Groundwater Level Prediction Using ARMA-ANN Model

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***Abstract*—At present, classic methods are often used to predict groundwater level, but the result is not ideal. Recent studies show that the combinational prediction methods have higher precison than single prediction methods. A combinational prediction model is presented bsed on ARMA and ANN neural network. And it is applied to comprehensive analysis and prediction of groundwater level. Case study indicates that precision of the model is rather high and its popularization significance is better than the other models, and has some practical value when being used in the dynamic groundwater level analysis**

***Index Terms*—groundwater level; ARMA; BP neural network**

I. INTRODUCTION

As everyone knows, groundwater is one of the foundations that all lives depend on. Groundwater trend is related to atmosphere precipitation. There is obvious seasonality in variation of atmosphere precipitation, so the groundwater level is changed by the seasonality and periodicity. In this case, when the groundwater level is predicted, the original data of groundwater need to be dealt with in general. In this course, the cycle one, trend one and random one need to be calculated separately after the three items isolated. We can get the final results. The traditional analysis and prediction method of groundwater is mainly the mathematics model of the determinacy and random statistical method, for such as finite element, finite difference, analyze, harmony wave analysis, time series analysis, probability statistic, etc. These methods are mainly based on linear theory. Since models are simple, precision is not high (Zhang et al., 2002; Luo et al., 2003). So in this paper, on the basis of thorough analyzing in depth the predicting method, adopting grey dynamic groups combined with neural network to predict the groundwater level, the more satisfied results may be concluded.

II. ARMA MODEL

Observed data sequence Produced and arranged is called the time series, according to the time sequence. If the sequence is continual, the serie is called the run-on time sequence. The time series analysis method's characteristic is that the relationship between the ordr of observed data and around the observed data is the statistical dependence. The data statistics are expressed with related or the autocorrelation function between observation value. according to this variable past change rule, its future change will be forecasted. The time series analysis establishes one kind of model, it can realize the transformation of the non-independent observed data turning to the independent data, then using statistical method to estimate, forecast and control.

The ARMA model (specially AR model) is most basic, the practical application broadest in the succession method, the ARMA model has the stochastic difference equation form.

A. AR model

If time series y_t is linear function of expired value and stochastic, it can be expressed as followings:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + e_t \tag{1}$$

Then the time serie y_t can be called as return sequence, equation (1) is autoreturn model, which is AR (p). Real variable $\phi_1, \phi_2, \ldots \phi_p$ can be called as autoreturn coefficient, which is the parameter to be estimated. Stochastic item e_t is mutually independent white noise sequence, which is normal distribution whose obedient average value is 0, variances is σ_e^2 . Stochastic item e_t is non-correlated with the lagged variable $y_{t-1}, y_{t-2}, \ldots y_{t-p}$. In equation (1) , the sequence average value of y_t is supposed as 0. If $Ey_t = \mu \neq 0$, then $y_t^{`} = y_t - \mu$, the $y_t^{`}$ can be written as equation (1). B^k is the k[th] Lag operator, $B^k y_t = y_{t-k}$.

Then the model(1) can be expressed as

$$y_t = \phi_1 B y_t + \phi_2 B^2 y_t + \ldots + \phi_p B^p y_t + e_t$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \tag{2}$$

The model can be simplified as:

$$\phi(B) y_t = e_t \tag{3}$$

The steady condition of AR (p) is the root Lag multinomial $\phi(B)$ outside unit circle. The root of $\phi(B) = 0$ is bigger than 1.

B. Running mean model

If time serie can be expressed as

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$$y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} \quad (4)$$

Then the time serie y_t can be called as running mean model，equation（4） is q step running mean model, MA（q）model. Real parameters $\theta_1, \theta_2, ..., \theta_q$ is running mean coefficient, which is the coefficient to be estimated.

Lag operator is introduced, and named $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q$

Then the model can be simplified as

$$y_t = \theta(B)\, e_t \quad (5)$$

The running mean process is unconditionally steady. But hoped that the AR process and the MA process can manifest mutually, namely the process is reversible. Therefore, lag multinomial's root is requested outside the unit circle, after the inferential reasoning, equation (5) can be got

$$\left(1 - \pi_1 B - \pi_2 B^2 - ...\right) y_t - \left(-\sum_{j=0}^{\infty} \pi_j B^j\right) y_t = e_t \quad (6)$$

Where $\pi_0 = -1, B^0 = 1$， other weights may be obtained by recursion. Equation (6) is the reversal form for model MA(q), it equals infinite step AR process

C. Autoreturn running mean model

If the time series y_t are stochastic error term of the time and the earlier period as well as the first age's linear function, then the expression is

$$y_t = \phi_1 y_t + \phi_2 y_t + ... + \phi_p y_t + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} \quad (7)$$

The time series can be expressed as ARMA（p，q）. $\phi_1, \phi_2, ... \phi_p$ are autoreturn coefficient, $\theta_1, \theta_2, ..., \theta_q$ are the running mean coeffiecient ， which are all the parameters to be estimated.

