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3) Θ - Theta Notation.
It represent average bond of algorithm running time.

4) Little oh notation.

Q. Explain the asymtotic Notation with
imp fun. write, graph. example.

1)* Find the upper bound of
$f(n) = 3n + 8$
→ Cond^n of Big O notation
$f(n) \leq c.g(n)$ where $n \geq n_0$ &
$c > 0$ & $n > 1$.
Assume $c = 4$
$f(n) \leq c.g(n)$
$3n + 8 \leq 4n$ is true.

Teacher's Signature

	F(n)	g(n)
	$3n+8$	$4n$
$n=1$	11	4
$n=5$	23	20
$n=6$	26	24
$n=7$	29	28
$n=8$	32	32
$n=9$	35	36

$\therefore 3n+8 = O(n)$ for
$c=4, \quad n_0 = 8$

The Condition is satisfied.

② $f(n) = n^2+10$, find the upper bound.

→ $F(n) = n^2+10$
$\therefore$ Cond^n Big O notation
$f(n) \leq c.g(n)$ where $n \geq n_0$.
c, n_0 are Constant

$F(n) = n^2+10$ Assume $g(n) = 2n^2$
$n^2+10 \leq 2n^2$? $c=2$.

	F(n)	g(n)
	n^2+10	$2n^2$
$n=1$	11	2
$n=3$	19	18
$n=4$	26	32
$n=5$	35	50

$n^2+10 \leq 2n^2$ is true
$\therefore$ For $c=2$ & $n_0=4$
$n^2+10 = O(n^2)$

③ $F(n) = 5n+50$, find the upper bound.

→ $f(n) = 5n+50$
$\therefore$ Cond^n Big O notation.
$f(n) \leq c.g(n)$ where $n \geq n_0$,
c, n_0 are Constant.

c की value +1 करायथे

$f(n) = 5n+50$ Assume $c=6$

$5n+50 \leq \cancel{8n}\ 6n$

	f(n)	g(n)
	$5n+50$	$\cancel{3n}$ $8n$
n=1	55	$\cancel{24}$ 8
n=2	60	16
n=30	200	240.

	f(n)	g(n)
	$5n+50$	$6n$
n=1	55	6
n=10	100	60
n=50	300	300.

$5n+50 \leq 6n$ is true for $n=50$, & $c=6$.

(4) Assume that the $f(n) = 5n+50$, $g(n) = 100\log_{10} n$ is g(n) upperbound of f(n).

→ $f(n) = 5n+50$

$5n+50 \leq 100\log_{10} n$.

	f(n)	g(n)
	$5n+50$	$100\log_{10} n$
n=1	55	0
n=2	60	30.10
n=5	75	69.89
n=8	90	90.30
n=7	85	84.50.
n=10	100	100
n=100	550	200
n=1000	5050	300.

$5n+50 \leq 100\log_{10} n$ is false for $n=100$ ₹.

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(5) find the upperbound of function. $f(n) = 2^n + 3n^3$.

→ $f(n) = 2^n + 3n^3$ $\qquad g(n) = 2 \cdot 2^n$

$f(n) \le c \cdot g(n)$ $\qquad = 2^{n+1}$

Assume = $c = \underline{2^{n+1}}$

2^{n+1}

$f(n) = 2^n + 3n^3$

$2^n + 3n^3 \le 2^{n+1}$

	$f(n)$ $2^n + 3n^3$	$g(n)$ 2^{n+1}
$n=5$	~~66~~ 407	64
$n=6$	712	128
$n=12$	9280	8192
$n=14$	24616	32768
* $n=13$	14783	16384.

$2^n + 3n^3 \le 2^{n+1}$ is true for $n = 13$, $c = 2^{\cancel{n+1}}$

dominating term आहे lower bound मध्ये तीच consider करायची

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(6) find lower bound of $f(n) = 10n^2 + 5$.

→ Consider the Big-Ω notation

$f(n) \ge c \cdot g(n)$ where $n \ge n_0$, $c > 0$, $n_0 \ge 1$.

~~We Assume~~

$f(n) = 10n^2 + 5$ $\qquad$ Assume $g(n) = n^2$.

$c = 1$.

$f(n) \ge c \cdot g(n)$.

$10n^2 + 5 \ge n^2$.

	$f(n)$ $10n^2 + 5$	$g(n)$ n^2
$n=1$	15	1
~~$n=5$~~		

$\therefore 10n^2 + 5 = \Omega(n^2)$

$c = 1$ & $n_0 = 1$.

$\therefore 10n^2 + 5 = \Omega(n^2)$

$c = 10$ & $n = 1$.

	$f(n)$ $10n^2 + 5$	$g(n)$ $10n^2$
$n=1$	15	10

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HW Show that the

(7) $f(n) = n^3 + 3n^2$ is $= \Theta(n^3)$. find the Average bound.

→ for upper bound

$f(n) \le c_2 g(n)$.

$f(n) = n^3 + 3n^2$ ∴ assume $g(n) = 2n^3$.

where $n \ge n_0$, $n \ge 1$, $c > 0$.

	n^3+3n^2	$2n^3$
$n=1$	4	2
$n=2$	20	16
$n=3$	54	54
$n=4$	112	128

∴ $n^3 + 3n^2 \le 2n^3$ is true for $n=3$ & $c=2$.

for lower bound assume $g(n) = n^3$

	$f(n)$	$g(n)$
	n^2+3n^2	n^3
$n=1$	4	1

∴ $n^3 + 3n^2 > n^3$ is true for $n=1$ & $c_1 = 1$

∴ $c_1 g(n) \le f(n) \le c_2 g(n)$

$n^3 \le n^3 + 3n^2 < 2n^3$

$c_1 = 1$ & $c_2 = 2$ & $n \ge 3$.

