Number System

Date :

* Surds → Any root of a number, which cannot be exactly found (irrational numbers)

* Perfect Number → Any no. that is equal to sum of its factors (6, 28, 496 ...)

① Which of the following is prime no.
31, 91, 87, 57
⇒ 31

⇒ Basic rules of Even & odd

odd ± odd = even
even ± even = even
odd ± even = odd
odd × odd = odd
even × even = even
odd × even = even

⇒ Divisibility Rule

2 → Unit digit is divisible by 2 → 0, 2, 4, 6, 8
3 → If sum of its digits is divisibly by 3
4 → last 2 digits in same order divisible by 4
5 → 0 or 5

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6 → divisible by both 2 and 3
7 → double last digit & subtract with the no.
8 → last 3 digits are divisible by 8
9 → sum of its digits are divisible by 9.

⇒ Division and remainder

Dividend = (Divisor × Quotient) + Remainder.

⇒ Rules of Sum of Numbers

* Sum of first n natural no. $= \frac{n(n+1)}{2}$
* ——— " ——— odd no. $= n^2$
* ——— " ——— even no. $= n(n+1)$
* Sum of Square of 1st n natural no. $= \frac{n(n+1)(2n+1)}{6}$
* — " — Cube — " — $= \left(\frac{n(n+1)}{2}\right)^2$

① Sum & product of two no. is 25 and 144. What is difference of no.?

Sol:

$x + y = 25 \Rightarrow x = 25 - y$

$xy = 25 \Rightarrow (25 - y)y = 25$

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$$25y - y^2 = 25$$

Short Cube : $D = \sqrt{s^2 - 4P}$

$$= \sqrt{625 - 4(144)}$$
$$= \sqrt{625 - 576}$$
$$= \sqrt{49} = 7$$

(2) Sum of 2 nos. is 15. Two times of first exceeds by 5 the 3 times of other. Then the nos. will be?

$$x + y = 15 \Rightarrow x = 15 - y$$
$$2x = 3y + 5 \Rightarrow 2(15 - y) = 3y + 5$$
$$30 - 2y = 3y + 5$$
$$25 = 5y$$
$$\boxed{y = 5}$$

$x = 15 - y$

$\boxed{x = 10}$

10 & 5

(03) A no. exceeds by 25 from its $\frac{3}{8}^{th}$ part. then no. is.

$$x = \frac{3}{8}x + 25$$

$$8x = 3x + 200 \qquad 5x = 200 \qquad \boxed{x = 40}$$

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⇒ Power Cycle

* basic rules for unit digit.

for 0, 1, 5 and 6:

These no. will have same unit digit

⇒ 156^4 ⇒ 6 , 1311^{25} ⇒ 1

* Rule for 2, 3, 7 and 8: (4 cycles)

$2^1 = 2$	$3^1 = 3$	$7^1 = 7$	$8^1 = 8$
$2^2 = 4$	$3^2 = 9$	$7^2 = 9$	$8^2 = 4$
$2^3 = 8$	$3^3 = 7$	$7^3 = 3$	$8^3 = 2$
$2^4 = 6$	$3^4 = 1$	$7^4 = 1$	$8 = 6$

* Rule for 4: (2 cycle)

$4^1 = 4$ $4^3 = 4$

$4^2 = 6$ $4^4 = 6$

odd = 4
even = 6

* Rule for 9: (2 cycle)

$9^1 = 9$

$9^2 = 1$

odd = 9
even = 1

Note → 4^n divided by 9 → 4^{3k+1} leaves remainder 4

4^{3k-1} leaves remainder 7

4^{3k} leaves " " 1

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① Remainder when 41^{43} is divided by 9?

Sol:- $4+1 \Rightarrow 5 \Rightarrow 5^{43} \bmod 9$

$5^1 = 5 \quad 5^2 = 7 \quad 5^3 = 8 \quad 5^4 = 4 \quad 5^5 = 2 \quad 5^6 = 1$ (6 cycles)

$\frac{43}{6} = 7$ [r = 1]

$5^1 = 5$

② $((32)^{32})^{32}$ divided by 7

Sol:- $32/7 = 4$ 3 Cycles

$32^2 = 2$

$32^3 = 1$

$\Rightarrow \frac{32}{3} = 1$ $\qquad 32^1 = 4$

Trailing Zeroes

Q1 What is the no. of trailing zeroes in 23!?

