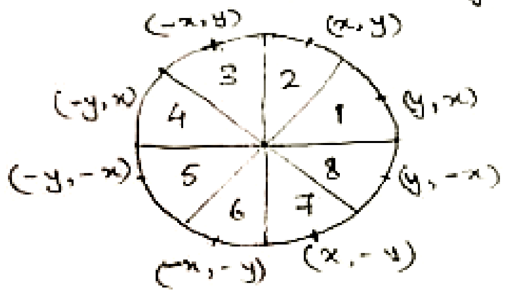
Bresenham’s mid-point circle drawing algorithm:

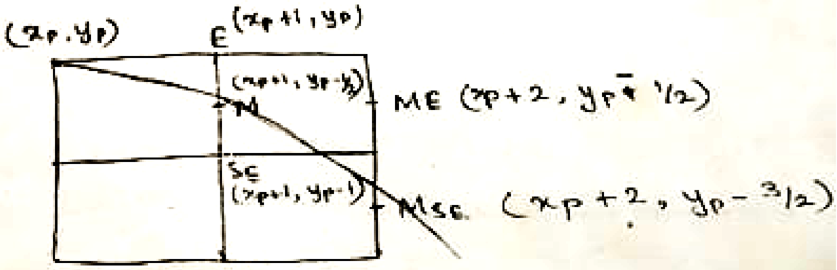
A circle can be represented mathematically by the 2nd order polynomial equation

$(x-x_c)^2 + (y-y_c)^2 = R^2$ where (x_c, y_c) is the centre of circle and R is the radius.

For a given radius R, and screen centre position (x_c, y_c), we can first calculate pixel positions around a circle path centered at origin (0, 0). Then each calculated position (x, y) is moved to proper screen position by adding x_c to x and y_c to y.



Due to symmetry in circle, we need to plot only octant 2. Other points can be plotted through mirror effect.



Now, for octant 2, choice is between E & S and the decision function is

$$F(x, y) = x^2 + y^2 - R^2 = 0$$

Decision variable $d = F(M)$

$$= F(x_p+1, y_p - 1/2)$$

$$= (x_p+1)^2 + (y_p - 1/2)^2 - R^2$$

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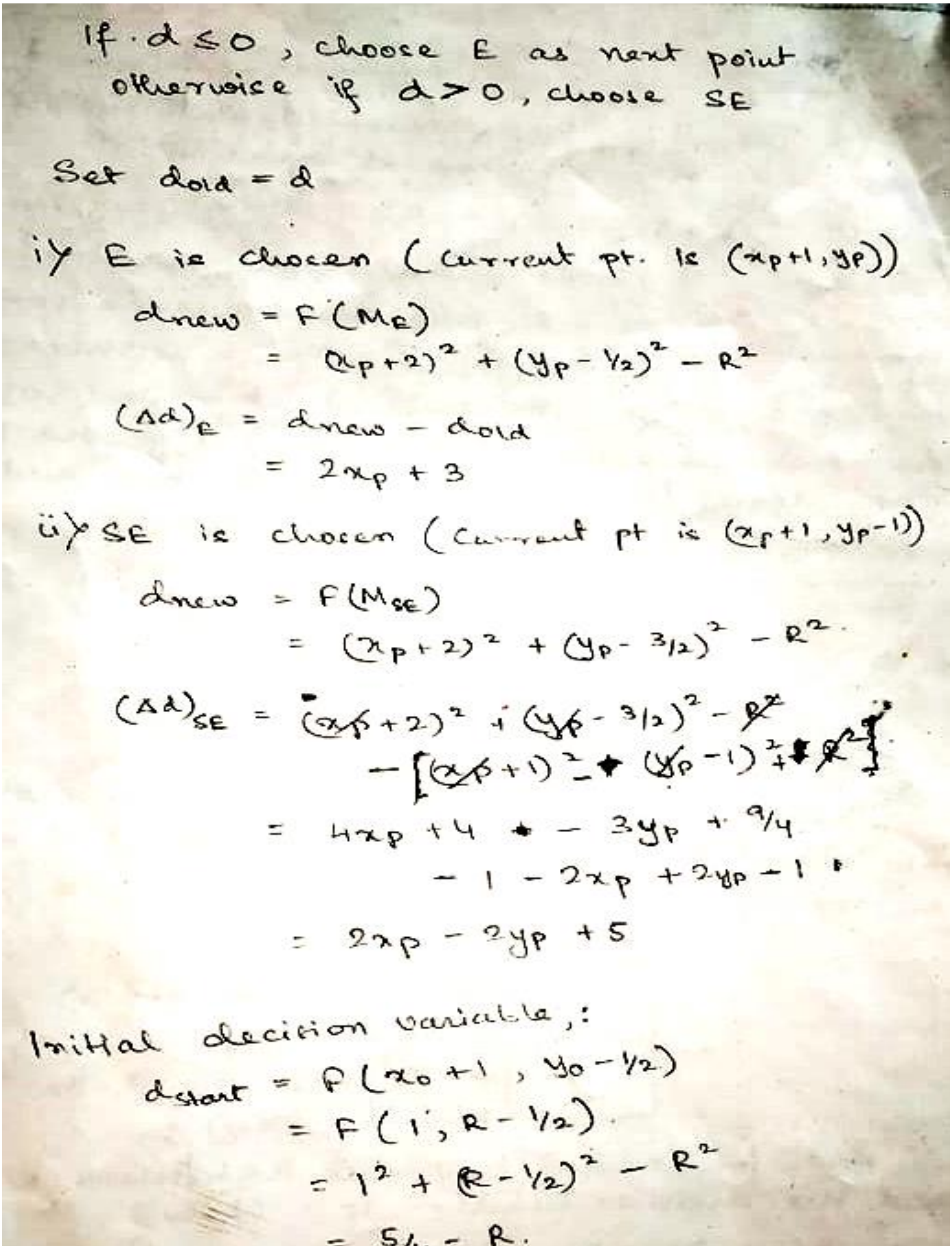


