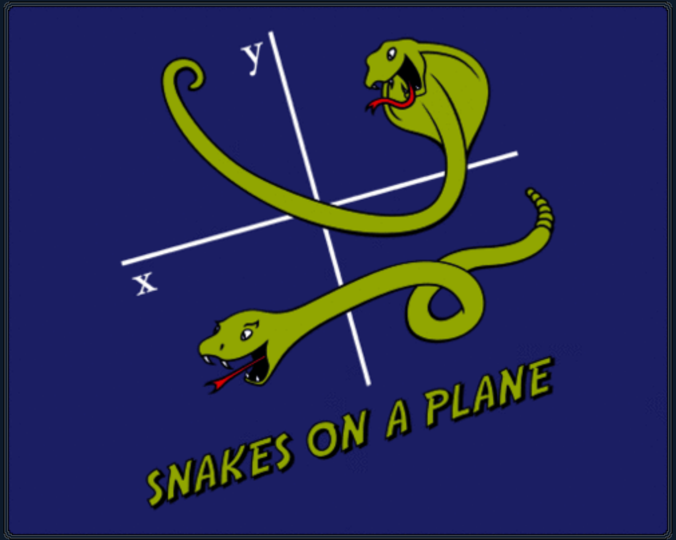
Vectors

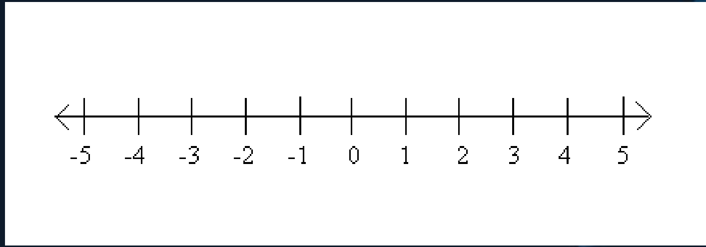


Topics

- 2D and 3D space
- 2D and 3D vectors
- Vector operations
  - addition, subtraction, scalar multiplication
- Magnitude
- Normalising vectors

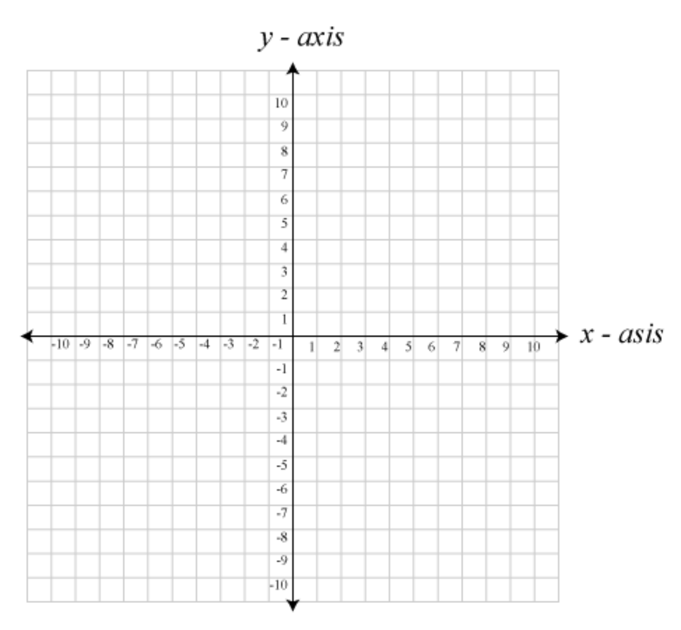
The Number Line

- Standard method of visualising numbers
- Single axis (often labelled x)
- Not particularly useful since we'll be working in 2 dimensions, hence…



