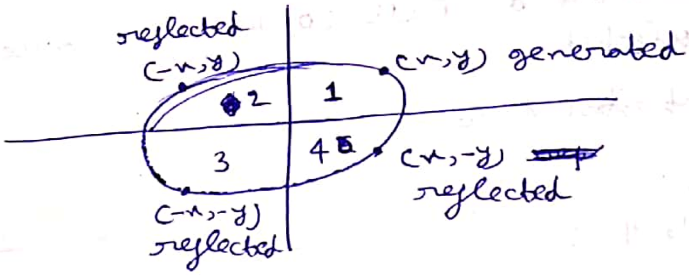
• Midpoint Ellipse Algorithm

Ellipse is a non-linear curve with finite curvature in 2D. For simplicity, ellipse having center at origin and axes (major and minor) parallel to the co-ordinate axes is considered. The algebraic expression of such an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a^2 \geq b^2)$$, where a = length of semi major axis
b = length of semi minor axis

$$\Rightarrow b^2x^2 + a^2y^2 - a^2b^2 = 0$$

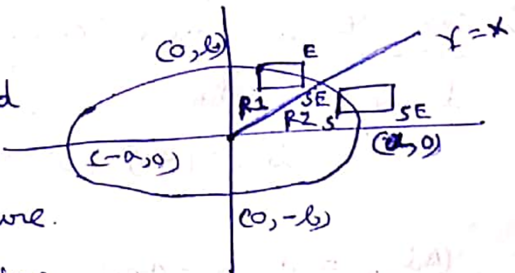


An ellipse can be divided into four parts (equally). So, if one part or quadrant can be generated then the other three parts can easily be replicated by mirroring the original part (4-way symmetry).

Thus, setEllipsePixel (xc, yc, x, y) // xc, yc → center of the ellipse

```
{
    setPixel (xc+x, yc+y);  → 1
    setPixel (xc-x, yc+y);  → 2
    setPixel (xc-x, yc-y);  → 3
    setPixel (xc+x, yc-y);  → 4
}
```

We need to generate only the 1st quadrant. For applying the Midpoint method 1st quadrant is logically divided into two regions as shown in the figure.



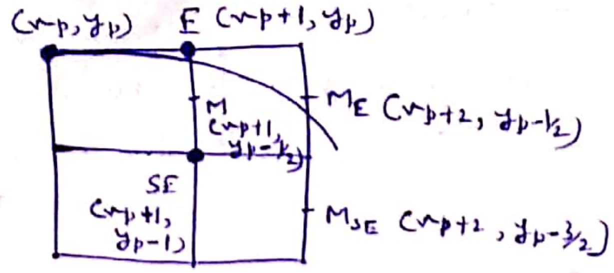
Starting at $(0,b)$ and moving clockwise along the ellipse path in region 1 we take unit steps in X direction until we reach the boundary between region 1 and 2. Then we switch to unit steps in Y direction while sampling region 2 of the curve.

Slope at a point (x,y) is $dy/dx = -b^2x/a^2y$

The magnitude of the slope is 0 at $(0,b)$ and gradually increments and finally becomes infinity at $(a,0)$.

Ellipse equation, $F(x,y) = b^2x^2 + a^2y^2 - a^2b^2 = 0$

Region 1



The selection is to be made between E and SE. The mid point M will decide the selection.

If, decision variable $d = F(M) < 0$ then E is selected else, SE is selected.

M_E = mid point after selecting E

M_{SE} = " " " " SE

$$d = F(x_p+1, y_p-\tfrac{1}{2})$$
$$= b^2(x_p+1)^2 + a^2(y_p-\tfrac{1}{2})^2 - a^2b^2$$

set, $d_{old} = d$

<i> Case E is chosen (current point is (x_p+1, y_p))

$$d_{new} = F(M_E)$$
$$= b^2(x_p+2)^2 + a^2(y_p-\tfrac{1}{2})^2 - a^2b^2$$

$$(\Delta d)_E = d_{new} - d_{old}$$
$$= b^2(x_p^2 + 4x_p + 4 - x_p^2 - 2x_p - 1)$$
$$= b^2(2x_p+3)$$

<ii> case SE is chosen (current point is (x_p+1, y_p+1))

$$d_{new} = F(M_{SE})$$
$$= b^2(x_p+2)^2 + a^2(y_p-\tfrac{3}{2})^2 - a^2b^2$$

$$(\Delta d)_{SE} = d_{new} - d_{old}$$
$$= b^2(2x_p+3) + a^2(y_p^2 - 3y_p + \tfrac{9}{4} - y_p^2 + y_p - \tfrac{1}{4})$$
$$= b^2(2x_p+3) + a^2(-2y_p+2)$$

