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The objective of this report is to investigate 3 different pricing models:

1) Binomial Model
2) Black–Scholes model
3) Leisen-Reimer Model

In order to do so we will compute the **price of a call option** using them.
We will use as a set of parameters for our option the following:

Underlying price S=100+5.3=105.3 $
Interest rate r=0.01-0.0003=0.0097=0.97%
Time period T=1 year
Strike price K=100+4.7=104.7 $
Volatility vol=0.2-0.008=0.192=19.2%

(I added a **randomly choosen value**, more or less **smaller by 2 orders of magnitude**, to the ones that were given in the instruction of the report in order to have a **unique** set of parameters)

I can now write a **VBA script** for each of the 3 models that will take as **inputs** the aformentioned **parameters** and the **number of steps n** for the **models 1)** and **3)**

Models descriptions:

1) Binomial Model

The **binomial model** operates in **discrete time**.
The **smallest possible time step** is given by **dt=T/n** and we assume that the **price of our underlying** can go either **up or down** at each **time step** by a factor **u=e^(vol*sqrt(dt))** or **d=1/u**.
The probability **q** of the **price** of our asset to **go up** is given by **q=(e^(r*dt)-d)/(u-d)**.
Thus there are **n+1** possible **price values** at the end given by S***u^j*d^(n-j)** where **j=0,…,n**.
In order to c**ompute the price** of an option call we need to **actualize the expected value** of our **payoff** which in our case is the **positive part of the final price minus the strike price**.
I decided to compute the option prices **recursively from time step n to time step 0**.
At **time step n** the option **price** is the corresponding **payoff**, then in order to compute the prices at **time step i** we calculate **e^(-r*dt)*(q*a+(1-q)*b)** where **a** and **b** are the **prices** of our **option** at **time i+1** if it **respectively** went **up or down**.
Continuing to do so **until n=0** give us the **option price at time n=0**.
As **n goes to infinity**, the **price converges** to the **one** calculated with the **Black-Scholes model**.

2) Black-Scholes model

The **Black-Scholes model** operates in **continuous time**.
Firstly we need to calculate the factors:

d1=(ln(S/K)+T(r+0.5vol^2))/(vol*sqrt(T))
d2=d1-vol*sqrt(T)

Then the **price of a call option** is given by the formula:

price=S*N(d1)-Ke^(-r*T)*N(d2) where **N(x)=Prob(N(0.1)<=x)**

3) Leisen-Reimer Model

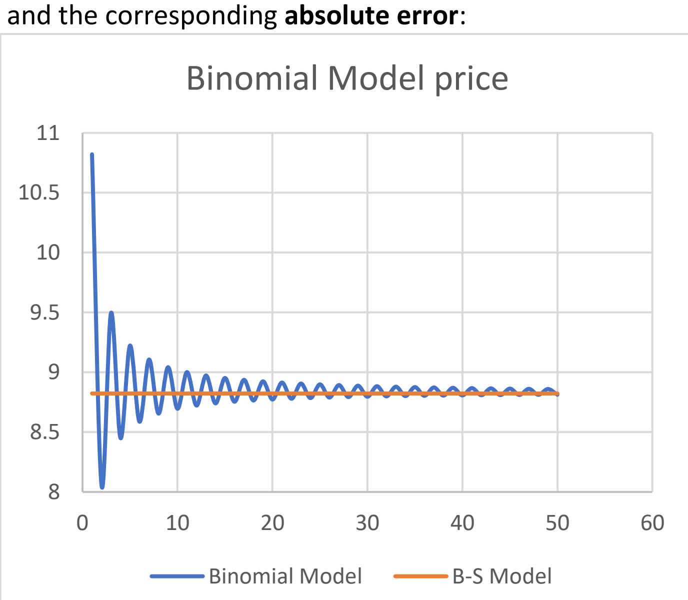
The **Leisen-Reimer model** is **similar** to the **Binomial model**, the only difference between the two is the **estimation** of the parameter **u,d,q**.
More precisely we have that, **given d1** and **d2** computed as in the **Black-Scholes model**:

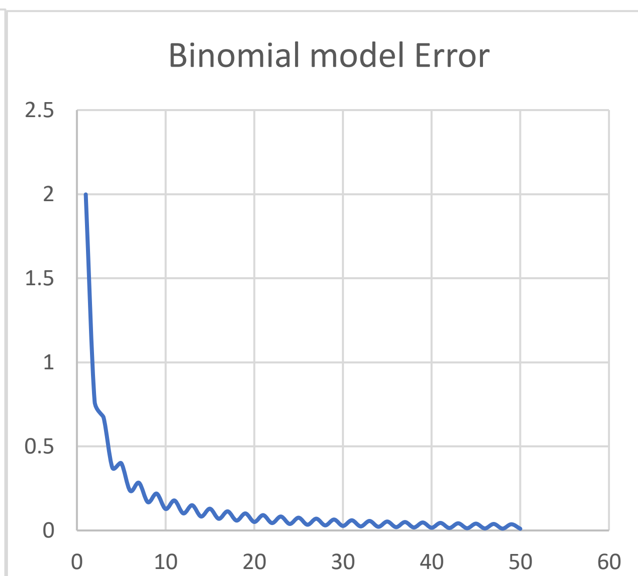
p=h^-1(d2)
p'=h^-1(d1)
where **h^-1(z)=0.5+(sign(z)/2)*sqrt(1-e^(-(z/(n+1/3+(0.1/n+1)))^2*(n+1/6)))**
is the **Peizer-Pratt inversion function**.
q=p
u=e^(r*dt)*(p'/p)
d=e^(r*dt)*((1-p')/(1-p))

This model was **created** in order to have a **faster convergence** as **n goes to infinity** to the **Black-Sholes formula** compared to the **binomial model**.

Conclusion:

I can plot here the **price of the binomial model** compared to the **fixed price of the Black-Scholes formula** and the corresponding **absolute error**:

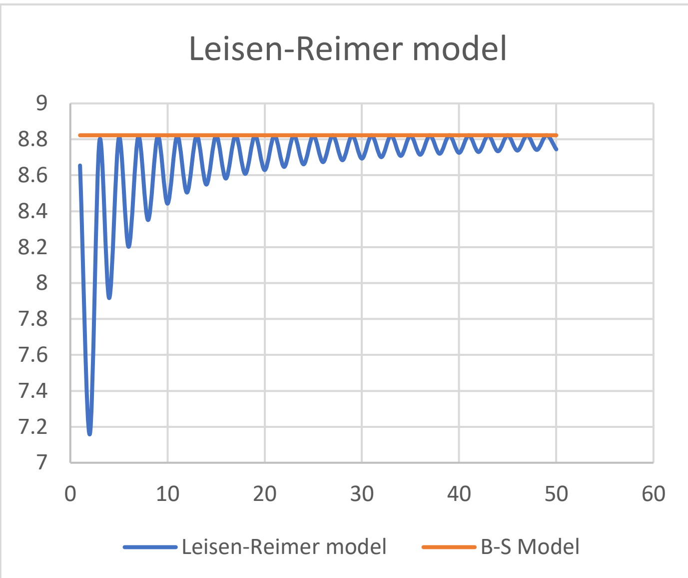


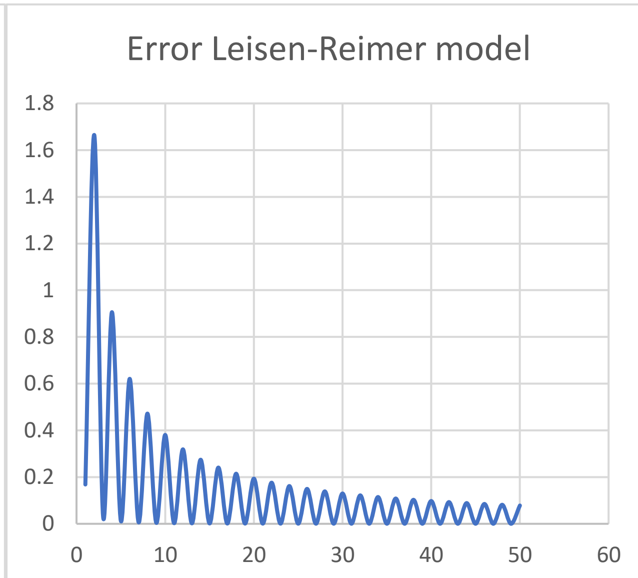


where in the **x-axis** corresponds to **n** in both plots.

