Patrick Ørsted Povey Andersen

Mandatory Exercise 3

Numeriske metoder

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Exercise 5

Problem I

For N = 2 (only one midpoint), state by hand computations an analytical expression for the approximation of the integral as obtained by the Extended Midpoint method.

Solution

We need to approximate the integral $\int_a^b \frac{\cos(x^3)\exp(-x)}{\sqrt{x}}\,dx$ using the extended midpoint method with N = 2 (only one midpoint).

First, we find the midpoint of the interval ([a, b]) given by $m = \frac{a+b}{2}$

Then we evaluate the integrand at the midpoint.

The integrand is $f(x) = \frac{\cos(x^3)\exp(-x)}{\sqrt{x}}$ at $x = m = \frac{a+b}{2}$ so we get:

$$f\left(\frac{a+b}{2}\right) = \frac{\cos\left(\left(\frac{a+b}{2}\right)^3\right)\exp\left(-\frac{a+b}{2}\right)}{\sqrt{\frac{a+b}{2}}}$$

Now er then apply the midpoint rule for the integral, which is: $(b-a) * f(\frac{a+b}{2})$.

Using this we get the approximation:

$$(b-a) * \frac{\cos\left(\left(\frac{a+b}{2}\right)^3\right)\exp\left(-\frac{a+b}{2}\right)}{\sqrt{\frac{a+b}{2}}}$$

Furthermore, we know that the error estimate is given by $O\left(\frac{1}{N^2}\right)$ and N = 2 so we get the final approximation:

$$(b-a) * \frac{\cos\left(\left(\frac{a+b}{2}\right)^3\right)\exp\left(-\frac{a+b}{2}\right)}{\sqrt{\frac{a+b}{2}}} + O\left(\frac{1}{4}\right)$$

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Problem II

With N = 2, 3, 5, 9, ... (corresponding to N - 1 = 1, 2, 4, 8,...) use the Extended Midpoint method to approximate the integral. Terminate the subdivisions when you reach a proven accuracy of better than 10-3. State your results in a table like those used during the course and state how you computed the accuracy. State also how many f-computations (Computations of the integrand for a given x) were needed. Submit the used code.

Solution

i	A(h_i)	A(h_(i-1)) - A(h_i)	alp^k	Rich-error	f-calculations	order-estimate
1	0.803856					
2	0.761251	0.042605				
3	0.981042	-0.219791	-0.193842	0.073264	4	0.000000
4	1.135919	-0.154877	1.419135	0.051626	8	0.505012
5	1.218004	-0.082086	1.886767	0.027362	16	0.915916
6	1.279150	-0.061146	1.342450	0.020382	32	0.424868
7	1.323213	-0.044063	1.387713	0.014688	64	0.472710
8	1.354464	-0.031251	1.409939	0.010417	128	0.495633
9	1.376589	-0.022124	1.412546	0.007375	256	0.498298
10	1.392241	-0.015652	1.413469	0.005217	512	0.499240
11	1.403312	-0.011071	1.413866	0.003690	1024	0.499645
12	1.411141	-0.007829	1.414048	0.002610	2048	0.499831
13	1.416677	-0.005536	1.414133	0.001845	4096	0.499918
14	1.420592	-0.003915	1.414174	0.001305	8192	0.499960
15	1.423360	-0.002768	1.414194	0.000923	16384	0.499980

Converged to desired accuracy.

Problem III

Use DErule to approximate the integral. State your result and how many f-computations were applied. Submit the used code.

Solution

Approximations	f-calculations
1.022004	3
1.433245	12
1.429545	48
1.430043	192
1.430043	768

convereged towards final result

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Full Terminal Solution

```
    i         A(h_i)      A(h_(i-1)) - A(h_i)     alp^k       Rich-error      f-calculations       order-estimate

    1        0.803856
    2        0.761251           0.042605
    3        0.981042          -0.219791       -0.193842       0.073264               4              0.000000
    4        1.135919          -0.154877        1.419135       0.051626               8              0.505012
    5        1.218004          -0.082086        1.886767       0.027362              16              0.915916
    6        1.279150          -0.061146        1.342450       0.020382              32              0.424868
    7        1.323213          -0.044063        1.387713       0.014688              64              0.472710
    8        1.354464          -0.031251        1.409939       0.010417             128              0.495633
    9        1.376589          -0.022124        1.412546       0.007375             256              0.498298
   10        1.392241          -0.015652        1.413469       0.005217             512              0.499240
   11        1.403312          -0.011071        1.413866       0.003690            1024              0.499645
   12        1.411141          -0.007829        1.414048       0.002610            2048              0.499831
   13        1.416677          -0.005536        1.414133       0.001845            4096              0.499918
   14        1.420592          -0.003915        1.414174       0.001305            8192              0.499960
   15        1.423360          -0.002768        1.414194       0.000923           16384              0.499980

Converged to desired accuracy.

Approximations      f-calculations

   1.022004                3
   1.433245               12
   1.429545               48
   1.430043              192
   1.430043              768

convereged towards final result
```