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ARTIFICIAL INTELLIGENCE ASSIGNMENT

8.FIRST ORDER LOGIC

1.Is the sentence ∃ x, y x = y valid? Explain.

Solution:

The sentence ∃ x, y x = y is valid. A sentence is valid if it is true in every model. An existentially quantified sentence is true in a model if it holds under any extended interpretation in which its variables are assigned to domain elements. According to the standard semantics of FOL as given in the chapter, every model contains at least one domain element, hence, for any model, there is an extended interpretation in which x and y are assigned to the first domain element. In such an interpretation, x = y is true

2.Does the fact ¬Spouse(George, Laura) follow from the facts Jim = George and Spouse(Jim, Laura)? If so, give a proof; if not, supply additional axioms as needed. What happens if we use Spouse as a unary function symbol instead of a binary predicate?

Solution:

The fact ¬Spouse(George, Laura) does not follow. We need to assert that at most one person can be the spouse of any given person: ∀ x, y, z Spouse(x, z) ∧ Spouse(y, z) ⇒ x = y . With this axiom, a resolution proof of ¬Spouse(George, Laura) is straightforward. If Spouse is a unary function symbol, then the question is whether ¬Spouse(Laura) = George follows from Jim≠ George and Spouse(Laura) = Jim. The answer is yes, it does follow. They could not both be the value of the function applied to the same argument if they were different objects.

3.Consider a vocabulary with the following symbols: Occupation(p, o): Predicate. Person p has occupation o. Customer (p1, p2): Predicate. Person p1 is a customer of person p2. Boss(p1, p2): Predicate. Person p1 is a boss of person p2. Doctor , Surgeon, Lawyer , Actor : Constants denoting occupations. Emily, Joe: Constants denoting people. Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.

b. Joe is an actor, but he also holds another job.

c. All surgeons are doctors.

d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).

e. Emily has a boss who is a lawyer.

f. There exists a lawyer all of whose customers are doctors.

g. Every surgeon has a lawyer.

Solution:

a. $O(E,S) \lor O(E,L)$.

b. $O(J, A) \land \exists\, p\; p \neq A \land O(J, p)$.

c. $\forall\, p\; O(p, S) \Rightarrow O(p, D)$.

d. $\neg\exists\, p\; C(J, p) \land O(p, L)$.

e. $\exists\, p\; B(p, E) \land O(p, L)$.

f. $\exists\, p\; O(p, L) \land \forall\, q\; C(q, p) \Rightarrow O(q, D)$.

g. $\forall\, p\; O(p, S) \Rightarrow \exists\, q\; O(q, L) \land C(p, q)$.

4.Consider a knowledge base containing just two sentences: P(a) and P(b). Does this knowledge base entail ∀ x P(x)? Explain your answer in terms of models.

Solution:

The knowledge base does not entail ∀ x P(x). To show this, we must give a model where P(a) and P(b) but ∀ x P(x) is false. Consider any model with three domain elements, where a and b refer to the first two elements and the relation referred to by P holds only for those two elements.

5.**Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., physical instances of CDs). The vocabulary contains the following symbols: Copy Of (d, a): Predicate. Disk d is a copy of album a. Owns (p, d): Predicate. Person p owns disk d. Sings (p, s, a): Album a includes a recording of song s sung by person p. Wrote (p, s): Person p wrote song s. McCartney, Gershwin, B Holiday, Joe, Eleanor Rigby, The Man I Love, Revolver : Constants with the obvious meanings. Exercises 321 Express the following statements in first-order logic:**

a. Gershwin wrote "The Man I Love."

b. Gershwin did not write "Eleanor Rigby."

c. Either Gershwin or McCartney wrote "The Man I Love."

d. Joe has written at least one song.

e. Joe owns a copy of Revolver.

f. Every song that McCartney sings on Revolver was written by McCartney.

g. Gershwin did not write any of the songs on Revolver.

h. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)

i. There is a single album that contains every song that Joe has written.

j. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."

k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

l. Joe owns a copy of every album on which all the songs are sung by Billie Holiday

Solution:

a. $W(G, T)$.

b. $\neg W(G, E)$.

c. $W(G, T) \lor W(M,T)$.

d. ∃ s W(J, s).
e. ∃ x C(x, R) ∧ O(J, x).
f. ∀ s S(M, s, R) ⇒ W(M,s).
g. ¬[∃ s W(G, s) ∧ ∃ p S(p, s, R)].
h. ∀ s W(G, s) ⇒ ∃ p, a S(p, s, a).
i. ∃ a ∀ s W(J, s) ⇒ ∃ p S(p, s, a).
j. ∃ d, a, s C(d, a) ∧ O(J, d) ∧ S(B,T, a).
k. ∀ a [∃ s S(M, s, a)] ⇒ ∃ d C(d, a) ∧ O(J, d).
l. ∀ a [∀ s, p S(p, s, a) ⇒ S(B, s, a)] ⇒ ∃ d C(d, a) ∧ O(J, d)