The Cartesian Plane

- Also known as the Number Plane

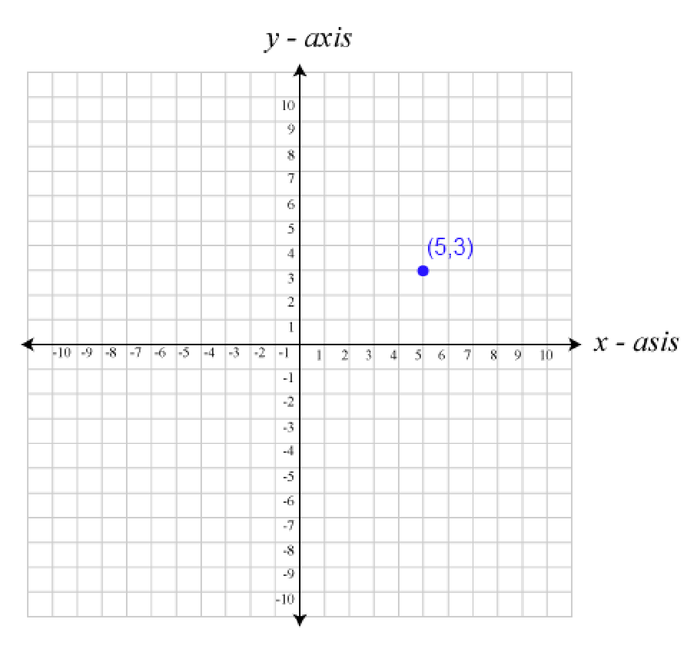


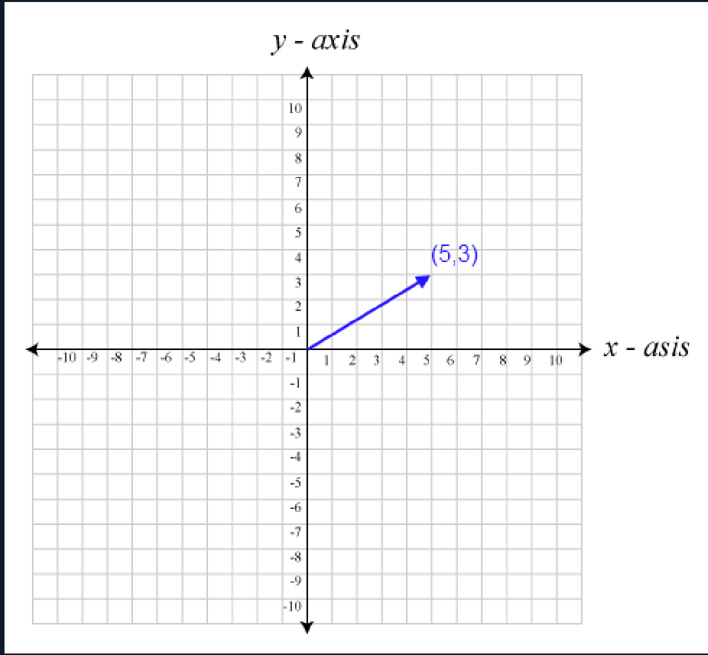
- Comprised of two axes at right-angles to one another (x and y)
- Easy way to visualise positions and lines in 2D space.
- Multi-dimensional values are known as vectors

Vectors

- A vector is a geometric object that has a magnitude and direction
- In programming we often use them to represent positions *or* directions in space.
- 2D vectors are made up of 2 elements – an x value and a y value
  - e.g. (5,3)