Introducing lag operator B, equation(7) can be simplified as followings:

$$\phi(B)\, y_t = \theta(B)\, e_t \quad (8)$$

Steady condition of ARMA(p，q) is lag multinomial's root outside the unit circle, while the reversible condition is the $\theta(B)$ root outside the unit circle.

Using B-J method to study time series, the most important tool is autocorrlations and partial autocorrlations

A autocorrelations： each successive value's simple correlation relations $y_t, y_{t-1}, ... y_{t-k}$ between the constitution time series' are called the autocorrelation. The autocorrelation degree measures by autocorrelation coefficient r_k,which expressed that relativity observed valuethe between the k period.

$$r_k = \frac{\sum_{i=1}^{n-k}(y_i - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad (9)$$

Where n -sample size; k -Lag phase; $\bar{y}$ arithmetic mean value of sample data.

b、partial autocorrelations：to time serie y_t, under the conditions of $y_{t-1}, y_{t-2}, ... y_{t-k+1}$ ， relative relationship of y_t and y_{t-k}. Relativity can be expressed by partial autocorrelations Φ_{kk}, $-1 \le \Phi_{kk} \le 1$,

$$\Phi_{kk} = \begin{cases} r_1 & k = 1 \\ \dfrac{r_k - \sum_{j=1}^{k-1} \Phi_{k-1,j} \cdot r_{k-j}}{1 - \sum_{j=1}^{k-1} \Phi_{k-1,j} \cdot r_j} & k = 2,3,... \end{cases} \quad (10)$$

Where r_k is the autocorrelations coefficient of lagged k period, $\Phi_{k,j} = \Phi_{k-1,j} - \Phi_{kk} \cdot \Phi_{k-1,k-j}$ j=1，2，...k-1

III. ANN MODEL

An artificial neural network (ANN), also called a simulated neural network (SNN) or commonly just neural network (NN) is an interconnected group of artificial neurons that uses a mathematical or computational model for information processing based on a connectionistic approach to computation. In most cases an ANN is an adaptive system that changes its structure based on external or internal information that flows through the network.

In more practical terms neural networks are non-linear statistical data modeling or decision making tools. They can be used to model complex relationships between inputs and outputs or to find patterns in data.

In recent years, neural network has been widely applied to the different scope, in which BP network is commonly used. The model created in this paper is a BP neural network with three-layer network (Figure1),

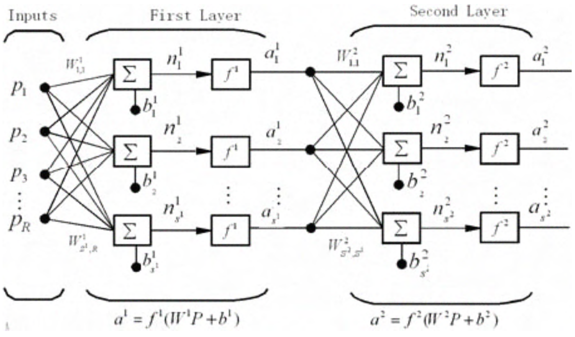

Figure 1. Three-layer BP network structure

In the Figure 1, p is input of neuron. Each layer has its own weight matrix $\vec{W}$, its bias vector $\vec{b}$, a net input vector $\vec{n}$, and an output vector $\vec{a}$. $\vec{W}$ is an $S \times R$ matrix, and $\vec{a}$ and $\vec{b}$ are vectors of length S respectively. The superscripts of symbols identify the layers. Also shown in Figure 1 are R input, S^1 neurons in the first layer, and S^2 neurons in the second layer. Different layers can have different numbers of neurons. The outputs of layers one and two are the inputs for layers two and three. Thus layer 2 can be viewed as a one-layer network with $R = S^1$ inputs, $S = S^2$ neurons, and an $S^1 \times S^2$ weight matrix $\vec{W}^1$. The input of layer 2 is $\vec{a}^1$, and the output is $\vec{a}^2$. The other layer also can be drawn using same abbreviated notation.

First, the output of the network will be computed. In the hidden and output layers, the net input to unit i is of the form:

$$s_i = \sum w_{ji} y_i + \theta_i \tag{11}$$

Several types of transfer functions are used; however, the most frequently used is the sigmoid function. This transfer function is usually a steadily increasing S-shaped curve. The sigmoid function is continuous, differentiable everywhere, and monotonically increasing. In this study, two S-shaped transfer functions in a MATLAB neural network toolbox were used: the tansig function and logsig function. The two functions are of the form:

$$\tan\ sig\ (n) = \frac{2}{1 + e^{-2n}} - 1 \tag{12}$$

$$\tan\ sig\ (n) = \frac{1}{1 + e^{-n}} \tag{13}$$

These accumulated inputs are then transformed to the neuron output. This output is generally distributed to various connection pathways to provide inputs to the other neurons; each of these connection pathways transmits the full output of the contributing neuron. Second, the error between the real output and the expected output will be computed. If the expected error is not satisfied, the precision, weights and biases will be adjusted according to the error.