Sol:- $\left(\frac{23}{5}\right) = 4 \rightarrow$ less than 5

So $= 4$

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02. Trailing zeroes in 123!?

Sol

$= \frac{123}{8}^{24} = \frac{24}{8_1}^{4} = 4 < 5$

$\Rightarrow 24 + 4 = 28$

03. Trailing zeroes in 1123!?

Sol;

$= \frac{1123}{8_1}^{224} = \frac{224}{8_1}^{44} = \frac{44}{8_1}^{8} = \frac{8}{8_1}^{1}$

$= 224 + 44 + 8 + 1 = 277$

LCM and HCF

01. LCM : 15 & 30

Sol:

3 | 15	3 | 30	5 | 15, 30
5 | 5	5 | 10	3 | 3, 6 $= 5 \times 3 \times 2$
1	2	2 | 1, 2 $= 30$
		1, 1

$= 3 \times 5 = 3 \times 5 \times 2$

$= 30$

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02. HCF → 27, 18, 36

3 | 27	3 | 18	3 | 36
3 | 9	3 | 6	3 | 12
3	2	2 | 4
		2

= 3 × 3 = 9

⇒ Basic Rules for LCM & HCF fractions

* LCM of fraction = $\frac{\text{LCM of numerators}}{\text{HCM of denominators}}$

* HCF of fraction = $\frac{\text{HCF of numerators}}{\text{LCM of denominator}}$

⇒ Simplification → BODMAS

01. 2960 ÷ 16 − 35 = ?

~~740~~ 185

Sol: ~~2960~~ / ~~16~~ 4, = 185 − 35 = 150

02. (1700 ÷ 68) * (135 ÷ 9) − (24 × 4) = ?

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Sol: $\left(\frac{1700}{68}\right) \times \left(\frac{135}{9}\right) - 96$

$25 \times 15 - 96$

$375 - 96 = 279$

⇒ Surds & Indices

* do not have proper roots.

3 types

01. Pure Surds :- Single Rational no. ⇒ $\sqrt{7}, \sqrt{x^3}$
02. Mixed Surds :- Mix of -11- ⇒ $x\sqrt{y}, 4\sqrt{3}$
03. Compound Surds :- Composed of 2 surds ⇒ $x+\sqrt{y}, 4+\sqrt{3}$

* Basic Rule

Addition → Surds cannot be added ⇒ $\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$
Subtraction → -11- Subtracted ⇒ $\sqrt{a}-\sqrt{b} \neq \sqrt{a-b}$
Multiplication → Surds can be multiplied ⇒ $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
Division → Surds can be divided ⇒ $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$
Exponent → Can be written ⇒ $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

Note: $m\sqrt{a} + n\sqrt{b}$ ×

$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

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01. Convert $\sqrt{80}$ into mixed Surds

Ans $= \sqrt{4 \times 4 \times 5} = 2\sqrt{5}$

02. $\sqrt{5+2\sqrt{6}} =$

$5+2\sqrt{6} \Rightarrow 3+2+2(\sqrt{3} \times \sqrt{2})$

$= (\sqrt{3})^2 + (\sqrt{2})^2 + 2(\sqrt{3} \times \sqrt{2})$

$= (\sqrt{3}+\sqrt{2})^2$

$\sqrt{5+2\sqrt{6}} \Rightarrow \sqrt{(\sqrt{3}+\sqrt{2})^2}$

$= \sqrt{3}+\sqrt{2}$

⇒ Basic Rules of Indices

3 Rules

* Multiplication Rule.

01. $a^m \times a^n = a^{m+n}$

02. $a^m + b^m = (ab)^m$

* Division Rule

01. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ 02. $\frac{a^m}{a^n} = (a)^{m-n}$

* Power Rule:

01. $(a^m)^n = a^{m \times n}$ 02. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

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3. $\sqrt[n]{a} = a^{\frac{1}{n}}$ 04. $a^{-m} = \frac{1}{a^m}$

01. Simplify $\left(\frac{27}{1331}\right)^{\frac{2}{3}} \times \left(\frac{121}{9}\right)^{\frac{1}{2}}$

Sol:-

$$= \left(\frac{3}{11}\right)^2 \times \left(\frac{11}{3}\right)^1 = \frac{9}{121} \times \frac{11}{3} = \frac{3}{11}$$

