Computer Graphics

Geometrical transformation of objects:

2D transformation means a change in either position or orientation or size or shape of graphic objects like line, circle, ellipse, polygon etc. All such graphic obj. are constructed with finite number of points. All sorts of obj. transformation result from simultaneous change of position of all/some of the pts. of an object.

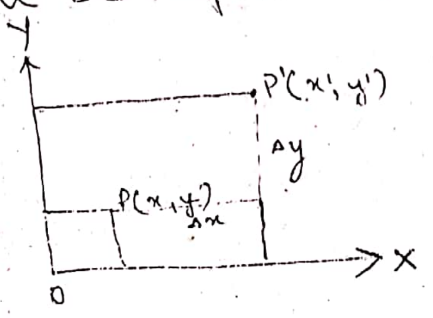
A point's coordinates can be expressed as elements of a matrix.

To identify position of a pt. uniquely, we need a reference frame. Each ref. frame has it's degree of freedom ..... min. no. of parameters req. to identify the position. Commonly, we use rectangular rectilinear cartesian coordinate system in right-handed form. Other coordinate systems are also used like – left-handed cartesian system, oblique C.S. Polar Coordinate system, Cylindrical C.S, Spherical C.S etc.

In Computer graphics, it is convention to consider topmost left corner of display screen as origin. Vertically downwards through it is +ve Y axis and horizontally rightward direction is +ve X axis

2D Transformation:

1) Translation – can be defined as rectilinear movements along respective axis. Geometrically, it can be represented by following figure:



(5) In matrix coordinate form, translation can represented as $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = T(\Delta x, \Delta y)$$

Some features of Translation :

1. It is additive in nature.
2. It can be applied in any order or partially
3. $T(\Delta x, \Delta y) \neq T(\Delta y, \Delta x)$
4. If any 1 of the parameters in Translation is zero, then is a pure translation along a particular direction.

2) Scaling — means either maximization or minimization along respective axes. Scaling matrix is given by $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$, where s_x and s_y are ~~scaling~~ 'scaling factors' along the direction of X and Y axis.

Features :

1. Scaling factors are always > 0
2. When both scaling factors are equal, then it is called uniform / homogenous / isotropic Scaling. Else it is heterogenous scaling.
3. If both scaling factors are equal to 1, then no change results.
4. If scaling factors are greater than 1, ~~and~~ then it is magnification else if they are less than 1, then it is minimization.
5. For uniform scaling, if both scaling factors are > 1, then the obj. becomes larger [Uniform Expansion]

   If both < 1, then uniform compression occurs and obj. becomes smaller.

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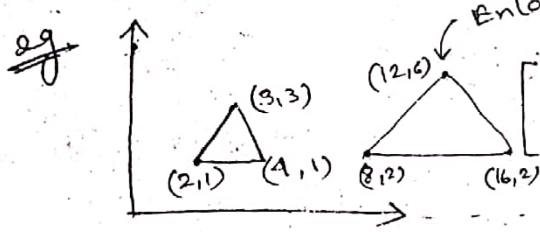
(b) Non-uniform expansions or compressions occur depending on whether S_x and S_y are individually >1 or <1 but unequal, such scaling is called 'Differential Scaling'.

The shape and size both changes in this case.

eg.



$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 16 & 12 \\ 2 & 2 & 6 \end{bmatrix}$$

3) Reflection – about Y-axis is given by =

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

About X axis $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

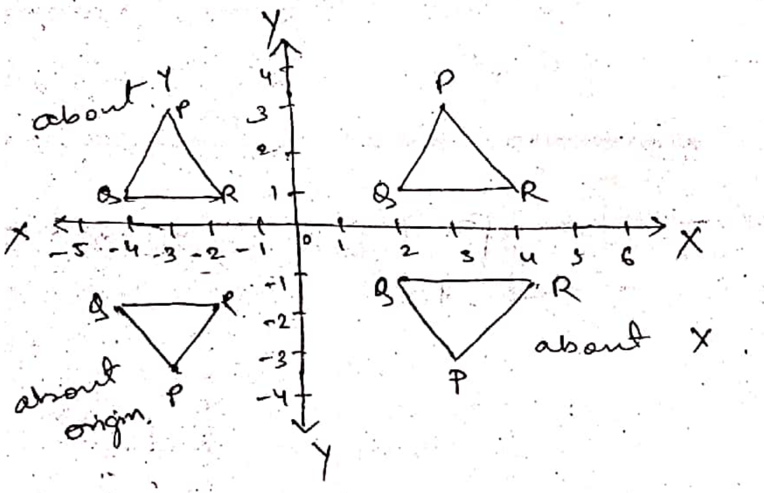
Taking above eg.

about y-axis. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -2 & -4 & -3 \\ 1 & 1 & 3 \end{bmatrix}$$

about x-axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 & 3 \\ -1 & -1 & -3 \end{bmatrix}$$



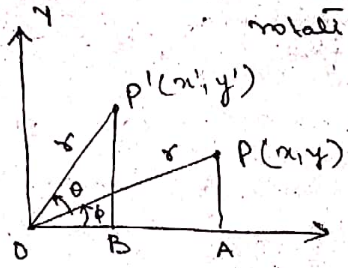
about origin:

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & -3 \\ -1 & -1 & -3 \end{bmatrix}$$

Even no. of consecutive reflections about a part. axis will result in same position.