Position vs Direction





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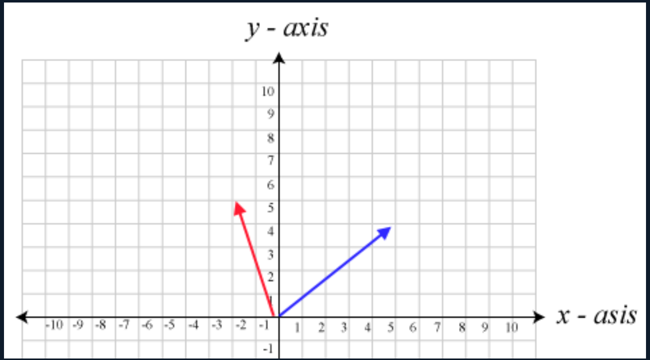
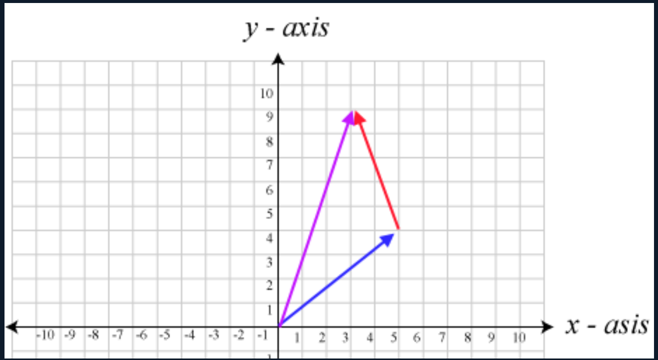
Addition of Vectors

- Element by element addition
- (x1, y1) + (x2,y2) = (x1+x2, y1+y2)

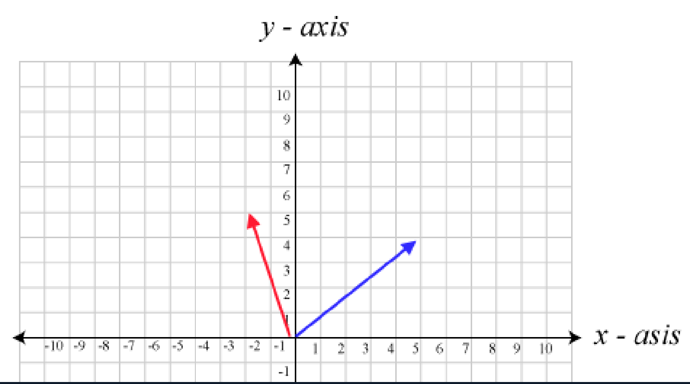


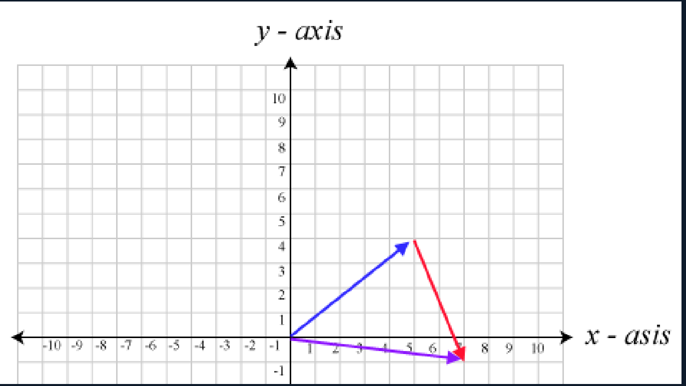


- (5,4) + (-2,5) = (3, 9)

Subtraction

- Similar to addition
- (x1, y1) – (x2, y2) = (x1 – x2, y1 – y2)





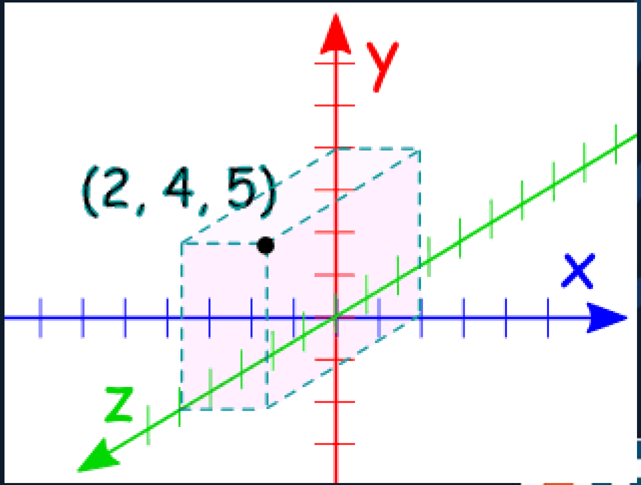
- (5, 4) – (-2, 5) = (7, -1)

