$$y = (x^3+3)^4 \; (2x^3-5)^3$$

$$f_1(x) = (x^3+3)^4 \qquad f_2(x) = (2x^3-5)^3$$

$$\Rightarrow \quad y = f_1(x) \, f_2(x)$$

$$\Rightarrow \quad \frac{dy}{dx} = f_1(x) f_2'(x) + f_2(x) f_1'(x)$$

[using the Product rule of differentiation]

$$\Rightarrow \frac{dy}{dx} = (x^3+3)^4 \cdot 3 \cdot (2x^3-5)^2 \frac{d}{dx}(2x^3-5)$$

$$+ (2x^3-5)^3 \cdot 4 \cdot (x^3+3)^3 \frac{d}{dx}(x^3+3)$$

$$\Rightarrow \frac{dy}{dx} = (x^3+3)^4 \cdot 3 \cdot (2x^3-5)^2 (6x^2)$$

$$+ (2x^3-5)^3 \cdot 4 \cdot (x^3+3)^3 (3x^2)$$

$$\Rightarrow \frac{dy}{dx} = 18x^2 (x^3+3)^4 (2x^3-5)^2$$

$$+ 12x^2 (2x^3-5)^3 (x^3+3)^3$$

$$\Rightarrow \frac{dy}{dx} = 6x^2 (x^3+3)^3 (2x^3-5)^2 \cdot$$

$$\left(3(x^3+3) + 2(2x^3-5) \right)$$

$$\Rightarrow \frac{dy}{dx} = 6x^2(x^3+3)^3(2x^3-5)^2$$
$$(3x^3+9+4x^3-10)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = 6x^2(x^3+3)^3(2x^3-5)^2(7x^3-1)}$$

Quest 7)) Find??

[Extrema for the following functions and classify them as Maxima & Minimum.]

(i) $f(x) = x^3 + 2x^2 - 4x - 8$

Sol^n :-> (i) $f(x) = x^3 + 2x^2 - 4x - 8 \rightarrow$ (A)*

Extrema occurs where $f'(x) = 0$,

here $f(x) = x^3 + 2x^2 - 4x - 8$ ----- (1)

$\Rightarrow f'(x) = 3x^2 + 4x - 4$ ----- (2)

$\Rightarrow \quad 3x^2 + 4x - 4 = 0 \quad (\because (1)) \rightarrow$ (B)*

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

where $\Rightarrow D = b^2 - 4ac$

$\Rightarrow \quad = (4)^2 (3)(-4)$

$= 16 + 48$

$= 64$

$$x = \frac{-(4) \pm \sqrt{64}}{2(3)}$$

$$= \frac{-4 \pm 8}{2(3)}$$

$$= \frac{-2 \pm 4}{3}$$

$x = \frac{-6}{3}$ and $x = \frac{2}{3}$

$= -2$

So extremum occur at

$\boxed{x = -2}$ and $\boxed{x = \frac{2}{3}}$

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Now to classify as minima or maxima, checking $f''(x)$: ··· from (2)

$$f'(x) = 3x^2 - 4x - 4$$

$$f''(x) = 6x + 4$$

Now, (I) When $x = -2$.

$$f''(x)$$
$$\Downarrow$$

then $f''(-2) = -12 + 4$

$$= -8$$

i.e. $f''(-2) < 0$

$\therefore$ $x = -2$ is a point of Maximum

(II) When $x = \frac{2}{3}$

then $f''\left(\frac{2}{3}\right) = 6\left(\frac{2}{3}\right) + 4$

$$= 8$$

i.e. $f''\left(\frac{2}{3}\right) > 0$

$\therefore$ $x = \frac{2}{3}$ is a point of Minimum

7 (ii)

$$f(x) = x^3 - 6x^2 + 9x - 8$$

Sol 7(ii) $f(x) = x^3 - 6x^2 + 9x - 8$

extrema occurs at $f'(x) = 0$

here $f(x) = x^3 - 6x^2 + 9x - 8$ ---- (1)

