Multi dimensional root finding

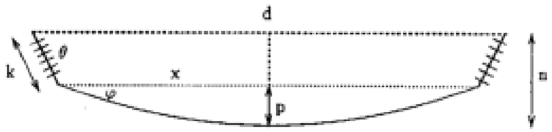

Figure 1: High voltage cables. Geometry and notation.

This exercise concerns high voltage cables suspended between two pylons as indicated in Figure 1. The cable is attached to insulators which in turn are attached to the pylons. The insulators are assumed to be attached in the same horizontal plane.

Material constants:

v	Insulators weight	$v = 120kg$
k	Length of the insulators	$k = 2.5m$
w	Weight of the cable/meter(resting)	$w = 4.0kg/m$
α	Elasticity coefficient of the cable	$\alpha = 2 \cdot 10^{-7}kg^{-1}$

Variables(see figure):

d	Distance between the pylons
L_0	Resting length of the cable
L	Suspended length of the cable
n	"Sagging" of the cable from the insulators attachment points
p	"Sagging" of the cable from the cables attachment points
x	Half the distance between the cables attachment points
θ	Angle between insulator and the horizontal plane
φ	Angle between cable and the horizontal plane, at the cables attachment points
a	Parameter in the catenary equation for the cable
H	String tension in the cable

Equations:

1) $p = a(\cosh \frac{x}{a} - 1)$
2) $L = 2a \sinh \frac{x}{a}$
3) $d = 2x + 2k \cos\theta$
4) $n = p + k \sin\theta$
5) $\tan\varphi = \sinh \frac{x}{a}$
6) $\tan\theta = (1 + \frac{v}{wL_0}) \tan\varphi$
7) $L = L_0(1 + \alpha H)$
8) $H = \frac{wL_0}{2 \sin\varphi}$

Task

Your are ordering cables, each with a resting length, L_0, to be between pylons that are $d = 30$ meters apart. Ideally, You would like no "sagging", $n = 0$, but that would cause infinite string tension in the cable, so you need to choose a compromise between sagging and tension. To inform the decision, you:

Determine L_0 and H for $n = 5.0$, $n = 2.0$, $n = 1.0$, $n = 0.5$, $n = 0.2$, and $n = 0.1$.

Help

Above, you are given 8 non-linear equations in 8 unknowns, $\mathbf{q} = (L_0, L, p, x, \theta, \phi, a, H)$, (since n and d are held constant). You should use Newton's method together with an initial guess to find the $\mathbf{q}$ that simultaniously satisfies all 8 equations.

The exercise starting point contains code to estimate the jacobian, $\mathbf{J}(\mathbf{q})$, by finite differences, so you don't need to find an expression for the jacobian. You should implement Newton yourself; iteratively updating the variables, $\mathbf{q} := \mathbf{q} + \mathbf{dq}$ by solving the set of linear equations, $\mathbf{J}(\mathbf{q}) \cdot \mathbf{dq} = -\mathbf{F}(\mathbf{q})$. You can use $\frac{1}{2}\mathbf{F}(\mathbf{q}) \cdot \mathbf{F}(\mathbf{q}) < 10^{-8}$ as the stopping condition. You don't need to implement backtracking.

You need to define a vector function, $\mathbf{F}(\mathbf{q})$, that takes the unknowns, $\mathbf{q}$, as a vector and returns a vector. $\mathbf{F}(\mathbf{q})$ should be the zero-vector, when all the equations are satisfied (You have to manipulate the 8 equations slightly). Make sure you understand that this is a multi dimensional root finding problem.

You need a starting guess for your variables. You can use $a = 40$ and $H = 100$. Estimate a guess for the other variables from Figure 1. If it doesn't converge, try reassesing your starting guess.