Scalar Multiplication

- Not much use for element by element multiplication
- A scalar is just another name for a single number
- So scalar multiplication means multiplying a vector by a single number
- n * (x, y) = (n*x, n*y)
  - e.g. 3 * (4, 5) = (12, 15)

The 3rd dimension

- In 2D Cartesian space we just have the X and Y axis to deal with.
- In 3D Cartesian space, we also have the Z axis.
- The Z axis is perpendicular (at right angles to) the X and Y axes.



3D Vectors

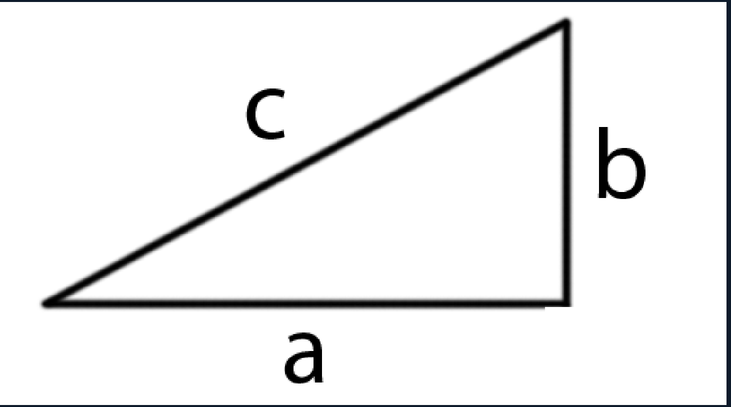
- 3D vectors are just like 2D vectors, except that they have a 3rd component: Z
  - (x, y, z)
- They can represent positions or directions in 3D space
- Addition, Subtraction and scalar multiplication are all the same as the 2D methods.

3D Vector Arithmetic

- Given the vectors $A(-7, 9,2)$ and $B(3, 14, -3)$ perform the following operations:
  - $A + B$
  - $A - B$
  - $3A \; - 2B$

Pythagoras' Theorem

- Pythagoras' theorem allows you to calculate a side of a right angle triangle given the two other sides.



$$a^2 + b^2 = c^2$$

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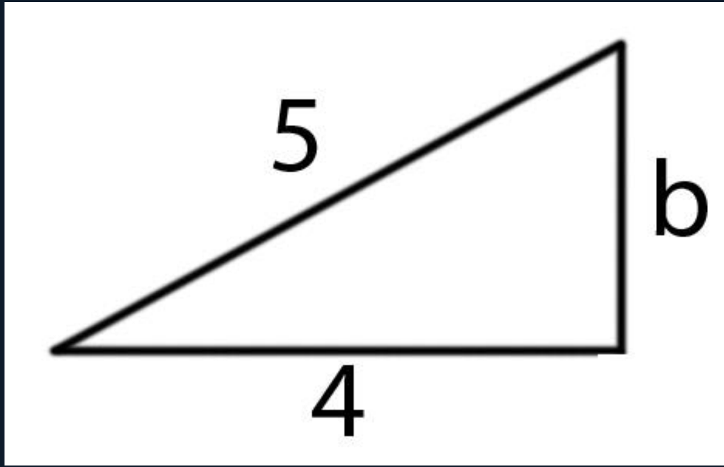
Solving a Problem with Pythagoras