9.INFERENCE IN FIRST ORDER LOGIC

1.Suppose a knowledge base contains just one sentence, ∃ x As High As(x, Everest). Which of the following are legitimate results of applying Existential Instantiation?
a. As High As(Everest, Everest).
b. As High As(Kilimanjaro, Everest).
c. As High As(Kilimanjaro, Everest) ∧ As High As(BenNevis, Everest).
Solution:
Both b and c are sound conclusions; a is unsound because it introduces the previously used symbol Everest. Note that c does not imply that there are two mountains as high as Everest, because nowhere is it stated that BenNevis is different from Kilimanjaro (or Everest, for that matter).

2. Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens: a. Horses, cows, and pigs are mammals. b. An offspring of a horse is a horse. c. Bluebeard is a horse. d. Bluebeard is Charlie's parent. e. Offspring and parent are inverse relations. f. Every mammal has a parent.
Solution:
We use a very simple ontology to make the examples easier: a. Horse(x) ⇒ Mammal(x) Cow(x) ⇒ Mammal(x) Pig(x) ⇒ Mammal(x). b. Offspring (x, y) ∧ Horse(y) ⇒ Horse(x). c. Horse (Bluebeard). d. Parent (Bluebeard, Charlie). e. Offspring (x, y) ⇒ Parent (y, x) Parent(x, y) ⇒ Offspring(y, x). (Note we couldn't do Offspring (x, y) ⇔ Parent (y, x) because that is not in the form expected by Generalized Modus Ponens.) f. Mammal(x) ⇒ Parent(G(x), x) .

3.One might suppose that we can avoid the problem of variable conflict in unification during backward chaining by standardizing apart all of the sentences in the knowledge base once and for all. Show that, for some sentences, this approach cannot work. (Hint: Consider a sentence in which one part unifies with another.)

<u>Solution:</u>

This would work if there were no recursive rules in the knowledge base. But suppose the knowledge base contains the sentences:

Member (x, [x | r])

Member (x, r) ⇒ Member (x, [y | r])

Now take the query Member (3, [1, 2, 3]), with a backward chaining system. We unify the query with the consequent of the implication to get the substitution θ = {x/3, y/1, r/ [2, 3]}. We then substitute this into the left-hand side to get Member (3, [2, 3]) and try to back chain on that with the substitution θ. When we then try to apply the implication again, we get a failure because y cannot be both 1 and 2. In other words, the failure to standardize apart causes failure in some cases where recursive rules would result in a solution if we did standardize apart.

4. Let L be the first-order language with a single predicate S (p, q), meaning "p shaves q." Assume a domain of people.

a. Consider the sentence "There exists a person P who shaves everyone who does not shave themselves, and only people that do not shave themselves." Express this in L.

b. Convert the sentence in (a) to clausal form.

c. Construct a resolution proof to show that the clauses in (b) are inherently inconsistent. (Note: you do not need any additional axioms.)

<u>Solution:</u>

a. ∃ p ∀ q S (p, q) ⇔ ¬S (q, q).

b. There are two clauses, corresponding to the two directions of the implication. C1: ¬S (Sk1, q) ∨ ¬S (q, q). C2: S (Sk1, q) ∨ S (q, q).

c. Applying factoring to C1, using the substitution q/Sk1 gives: C3: ¬S (Sk1, Sk1). Applying factoring to C2, using the substitution q/Sk1 gives: C4: S (Sk1, Sk1). Resolving C3 with C4 gives the null clause.

5. How can resolution be used to show that a sentence is valid? Unsatisfiable?

<u>Solution:</u>

This question tests both the student's understanding of resolution and their ability to think at a high level about relations among sets of sentences. Recall that resolution allows one to show that KB |= α by proving that KB ∧ ¬α is inconsistent. Suppose that in general the resolution system is called using ASK (KB, α). Now we want to show that a given sentence, say β is valid or unsatisfiable. A sentence β is valid if it can be shown to be true without additional information. We check this by calling ASK (KB0, β) where KB0 is the empty knowledge base. A sentence β that is unsatisfiable is inconsistent by itself. So if we empty the knowledge base again and call ASK (KB0, ¬β) the resolution system will attempt to derive a contradiction starting from ¬¬β. If it can do so, then it must be that ¬¬β, and hence β, is inconsistent.