Sol^n 2 :- by sir.

→ Condition for Average bound (Θ notation)

$c_1 g(n) \le f(n) \le c_2 . g(n)$

where $n \ge n_0$ and c & n_0 are constants.

(1) $f(n) \le c_2 g(n)$

$f(n) = n^3 + 3n^2$

Assume $g(n) = 2n^3$

$c = 2$

~~n^3~~ $f(n) \le c_2 . g(n)$

$n^3 + 3n^2 \le 2n^3$

table लिहायचा मागचा

$n^3+3n^2 = O(n^2)$ for $c=2$ & $n_0=3$.

(3) $f(n) \ge c.g(n)$

$f(n) = n^3+3n^2$

Assume $g(n) = n^3$, $c=1$

$n^3+3n^2 \ge n^3$?

table मिलाओ

$n^3+3n^2 = \Omega(n^3)$

for $c=1$ & $n_0=1$.

Condition Θ notation

$c_1.g(n) \le f(n) \le c_2.g(n)$

for $c_1=1$, $n_0=3$, $c_2=2$.

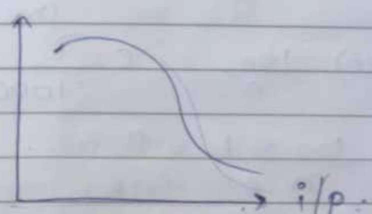
* Decrement function

a function which has a Numerator value is less then the denominator

$\frac{10}{n}, \frac{n}{n^2}, \frac{n}{2^n}, \frac{n^4}{5^n}, \frac{n^5}{5^n}$

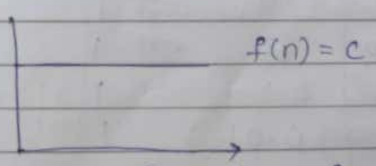
$= \frac{10}{n}, \frac{1}{n}, \frac{n}{2^n}, \frac{n^4}{5^n}, \frac{n^5}{5^n}$

$= \frac{10}{n}, \frac{1}{n}, \frac{n}{2^n}, \frac{n^5}{5^n}, \frac{n^4}{5^n}$.



* Constant function.

ex:- 100, 2^{10}, 5, 1 thousand.



Constant function is asymptotically greated or bigger than decrement function.

eg:- $f(n) = \frac{100}{n}$ $g(n) = 1000$

is $f(n) = O(g(n))$.

→ $f(n) \leq c \cdot g(n)$ where $n \geq n_0$ and c & n_0 are constants.

$f(n) = \frac{100}{n}$, $c = \frac{1}{1000}$

$\frac{100}{n} \leq 1 = ?$

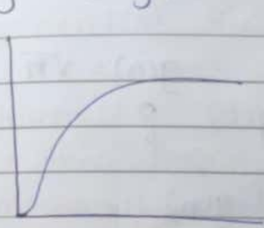
	f(n) $\frac{100}{n}$	g(n) 1
n=10	10	1
n=100	1	1
n=1000	0.1	1
n=10000	0.01	1

$\frac{100}{n} \leq 1$ is true

$\frac{100}{n} = O(1)$ for $c = \frac{1}{1000}$, $n = 100$.

* Logarithmic function.

eg $\log n$ $(\log n)^5$, $\log\log n$, $\log n . \log\log$



$f(n) = \log$.

ex:- $f(n) = 100$, $g(n) = \log_{10} n$.

→ $f(n) \leq c.g(n)$

$f(n) = 100$, $g(n) = \log_{10} n$.

	f(n) 100	g(n) $\log_{10} n$	
n=10	100	1	
$n=10^{100}$	100	100	2
$n=10^{1000}$	100	1000	·3

Logarithmic function value is greater than the constant function

* polynomial function.

eg. n^2, $\sqrt{n}$, n^{10}, n^k

$f(n) = \log_{10} n$, $g(n) = \sqrt{n}$

is $f(n) = O\ g(n)$?

$f(n) \le c \cdot g(n)$.

	$f(n)$ $\log_{10} n$	$g(n)$ $\sqrt{n}$
n=10	1	3.16
n=100	2	10

* Exponential function.

eg. 2^n, $n!$, n^n, 5^n

exponential fun is bigger than the polynomial function

$f(n) = \sqrt{n}$, $g(n) = 2^n$.

	$f(n)$ $\sqrt{n}$	$g(n)$ 2^n
n=2	1.41	4
n=4	2	16

Decrement fun < constant fun < logarithmic fun < polynomial fun < exponential fun.

* Problem

write the following function is a asymtotacally increasing order of their growth rate. ~~loglogn~~

→ n^4, $n^{4.5}$, $n^2 \log n$, $\log n$, $\log n \cdot (\log n)^{10}$, 100, 2^{52}, $(0.5)^n$, $(5.5)^n$.