Initial value of d,

$$d_{start} = F(1, b - 1/2) \quad [x_p = 0, y_p = b]$$
$$= b^2 + a^2(b - 1/2)^2 - a^2b^2$$
$$= b^2 + \cancel{a^2b^2} - a^2b + a^2/4 - \cancel{a^2b^2}$$
$$= b^2 - a^2b + \frac{a^2}{4}$$
$$= b^2 + a^2(1/4 - b)$$

Loop condition

Region 1 is above the st. line $y = x$ so we must have $y > x$ in region 1.

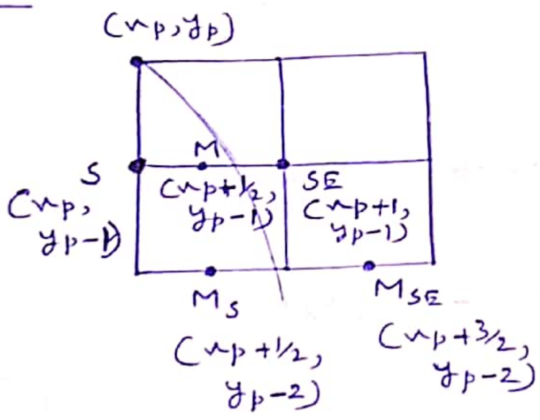
$\Rightarrow a^2y > b^2x$ [$\because a^2 \geq b^2$ for the ellipse]

In particular, this relation will also be true for $M(x_p+1, y_p - 1/2)$

$\therefore$ we can modify to,

$$a^2(y - 1/2) > b^2(x+1)$$

Region 2



If, $d = F(M) < 0$ then SE is selected
else, S is selected

$$d = F(x_p + 1/2, y_p - 1)$$
$$= b^2(x_p + 1/2)^2 + a^2(y_p - 1)^2 - a^2b^2$$

set, $d_{old} = d$

<i> Case SE is chosen (current point is (x_p+1, y_p-1))

$$d_{new} = F(M_{SE})$$
$$= b^2(x_p+3/2)^2 + a^2(y_p-2)^2 - a^2b^2$$

$$(\Delta d)_{SE} = d_{new} - d_{old}$$
$$= b^2(\cancel{x_p^2} + 3x_p + 9/4 - \cancel{x_p^2} - x_p - 1/4)$$
$$+ a^2(\cancel{y_p^2} - 4y_p + 4 - \cancel{y_p^2} + 2y_p - 1)$$
$$= b^2(2x_p+2) + a^2(-2y_p+3)$$
$$= b^2(2x_p+2) - a^2(2y_p-3)$$

<ii> Case S is chosen (current point is (x_p, y_p-1))

$$d_{new} = F(M_S)$$
$$= b^2(x_p+1/2)^2 + a^2(y_p-2)^2 - a^2b^2$$
$$(\Delta d)_S = d_{new} - d_{old}$$
$$= a^2(-2y_p+3)$$
$$= -a^2(2y_p-3)$$

Initial value of d,

$$d = F(\text{first mid point in Region 2})$$
$$= F(x+1/2, y-1)$$
$$= b^2(x+1/2)^2 + a^2(y-1)^2 - a^2b^2$$

Loop condition

If we consider all those points in 1st quadrant which are not in region 1 therefore the simplest condition is $y>0$

```
Bresenham_ellipse (int a, int b, int xc, int yc)
{

    d = b^2 + a^2 (1/4 - b);  // Initialize decision variable in Region 1
    set EllipsePixel (xc, yc, 0, b);
    while (a^2 (y - 1/2) > b^2 (x+1)) /* Loop condition for Region 1 */
    {
        if (d<0) // select E
            d = d + b^2 (2x + 3);
        else // select SE
        {
            d = d + b^2 (2x+3) + a^2 (-2y+2)
            y = y - 1;
        }
        x = x+1;
        set EllipsePixel (xc, yc, x, y);
    }

    d = b^2 (x+1/2)^2 + a^2 (y-1)^2 - a^2 b^2;  // Initialize decision variable in Region 2

    while (y > 0)
    {
        if (d<0) // select SE
        {
            d = d + b^2 (2x +2) - a^2 (2y-3);
            x = x+1;
        }
        else // select S
            d = d - a^2 (2y-3);

        y = y-1;
        set Ellipse Pixel (xc, yc, x, y);

    }
}
```

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