$f'(x) = 3x^2 - 12x + 9$ ---- (2)

So,

$\Rightarrow$ $3x^2 - 12x + 9 = 0$ --- ($\because$ (1))

$\Rightarrow$ whole eqt^n devide by 3,

$$x^2 - 4x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

where $D = b^2 - 4ac$

$$x = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$= \frac{4 \pm 2}{2}$$

$\boxed{\text{Extrema Points} \Rightarrow x = 3 \text{ or } 1}$

There

Questio

Sol

$$f''(x) = 6x - 12$$

$$f''(3) = 18 - 12$$ [substituting the value as 3]

$$= 6$$

$$\boxed{f''(3) > 0 \quad \therefore \text{Minimum}}$$

[Substituting the value of x as 1

$$f''(1) = 6 - 12$$

$$= -6$$

$$\boxed{f''(1) < 0 \quad \therefore \text{Maximum}}$$

Therefore, at $x = 3$ is the local Minimum and $x = 1$ is the local Maximum.

Question 3: Check wether vectors are linearly dependent or independent.

(i) $(1,1,1), (1,2,1), (2,3,4)$ in R^3 over R.

Soln (i) $(1,1,1), (1,2,1), (2,3,4)$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 4 \end{vmatrix} \quad R_2 \to R_2 - R_1$$

ALLVA's

VA-1

10/10

AA-1, Q1->

Ex 1 Vector Spaces

-> Natural operations turning into vector space.

(I) $R^3 := \{(x, y, z) \mid x, y, z \in R\}$ over R

(I) Addition ->

=> $(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$

II Scalar Multiplication

=> $\lambda(x, y, z) = (\lambda x, \lambda y, \lambda z)$ for any $\lambda \in R$

The Zero Vector is $(0, 0)$.

Ex1 (II) C over R

I) Standard Addition of complex No.

$(a+bi) + (c+di) = (a+c) + (b+d)$

along with the usual multiplication

II) Multiplication

$\lambda(a+bi) = \lambda a + \lambda bi$ make C into a vector space

The zero vector is $0 + 0i$ (As a vector space, C looks R^2)

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Ex (1) (iii) Rover Q

(*) Ref: → Sir's answer.

(i) Similar to above question
The usual addition of Real No. will work

(ii) Multiply a Real No. by Rational No. gives back a Real No.

(iii) The zero vector is the no. 0.

(iv) $C(\mathbb{R}, \mathbb{R})$ (the space of continuous function from R to R) over R

(I) $C(R, R)$
for addition
If f & g 2 functions
then f+g defined as

$$(f+g)(x) = f(x) + g(x)$$

(II) Multiplication
λf
$$(\lambda f)(x) = \lambda(f(x))$$

(iii) The zero vector is the $f x^n$ $x \to 0$

Ex 2 LINEAR INDEPENDENCE

(1) $\{(0,i), (1,0), (4,8)\}$ in $\mathbb{C}^2$ over R

$$\begin{pmatrix} 0 & i \\ 1 & 0 \\ 4 & 8 \end{pmatrix}$$ SWAPPING R_1 & R_2

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 4 & 8 \end{pmatrix} \quad R_3 \Rightarrow R_3 - 4R_1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & 8 \end{pmatrix} \quad R_3 \to R_3 / 8$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & 1 \end{pmatrix}$$

We can't make rows of zero in R_2 & R_3 because i is not in real Numbers

Hence linearly Independent

(2) $\{(0,i),(1,0),(4,8)\}$ in $\mathbb{C}^2$ on $\mathbb{C}$

$$\begin{pmatrix} 0 & i \\ 1 & 0 \\ 4 & 8 \end{pmatrix} \quad \text{Swap } R_1 \text{ \& } R_2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 4 & 8 \end{pmatrix} \quad R_3 \to R_3 - 4R_1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & 8 \end{pmatrix} \quad R_3 \to R_3 / 8$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & 1 \end{pmatrix} \quad R_3 \to R_3 \times -i$$

$$\cancel{\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & -i \end{pmatrix}} \quad \cancel{R_3 = R_3 - R_2}$$

$$\cancel{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & -i \end{pmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \\ 0 & 0 \end{pmatrix}$$

Linearly dependent.

