<u>Design of IIR filters using Bilinear Transformation:</u>

Steps to design digital filter using bilinear transform technique:

1. From the given specifications, find prewarping analog frequencies using formula
   $$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$
2. Using the analog frequencies find *H(s)* of the analog filter.
3. Select the sampling rate of the digital filter, call it T seconds per sample.
4. Substitute $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ into the transfer function found in step2.

Apply bilinear transformation of $H(s) = \frac{2}{(s+1)(s+2)}$ with *T=1 sec* and find *H(z)*.[Nov/Dec-13]

Solution:

Given: The system function $H(s) = \frac{2}{(s+1)(s+2)}$

$$\text{Substitute } s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \text{ in } H(s) \text{ to get } H(z)$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{2}{(s+1)(s+2)}\Bigg|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Given *T=1 sec.*

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2\left(1+z^{-1}\right)^2}{\left(3-z^{-1}\right)(4)}$$

$$= \frac{\left(1+z^{-1}\right)^2}{6-2z^{-1}}$$

$$H(z) = \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

Using the bilinear transformation, design a high pass filter, monotonic in pass band with cut off frequency of 1000Hz and down 10dB at 350 Hz. The sampling frequency is 5000Hz. [May/June-16]

Solution:

Given: Pass band attenuation $\alpha_P = 3dB$; Stop band attenuation $\alpha_S = 10dB$

Pass band frequency $\omega_P = 2\pi * 1000 = 2000\pi$ *rad/sec.*

Stop band frequency $\omega_S = 2\pi * 350 = 700\pi$ *rad/sec.*

$$T = \frac{1}{f} = \frac{1}{5000} = 2*10^{-4}\ \text{sec}.$$

Prewarping the digital frequencies, we have

$$\Omega_P = \frac{2}{T}\tan\frac{\omega_P T}{2} = \frac{2}{2*10^{-4}}\tan\frac{\left(2000\pi*2*10^{-4}\right)}{2} = 10^4\ \tan(0.2\pi) = 7265\, rad/sec.$$

$$\Omega_S = \frac{2}{T}\tan\frac{\omega_S T}{2} = \frac{2}{2*10^{-4}}\tan\frac{\left(700\pi*2*10^{-4}\right)}{2} = 10^4\ \tan(0.07\pi) = 2235\, rad/sec.$$

The order of the filter

$$N \geq \frac{\log\sqrt{\dfrac{10^{0.1\alpha_S}-1}{10^{0.1\alpha_P}-1}}}{\log\dfrac{\Omega_S}{\Omega_P}}$$

$$= \frac{\log\sqrt{\dfrac{10^{0.1\times10}-1}{10^{0.1\times3}-1}}}{\log\dfrac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

$$N = 1$$

The first order Butterworth filter for Ω_C*=1 rad/sec is H(s) = 1/S+1*

The high pass filter for $\Omega_C = \Omega_P$*=7265 rad/sec can be obtained by using the transformation.*

$$S \rightarrow \frac{\Omega_C}{s}$$

$$S \rightarrow \frac{7265}{s}$$

The transfer function of high pass filter

$$H(s) = \left.\frac{1}{s+1}\right|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

$U \sin g\ bilinear\, transformation$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Bigg|_{s=\frac{2}{2*10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$= \frac{0.5792\left(1-z^{-1}\right)}{1-0.1584z^{-1}}$$