→ $(0.5)^n \longrightarrow \frac{5^n}{10^n}$ decrement fun

$(5.5)^n \longrightarrow \cancel{5.5}\ \frac{(55)^n}{10^n}$ exponential fun

Numerator is greater. (55>10)

polynomial function:- n^4, $n^{4.5}$.

constant function, 100, 2^{52}

logarithmic fun:- $n^2 \log n$, $\log n$, $\log n (\log n)^{10}$

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= ~~$(0.5)^n$, 100 2^{5^2}, loglogn,~~
$\log n (\log n)^{10}$, $n^2 \log n$, n^4, $n^{4.5}$, $(5.5)^n$

1

2

$(0.5)^n$, 100, 2^{5^2}, $\log n (\log n)^{10}$,
$\log n$, $n^2 \log n$, n^4, $n^{4.5}$, $(5.5)^n$.

1

* Find Time Complexity.

1) Algo sum (a, n)

```
{
   sum = 0;                      ——  1 unit
   for (i=1; i<=n; i++)          ——  n+1 times
   {
      sum = sum + a[i];          ——  n+1 times
      return sum;                ——  1 times
}
                                 2n+3 4 units.
```

Time Complexity = $O(n)$

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2)
```
for (i=1; i<=n; i+2)
{
   printf (" Computer");
}
```

$i=1$, $1+2$, $1+2\times2$, $1+3\times2$ ---- $1+k\times2$

Assume

$1+k\times2 = n$

$k = \frac{n-1}{2}$

$\boxed{O(n)}$ --- Complexity.

3)
```
for (i=n, i>=1, i--)
   printf ("Computer");
```

$i=n$, $n-1$, $n-2$, ---- $n-k$

$n-k=1$

$k = n+1$

$\boxed{O(n)}$ --- Complexity.

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4) for (i=n, i>=1, i-2)
printf("computer");

i=n, n-2, n-4, ... n-k.

= n-k= 2

k= n-2

$\boxed{\text{complexity} = O(n).}$

5) for (i=1; i<=n; i*2)
printf ("computer");

i=1, 1x2, 1x2^2, 2^3 .... 2^k

Assume $2^k = n$

Apply log ~~log~~ on both side

$\log_2 2^k = \log_2 n$

$k = \log_2 n$

$\boxed{\text{complexity} = O(\log n)}$

6) for (i=n, i>=1, i/2)
printf (" Computer");

$i = n, \frac{n}{2}, \frac{n}{2^2}, \frac{n}{2^3}, \dots \frac{n}{2^k}$

Assume $\frac{n}{2^k} = 1$

$2^k = n$

Apply log on both side

$k = \log_2 n$.

$\boxed{\text{Complexity} = O(\log n)}$

7) For (i=3, i<=n; i= i2)
a = a+5;

$i = 3, 3^2, 3^{2^2}, 3^{2^3} \dots, 3^{2k}$.

Assume $3^{2k} = n$

Apply $\log_3$ on both side

$2^k \log_3 3 = \log_3 n$

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$2^k = \log_3 n$

Apply $\log_2$ to both side

$\log_2 2^k = \log_2 \log_3 n$

$\boxed{k = \log_2 \log_3 n}$

$\boxed{\text{Complexity} = \Theta(\log\log n)}$

8) For (i=n; i>=5, i=√i)
printf(" T.Y. Computer");

$i = n, (n)^{\frac{1}{2}}, (n)^{\frac{1}{2^2}}, (n)^{\frac{1}{2^3}}, \cdots (n)^{\frac{1}{2^k}}$

Assume $(n)^{\frac{1}{2^k}} = 5$

Apply $\log_5$ on both side

$\log_5 (n)^{\frac{1}{2^k}} = \log_5 5$

$\frac{1}{2^k} \log_5 n = 1$

$2^k = \log_5 n$

Apply $\log_2$ on both side

$\log_2 2^k = \log_2 \log_5 n$

$k = \log_2 \log_5 n$ $\boxed{\text{Complexity} = \log\log n}$

9) For (i=1; i<=2^n; i++)
printf(" T.Y. Computer");
Complexity = $\Theta(2^n)$

10) for (i=1; i<=√n; i++)
Complexity = $\Theta(\sqrt{n})$

11) for (i = n^2; i>=1; i = i/2)
printf(" Computer");

$i = n^2, \frac{n^2}{2}, \frac{n^2}{2^2}, \frac{n^2}{2^3}, \cdots \frac{n^2}{2^k}$

Assume $\frac{n^2}{2^k} = 1$

$n^2 = 2^k$

Taking log on both side.

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$\log n^2 = \log_2 2^k$

$k = \log_2 n^2$

$k = 2\log_2 n$

$\boxed{\text{Complexity} = O(\log n)}$

12) For (i = 5^n ; i >= 5 ; i = $\sqrt[5]{i}$)
printf("Computer");

$i = 5^n,\ 5^{n/5},\ 5^{n/5^2},\ 5^{n/5^3} \dots 5^{n/5^k}$

$5^{\frac{n}{5^k}} = 5$

$\log_5 5^{\frac{n}{5^k}} = \log_5 5$ $\qquad \log_5 n = \log_5 5^k$

$\frac{n}{5^k} = 1$ $\qquad k = \log_5 n$

$n = 5^k \longrightarrow$ $\qquad \boxed{(O = \log n)}$

13) For (i = 2 ; i <= 2^n ; i = i^2).