(3) $\{(1,2,3), (0,0,0)(3,2,1)\}$ R^3 over R Ex 3

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

The matrix already has a row of zeros in R_2

Its Linear Dependent //

(4) $\{(1,2,3), (0,1,0), (3,2,1)\}$ in R^3 over R

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \quad R_3 = R_3 - 3R_1$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 7 & 0 \\ 0 & -4 & -8 \end{pmatrix} \quad R_3 \rightarrow R_3 / -4$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \quad R_3 = R_3 - R_2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad R_3 = R_3 / 2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Matrix has achieved Row echelon form
LINEARLY INDEPENDENT //

1)

Exc 3 $f(1, 2, 2) = (1, 1, 1)$ ...

As

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 2 & 2 & -3 \end{pmatrix}$$ and they are equal to

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix}$$

we can write it as.

$$A \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 2 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 2 & 2 & -3 \end{pmatrix}^{-1}$$

1) $\det(C) = 1 \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} + 0 \begin{vmatrix} 2 & 2 \\ 2 & -3 \end{vmatrix} + -1 \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix}$

$(-3-4) + (0) + -1(4-2)$

$-7 + 0 - 2$

$= -9$

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix} \left[\begin{array}{ccc} +\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 2 & -3 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ 2 & -3 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} \\ +\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \end{array} \right]$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} -1 & -2 & 2 \\ -2 & -1 & -2 \\ -1 & 1 & 1 \end{pmatrix} 1/-9$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} 1/9 & 2/9 & -2/9 \\ 2/9 & 1/9 & 2/2 \\ 1/9 & -1/9 & -1/9 \end{pmatrix}$$

The sol^n is not

10/10

VA2

$$A_1 = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad B: \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$v = (1, 1, -2)^t, \quad v_2 = (1, 1, 1)^t$$

$$v_3 = (5, 4, 7)^t$$

Solve $Ax_1 = v_1$, $Bx_2 = v_2$, $Bx_3 = v_3$

$$(B \mid V_1) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & 0 & -2 \end{array}\right) \quad R_2 \to R_2 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & -2 & -1 & -3 \end{array}\right) \quad R_2 \to R_2 \times -1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & -1 & -3 \end{array}\right) \quad R_3 \to R_3 + 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & -1 \end{array}\right) \quad R_3 \to R_3/3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1/3 \end{array}\right) \quad \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - 2R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 4/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -1/3 \end{array}\right) \quad R_1 \to R_1 - 2R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -6/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -1/3 \end{array}\right) \quad X_1 = \frac{1}{3}(-6, 5, -1)^t$$

For $B X_2 = V_2$

$$(B \mid V_2) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 0 & 1 \\ 0 & 0 & 7 & 1 \end{array}\right) \quad R_2 = R_2 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & -6 & -1 \\ 0 & 0 & 7 & 1 \end{array}\right) \quad R_2 = R_2 + \frac{6}{7} R_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & -1/7 \\ 0 & 0 & 7 & 1 \end{array}\right)$$

As Row is equal two

(i.e) $a = 1/7$

$$(B/v_3) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 4 & 0 & 4 \\ 0 & 0 & 7 & 7 \end{array}\right) \quad R_2 \to R_2 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & -6 & -6 \\ 0 & 0 & 7 & 7 \end{array}\right) \quad R_2 \to R_2 / -6$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 7 & 7 \end{array}\right) \quad R_3 = R_3 - 7R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad R_1 = R_1 - 3R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$c = -1$, $a + 2b = 8$

$x_1 = (8 - 2b, b, -1)$

As whole Row $= 0$ so Infinite Possib.

