United Grey System-Neural Network Model and its Application in Prediction of Groundwater Level

Changjun Zhu
College of Urban Construction
Hebei University of Engineering
Handan, 056038, China
christorf@126.com

Qin Ju
State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering
Hohai University
Nanjing, 210098, China

***Abstract*— At present, classic methods are often used to predict groundwater level, but the result is not ideal. Though GM(1,1) and neural network are applied in this field, some limits have been existed. In view of the difficulty to predict groundwater level, in this paper, a grey system-neural network united model is developed based on the grey theory and neural network method. It predicts various tendency of groundwater in this area in the future. Case study indicates that precision of the model is rather high and its popularization significance is better than the other models, and has some practical value when being used in the dynamic groundwater level analysis.**

Keywords- grey model, neural network, groundwater level, prediction, united grey ANN

I. INTRODUCTION

As everyone knows, groundwater is one of the foundations that all lives depend on. Groundwater trend is related to atmosphere precipitation. There is obvious seasonality in variation of atmosphere precipitation, so the groundwater level is changed by the seasonality and periodicity. In this case, when the groundwater level is predicted, the original data of groundwater need to be dealt with in general. In this course, the cycle one, trend one and random one need to be calculated separately after the three items isolated. We can get the final results. The traditional analysis and prediction method of groundwater is mainly the mathematics model of the determinacy and random statistical method, for such as finite element, finite difference, analyze, harmony wave analysis, time series analysis, probability statistic, etc. These methods are mainly based on linear theory. Since models are simple, precision is not high (Zhang et al., 2002; Luo et al., 2003). So in this paper, on the basis of thorough analyzing in depth the predicting method, adopting grey dynamic groups combined with neural network to predict the groundwater level, the more satisfied results may be concluded.

II. COMBINED GREY NEURAL NETWORK MODEL

A. . Principle of GM (1,1)

Essence of GM (1,1)(Li *et al*., 2002; Wang *et al*., 2002) is to make original data accumulate to get a series of regular data. Through setting up differential equation model, we can get the fitted curve to predict the system. Making the given the primitive time array

$$\left\{x^{(0)}(t)=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\}$$

1-AGO:

$$\left\{x^{(1)}(t)=(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\right\} \tag{1}$$

Where $x^{(1)}(k)=\sum_{t=1}^{k}x^{(0)}(i)$, $k=1,2,\cdots n$;

Mean value of $x^{(1)}$: $x=(x(2),x(3),\cdots x(n))$;

Where: $x(t)=\frac{1}{2}(x^{(1)}(t)+x^{(1)}(t-1))$ $t=2,3,\cdots,n$;

Corresponding differential equation: $\frac{dx^{(1)}}{dt}+ax^{(1)}=u$, utilizing least square method to solve parameters a,u .

$$\hat{a}=\begin{bmatrix}a\\u\end{bmatrix}=(B^TB)^{-1}B^TY_N \tag{2}$$

$$B=\begin{bmatrix}-x(2) & 1\\-x(3) & 1\\ \vdots & \vdots\\-x(n) & 1\end{bmatrix} \qquad Y_N=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\ \vdots\\x^{(0)}(n)\end{bmatrix}$$

Grey predicting model of $x^{(1)}$:

$$\hat{x}^{(1)}(t+1)=(x^{(0)}(1)-\frac{u}{a})e^{-at}+\frac{u}{a} \tag{3}$$

Grey predicting model of $x^{(0)}$:

$$\hat{x}^{(0)}(t+1)=(1-e^{a})(x^{(0)}(1)-\frac{u}{a})e^{-at}$$

(4)