If $d \le 0$, choose E as next point
otherwise if $d > 0$, choose SE

Set $d_{old} = d$

i) E is chosen (current pt. is (x_p+1, y_p))

$$d_{new} = F(M_E)$$
$$= (x_p+2)^2 + (y_p - 1/2)^2 - R^2$$

$$(\Delta d)_E = d_{new} - d_{old}$$
$$= 2x_p + 3$$

ii) SE is chosen (current pt is (x_p+1, y_p-1))

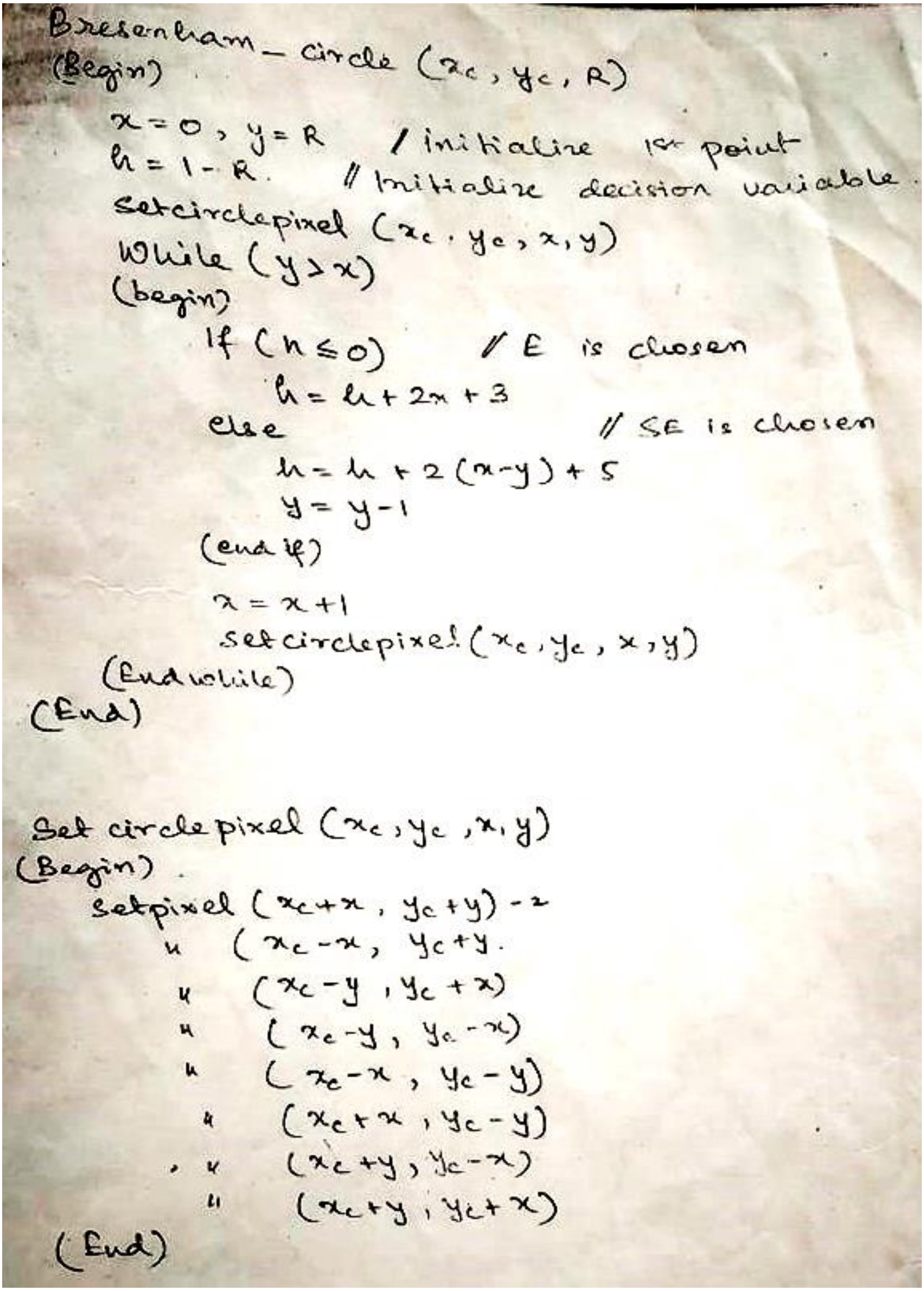
$$d_{new} = F(M_{SE})$$
$$= (x_p+2)^2 + (y_p - 3/2)^2 - R^2.$$

