* Conditional Probability *

- Conditional probability is used to determine the likelihood of an event when a partial information about the event is known.

e.g ① In an experiment of rolling a die, the die is rolled twice. It was observed that the sum of the outcomes was even. so, what is the chance that the first outcome was 2 ?

② In an experiment where a word is to be guessed. It was observed that the first letter of the word is 'A'. What is the chance that the second letter is 'b' ?

→ We have two events in 1st example. - A & B

P(A|B) = (Event A) | (Event B)

P(outcome being 2 | Given that outcome is even)

↓ Event A ↓ Event B

Scenario : In an experiement of rolling a die, what is the likelihood that the outcome is 2, given that the outcome is even ?

- It is given that the outcome is Even (Event B) - so possible values are 2, 4, 6 from die, therefore the total no. of elements for event B = 3

- Event A can take any one value amongst 2, 4, 6. Therefore the total number of elements for event A = 1

$$P(A|B) = \frac{\text{No. of elements of } A \cap B}{\text{No. of elements of } B} = \frac{1}{3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$$

Note: Conditional probability is undefined if the conditioning event has zero probability i.e. $P(B) > 0$ is mandatory.

* Examples *

x: (1) Consider the following contingency table

	Right handed	Left handed	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

Find the probability that a randomly selected person is

(a) A male given that he is right handed

→

$$P(M|R) = \frac{P(M \cap R)}{P(R)} = \frac{0.41}{0.86} \approx 0.477$$

(b) Right-handed given that he is male.

$$P(R/M) = \frac{P(R \cap M)}{P(M)} = \frac{0.41}{0.49} = 0.837$$

(c) A female given that she is left-handed.

$$P(F/L) = \frac{P(F \cap L)}{P(L)} = \frac{0.06}{0.14} = 0.429$$

(d) Are the events being a female and being left-handed independent ? Justify.

$P(F/L) = 0.429$
$P(F) = 0.51$
$P(L) = 0.14$
} dependent

Ex2

	Blue	Yellow
A Bowl	1	4
B Bowl	3	2

a) $P(\text{Blue}) = \frac{4}{10}$

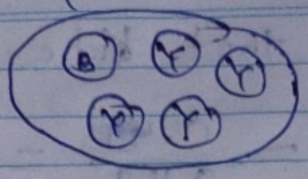
$P(\text{Yellow}) = \frac{6}{10}$

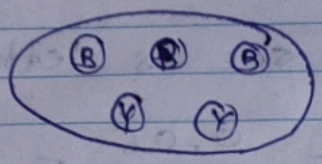
b) What is the probability of blue marble, given bowl A.

$$P(\text{Blue}/A) = \frac{P(\text{Blue} \cap A)}{P(A)} = \frac{1}{5}$$

where, B → Blue
Y → Yellow

Ex · 3

A 

B 

a) $P(\text{Blue}) = \frac{4}{10}$

$P(\text{Yellow}) = \frac{6}{10}$

b) What is the probability of blue marble, given Bowl A

$$P(\text{Blue} \mid A) = \frac{P(\text{Blue} \cap A)}{P(A)} = \frac{1}{5}$$

c) What is the probability ~~that~~ the marble is selected from bowl A, given that the marble is Blue

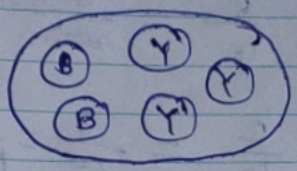
$$P(A \mid \text{Blue}) = \frac{P(A \cap \text{Blue})}{P(\text{Blue})} = \frac{1}{4}$$

B.M.I.T.