02. Value of x in $(7)^{3x-2} = 2401$

By:

$$7^{(3x-2)} = 2401 \Rightarrow 7^{(3x-2)} = 7^4$$

$$3x - 2 = 4$$

$$3x = 6$$

$$\boxed{x = 2}$$

Progression

→ Arranging nos. in particular Order based on repitation

01. AP
02. GP
03. HP

Date :

* Basic Rules of AP

→ $a_n = a + (n-1)d$

→ $S_n = \frac{n}{2}[2a + (n-1)d]$

(or)

→ $S_n = \frac{n}{2}[a + l]$

→ $A_n = S_n - S_{n-1}$

* Basic Rules of GP

→ $a, ar, ar^2, ar^3 \ldots$

→ n^{th} term, $T_n = ar^{n-1}$ $\quad r = \frac{T_n}{T_{n-1}}$

→ Sum of 1^{st} n terms $(r<1)$ = $\frac{a(1-r^n)}{1-r}$

→ ——— " ——— $(r>1)$ = $\frac{a(r^n-1)}{r-1}$

→ — " — infinite terms $(r<1)$ = $\frac{a}{1-r}$

* Basic Rules of HP

* $HP = \frac{1}{AP}$

* HP of 2 nos. = $\frac{2ab}{a+b}$

* n^{th} terms of GP and AP → $\frac{GP^2}{AP}$

Date :

01. Find Sum of Sequence 10, 15, 20, 25, 30, . . . , 100

Sol:

$a_n = a + (n-1)d$

$100 = 10 + (n-1)5$

$\boxed{n = 19}$

$S_n = \frac{n}{2}[a+l]$

$= \frac{19}{2}[10+100]$

$\boxed{S_n = 1045}$

02. A ball is dropped from a height of 128 m. It bounces back rising to a height of 64m. Each time it further touches the floor, it rises to the height of half the height it fell from before the previous bounce. Find total distance travelled by the ball.

Sol:

GP $(r<1)$: $S = \frac{a}{1-r}$

total distance = distance covered in first $+ \left(2 \times \frac{a}{1-r}\right)$

$a = 128 \quad r = 0.5$

$= 128 + \left(2 \times \frac{128}{1-0.5}\right) = 384$

Assessment

① A Number exceeds by 25 from its 3/8 th part. Then the Number is?

Sol:

$$x = \frac{3}{8}x + 25$$

$$x - \frac{3}{8}x = 25 \qquad \times 8$$

$$\frac{5}{8}x = 25$$

$$5x = 200$$

$$\boxed{x = 40}$$

② The Six digit number 54321A is divisible by 9 where A is a Single digit whole number. find A

Sol: $A = 3$

③ find the greatest 6-digit number, which is a multiple of 12

Sol: $999999 \rightarrow \frac{999999}{12} = 999996$

④ Simplify the Expression using BODMAS Rule

$$(105 + 206) - 550 \div 85^2 + 10$$

$$\Rightarrow 311 - 550 \div 25 + 10$$

$$311 - 32 + 10 \Rightarrow 279$$

$$\begin{array}{r} 22 \\ 25\overline{)550} \\ \hline \end{array}$$

$$\begin{array}{r} 311 \\ 32 \\ \hline 9 \end{array}$$

⑤ What is the difference b/w the greatest 5 digit & 4 digit no. & the Smallest 5 digit no.?

Sol: $99999 - 10000 \Rightarrow 89999$

⑥ what is the Unit digit in the product $(365 \times 659 \times 771)$?

Sol: Unit digit $\rightarrow \boxed{5}$

⑦ There are 20 people in the party. If every person shakes hand with every other person, what is the total no. of hand shakes?

Sol: $= \frac{n(n-1)}{2} = \frac{20 \times 19}{2} = 190$

⑧ The Unit digit in the sum of (124) to the power of $372 + (124)$ to the power of 373 is?

Sol: $124^{372} + 124^{373} \Rightarrow 124^{745}$

$4^2 \rightarrow$ even $\rightarrow 6$

$4^3 \rightarrow$ odd $= 4$

$\Rightarrow \boxed{6+4=10}$

⑨ Find Unit place digit in $71 \times 72 \times 73 \times 74 \times 76 \times 77 \times 79 \times 78$

Sol: $\Rightarrow 6$

⑩ Find the remainder $\rightarrow 19^{77}$ divided by $7 \Rightarrow$

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