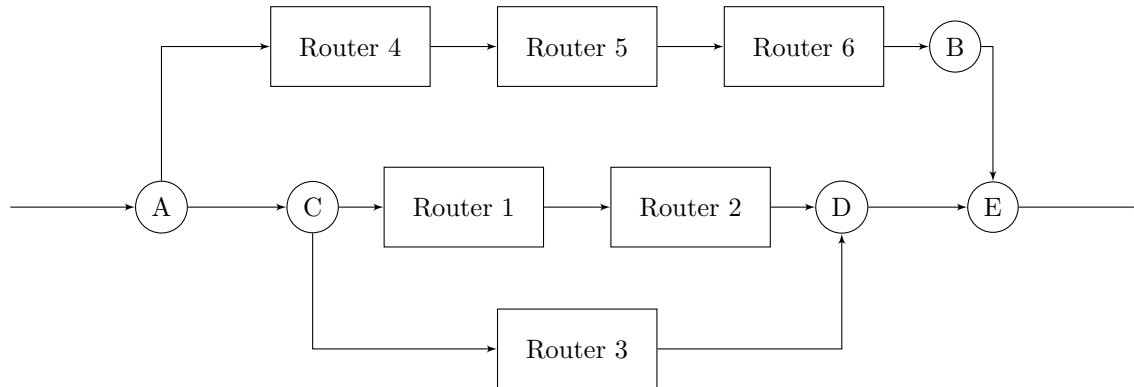


Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet.

1. Consider the system below with all routers acting independently and each having probability 0.80 of successfully transmitting a signal.



- Calculate the probability of a signal passing from A to B.
  - Calculate the probability of a signal passing from C to D.
  - Calculate the system reliability.
  - If you could replace one router with a perfect router that has 100% success which router would it be and why?
2. A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following information:
- 85% of the cabs in the city are Green and the rest are Blue.
  - An eye witness identified the cab as Blue.
  - The court tested the reliability of the witness under the same conditions that existed on the night of the accident and concluded that the witness correctly identified Green cars 90% of the time and correctly identified Blue cars 80% of the time.

What is the probability that the cab involved in the accident was Blue?

3. Let  $X$  be a random variable with image  $Im(X) = \{0, 1, 2, 3\}$ .

- (a) Fill in the blank in the table below to make it a valid probability mass function:

$x$	0	1	2	3
$p_X(x)$	0.5	0.25	0.1	

- Derive the cumulative distribution function for  $X$  and draw it in a table.
- Determine the probabilities that...
  - $X$  is at least 2.
  - $X$  is neither 0 nor 2.
  - $X$  is non-negative.
- Find the expected value and variance of  $X$ .
- Let  $Y$  be a random variable with  $Y = 5 - 2X$  where  $X$  has the *pmf* from above;
  - Determine the image of  $Y$ .
  - Using the rules for computing expected values and variances of a linear function of a random variable, find the expected value and variance of  $Y$ .

- (f) Suppose we have a collection of independent and identically distributed  $X_i$  according to a) and  $Z = \sum_{i=1}^n X_i$ . (Hint: the answer below will depend on  $n$ .)
- What is  $E[Z]$ ?
  - What is  $Var[Z]$ ?
4. On average, 8% of circuit boards are defective.
- What is the probability of finding exactly 2 defective boards in a batch of 10 randomly chosen circuit boards?
  - What is the probability of finding at most 2 defective boards in a batch of 10 randomly chosen circuit boards?
  - What is the probability of finding more than 1 but less than 4 defective boards in a batch of 10 randomly chosen circuit boards?
5. On average, a particular web page is accessed 10 times an hour. Let  $X$  be the number of times this web page will be accessed in the next hour.
- Of the discrete distributions you have learned, which one is the most suitable for modeling  $X$ ? (Hint: it means *fish* in French.)
  - What is  $E[X]$  and  $Var[X]$ ?
  - What is the probability there is at least one access in the next hour?
  - What is the probability there are between 8 and 12 (inclusive) accesses in the next hour?