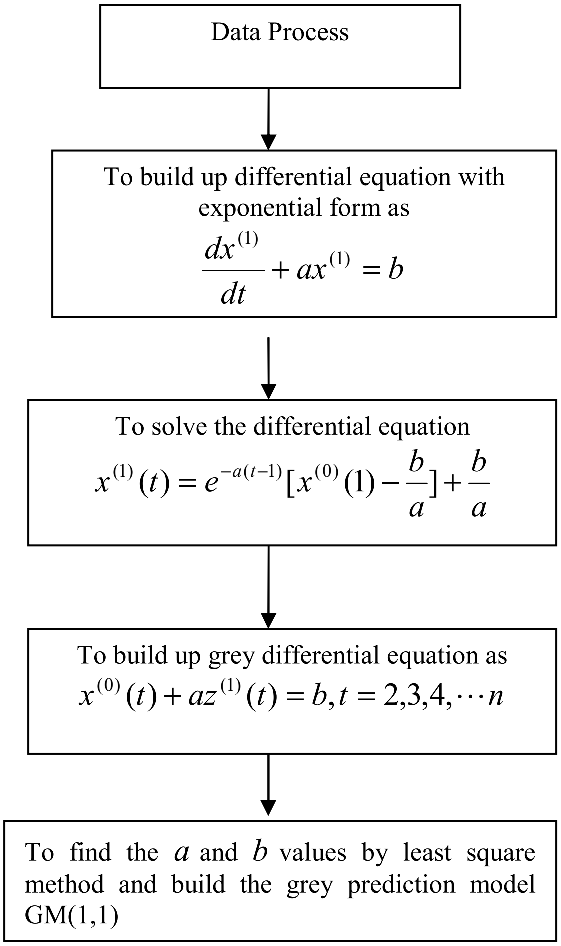

Figure 1. Grey calculation process

In order to differentiate models good or bad.

$$e_k = x^{(0)}(k) - \hat{x}^{(0)}(k) \quad k = 1,2,\cdots,n)$$

$$\bar{e} = \frac{1}{n}\sum_{k=1}^{n} e_k \qquad \bar{x}^{(0)} = \frac{1}{n}\sum_{t=1}^{n} x^{(0)}(t)$$

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n}[x^{(0)}(k) - \bar{x}]^2 \qquad S_2^2 = \frac{1}{n}\sum_{k=1}^{n}[e(k) - \bar{e}]^2$$

While $c = \frac{S_2}{S_1} < 0.35$

$P = \{|e_k - \bar{e}| < 06745 S_1\} > 0.95$, the precision of models can meet the first class precision.

B. . Neural Network Prediction Model

The artificial neural network model is an intellectual discipline that rose rapidly in the eighties, it is one non-linear system whose operation runs side by side and which can simulate human brain structure and encourage behavior. It has been successfully applied to each field. In recent years, artificial neural network is introduced as a method in predicting science, has achieved better result (Han *et al.*, 2001).

Back-Propagation(BP) was created by generalizing the Widrow-Hoff learning rule to multiple-layer networks and nonlinear differentiable transfer functions. Input vectors and the corresponding target vectors are used to train a network until it can approximate a function, associate input vectors with specific output vectors, or classify input vectors in an appropriate way as defined by you. Networks with biases, a sigmoid layer, and a linear output layer are capable of approximating any function with a finite number of discontinuities (Xu *et al.*, 2002). The encourage function is:

$$f(x) = \frac{1}{1 + \exp(-x)} \tag{5}$$

Input node of BP network has no threshold value and encouraged function, input of input node is the output of input node. Threshold values of hiding layer and output layer are dealt with according to right value. Their neuron model is adopted hide layer.

$$O_j = f(\sum_{k=0}^{N} W_{jk} \cdot O_k) \tag{6}$$

Output layer:

$$O_i = f(\sum_{j=0}^{L} W_{ij} \cdot O_j) \tag{7}$$

Where $W_{i0} = \theta_i, W_{j0} = \theta_j$ The common error estimates criterion is that the variance is minimum. That is to say energy function: $E = \frac{1}{2}\sum_{i=1}^{M}(y_i - O_i)^2$

The study of network is to utilize the technology of gradient search to adjust W_{ij} and W_{jk} to make E minimum. Regulated formula of right value is:

$$\begin{cases} \Delta W_{ij}(t) = \eta \cdot (y_i - O_i) \cdot O_i \cdot (1 - O_i) \cdot O_j + \alpha \Delta W_{ij}(t-1) \\ \Delta W_{jk}(t) = \eta \cdot O_j \cdot (1 - O_j) \sum_{i=0}^{M} \delta_i W_{ij} + \alpha \Delta W_{jk}(t-1) \end{cases} \tag{8}$$

η, α called separately studying ratio and amount factor. The range of both is in (0,1), the bigger η is, the faster disappear, but unstable, may shake. The function of α is just opposite with η.

Because the output value of node is in [0,1], input and output value of training sample are dealt with the following method (Fu *et al.*, 2002):

$$x_n = \frac{(x_p - a)}{(b - a)} \tag{9}$$

Where x_p -surveying value; x_n -value after initialing; the value of a is between 0 and x_p ,b is the maximum of x_p (Xu et al., 2002; Lou et al., 1998).