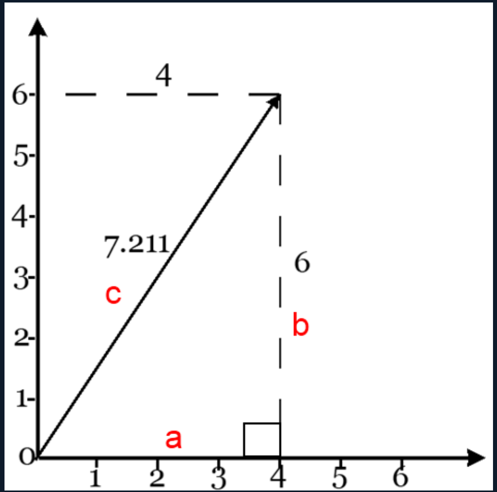
- Let’s find the value of **b**

- $b^2 + 4^2 = 5^2$
- $b^2 + 4^2 - 4^2 = 5^2 - 4^2$
- $b^2 = 5^2 - 4^2$
- $b^2 = 25 - 16$
- $\sqrt{b^2} = \sqrt{25 - 16}$
- $b = \sqrt{25 - 16}$
- $b = \sqrt{9}$
- $b = 3$



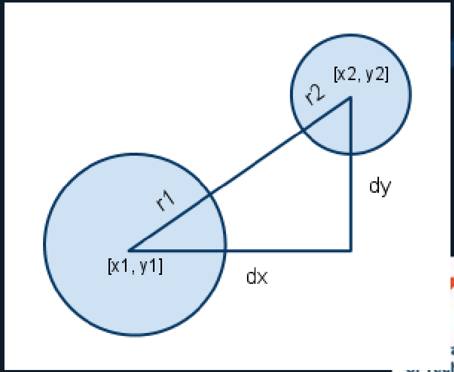
Magnitude

- The magnitude is the length of the vector
- Remember Pythagoras?
- $a^2 + b^2 = c^2$
- Thus the (2D) formula for magnitude is:
  - $m = \sqrt{x^2 + y^2}$
- In 3D:
  - $m = \sqrt{x^2 + y^2 + z^2}$



A Magnitude of Uses

- But I’ll never have to use all this maths right…?
  - Finding the distance between two objects (how far away something is)
  - Circle collision
  - If you are travelling at (3,4) units per second, what is your speed?
    - Your speed is the magnitude of the velocity vector, which is 5.

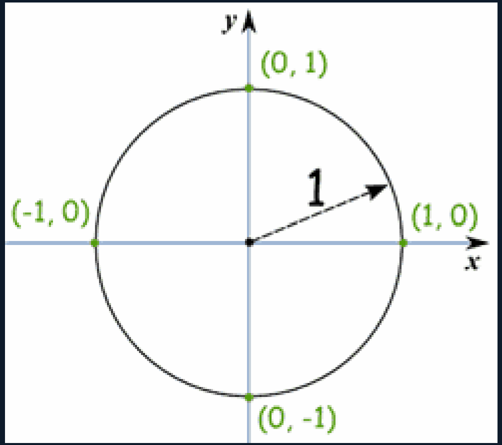


Magnitude - practice

- Calculate the magnitude of the following vectors:
  - (6, 12)
  - (-3, 1)
  - (12, -3, -24)
  - (-3, 4, -2)
  - (-9, -4, 2)

The Unit Circle

- A circle with a radius of 1
- The magnitude of any vector (x, y) is 1
- The vector is said to be of unit length
- These vectors are also called *unit vectors*



Normalise

- Normalising a vector reduces the vector in length, so its magnitude is 1.
  - The proportions of the vector are maintained
  - This is called a unit vector
- The formula is:

$$Nx = \frac{x}{length}$$

$$Ny = \frac{y}{length}$$

$$Nz = \frac{z}{length}$$

Why would I want to have normal vectors?

