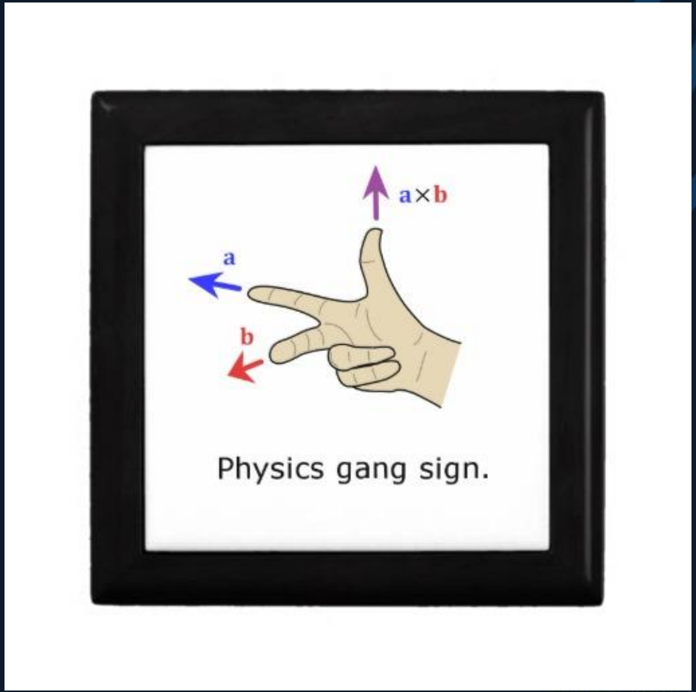
Vectors – Part II

Dot and Cross Product

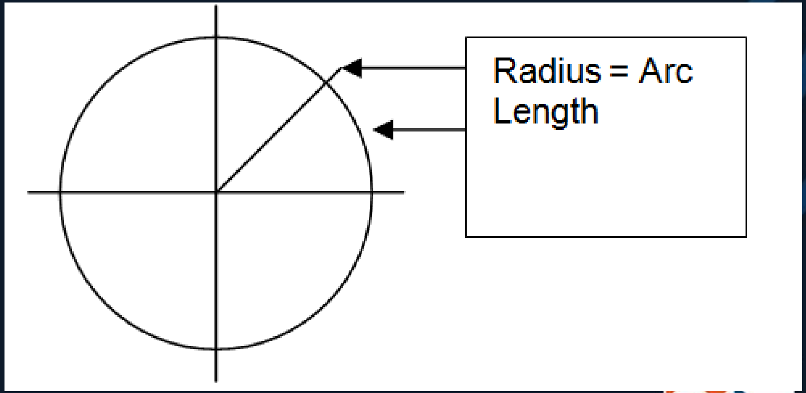


Topics

- Trigonometry Review
- Dot product
- Cross product

Radians

- 2π radians in 360°
- Radians = (Degrees/180)*PI
- Degrees = (Radians/PI)*180

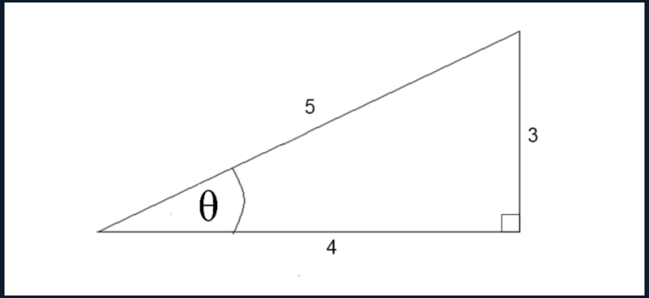


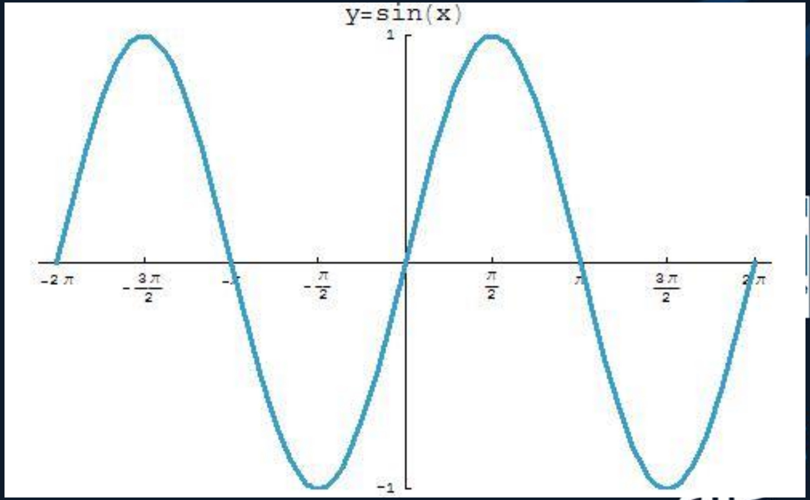
Trigonometry

- There are three base trig functions:
  - Sine
  - Cosine
  - Tangent

Sine

- Sin(θ) = Opposite/hypotenuse
- Eg. Sin(θ) = 3/5 = 0.6
