Discuss Commutative property of Scaling and Rotation:

If we consider Scaling followed by rotation, then the composite matrix will be

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{S_x} S_x \cos\theta & -S_{\cancel{y}}^{x} \sin\theta \\ S_{\cancel{xy}} \sin\theta & S_y \cos\theta \end{bmatrix}$$

= Scaling followed by rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= \begin{bmatrix} S_x \cos\theta & -S_{\cancel{x}y} \sin\theta \\ S_x \sin\theta & S_y \cos\theta \end{bmatrix}$$

In general, these two ~~operation~~ transformations are not commutative, but for some spl cases, they will be commutative as follows.

Case 1: [when scaling is uniform]

$S_x = S_y = A$ (say)

then the transformation reduces to

$$\begin{bmatrix} a\cos\theta & -a\sin\theta \\ a\sin\theta & a\cos\theta \end{bmatrix}$$

Case 2: $[\theta = n\pi]$

In this case, $\sin\theta = 0$, $|\cos\theta| = 1$.

$\therefore$ Transformation matrix reduces to $\begin{bmatrix} \cancel{0} S_x & 0 \\ 0 & S_y \end{bmatrix}$

Find the composite transformation matrix for reflection wrt st. line $y = x$.
Also find for st. line $y = -x$. $\Rightarrow \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

This can be done using a series of consecutive transformation as follows:

i) Rotation with 45° in clockwise direction
ii) Reflection about X-axis.
iii) Inverse rotation with 45°

1st

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

About Y-axis:

① $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

③ $\begin{bmatrix} 1/\sqrt{2} & +1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

② $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

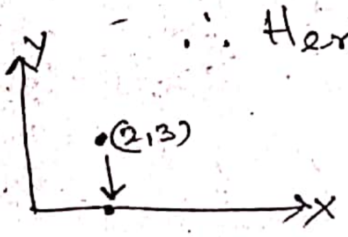
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- What is the significance of the transformation (1)
Matrix,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(2,3) is a random pt.
resultant matrix gives pt. (2,0)

$\therefore$ Hence it is a projection on x - axis.



$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ shows horizontal translation along x axis.

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} =$ shows vertical translation along Y axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \ldots$$

Prove that reflection about Y-axis followed by reflection about Y=-x equivalent to a rotation about origin.

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Comparing the above with rotation matrix

$\therefore \theta = \pi/2$

$\theta = (2n\pi + \pi/2)$ where ~~Gen~~ $n = 0, 1, 2 \ldots$

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Show that a 2D reflection through X-axis

11) followed by ~~xxxx~~ reflection about ~~xxx~~ Y=-X equivalent to a rotation about the origin

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Comparing with rotation matrix, we get

$\theta = 3\pi/2$. (Ans)

Successive two rotations are additive or not

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

=

Successive two translations are additive or not

$$\begin{bmatrix} 1 & 0 & \Delta x_2 \\ 0 & 1 & \Delta y_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \Delta x_1 \\ 0 & 1 & \Delta y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \Delta x_1 + \Delta x_2 \\ 0 & 1 & \Delta y_2 + \Delta y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ It is additive.

Successive two scaling

$$\begin{bmatrix} S x_2 & 0 \\ 0 & S y_1 \end{bmatrix} \begin{bmatrix} S x_1 & 0 \\ 0 & S y_1 \end{bmatrix}$$

$\therefore$ It is multiplicative.

24.7.18.

Homogeneous Coordinate System:

To define composite matrix of transformation, we have to multiply a series of matrices. It can only be done when all elementary transformations are expressed in multiplicative form. But since translation is additive, this is why H.C. System is used to represent translation matrix in such a form that it will be suitable for multiplication.

In H.C. System, the ~~us~~ usual 2x2 matrix can be represented as 3x3 matrix form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & m \\ c & d & n \\ 0 & 0 & 1 \end{bmatrix}$$

But such 3x3 matrices are not comfortable for multiplication with 2x1 2D position matrices. Thus, we need to introduce dummy coordinates to make 2x1 position vector matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ to a 3x1 matrix $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ where the 3rd coordinate is a dummy.

Now, if we multiply $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ with a non-zero scalar value h, then the matrix form is $\begin{bmatrix} xh \\ yh \\ h \end{bmatrix}$. This is known as the homogenous coordinate of the same point $\begin{bmatrix} x \\ y \end{bmatrix}$ in 2D plane. The extra coordinate h, which is homogeneously is known as the weight, applied to the cartesian coordinate. As h can have any non-zero value, there can be infinite no. of equivalent homogeneous representation of any given pt. in space. We will use h=1 ~~&~~ coordinate $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ to represent corresponding homogenoous points $\begin{bmatrix} x \\ y \end{bmatrix}$ in x-y plane.

Expressing positions in homogeneous coordinates allow us to represent all geometric transformation as matrix multiplication. Using homogeneous coordinate the translation matrix will be of the form:

$$\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longrightarrow \text{for Translation}$$

Similarly, for rotation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

For scaling:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

For reflection:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longrightarrow \text{wrt } y\text{-axis}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longrightarrow \text{wrt } X\text{-axis}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longrightarrow \text{wrt origin.}$$

Homogeneous coordinate $(8, 12, 4) \longrightarrow (hx, hy, h)$

$\therefore$ Corresponding xy coordinates $= (2, 3)$

$$= \left(\frac{hx}{h}, \frac{hy}{h}\right)$$