4) Rotation — We consider rotation of a point P in anticlockwise direction about origin.
rotation by angle θ.



To find new coordinates, we do the following:

From $\Delta OBP'$: $\cos(\theta+\phi) = \frac{OB}{OP'} = \frac{x'}{r}$

$$\therefore x' = r\cos(\theta+\phi)$$

$$= r\cos\theta \cdot \cos\phi - r\sin\theta \sin\phi \quad \text{——— (1)}$$

$$y' = r\sin\theta\cos\phi + r\cos\theta \cdot \sin\phi \quad \text{——— (2)}$$

From ΔOAP : $\cos\phi = \frac{x}{r}$

$$\therefore x = r\cos\phi \quad \text{——— (3)}$$

$$y = r\sin\phi \quad \text{——— (4)}$$

From (1), (3) and (4)

$$x' = x\cos\theta - y\sin\theta.$$

From (2), (3), and (4)

$$y' = x\sin\theta + y\cos\theta.$$

We can represent this in matrix form as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Features :

(1) Since successive rotations are additive in nature, we can write $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^n$

$$= \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$$

(2) Very small rotation will reduce $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1$.

For clockwise, θ is replaced by $-\theta$.

Discuss commutative property of Scaling and Rotation:

If we consider scaling followed by rotation, then the composite matrix will be

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} S_x\cos\theta & -S_x\sin\theta \\ S_y\sin\theta & S_y\cos\theta \end{bmatrix}$$

= scaling followed by rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= \begin{bmatrix} S_x\cos\theta & -S_y\sin\theta \\ S_x\sin\theta & S_y\cos\theta \end{bmatrix}$$

In general, these two ~~operations~~ transformations are not commutative, but for some spl. cases, they will be commutative as follows.

__Case 1__: [When scaling is uniform]

$S_x = S_y = A$ (say)

then the transformation reduces to

$$\begin{bmatrix} a\cos\theta & -a\sin\theta \\ a\sin\theta & a\cos\theta \end{bmatrix}$$

__Case 2__: [$\theta = n\pi$]

In this case, $\sin\theta = 0$, $|\cos\theta| = 1$.

$\therefore$ Transformation matrix reduces to $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

7)

Find the composite transformation matrix for reflection wrt st. line $y = x$.
Also find for st. line $y = -x$. $\Rightarrow \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

This can be done using a series of consecutive transformations as follows:

i) Rotation with 45° in clockwise direction
ii) Reflection about X-axis.
iii) Inverse rotation with 45°

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^{1st}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$\frac{1}{2}$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

About Y-axis:

① $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

③ $\begin{bmatrix} 1/\sqrt{2} & +1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

② $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

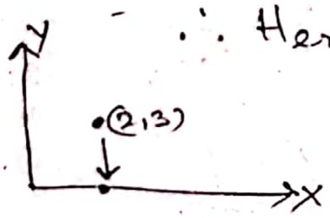
What is the significance of the transformation Matrix, 30.7.18 (10

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(2,3) is a random pt.

resultant matrix gives pt. (2,0)

∴ Hence it is a projection on x-axis.



$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ shows horizontal translation along x axis.

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} =$ shows vertical translation along Y axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \ldots$$

Prove that reflection about Y-axis followed by reflection about $Y=-x$ equivalent to a rotation about origin.

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Comparing the above with rotation matrix

$\therefore \; \theta = \pi/2$

$\theta = (2n\pi + \pi/2)$ where $n = 0, 1, 2 \ldots$

Show that a 2D reflection through X-axis (11) followed by ~~[illegible]~~ reflection about ~~[illegible]~~ Y=-X equivalent to a rotation about the origin

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Comparing with rotation matrix, we get

$\theta = 3\pi/2$ (Ans)

Successive two rotations are additive or not

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

=

Successive two translations are additive or not

$$\begin{bmatrix} 1 & 0 & \Delta x_2 \\ 0 & 1 & \Delta y_2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & \Delta x_1 \\ 0 & 1 & \Delta y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \Delta x_1 + \Delta x_2 \\ 0 & 1 & \Delta y_2 + \Delta y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ It is additive.

Successive two scaling

$$\begin{bmatrix} Sx_2 & 0 \\ 0 & Sy_1 \end{bmatrix}\begin{bmatrix} Sx_1 & 0 \\ 0 & Sy_1 \end{bmatrix}$$

$\therefore$ It is multiplicative

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