- All values between -1 and 1





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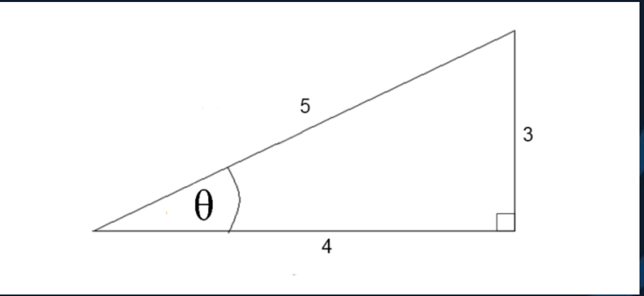
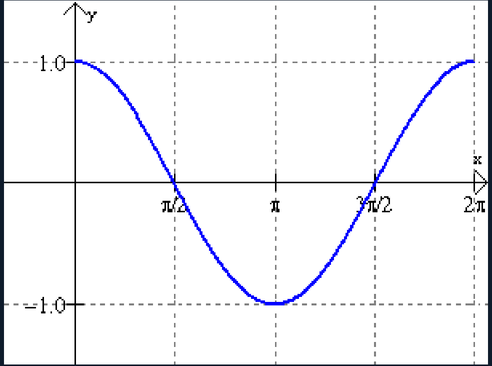
Cosine

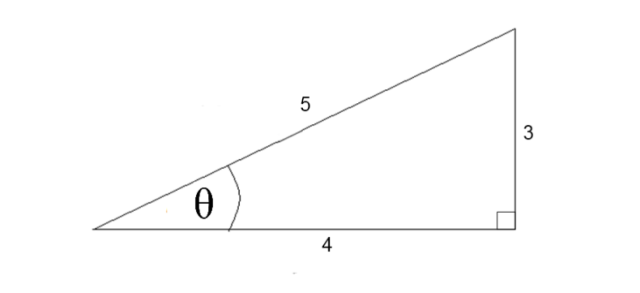
- $\text{Cos}(\theta) = \text{Adjacent}/\text{Hypotenuse}$
- $\text{Cos}(\theta) = 4/5 = 0.8$

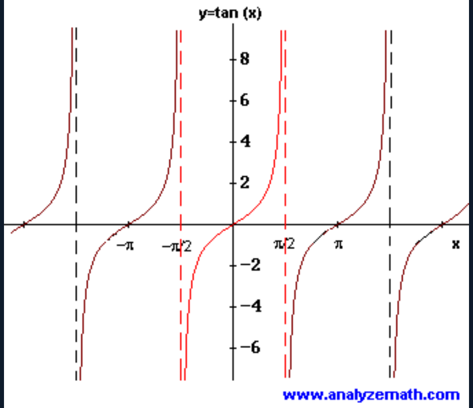




Tangent

- Tan(θ) = Opposite / Adjacent
- Tan(θ) = ¾ = 0.25
- Values approach infinity and are undefined at certain points