$$(\Delta d)_{SE} = (x_p+2)^2 + (y_p - 3/2)^2 - \cancel{R^2}$$
$$- [(x_p+1)^2 + (y_p-1)^2 - \cancel{R^2}]$$
$$= 4x_p + 4 - 3y_p + 9/4$$
$$- 1 - 2x_p + 2y_p - 1$$
$$= 2x_p - 2y_p + 5$$

Initial decision variable,:

$$d_{start} = F(x_0+1, y_0 - 1/2)$$
$$= F(1, R - 1/2)$$
$$= 1^2 + (R - 1/2)^2 - R^2$$
$$= 5/4 - R.$$

∴ To get rid of the fraction,
$$h = d - 1/4 = (1 - R)$$



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Bresenham_ circle ($x_c$, $y_c$, R)
(Begin)
    $x = 0$, $y = R$        / initialize 1st point
    $h = 1 - R$.       // Initialize decision variable.
    setcirclepixel ($x_c$. $y_c$, x, y)
    While ($y > x$)
    (begin)
        If ($h \leq 0$)        / E is chosen
            $h = h + 2x + 3$
        else                   // SE is chosen
            $h = h + 2(x-y) + 5$
            $y = y - 1$
        (end if)
        $x = x + 1$
        setcirclepixel ($x_c$, $y_c$, x, y)
    (Endwhile)
(End)


Set circlepixel ($x_c$, $y_c$, x, y)
(Begin)
    setpixel ($x_c+x$, $y_c+y$) -2
        "   ($x_c-x$, $y_c+y$.
        "   ($x_c-y$, $y_c+x$)
        "   ($x_c-y$, $y_c-x$)
        "   ($x_c-x$, $y_c-y$)
        "   ($x_c+x$, $y_c-y$)
        "   ($x_c+y$, $y_c-x$)
        "   ($x_c+y$, $y_c+x$)
(End)
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Functions used:

SetCirclepixel(x_c, y_c, x, y) is used to plot the corresponding pixels of the circle with centre (x_c, y_c) on the proper position of the screen.

SetPixel(x, y) is used to plot the corresponding pixel defined by co-ordinate (x, y) on the screen.

Q. Find plotted pixels or plotted points of circle with centre (0, 0) and radius 5 using Bresenham's mid-point circle drawing algorithm.

Answer:

h	x	y	Octant1	Octant2	Octant3	Octant4	Octant5	Octant6	Octant7	Octant8
	0	5	(5,0)	(0,5)	(0,5)	(-5,0)	(-5,0)	(0,-5)	(0,-5)	(5,0)
-4	1	5	(5,1)	(1,5)	(-1,5)	(-5,1)	(-5,-1)	(-1,-5)	(1,-5)	(5,-1)
-1	2	5	(5,2)	(2,5)	(-2,5)	(-5,2)	(-5,-2)	(-2,-5)	(2,-5)	(5,-2)
4	3	4	(4,3)	(3,4)	(-3,4)	(-4,3)	(-4,-3)	(-3,-4)	(3,-4)	(4,-3)
3	4	3	(3,4)	(4,3)	(-4,3)	(-3,4)	(-3,-4)	(-4,-3)	(4,-3)	(3,-4)

Initial plotting of pixels

Plotting of pixels through iterations

HW. Find plotted pixels or plotted points of circle with centre (2, 4) and radius 5 using Bresenham's mid-point circle drawing algorithm.

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