C. Optimum model of Combined Grey Neural Network

if f_1 is the predicting value by grey groups , f_2 is the value by neural network and f_c is the value by optimally combined grey neural network method, the predicting error is respectively: e_1 and e_2 , w_1 and w_2 are the corresponding right coefficient, and $w_1 + w_2 = 1$,so

$$f_c = w_1 f_1 + w_2 f_2 \tag{10}$$

Error: $e_c = w_1 e_1 + w_2 e_2$

Variance:

$$Var(e_c) = w_1{}^2 Var(e_1) + w_2{}^2 Var(e_2) + 2 w_1 w_2 \operatorname{cov}(e_1, e_2)$$

Solving the minimum of $Var(e_c)$, we can get:

$$w_1 = \frac{V\operatorname{ar}(e_2) - \operatorname{cov}(e_1 e_2)}{Var(e_1) + Var(e_2) - 2\operatorname{cov}(e_1 e_2)}$$

And $w_2 = 1 - w_1$, obviously $\operatorname{cov}(e_1, e_2) = 0$, so

$$w_1 = \frac{V\operatorname{ar}(e_2)}{Var(e_1) + Var(e_2)} , w_2 = \frac{V\operatorname{ar}(e_1)}{Var(e_1) + Var(e_2)}$$

From the minimum theory of statistical variance, we know that this method is superior to every single method (Niu et al., 2001; Xing, 2001). So the optimally combined model can be got.

III. INSTANCES

Using the mean value of groundwater level in February 、June and October from 1985 to 1995, Table 1 is the mean value and Figure 1 is the changing curve of groundwater level. From the Figure 1,we can see that the groundwater level changes with the season. The level in June is the lowest, while in October is the highest, the level in February lies between June and October. And groundwater level has the downward trend. In the course of data processing, when the GM (1,1) model is applied, the model group of GM (1,1) is set up in dry period, raining period, and the normal period. When the neural network model is used, we don't plan to separate the cycle item according to the routine, but want to make the cycle phase reflect to some extent, so we adopt the method that next cycle data can bet got from the last cycle(Zhang *et al*., 2002; Wang *et al*., 2002). That is to say, the neural network is the three layer BP network which has three inputting nodes, five hiding nodes, and one outputting node. 1995-1994 data is used to train and 1995 data is to examine (Zhang *et al*., 2002). Finally, the predicting value can be got by the combined method. The results are shown in Table 2 and Figure 2.

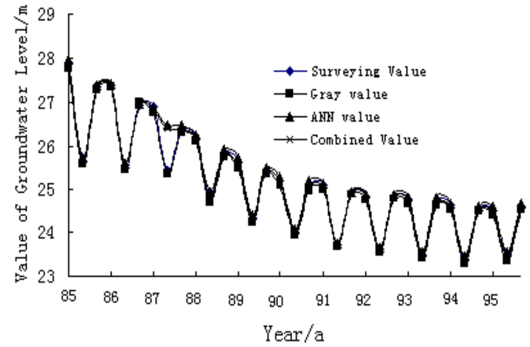

Figure 2. Contrast curve of Surveying value and simulation value and predicted value.

IV. CONCLUSIONS

The predicted method of united grey system-neural network is proposed by virtual of the dynamic characters of groundwater. Examined by the instance and draft, the precision are high. So the method is reliable and effective.

The united grey system-neural network is the improvement of the groundwater predicting method. Its analysis course and result have the advantage that geological method of traditional mathematics does not. It can deal with the nonlinear and Periodical issues. Through the optimum technology, the precision of groundwater prediction is improved.

This method is not only suitable for the dynamic prediction of the groundwater but also suitable for other respects such as river water quality, prediction of quality of the atmospheric environment *et al*. The method has the advantage that the concrete form of nonlinear function does not need to confirm. And it can overall search and fit the minimum error, precision of prediction is improved greatly.

REFERENCES

[1] Li, R., Wang, J., Qian, J., 2002. Prediction of River Water Quality Based on Grey Dynamic Model Group. Bulletin of Soil and Water Conservation, 22(4),10-12.

[2] Niu, D., Chen, Z., Xie, H., 2000. Combined Grey Neural Network Model and its Application in the Seasonal Power Load Forecasting. Journal of North China Electric Power University, 27(4),1-6.

[3] Zhang, Z., 2002. Application of Artificial Neural Network in the prediction of Variation of Groundwater. Journal of Liaoning Technical University, 21(4),504-506.

[4] Xu, D., Wu, Z., 2002. System Analysis and Design based on MATLAB6.X−Neural Network. Xi'an Electron University Press, 19-24.

[5] Xing, M., 2001. Research On Combined Grey Neural Network Model of Seasonal Forecast. Systems Engineering-Theory & Practice, (1), 31-35.