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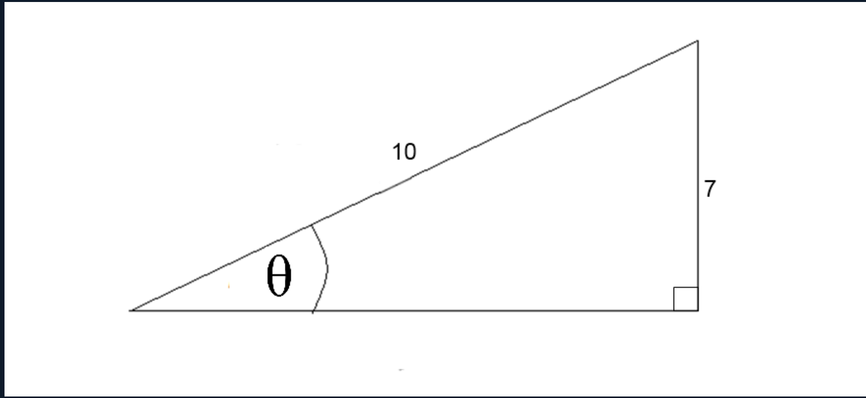
Inverse Trigonometric Functions

- We can use the inverse functions to find the size of an angle
  - arcsine
  - arccos
  - arctan
- For example,

  $$\begin{aligned} \text{Sin}(\theta) &= \text{Opp./Hyp.} \\ \sin(\theta) &= 7/10 \\ \theta &= \arcsin(7/10) \\ &= 0.77 \text{ radians} \\ &= 44 \text{ degrees} \end{aligned}$$



But…

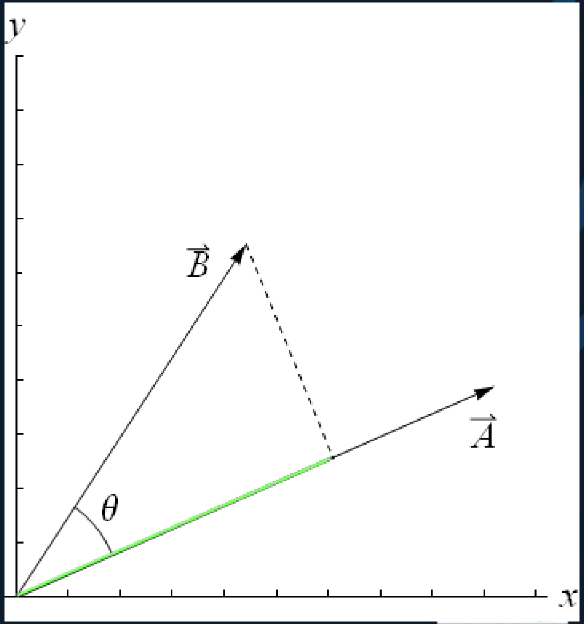
- This only works for right-angle triangles
- What if we want to find the angle between two vectors?

Vector maths to the rescue!

- There is a method to calculate the angle between **any** two vectors
- But first we need to learn about the dot product
- The dot product is a scalar that represents the number of units one vector projects onto another

The Dot Product

- $A \cdot B = AxBx + AyBy + AzBz$
- image one vector is casting a shadow onto the other; how far does the shadow reach?