$i = 2,\ 2^2,\ 2^{2^2},\ 2^{2^3},\ 2^{2^k}.$

Assume $2^{2^k} = 2^n$

Apply $\log_2$ both side $\qquad \log_2 2^k = \log_2 n$

$\log_2 2^{2^k} = \log_2 2^n$ $\qquad k = \log_2 n$

$(O = \log n)$

$2^k \log_2 2 = \log_2 n \rightarrow$ $\longrightarrow \boxed{O = \log n}$

$2^k \dots \quad 2^k = n$

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14) For (i = 3 ; i <= n ; i = i^3) — $\log_3 \log_3 n$
For (j = 1 ; j >= 1 ; j = j/2) — $\log_2 n$
For (k = 1 ; k <= n ; k = k * 4) — $\log_4 n$
a = a + 5

Complexity $O = (\log^2 n \log\log n)$

15) For (i = 1 ; i <= n ; i++) — n+1
For (j = 1 ; j <= i ; j++) — n
a = a + 5; — n^2

$\boxed{\text{Complexity } O = n^2}$

16) For (i = 1 ; i <= n ; i++)
For (j = 1 ; j <= n ; j = j + i)
a = a + 5 ;

$n \left(n + \frac{n}{2} + \frac{n}{3} + \dots + \frac{n}{n} \right)$

$n \left(1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n} \right) (n \log n)$

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* Recurrence Relation.

A Recurrence Relation is a mathematical expression that describe the overall cost of the problem in terms of cost of solving the smaller sub problem. It is called as Recurrence Relation.

*. Algo fact (n) $\longrightarrow T(n)$
  if (n==1)
    return 1 $\longrightarrow$ 1 unit
  else
    return n*fact(n-1) —— $n \times T(n-1)$

$$T(n) = \begin{cases} n \times T & n > 1 \\ 1 & n = 1 \end{cases}$$

This is a Recurrence Relation.

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* Substitution method.

ex. (1) $T(n) = \begin{cases} T(n-1) + c. & \text{if } n > 1 \\ c & n = 1 \end{cases}$

$T(n) = T(n-1) + c$ ... (1)

put n as n-1 in eqn (1)

$T(n-1) = T(n-1-1) + c$

$T(n-1) = T(n-2) + c$

put the value of $T(n-1)$ in eqn (1)

$T(n) = T(n-2) + c + c$

$T(n) = T(n-2) + 2c$ ... (2)

put n as n-2 in eqn (1)

$T(n-2) = T(n-2-1) + c$

$T(n-2) = T(n-3) + c$

put $T(n-2)$ in eqn (2)

$T(n) = T(n-3) + c + 2c$

$T(n) = T(n-3) + 3c$ ... (3)

:

$T(n) = T(n-k) + kc$ ... (4)

Assume $= n-k = 1$

$k = n-1$

put the value of k in eq^n 4

$T(n) = T\ (n-[n-1)]\ +(n-1)C$

$= T(1) + n.c - c$

$= \cancel{c} + n.c - \cancel{c}$

$= n.c$

$\boxed{\text{Complexity } \Theta(n)}$

2) $T(n) = \begin{cases} T(n-1) + n & \text{if } n>1 \\ 1 & n=1 \end{cases}$

$T(n) = T(n-1) + n \quad \cdots \text{ (1)}$

put n as n-1 in eq^n (1)

$T(n-1) = T(n-1-1) + (n-1)$

$T(n-1) = T(n-2) + (n-1)$

put T(n-1) in eq^n (1)

$T(n) = T(n-2) + (n-1) + n \quad \cdots \text{ (2)}$

$T(n) = T(n-3) + (n-2) + (n-1) + n \cdots \text{ (3)}$

$\vdots$

$T(n) = T(n-k) + T(n-k-1) + \cdots (n-1) + n$

Assume $n-k=1$

$k = n-1$

$T(n) = T(n-(n-1)) + T(n-((n-1)-1)) + \cdots (n-1) + n$

$= T(1) + T(2) + \cdots + (n-1) + n$

$= 1+2+3+\cdots+(n-1)+n \quad \cdots \text{ (4)}$

$\frac{n \times (n+1)}{2} = \frac{n^2+n}{2} = n^2$

$\boxed{\text{Complexity } \Theta = n^2}$

3) $T(n) = \begin{cases} T(n-1) + \log n & \text{if } n>1 \\ 1 & n=1 \end{cases}$

Ans:- $\Theta = n \log n$

$T(n) = T(n-1) + \log n$

put n as n-1 in eq^n (1)

$T(n-1) = T(n-1-1) + \log(n-1)$

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factorial series.

1) $T(n) = \begin{cases} T(n-1) * n & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$

$T(n) = T(n-1) * n \quad \cdots (1)$

$T(n-1) = [T(n-1-1) * (n-1)]$

put $T(n-1)$ in eqn (1)

$T(n) = T(n-2) * (n-1) * n \quad \cdots (2)$

$T(n) = T(n-3) * (n-2) * (n-1) * n \cdots (3)$

$\vdots$

$T(n) = T(n-k) * (n-k) * (n-1k-1) * \cdots * n \quad (4)$

Assume $n-k = 1$

$k = n-1$

$T(n) = T(n-(n-1)) * (n-(n-1)) * (n-(n-2)) * \cdots * n.$

$= T(1) * 1 * 2 * 3 * \cdots * n$

$T(n) = 1 * 1 * 2 * 3 * \cdots * n$

$T(n) = n!$

$\boxed{O = (n!).}$

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Geometric P series.