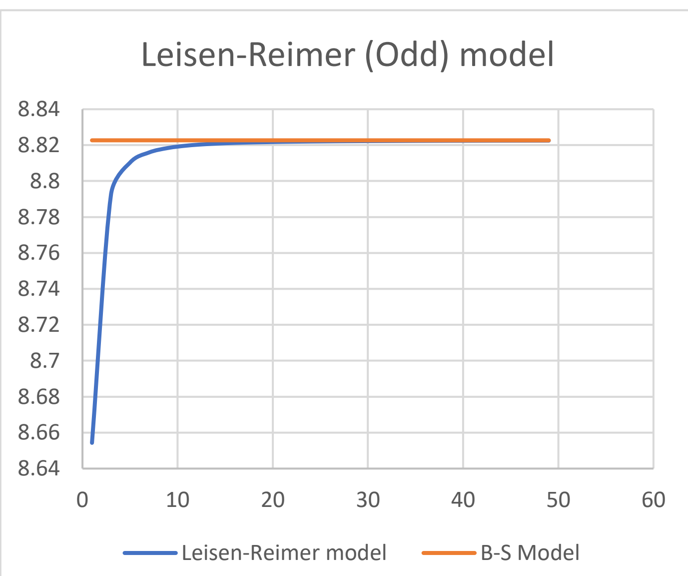
As we can see as **n goes to infinity** the **price of the binomial model** converges to the **one** given by the **Black-Scholes formula**.

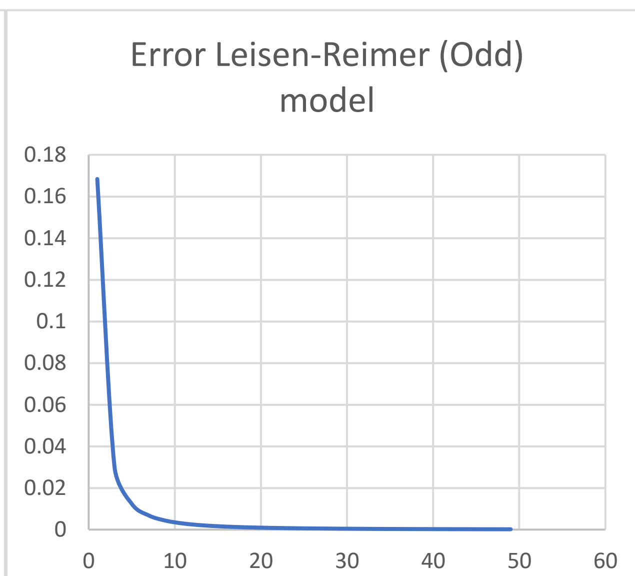
We can now **compare** the **two plot above** with the **corresponding one** made by the **Leisen-Reimer model**:





As we can see the **price** of the **Leiner-Reimer model oscillates** between being **almost equal** to the **Black-Sholes price** and **converging to it**. This is because the **Leiner-Reimer model** works much **better** when **n is an odd number**. By **only plotting them** we get the **following** result:





We can see, by comparing the **y-axis of the error plots**, how the **Leisen-Reimer model** with **odd** numbers **converges much faster** than the **binomial model**. I **reported** some **values** of the **error** in the table below:

n	**1**	**9**	**19**	**29**	**39**	**49**
Error Binomial	1.998421636	0.219178	0.101374	0.065067	0.047429	0.037006376
Error Leisen-Reimer	0.168376109	0.004255	0.001033	0.000455	0.000255	0.000162816