* Permutation & Combinations *

B.M.I.T.

* Factorial : (! or L)

e.g: 5! or $\lfloor 5$

→ 5 * 4 * 3 * 2 * 1 = 120

Note: $\boxed{\begin{array}{l} 0! = 1 \\ 1! = 1 \end{array}}$

* Permutations : (Arrangement / Giving)

① Items → persons
Or
② Items → places
Or
③ Persons → places

} ① Or items can be given to 'n' persons in how many ways.

② r items can be put in 'n' places in how many ways

③ r persons can sit in 'n' places in how many ways

Ans ⇒ $^{n}P_{r} = \frac{n!}{(n-r)!}$

Important Rules for $^{n}P_{r}$ to use above formula

① $r \leq n$

② For 'r' items - 'n' Persons
→ No person should be given more than one item
For 'r' items - 'n' places
→ NO place should be more than one item
For 'r' person - 'n' places
→ No place is seated by more than one person

* How to select n & r

If you find items ⟹ r
If you find places ⟹ n

Eg:

① How many ways 4 books can be arranged in 6 shelves?

→ r = 4 (books - items)
n = 6 (shelves - places)

$${}^{n}P_r = \frac{n!}{(n-r)!} = \frac{6!}{2!} = 360$$

4 books can be put in 6 shelves in 360 ways

② How many ways 3 persons can be arranged in 5 chairs

→ ~~r = 5 (chairs - items)~~
~~n = 3~~

~~${}^{n}P_r = \frac{n!}{(n-r)!} = \frac{3!}{}$~~

n = 5 (chairs - places)
r = 3

$r \leq n$
$3 \leq 5$

$${}^{n}P_r = \frac{5!}{2!} = 60$$

3 persons can be arranged in 5 chairs in 60 ways

* Arrangement of letters of given word

(I) Without conditions

(1) Non-repeated letters

Eg- How many ways the letters of the word 'DURGIASOFT' can be arranged.

⇒ No of letters = 9

$9! = 362880$

Eg- How many ways the letters of the word 'MARKET' can be arranged

→ No. of letters ⇒ $6!$

(2) Repeated letters

E.g- How many ways the letters of the word 'PROFESSIONAL' can be arranged?

→ No. of letters $= \frac{12!}{2! \times 2!} = \boxed{\frac{12!}{2! * 2!}}$

$= 119750400$

(O is repeated 2 times, S is repeated 2 times)

'O' is repeated two times

'S' is repeated two times

(II) With Conditions

① Specific letters - specified places

E.g - How many ways the letter of the word 'MARKER' can be arranged such that A at 4th place and 'R' at ~~place~~ 1st place

→ No. of letters = 5

R _ _ A _ _

step 1 result = 1

MARKER

MRKE = 4!

step 2 result = 4!

1 * 4! = 4!

steps
① work on conditions * (multiple)
② on Remaining problem

E.g: How many ways the letter of the word 'CERTIFICATION' can be arranged such that C and N at end places and F at 5th place.

→ CERTIFICATION = 13

step1 =

N _ _ _ F _ _ _ _ _ _ _ C

C _ _ _ F _ _ _ _ _ _ _ N

= 2!

step 2 = $\frac{10!}{2!(T) \times 3!(I)} \times 2! =$

$= \frac{10!}{2! \times 3!} * 2 = \boxed{\frac{10!}{3!}}$

(2) Specific letters - Together

E.g: How many ways the letter of the word 'DURGASOFT' can be arranged such that all vowels together ⟹ +1 Rule

→ DURGASOFT → UAO (vowels)

⇓

3!

DRGSFT

⇓

6! +1

steps
① Work on condition
② Remaining problem

whenever it is together problem add +1 to the remaining problem

3! * 7!

E.g: How many ways the letters of the word 'WHITEBOARD' can be arranged such that W, H, B & D together

→ WHITEBOARD ⟹ WHBD

⇓

4!

ITEOAR

⇓

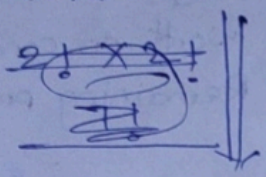
6! +1

⟹ 7! × 4!