5) $T(n) = \begin{cases} 2T(n-1)+n & \text{if } n>1 \\ 1 & \text{if } n=1 \end{cases}$

$\rightarrow T(n) = 2T(n-1)+n \cdots$ ①

Substitute n as $n-1$

$T(n-1) = 2T(n-1-1)+(n-1)$

$T(n-1) = 2T(n-2)+(n-1)$

put the value of $T(n-1)$ in eqn ①

$T(n) = 2[2T(n-2)+(n-1)]+n$

$T(n) = 2^2T(n-2)+2(n-1)+n \cdots$ ②

put n as $n-2$ in eqn ①

$T(n-2) = 2T(n-2-1)+(n-2)$

$T(n-2) = 2T(n-3)+(n-2)$

put $T(n-2)$ in eqn ②

$T(n) = 2^2[2T(n-3)+(n-2)]+2(n-1)+n$

$T(n) = 2^3T(n-3)+2^2(n-2)+2(n-1)+n$ - ③

⋮

$T(n) = 2^kT(n-k)+2^{k-1}(n-(k-1))+2^{k-2}(n-(k-2)) \cdots +n.$

Assume $n-k=1$

$k=n-1$

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$GP = A\dfrac{(r^n-1)}{r-1}$

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$T(n) = 2^{n-1}+2^{n-2}(n-(n-1-1))+2^{n-3}(n-(n-1-2))+\cdots+n$

$T(n) = 2^{n-1}+2^{n-2}(2)+2^{n-3}.3+\cdots 2^2(n-2)+2(n-1)+n.$

$T(n) = 2^0(n)+2^1(n-1)+2^2(n-2)+\cdots+2^{n-2}(2)+2^{n-1}(1) \cdots$ ④

Multiply 2 to both side

$2T(n) = 2^1(n)+2^2(n-1)+2^3(n-2)+\cdots 2^{n-1}(2)+2^n.1 \cdots$ ⑤

Substract ④ from ⑤

$2T(n)-T(n) = -n+2^1+2^2+2^3+\cdots+2^{n-1}+2^n \cdots$

$= -n+2\frac{(2^n-1)}{2-1} = -n+2^{n+1}-2$

$T(n) = 2\,2^n-(n+2)$

$\boxed{\text{ⓒ } (2^n)}$

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6) $T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n & \text{if } n>1 \\ 1 & \text{if } n=1 \end{cases}$

$\rightarrow T(n) = 2T\left(\frac{n}{2}\right) + n$ --- (1)

Substitute n as $\frac{n}{2}$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)$ --- (1)

put $T\left(\frac{n}{2}\right)$ in eq^n (1)

$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$

$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2n$ --- (2)

put n as $n/2^2$ in eq^n (1).

$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$

put $T\left(\frac{n}{2^2}\right)$ in eq^n (2)

$T(n) = 2^2\left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n$

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$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n$ --- (3)

$\vdots$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$ --- (4)

Assume $\frac{n}{2^k} = 1$

~~Apply $\log_2$ on both~~ side

~~$2^k = \log_2 n$~~ Apply log on bothside

$2^k = n$

$k = \log n$

$T(n) = 2^k T(1) + n\log n$

$T(n) = n + n\log n$

$\boxed{O(n\log n)}$

7) $T(n) = \begin{cases} \sqrt{n}\, T(\sqrt{n}) + n & \text{if } n>2 \\ 2 & \text{if } n=2 \end{cases}$

$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

$T(n) = (n)^{\frac{1}{2}} T(n^{\frac{1}{2}})$ --- (1)

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H.W Q. $T(n) = \begin{cases} 2T(\sqrt{n}) + 1 & \text{if } n > 2 \\ 2 & \text{if } 0 < n \le 2 \text{ or } (n=2) \end{cases}$

Substitute n as $n^{\frac{1}{2}}$

$T(n^{\frac{1}{2}}) = (n)^{\frac{1}{4}} T(n^{\frac{1}{4}}) + n^{\frac{1}{2}}$

put the value of $T(n)^{\frac{1}{2}}$ in eqn ①

$T(n) = (n)^{\frac{1}{2}} \left[n^{\frac{1}{4}} T(n^{\frac{1}{4}}) + n^{\frac{1}{2}} \right] + n$

$T(n) = n^{\frac{3}{4}} T(n^{\frac{1}{4}}) + 2n$ --- ②

$T(n) = n^{\frac{3}{4}} \left[n^{\frac{1}{8}} T(n^{\frac{1}{8}}) + n^{\frac{1}{4}} \right] + 2n$

$T(n) = n^{\frac{7}{8}} T(n^{\frac{1}{8}}) + 3n$ --- ③

⋮

$T(n) = n^{2k-1/2^k} T(n^{\frac{1}{2^k}}) + kn$ ---- ④

Assume

Apply $\log_2$ to both side

$\frac{1}{2^k} \log_n = \log_2 2$

$2^k = \log n$

Apply log

$k = \log\log n$

Formula.

$T(n) = n^{\log n - 1/\log n} . 2 + n\log\log n$

$= (n)^{1-1/\log n} . 2 + n \log\log n.$

$= \frac{n}{n^{1/\log n}} . 2 + n\log\log n.$

$= \frac{n}{\cancel{2}} . \cancel{2} + n\log\log n$

$\boxed{O(n\log\log n)}$

8) $T(n) = \begin{cases} 2T\sqrt{n} + 1 & \text{if } n > 2 \\ 2 & n = 2. \end{cases}$