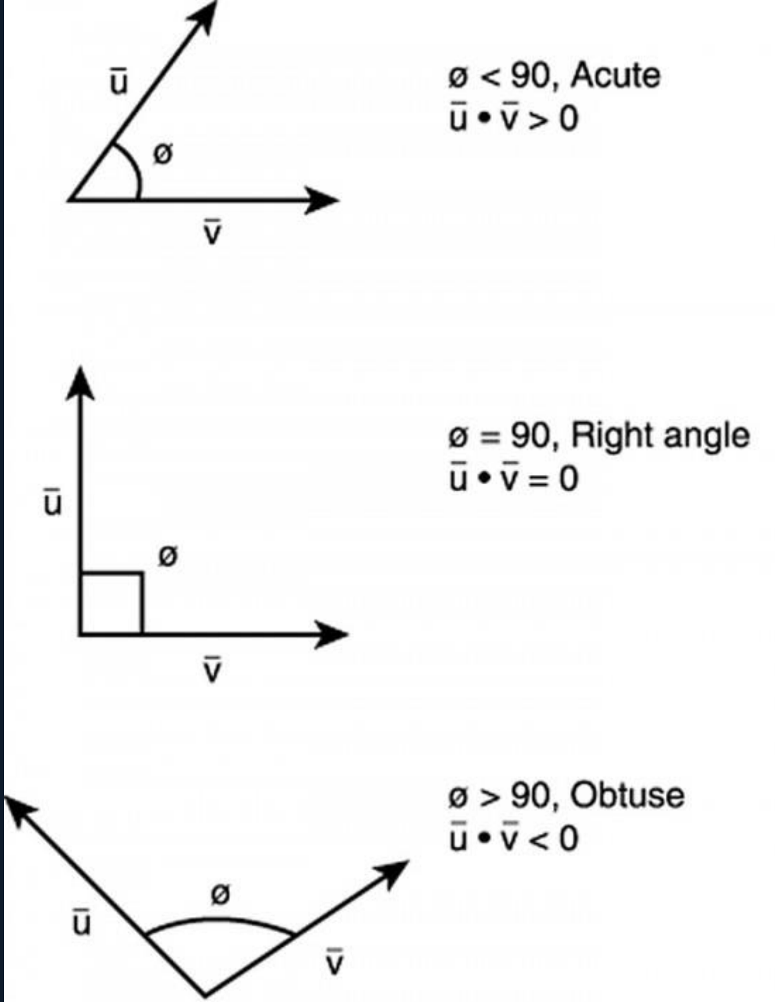


Example

- Find the dot product of A {5, 2, 1}, and B {-10, 4, 1}
  - A.B = Ax*Bx + Ay*By + Az+Bz
    = 5*-10 + 2*4 + 1*1
    = -50 + 8 + 1
    = -41
- Your turn:
  - find the dot product of A {-3, 10, 2} and {2, -4, 12}

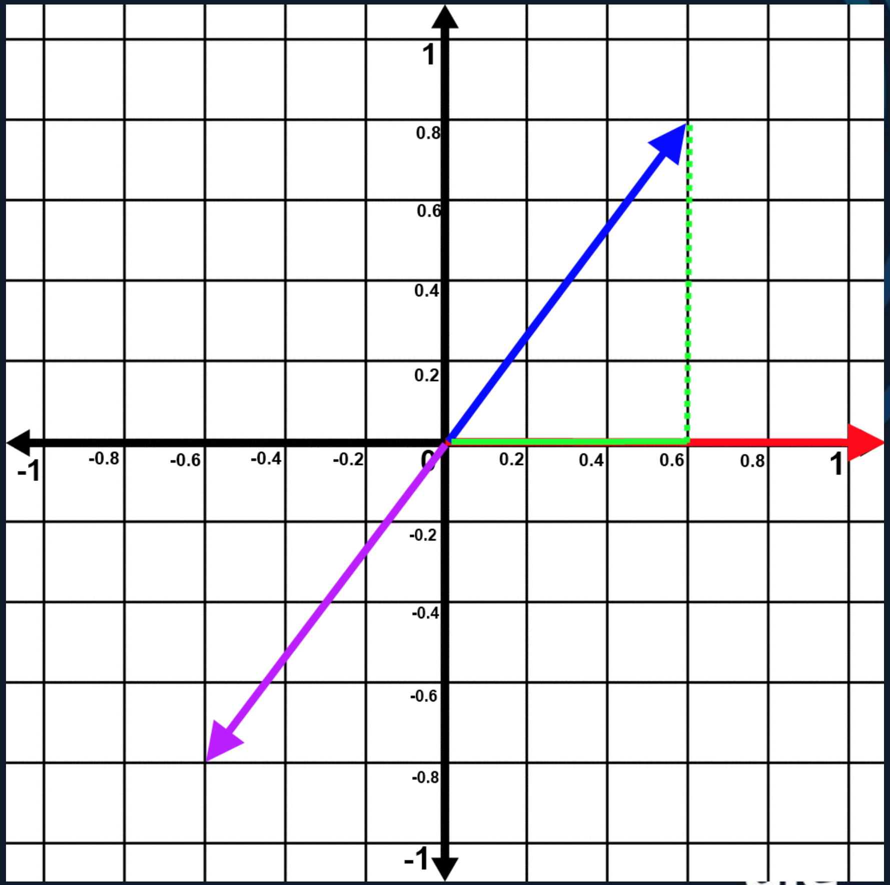
Dot product properties

- If 2 vectors are pointing in the same direction, the dot product will be a positive number
- If 2 vectors are in opposite directions then the dot product will be negative
- If the 2 vectors are perpendicular then the dot product will be zero