- To calculate an angle between two vectors
- Represent a direction (without the magnitude)
- Represent base vectors (1,0) and (0,1)
- Surface normals – used in lighting calculations

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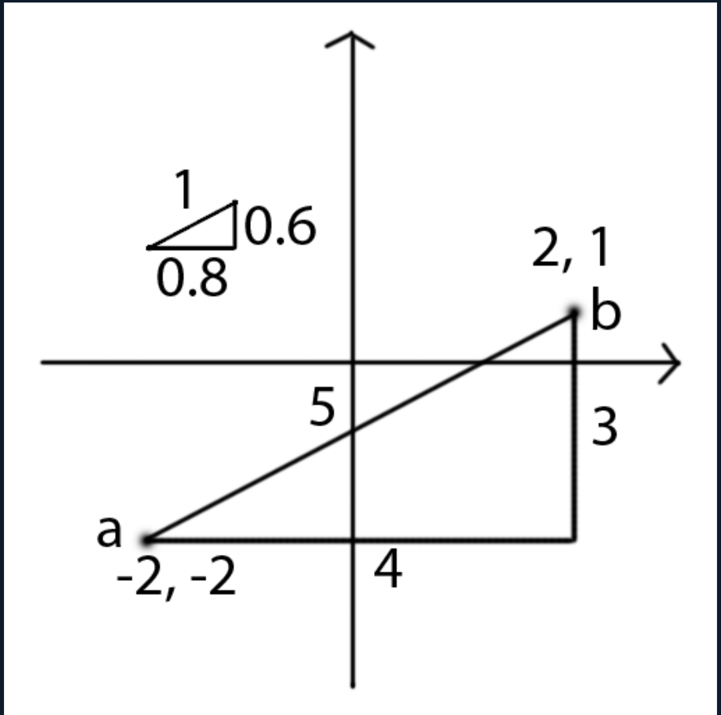
Normalising the vector **(a, b)**



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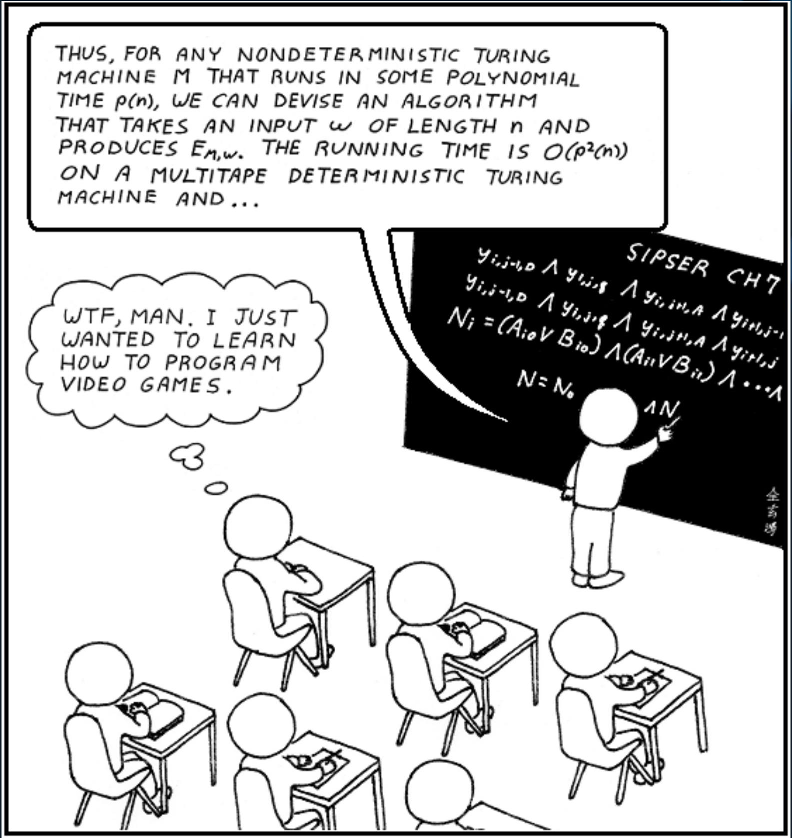
Summary

- Vectors can specify either a position or a direction
- We can add/subtract vectors, or multiply/divide by a scalar
- The magnitude is the length of a vector
- Normalizing a vector will convert it to unit length
- All these operations have applications in computer games



References

- Fletcher Dunn, 2002. *3D Math Primer For Graphics And Game Development (Wordware Game Math Library)*. 1 Edition. Jones & Bartlett Learning.