$T(n) = 2T(\sqrt{n}) + 1$ ---- ①

put $\sqrt{n}$ n as $\sqrt{n}$

$T(\sqrt{n}) = 2T(n^{\frac{1}{2^2}}) + 1$

put in eqn ①

$T(n) = 2(2T(n^{\frac{1}{2^2}}) + 1) + 1$

$T(n) = 2^2 T(n^{\frac{1}{2^2}}) + 2 + 1.$

$T(n) = 2^2 T(n^{\frac{1}{2^2}}) + 2 + 1.$

put n as $n^{\frac{1}{2^2}}$

$T\left(n^{\frac{1}{2^2}}\right) = 2T\left(n^{\frac{1}{2^3}}\right) + 1$

$T(n) = 2^2\left(2T\left(n^{\frac{1}{2^3}}\right) + 1\right) + 2 + 1$

$= 2^3 T\left(n^{\frac{1}{2^3}}\right) + 2^2 + 2 + 1$

$T(n) = 2^k T\left(n^{\frac{1}{2^k}}\right)$

Assume. $n^{\frac{1}{2^k}} = 2$

$2^{k-1} + 2^{k-2} + \cdots + 2^2 + 2^1 + 2^0$

$n^{\frac{1}{2^k}} = 2$

$\frac{1}{2^k} \log n = \log 2$

$\log n = 2^k$

$\log\log n = k$

$T(n) = \log n \, T(2) + \log\log n + (\log\log n - 1) + \cdots$

$= 2 \log n + \log\log n + (\log\log n - 1) \cdots + 2 + 1$

$T(n) = 2^k T\left(n^{\frac{1}{2^k}}\right) + 2^{k-1} + 2^{k-2} + \cdots + 2^2 + 2^1 + 2^0$

$= 2^k \cdot 2 + \frac{1(2^k - 1)}{2 - 1} = 2 \cdot 2^k + (2^k - 1)$

$= 2 \cdot 2^k + 2^k - 1$

$2^k(2+1) - 1 = 3 \cdot 2^k - 1$

$= 3 \cdot 2^k = 3 \log n$

$\boxed{O(\log n)}$

9) $T(n) = \begin{cases} \sqrt{2}\, T\left(\frac{n}{2}\right) + \sqrt{n} & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$

$\rightarrow T(n) = \sqrt{2}\, T\left(\frac{n}{2}\right) + \sqrt{2}$

put n as $\frac{n}{2}$

$T\left(\frac{n}{2}\right) = \sqrt{2}\, T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^{\frac{1}{2}}$ --- ①

Put the value of $T(n/2)$ in eqn ①

$a^{\log_2 b} \quad b^{\log_2 a}$

$$T(n) = \sqrt{2}\left[\sqrt{2}\,T\left(\frac{n}{2^2}\right) + \sqrt{\frac{n}{2}}\right] + \sqrt{n}$$

$$T(n) = (\sqrt{2})^2\left[T\left(\frac{n}{2^2}\right) + \sqrt{2}\cdot\frac{\sqrt{n}}{\sqrt{2}}\right] + \sqrt{n}$$

$$T(n) = (\sqrt{2})^2\,T\left(\frac{n}{2^2}\right) + 2\sqrt{n} \quad \cdots ②$$

$$T(n) = (\sqrt{2})^3\,T\left(\frac{n}{2^3}\right) + 3\sqrt{n} \quad \cdots ③$$

$\vdots$

$$T(n) = (\sqrt{2})^k\,T\left(\frac{n}{2^k}\right) + k\sqrt{n} \quad \cdots ④$$

Assume $\frac{n}{2^k} = 1$

$$n = 2^k$$

Apply log on both side

$$k = \log n$$

$$T(n) = (\sqrt{2})^{\log n}\cdot 1 + \sqrt{n}\log n.$$

$$T(n) = (2)^{\log n/2} + \sqrt{n}\log n$$

$$= (n)^{\log(2)^{\frac{1}{2}}} + \sqrt{n}\log n$$

$$= \sqrt{n} + \sqrt{n}\log n$$

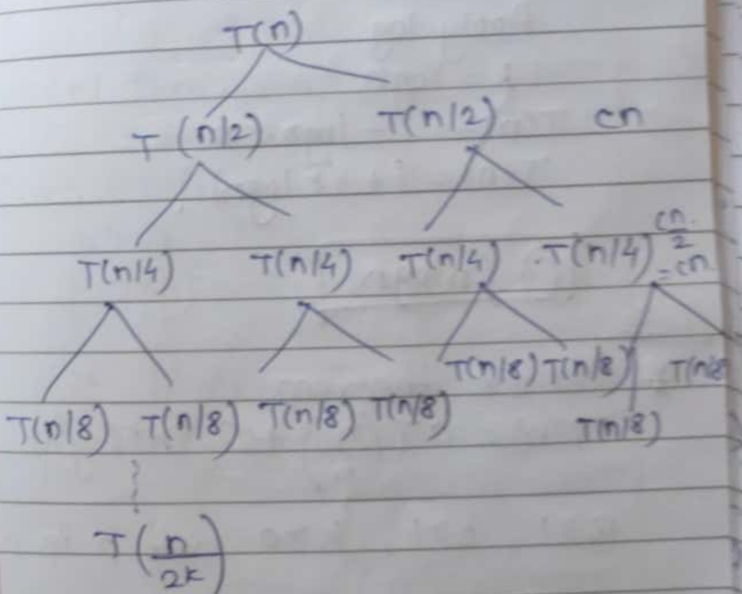
$\boxed{O\ \sqrt{n}\log n}$

Recurrance Tree Method.

$$T(n) = \begin{cases} T(n-1)+1 & n>1 \\ 1 & n=1 \end{cases}$$

① $$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + cn & \text{if } n>1 \\ 1 & \text{if } n=1 \end{cases}$$

$$\rightarrow T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + cn$$



$T(n) = T\left(\frac{n}{2^k}\right) + cn + cn + cn \cdots k$

$= T\left(\frac{n}{2^k}\right) + kcn$

Assume $\frac{n}{2^k} = 1$

$2^k = n$

Apply log

$k = \log n$

$T(n) = 1 + \log n \cdot c \cdot n$

$T(n) = 1 + (n \log n)$.