IV. APPLICATION

Using the mean value of groundwater level in February、June and October from 1985 to 1995, Table 1 is the mean value and Figure 1 is the changing curve of groundwater level. From the Figure 1,we can see that the groundwater level changes with the season. The level in June is the lowest, while in October is the highest, the level in February lies between June and October. And groundwater level has the downward trend. In the course of data processing, when the GM (1,1) model is applied, the model group of GM (1,1) is set up in dry period, raining period, and the normal period. When the neural network model is used, we don't plan to separate the cycle item according to the routine, but want to make the cycle phase reflect to some extent, so we adopt the method that next cycle data can bet got from the last cycle(Zhang *et al.*, 2002; Wang *et al.*, 2002). That is to say, the neural network is the three layer BP network which has three inputting nodes, five hiding nodes, and one outputting node. 1995-1994 data is used to train and 1995 data is to examine (Zhang *et al.*, 2002). Finally, the predicting value can be got by the combined method. The results are shown in Figure 2.

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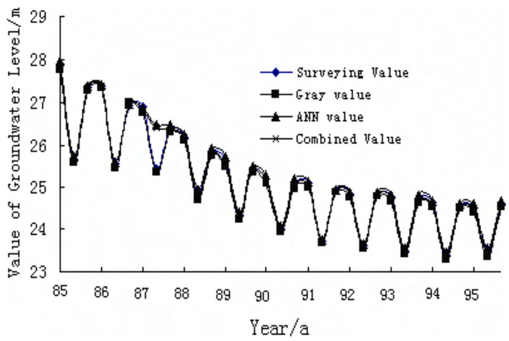

Figure 2. Contrast curve of Surveying value and simulation value and predicted value.

VI. CONCLUSIONS

The predicted method of united grey system-neural network is proposed by virtual of the dynamic characters of groundwater. Examined by the instance and draft, the precision are high. So the method is reliable and effective.

The united ARMA-ANN model is the improvement of the groundwater predicting method. Its analysis course and result have the advantage that geological method of traditional mathematics does not. It can deal with the nonlinear and Periodical issues. Through the optimum technology, the precision of groundwater prediction is improved.

This method is not only suitable for the dynamic prediction of the groundwater but also suitable for other respects such as river water quality, prediction of quality of the atmospheric environment *et al.* The method has the advantage that the concrete form of nonlinear function does not need to confirm.

And it can overall search and fit the minimum error, precision of prediction is improved greatly.

REFERENCES

[1] Li, R., Wang, J., Qian, J., 2002. Prediction of River Water Quality Based on Grey Dynamic Model Group. Bulletin of Soil and Water Conservation, 22(4),10-12.

[2] Niu, D., Chen, Z., Xie, H., 2000. Combined Grey Neural Network Model and its Application in the Seasonal Power Load Forecasting. Journal of North China Electric Power University, 27(4),1-6.

[3] Zhang, Z., 2002. Application of Artificial Neural Network in the prediction of Variation of Groundwater. Journal of Liaoning Technical University, 21(4),504-506.

[4] Xu, D., Wu, Z., 2002. System Analysis and Design based on MATLAB6.X–Neural Network. Xi'an Electron University Press, 19-24.

[5] Xing, M., 2001. Research On Combined Grey Neural Network Model of Seasonal Forecast. Systems Engineering-Theory & Practice, (1), 31-35.

[6] Wang, J., Liu, W., Qian, J., Sun, S., 2002. Grey Model of Groundwater Quality Assesment Based on Single Factor Contaminant Index. Journal of Hefei University of Technology, 25(5), 697-702.

[7] Han, F., Xue, Y., Zhang, Y., 2001. State Equation of Groundwater Based on Artificial Neural Network. Geological Journal of China Universities, 7(4), 483-487.

[8] Lou, S., Shi, Y., 1998. Systematic Analyze and Design based on Matlab-Neural Network Xi'an Electron University Press, 20-114.

[9] Luo, D., Guo, Q., Wang, X., 2003. Simulation and Prediction of Underground Water Dynamics Based on RBF Neural Network. Acta Geoscientia Sinica, 24(5),475-478.

[10] Fu, Q., Liu, J., Wang, L., 2002. Based on ANN Model to Forecast the Groundwater Level on the Area of Well Irrigation Rice.Journal of Northeast Agricultural University, 33(2),152-159.