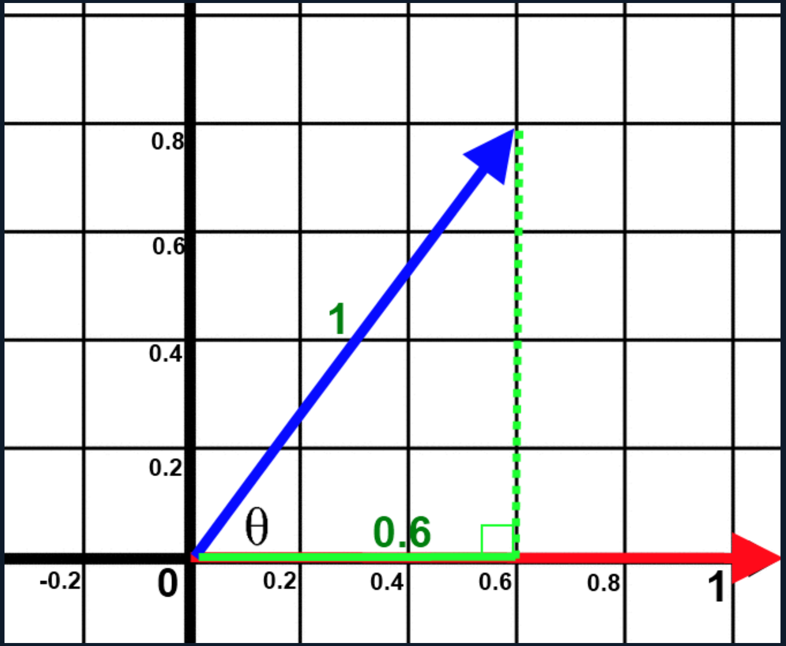


MOAR properties

- If the vectors are both normalised (have a length of 1) then:
  - The result will be between -1 and 1
  - If the vectors are equal the result will be 1
  - If the vectors are completely opposite the result will be -1



Dot product to calculate angles



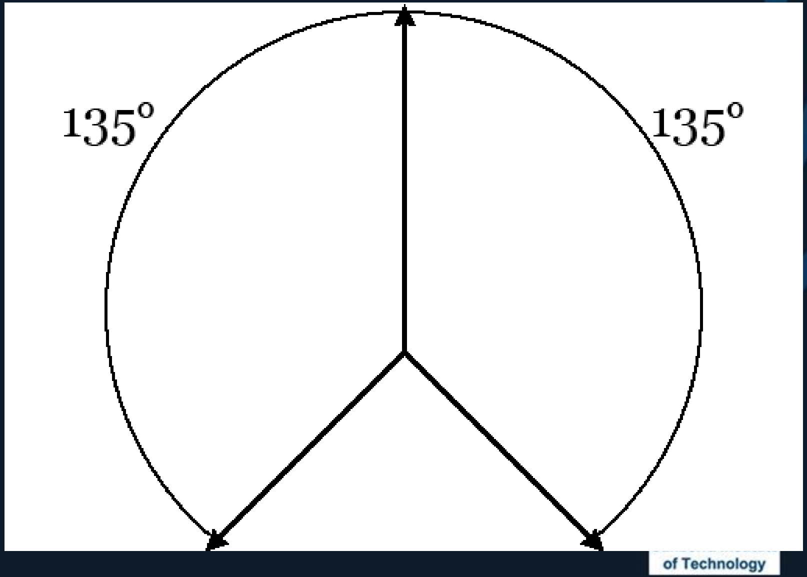
- We can use inverse cos with the dot product in order to work out the angle:

  Angle between A and B = arccos(A.B)

- This only works with normalised vectors!

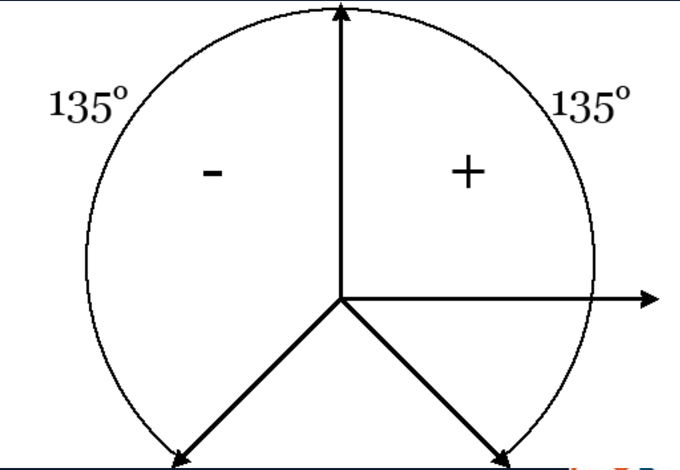
But which way does the angle go?