$\boxed{O(n \log n)}$

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* Master theorem

$T(n) = aT\left(\frac{n}{b}\right) + O(n^k \log^p n)$

$a \geq 1$, $b > 1$, $k \geq 0$ and p is a real no.

Case 1 :- if $a > b^k$ then

$T(n) = O(n^{\log_b a})$

Case 2 :- if $a = b^k$ then

a) $p > -1$ then $T(n) = O(n^{\log_b a} \log^{p+1} n)$

b) $p = -1$ then $T(n) = O(n^{\log_b a} \log \log n)$

c) $p < -1$ then $T(n) = O(n^{\log_b a})$

Case 3 :- if $a < b^k$ then

a) if $p \geq 0$ then

$T(n) = O(n^k \log^p n)$

b) if $p < 0$ then

$T(n) = O(n^k)$.

ex ① $T(n) = 4T\left(\frac{n}{2}\right) + n$

$\rightarrow$ $a=4$, $b=2$, $k=1$, $p=0$.

$a \quad b^k$

$4 > 2^1$

then $T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_2 4})$

$T(n) = \Theta(n^2)$

② $T(n) = 2T\left(\frac{n}{2}\right) + n$.

$\rightarrow$ $a=2$, $b=2$, $k=1$, $p=0$.

$a \quad b^k$

$2 = 2^1$

then $p > -1$

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$T(n) = \Theta(n^{\log_2 2} \log^{0+1} n)$

$T(n) = \Theta(n^{\log_2 2} \log^1 n)$

$T(n) = \Theta(n \log n)$.

③ $T(n) = 4T\left(\frac{n}{2}\right) + n^2$.

$a=4$, $b=2$, $k=2$, $p=0$.

$a \quad b^k$

$4 \quad 2^2$

$4 = 4$

then $p > -1$

then $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$T(n) = \Theta(n^{\log_2 4} \log^{0+1} n)$

$T(n) = \Theta(n^{\log_2 4} \log^1 n)$

$T(n) = \Theta(n^2 \log n)$.

④ $T(n) = 8T\left(\frac{n}{2}\right) + n^2$.

$\rightarrow$ $a=8$, $b=2$, $k=2$, $p=0$.

$a \quad b^k$

$8 \quad 2^2$

$8 > 4$

then $T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_2 8})$

$T(n) = \Theta(n^3)$.

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(5) $T(n) = 4T\left(\frac{n}{2}\right) + n\log n$

→ $a=4,\ b=2,\ k=1,\ p=1.$

$a \quad b^k$

$4 \quad 2^1$

$4 > 2$

then $T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_2 4})$

$T(n) = \Theta(n^2).$

(6) $T(n) = 4T\left(\frac{n}{2}\right) + n^2\log n.$

→ $a=4,\ b=2,\ k=2,\ p=1$

$a \quad b^k$

$4 \quad 2^2$

$4 = 4$

then $p > -1$

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$T(n) = \Theta(n^{\log_2 4} \log^{1+1} n)$

$T(n) = \Theta(n^{\log_2 4} \log^2 n)$

$T(n) = \Theta(n^2 \log^2 n).$

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(7) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n.$

→ $a=2,\ b=2,\ k=1,\ p=-1.$

$a \quad b^k$

$2 \quad 2^1$

$2 = 2.$

then $p = -1$

$T(n) = \Theta(n^{\log_b a} \log\log n)$

$T(n) = \Theta(n^{\log_2 2} \log\log n).$

$T(n) = \Theta(n \log\log n).$

(8) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log^2 n.$

→ $a=2,\ b=2,\ k=1,\ p=-2$

$a \quad b^k$

$2 \quad 2^1$

$2 = 2$

then $p < -1$

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 2})$

$T(n) = \Theta(n).$

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(9) $T(n) = 2T\left(\frac{n}{2}\right) + n^2$.

→ $a = 2,\ b = 2,\ k = 2,\ p = 0$.

$a \quad b^k$

$2 \quad 2^2$

$2 < 4$

then $p \geq 0$.

$T(n) = \Theta(n^k \log^p n)$

$T(n) = \Theta(n^2 \log^0 n)$

$T(n) = \Theta(n^2)$.

* $T(n) = 4T\left(\frac{n}{2}\right) + n \quad = n^2$

$T(n) = 2T\left(\frac{n}{2}\right) + 1 \quad = n$

$T(n) = 2T\left(\frac{n}{2}\right) + \log n = n \log n$.

$T(n) = 2T\left(\frac{n}{2}\right) + n \quad = n \log n$

$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \log^2 n \quad = n^2 \log^3 n$.

(10) $T(n) = 2T\left(\frac{n}{2}\right) + n / \log^4 n$.

→ $a = 2,\ b = 2,\ k = 1,\ p = -4$.

$a \quad b^k$

$2 \quad 2^1$

$2 = 2$

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then $p < -1$

$T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_2 2})$

$T(n) = \Theta(n)$.