Eg: How many ways the letter of the word 'TRAINING' can be arranged such that T & G are together

⇒ TRAINING → TG

RAININ



$$\frac{6!+1}{2!\times 2!} = \frac{7!}{2!\times 2!} \times 2! = \frac{7!}{2!}$$

(3) Specific letters - Not Together

Eg: How many ways the letter of the word 'TRAINING' can be arranged such that T, I and G never together ⇒ Rule ⇒ All - Together = Not together

→

Part 1 - without condition

TRAINING

$$\frac{8!}{2!\times 2!}$$

Part 2 - With condition of together

TRAINING → TIG

RANIN

$$\frac{5!+1=6!}{2!} \times 3!$$

$$\frac{6!\times 3!}{2!}$$

$$\frac{8!}{2!\times 2!} - \frac{3!\times 6!}{2!}$$

* Often confused questions in permutations

Q.) How many ways 4 Rings can be put on 3 fingers ?

→ $n = 3$ (place)
$r = 4$ (items)

but

$\therefore r > n$ so, the formula ^nP_r will not work, $r \leq n$ then only we can use ^nP_r formula.

$r > n \rightarrow {}^nP_r$ ✗

$r \leq n \rightarrow {}^nP_r$ ✓

Answer is $= 3^4$ (Think practically)

Q) How many ways 10 letters can be posted in 6 post boxes ?

→ $n = 6$ (place)
$r = 10$ (item)

$r > n$, we cant use ^nP_r formula, so think practically solve this question

Answer $= 6^{10}$

Q) How many ways 4 Hats can be put on 3 heads ?

→ $n = 3$ (place)
$r = 4$ (item)
$r > n$ we cant use ^nP_r formula

Answer $= 3^4$

* Combinations *

Combination : selection / Taking
Permutation : Arrangement / Giving

Combination			Permutation		
r		n	r		n
Items	—	Items	Items	—	Persons
persons	—	Persons	Items	—	places
			~~Items~~ Persons	—	places

- If ~~we~~ r & n are same, we can understand that it is combination problem
- If ~~we~~ r & n are different, then we can understand that it is permutation problem.

Q) How many ways 'r' items can be selected from available 'n' items

Items — Items

Q) How many ways 'r' persons can be selected from a group of 'n' persons?

Persons — Persons

Formula for above two type of question is

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Rules for ^nC_r formula.

① $r \leq n$

where r = Required number
n = Available number

Example: ① How many ways 4 persons can be selected from a group of 10 people

→ $r = 4$, $n = 10$, $r \leq n \Rightarrow \checkmark$ we can use ^nC_r formula

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$^{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4! * 6!}$$

$$= \frac{10 \times 9! \times 8! \times 7! \times 6!}{4! \times 6!}$$

$$\boxed{^{10}C_4 = 210}$$

② How many ways 4 books can be selected from 8 books

→ $r = 4$, $n = 8$, $r \leq n \Rightarrow \checkmark$

$$^8C_4 = \frac{8!}{4!(4)!} = \frac{8! \times 7! \times 6! \times 5! \times 4!}{4! \times 4!}$$

$$\boxed{^8C_4 = 70}$$

(3) How many ways 5 balls can be selected/ drawn from a box containing 7 Red, 8 blue, & 6 green

→ $r = 5$

$n =$ 7 Red,
8 Blue
6 green
21 Total balls

$r \leq n$ ✓

$${}^{21}C_5 = \frac{21}{5!(21-5)!} = \frac{21!}{5! \times 16!}$$

$$= \frac{21 \times 20! \times 19! \times 18! \times 17!}{5!}$$

(4) How many ways 5 balls can be drawn from a box containing 7 Red, 8 Blue & 6 green such that 3 are red balls?

→ $r = 5$

- In selection of 5 balls 3 balls must be red.
- 3 Balls from red & remaining 2 balls are from green or blue.

Red	Remaining
$r = 3$	$r = 2$
$n = 7$	$n = 8+6 = 14$

$\Rightarrow {}^{7}C_3 \ * \ {}^{14}C_2$

* → AND
+ → OR