[6] Wang, J., Liu, W., Qian, J., Sun, S., 2002. Grey Model of Groundwater Quality Assesment Based on Single Factor Contaminant Index. Journal of Hefei University of Technology, 25(5), 697-702.

[7] Han, F., Xue, Y., Zhang, Y., 2001. State Equation of Groundwater Based on Artificial Neural Network. Geological Journal of China Universities, 7(4), 483-487.

[8] Lou, S., Shi, Y., 1998. Systematic Analyze and Design based on Matlab-Neural Network Xi'an Electron University Press, 20-114.

[9] Luo, D., Guo, Q., Wang, X., 2003. Simulation and Prediction of Underground Water Dynamics Based on RBF Neural Network. Acta Geoscientia Sinica, 24(5),475-478.

[10] Fu, Q., Liu, J., Wang, L., 2002. Based on ANN Model to Forecast the Groundwater Level on the Area of Well Irrigation Rice.Journal of Northeast Agricultural University, 33(2),152-159.

TABLE I. SURVEYING VALUE OF GROUNDWATER LEVEL.

Year		1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Month	2	27.89	27.38	26.88	26.17	25.64	25.22	25.10	24.84	24.75	24.61	24.50
	6	25.64	25.55	25.42	24.876	24.33	24.00	23.70	23.61	23.49	23.40	23.48
	10	27.33	26.91	26.40	25.87	25.42	25.12	24.93	24.83	24.73	24.56	24.60

TABLE II. CONTRAST OF PREDICTING VALUE.

NO.	Surveying Value/m	Simulation value and Predicting value/m			Relative error %		
		Grey dynamic model	Neural network	Combined grey neural network	Grey dynamic model	Neural network	united grey Neural network
1	27.89	27.78	27.96	27.85	-0.3944	0.2509	-0.1434
2	25.67	25.6	25.71	25.64	-0.2726	0.1558	-0.1169
3	27.33	27.28	27.39	27.34	-0.1829	0.2196	0.0366
4	27.38	27.35	27.42	27.39	-0.1095	0.1461	0.0365
5	25.55	25.45	25.58	25.48	-0.3914	0.1174	-0.2739
6	26.91	26.98	26.97	26.97	0.26013	0.2229	0.2429
7	26.88	26.77	26.85	26.78	-0.4092	-0.1116	-0.3455
8	25.42	25.36	26.48	26.42	-0.2360	4.1699	3.9339
9	26.40	26.3	26.46	26.36	-0.3788	0.2273	-0.1515
10	26.17	26.1	26.25	26.18	-0.2675	0.3057	0.03821
11	24.86	24.69	24.95	24.78	-0.6838	0.3620	-0.3218
12	25.87	25.77	25.93	25.83	-0.3866	0.2319	-0.1546
13	25.64	25.49	25.72	25.57	-0.5850	0.31201	-0.2730
14	24.33	24.24	24.38	24.29	-0.3699	0.2055	-0.1644
15	25.42	25.37	25.49	25.44	-0.1967	0.2754	0.0787
16	25.22	25.12	25.3	25.2	-0.3965	0.3172	-0.0793
17	24.00	23.94	24.06	24.00	-0.2500	0.2500	0
18	25.12	24.99	25.19	25.06	-0.5175	0.2787	-0.2388
19	25.10	25	25.13	25.03	-0.3984	0.1195	-0.2789
20	23.70	23.68	23.75	23.73	-0.0844	0.2109	0.1266
21	24.93	24.87	24.96	24.90	-0.2407	0.1203	-0.1203
22	24.84	24.76	24.88	24.80	-0.3221	0.1610	-0.16103
23	23.61	23.56	23.65	23.60	-0.2118	0.1694	-0.0424
24	24.83	24.79	24.87	24.83	-0.1611	0.1611	0
25	24.75	24.65	24.82	24.72	-0.4040	0.2828	-0.1212
26	23.49	23.41	23.56	23.48	-0.3406	0.2979	-0.0426
27	24.73	24.62	24.81	24.7	-0.4448	0.3235	-0.1213
28	24.61	24.51	24.67	24.57	-0.4064	0.2438	-0.1625
29	23.40	23.29	23.46	23.35	-0.4701	0.2564	-0.2137
30	24.56	24.5	24.59	24.53	-0.2443	0.1222	-0.1222
31	24.50	24.39	24.58	24.47	-0.4489	0.3265	-0.1225
32	23.48	23.36	23.56	23.44	-0.5111	0.3407	-0.1704
33	24.60	24.52	24.64	24.56	-0.3252	0.1626	-0.1626