- There is a limitation
- This method will return an angle of 135 for both these vectors

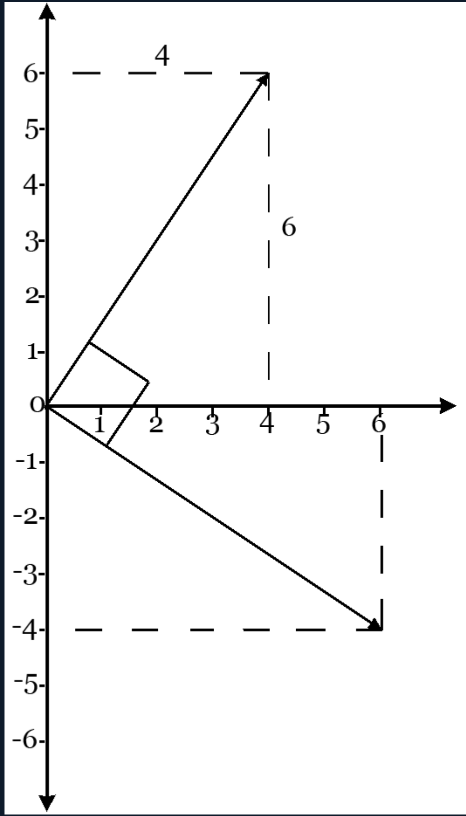


Let's get Perpendicular

- If a second vector is used perpendicular to the first
- We use dot product of the perp and the other vector
- The sign of this dot product simply tells us whether our angle should be positive or negative.



How to get a perpendicular vector?



- Swap the components and negate one of them:
  - Px = y
  - Py = -x
- This only works in 2D!

Cross Product

- To find the perpendicular vector in 3D we need to use a new operation: the cross product
- The cross product takes two input vectors and yields a vector result which is perpendicular to both.

Cross Product Formula

$$\boxed{a \times b = \; \|a\|\|b\| \sin\theta \; n}$$

- The cross product of vectors a and b is the magnitudes of the vectors multiplied by sin of the angle between the vectors multiplied by a unit vector perpendicular to the plane that contains vectors a and b.

Uh, what?

- We can simplify that into a formula more useful for programmers:

$$a \times b = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$

- The order we cross in is important as ***a x b*** will result in a vector that is 180 degrees opposed to ***b x a***

a1 a2 a3

b1 b2 b3

$$a \times b = (a_2 b_3\ )$$

a1 a2 a3

b1 b2 b3

$$a \times b = (a_2 b_3 - a_3 b_2)$$

a1	a2	a3
b1	b2	b3

$$a \times b = (a_2 b_3 - a_3 b_2, a_3 b_1 \;)$$

a1 a2 a3

b1 b2 b3

$$a \times b = (a_2 b_3 - a_3 b_2, a_3 b_1 - b_3 a_1)$$

a1 a2 a3

b1 b2 b3

$$a \times b = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2)$$

a1 a2 a3

b1 b2 b3

$$a \times b = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

Cross Product Uses

- We've already talked about needing to find the perpendicular for determining the sign of an angle
- What else can we use the cross product for?
  - Lighting calculations
  - Collision detection
  - Enemy detection
  - Camera movement
- It is wise to normalise the vector that is returned by the cross-product.

Summary

- The dot product tells us how much one vector projects onto another
- The cross product gives us the vector perpendicular to two vectors
- The dot and cross produces can be used to calculate the angle between two vectors

Further Reading

- http://demonstrations.wolfram.com/VectorProjection/
- http://demonstrations.wolfram.com/DotProduct/
- http://demonstrations.wolfram.com/CrossProductOfVectors/

References

- Fletcher Dunn, 2002. *3D Math Primer For Graphics And Game Development (Wordware Game Math Library)*. 1 Edition. Jones & Bartlett Learning.

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