Patrick Ørsted Povey Andersen

Mandatory Exercise 1

Numeriske metoder

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Exercise 1

Problem I

Find the Singular Value Decomposition $A = UWV^T$. State the diagonal elements in W. Code main.cpp in zip.file.

```
Problem I
Diagonal elements of W:
        4752.37         806.789         58.1826         16.9671          3.5654          3.02446
```

Problem II

Use the Singular Value Decomposition to compute the solution x to Ax = b. State the solution x. Code main.cpp in zip.file.

```
Problem II
Best fit solution parameters x:
       -24.3946         72.8921          1.7861        -63.2459        -76.9598        -17.3859
```

Problem III

State an estimate of the accuracy on the solution x. State an explanation of how you computed the accuracy. Code main.cpp in zip.file.

```
Problem III
Relative residual error epsilon_residual = 0.000162328
Expected error for a random model = 0.921954
Accuracy: Standard deviations of x:
       0.322198         0.27585       0.0960741          0.0509      0.00522238      0.00210278
```

The relative residual error measures how well the solution fits the data, while the standard deviations indicate the precision of each component of x.
The expected error for a random model provides a benchmark for comparison.

Problem IV

Compute and state the residual vector $r \equiv Ax - b$. Code main.cpp in zip.file.

```
Problem IV
Residual vector:
      -0.55671        0.527233       -0.673201        -1.26463       -0.831363        0.464874         7.6186         1.13496       -0.401232       -0.325707       -0.217701        0.230878       -0.218717        0.656575
    -0.0388285       -0.535747        0.752515        0.561871        0.324391       -0.743685       -0.744122         1.0811       -0.605121       -0.362054        0.341041       -0.89615       -0.411017       -0.0485407
       1.13667         1.08557       -0.627664       -0.400131         1.06093        0.132202         0.55506        0.408264         0.87311       -0.680265        0.400403       -0.763662
```

Problem V

Compute the new σ_i's and then the new design matrix A and new right hand side b. State $[A]_{0,0}$ and $[b]_6$. Code main.cpp in zip.file.

```
Problem V
New A[0][0] = 0.116867
New b[6] = 284.403
```

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Problem VI

Compute and use the Singular Value Decomposition to compute the solution x to Ax = b with the new design matrix and new right-hand side. State the solution x.

```
Problem VI
New solution x:
       -25.3757        72.8928        2.01114        -63.252       -76.9682       -17.3859
```

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Full Terminal Solution

```
Matrix A:
           1       0.508087       0.258152       0.131164      0.0666426      0.0338602
           1       -1.55734        2.42532       -3.77706        5.88218       -9.16058
           1       -4.70875        22.1723       -104.404        491.611       -2314.87
           1       -4.15205        17.2395       -71.5792          297.2       -1233.99
           1       -3.64325        13.2733       -48.3577        176.179       -641.864
           1       0.479769       0.230178       0.110433      0.0529821      0.0254192
           1         -4.0541        16.4357       -66.6322        270.134       -1095.15
           1      0.0552795     0.00305583     0.000168925   9.33809e-06    5.16205e-07
           1       -4.33758        18.8146       -81.6099          353.9       -1535.46
           1         3.5155        12.3587        43.4471        152.738        536.952
           1        2.22796        4.96381        11.0592        24.6394        54.8956
           1        4.33036         18.752         81.203        351.638         1522.72
           1        3.85598        14.8686        57.3332        221.076        852.465
           1        2.26466        5.12866        11.6147        26.3032        59.5676
           1        2.03277        4.13214        8.39968        17.0746        34.7087
           1       -4.18942        17.5512       -73.5294        308.045       -1290.53
           1        1.67444        2.80375        4.69471        7.86101        13.1628
           1        1.96257        3.85169        7.55923        14.8355        29.1158
           1       -1.70694        2.91364        -4.9734        8.48929       -14.4907
           1       -2.83137        8.01665       -22.6981        64.2667       -181.963
           1       -3.42315         11.718       -40.1125        137.311       -470.038
           1       0.124641      0.0155353     0.00193632    0.000241344    3.00813e-05
           1       -2.85867        8.17197       -23.3609        66.7811       -190.905
           1        3.03795        9.22912        28.0376        85.1767        258.762
           1        1.06876        1.14224        1.22078        1.30472        1.39443
           1       -3.31802        11.0092       -36.5288        121.203       -402.153
           1       -3.14992        9.92199       -31.2535        98.4459       -310.097
           1        2.18999        4.79608        10.5034        23.0023          50.375
           1      -0.287994      0.0829406     -0.0238864     0.00687915     -0.00198115
           1        1.20222        1.44532         1.7375         2.08896         2.51138
           1       -4.09582        16.7757       -68.7103        281.425       -1152.66
           1       -2.86528        8.20986       -23.5236        67.4018       -193.125
           1      -0.950413       0.903285      -0.858494       0.815924      -0.775465
           1        2.68672        7.21844        19.3939        52.1059         139.994
           1       -1.32378        1.75239       -2.31977         3.0707       -4.06514
           1       -2.19564        4.82085       -10.5849        23.2406       -51.0282
           1       0.193602      0.0374817     0.00725653     0.00140488     0.000271987
           1       -4.57794        20.9575       -95.9423        439.218       -2010.71
           1        1.64346        2.70094        4.43888         7.2951        11.9892
           1       -3.00623         9.0374       -27.1685        81.6745       -245.532

Vector b:
         7.6456       -188.651        8687.56        2813.63        393.675      -0.980857        2166.75       -21.5061        4307.27       -23583.7       -3403.01         -58347       -35177.5       -3645.32
      -2317.55        3082.46       -1028.84       -2001.06       -230.79       -562.523       -110.716       -16.5042       -560.494       -12613.4       -146.656       -271.362       -444.284       -3166.51
      -45.3599       -249.59        2434.99       -559.99       -88.1348       -7486.35       -137.254       -408.212       -11.6606        6903.88       -950.789       -525.187

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