G2

1)

02 LIMITS

$$\lim_{x \to 3} 2x$$

1) $= 2(3)$
$= 6$

2) $\lim_{x \to 0} \frac{2}{x}$

$x \to 0 \Rightarrow 2/0$
$= \infty$
that is underivable.

3) $\lim_{x \to 0} \frac{2\cancel{x}}{\cancel{x}}$

$\Rightarrow \lim_{x \to 0} \{2\}$

4) let $f(x) = x$ if $x \in Q$ & $f(x) = 2x$ if $x \in R / Q$

what about $\lim_{x \to 1} f(x)$?

what about $\lim_{x \to 0} f(x)$?

As $x=1$ the function might be near 1 & 2 so not unique

$\lim_{x \to 0} f(x) = 0$ because for any $\epsilon > 0$, choosing

ϵ / L gurantees $|f(0+\delta) - 0|$
$= |f(x)| < \epsilon$ where

(5) $\lim_{x \to 2} (x^2 - x)$
$= 2^2 - 2$
$= 4 - 2$
$= 2$

10/10 UA3

Ex 1

(1) $\frac{d}{dx} 5 = 0$

(2) $\frac{d}{dx} 2x = 2$

(3) $\frac{d}{dx} x^2 - x = 2x - 1$

(4) $\frac{d}{dx} e^{2x} \Rightarrow \frac{d}{dx}(2x) \frac{d}{dy} e^{y}$
$= 2e^{y}$
$= 2e^{2x}$

5) $\frac{d}{dx} (\sin x)(\cos x)$

$= \frac{d}{dx}(\sin x)(\cos x) + \sin x \frac{d}{dx} \cos x$

$= \cos^2 x - \sin^2 x$

6) $\frac{d}{dx} |x|$

$\frac{d}{dx} |x| = 1$ when $x > 0$;

$\frac{d}{dx} |x| = -1$ when $x < 0$

and doesn't exist when $x = 0$

Q2 in last //

Q3) Newton's method

- Find 3 steps of Newton's method for $x \to x^2 - 2$.

Starting with $x_0 = 1$

If $f(x) = x^2 - 2$ then $f'(x) = 2x$

formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

n	x_n	$f(x_n)$	$f'(x_n)$
0	1	-1	2
1	$1 - -1/2 = 1.5$	.25	3
2	$1.5 - .25/3 = 1.4167$	.0070	2.8334
3	$1.4167 - .0070/2.8334 = 1.4142$		

(4) GRADIENTS

(I) $x^2 \cos y$:

$\frac{d}{dx}(x^2 \cos y) = 2x\cos y$, while

$\frac{d}{dy}(x^2 \cos y) = -x^2 \sin y$

So the gradient is $(2x\cos y, -x^2 \sin y)$.

(II) $x+y$: $\frac{d}{dx}(x+y) = 1$,

$\frac{d}{dy}(x+y) = 1$ so the gradient is $(1, 1)$

(III) $x^2 + 2xy + y^2$:

Chain Rule can be used.

$f(x) = x^2 + 2xy + y^2 = (x+y)^2$

so the gradient

$\nabla f = 2(x+y)\,\nabla(x+y) = 2(x+y)(1,1)$

$= (2(x+y), 2(x+y))$.

Ex 2

(1)

(2)

Alternatively,

$$\frac{d}{dx} f = 2x+2y \ \& \ \frac{d}{dy} f = 2x+2y.$$

Ex 2 Extrema

(1) $x \to x^2 - 2x + 5$

$\frac{d}{dx} \quad x^2 - 2x + 5$

$= 2x - 2$

It is equal to zero when $X = 1$
The local extrema are

$0 \to 5$ a maximum, $1 - 4$ a minimum,
$10 \to 85$ a max

(2) $x \to \cos x$

$\frac{d}{dx} \quad \cos x$

It is equal to zero when $x = \pi$
Then extrema are Maximum at $0, 2\pi$
Then " — " Minimum at $\pi, 3\pi$
Local Maximum is at 10