(11) $T(n) = 3T\left(\frac{n}{2}\right) + n^2 / \log n)$

→ $a = 3,\ b = 2,\ k = 2,\ p = -1$

$a \quad b^k$

$3 \quad 2^2$

$3 < 4$

case3:- then $p < 0$.

$T(n) = \Theta(n^k)$

$T(n) = \Theta(n^2)$

$T(n) = \Theta(n^2)$.

(12) $T(n) = \sqrt{2}T\left(\frac{n}{2}\right) + \log n$.

→ $a = \sqrt{2},\ b = 2,\ k = 0,\ p = 1$.

$a \quad b$

$\sqrt{2} \quad 2^0$

$\sqrt{2} > 1$

then $T(n) = \Theta(n^{\log_b a}) =$

$T(n) = \Theta(n^{\log_2 \sqrt{2}})$

$T(n) = \Theta(n^{\log_2 2^{\frac{1}{2}}})$

$T(n) = \Theta(n^{\frac{1}{2}\log_2 2})$

$T(n) = \Theta(\sqrt{n})$.

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(15) $T(n) = T(\sqrt{n}) + \log n$

→ ~~$a=1$, $b=1$, $k=0$, $p=1$~~

Convert the expression in Master th^m

$T(n) = T(n^{1/2}) + \log n$

Assume $n = 2^m$

$(2^m)^{1/2} = 2^{m/2}$

$T(2^m) = T(2^{m/2}) + \log 2^m$

$T(2^m) = T(2^{m/2}) + m$

$T(2^m) = S(m)$

$S(m) = S\left(\frac{m}{2}\right) + m$

$a=1, b=2, k=1, p=0.$

$a \quad b^k$

$1 \quad 2^1$

$1 < 2$

then $p \geq 0$

$T(n) = \Theta(n^k \log^p n)$

$= \Theta(m^1 \log^0 n)$

$T(n) = \Theta(m)$

$T(n) = \Theta(\log n)$

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(14) $T(n) = 2T(\sqrt{n}) + \log n$

→ convert the expression in Master th^m

$T(n) = 2T(n^{\frac{1}{2}}) + \log n$

Assume = $n = 2^m$.

Apply log $m = \log n$

~~$T(n) = S(m)$~~

$T(2^m) = 2T(2^{m/2}) + m$

Assume $T(2^m) = S(m)$

$S(m) = 2S(m/2) + m$

Apply master theorem

$a=2, b=2, k=1, P=0$

$a \quad b^k$

$2 = 2^1$

Case 2 a) $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$T(n) = \Theta(m^{\log_2 2} \log m)$

$T(n) = \Theta(m \log n)$

$T(n) = \Theta(\log n \log\log n)$

(15) $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

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→ Convert expression in Master th^m.

$T(n) = \sqrt{n}\,T(n^{\frac{1}{2}}) + n$

$T(n) = (n)^{\frac{1}{2}}\,T(n^{\frac{1}{2}}) + n$.

Assume $n = 2^m$

Apply log $m = \log n$.

$T(2^m) = 2^{m/2}\,T(2^{m/2}) + 2^m$

Divide 2^m to both side

$\frac{T(2m)}{2m} = \frac{T(2m/2)}{2m/2} + 1$

Assume $\frac{T(2m)}{2m} = S(m)$

$S(m) = S\left(\frac{m}{2}\right) + 1$

Apply master theorem

$a = 1,\ b = 2,\ k = 0,\ p = 0$.

$a \quad b^k$

$1 = 2^0$.

Case 2 a) $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$S(m) = \Theta(m^{\log_2 1} \log m)$

$= \Theta(\log m)$

$\frac{T(2m)}{2^m} = \Theta(\log\log n)$

$T(2m) = 2^m \log\log n$.

$T(n) = n\log\log n$.

$\boxed{T(n) = \Theta(n\log\log n)}$

16) $T(n) = T(\sqrt{n}) + 1$.

→ Convert expression in Master th^m.

$T(n) = T(n^{\frac{1}{2}}) + 1$

Assume $= n = 2^m$

Apply log $m = \log n$.

$T(2^m) = T(2^{m/2}) + 1$. $T(2m) = S(m)$

$S(m) = S\left(\frac{m}{2}\right) + 1$.

Apply Master th^m.

$a = 1,\ b = 2,\ k = 0,\ p = 0$.

$a \quad b^k$

$1 \quad 2^0$

$1 \cancel{\leq} 1$

~~Case 3 a) $p \geq 0$~~

~~$T(n) = \Theta(n^k \log^p n)$~~

$= \Theta(n^0 \log^0 n)$

$\boxed{T(n) = \Theta(n \log n)}$

Case 2 a) $\Theta(n^{\log_b a} \log^{p+1} n)$

$\Theta = (m^{\log 2^1} \log^1 m)$

$\Theta = (\log m)$

$\boxed{\Theta = \log(\log n)}$

17) $T(n) = 2T(\sqrt{n}) + 1$

$\rightarrow$ $T(n) = 2T(n^{1/2}) + 1$

Assume $n = 2m$

$m = \log n$.

$T(2m) = 2T(2^{m/2}) + 1$

Assume $T(2m) = S(m)$

$T(2m) = 2S(\frac{m}{2}) + 1$

Here $a=2$, $b=2$, $k=0$, $p=0$.

$a > b^k$

$2 > 2^0$

Case 1

$T(m) = \Theta(m^{\log_b a}) = \Theta(m)$

$= \Theta(\log n)$.