* Extended Conditional Probability *

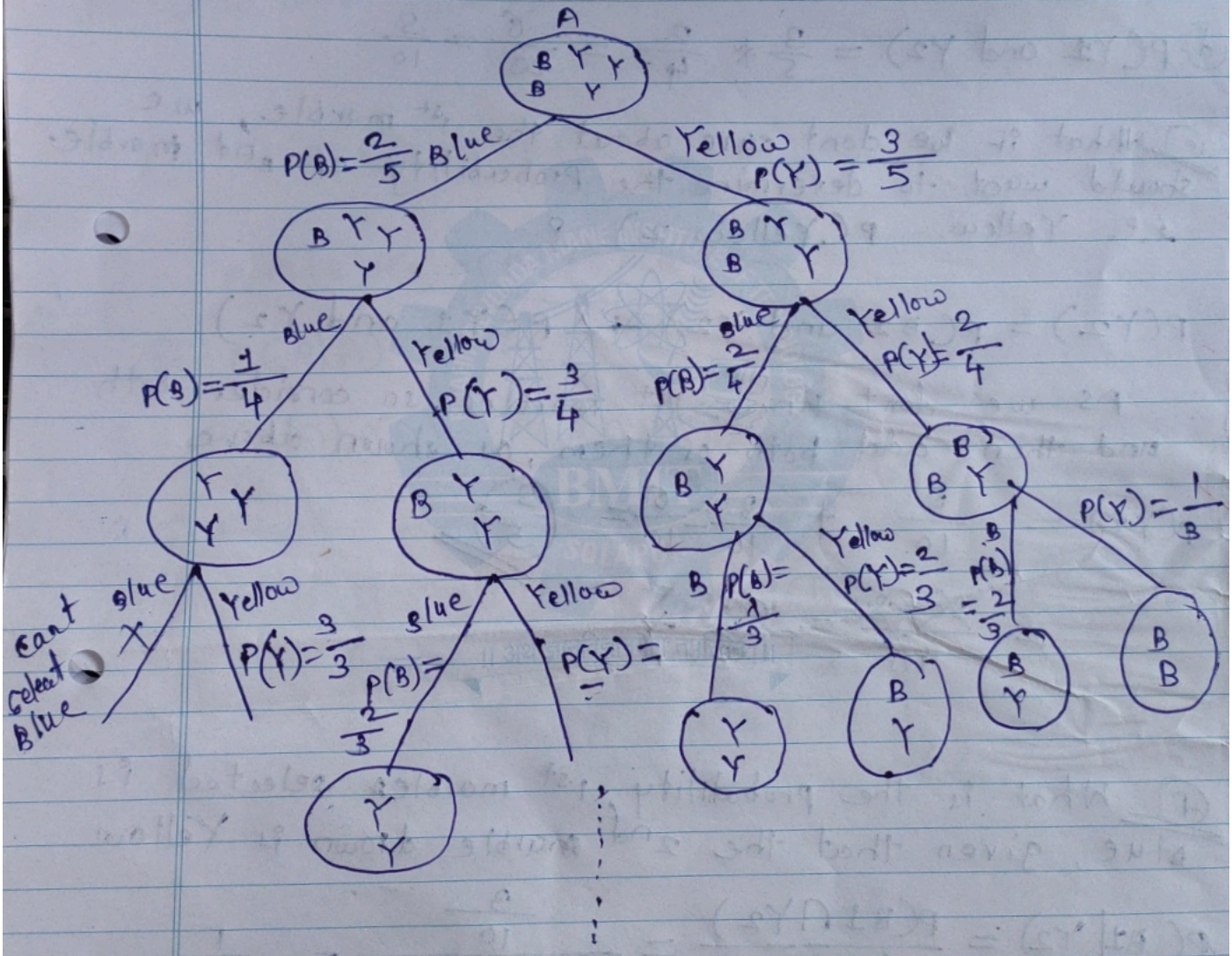
B → Blue
Y → Yellow

Ex. (3) Bowl A



⟹

Dependent Events



Q → (a) What is the probability of selecting two blue marble in a row [or]
P(Blue 1st and Blue 2nd) = ?

$P(B1 \text{ and } B2) = \frac{2}{5} * \frac{1}{4} = \frac{1}{10}$

(b) $P(B1 \text{ and } Y2) = \frac{2}{5} * \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

(c) $P(Y1 \text{ and } B2) = \frac{3}{5} * \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

(d) $P(Y1 \text{ and } Y2) = \frac{3}{5} * \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

(e) What if we dont care about the 1st marble, we should want to determine the probability of 2nd marble i.e. Yellow. $P(\text{Yellow } 2)$?

$P(Y2) = P(B1 \text{ and } Y2) + P(Y1 \text{ and } Y2)$

As we dont know, 1st marble's, so consider both and then add both of them, as shown above

$P(Y2) = \frac{3}{10} + \frac{3}{10} = \frac{6}{10} = \frac{3}{5}$

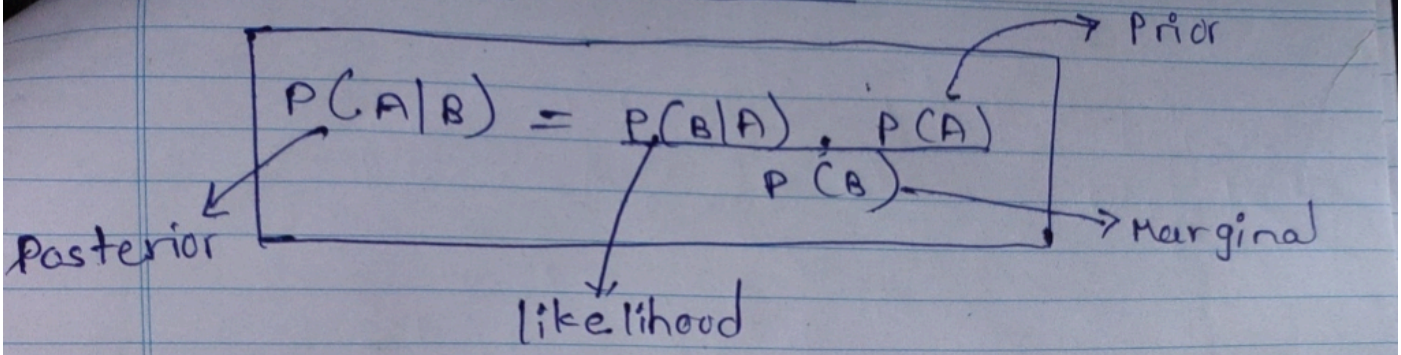
$\neq \frac{30+30}{100} = \frac{60}{100} \neq$

(f) What is the probability, 1st marble selected is Blue, given that the 2nd marble drawn is Yellow

$P(B1 | Y2) = \frac{P(B1 \cap Y2)}{P(Y2)} = \frac{\frac{3}{10}}{\frac{3}{10} + \frac{3}{10}} = \frac{1}{2}$

* Baye's Theorem *

B.M.I.T. 21/10



① $P(A|B) = \frac{P(A \cap B)}{P(B)}$

- $P(A|B)P(B) = P(A \cap B)$

② $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(B|A).P(A) = P(B \cap A)$

Almost Equal

Justification for baye's thm

$$P(A|B).P(B) = P(B|A).P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A).P(A)}{P(B)}}$$