Report 4 Palmeri Claudio N°2062671

The **objective** of this report is to **plot the behaviour** of the **5 Greeks** that we studied in class and observing how the parameters influences them.
The aformentioned parameters are:
K=100 strike price
r=0.01 interest rate
vol=20%,10%,40%
S=60,65,…,135,140 underlying price
Tau=0.1,0.2,…,4.9,5 Time to maturity
Both **K** and **r** will remain **fixed** during these report; **S** and **Tau** will be used to **plot 3** (one for **each volatility** value) **3D graphs** for each **greek**.
We will only focus on **Call options**.

The Black-Scholes model

The **Black-Scholes model** operates in **continuous time**.
Firstly we need to calculate the factors:

d1=(ln(S/K)+(Tau-t)(r+0.5vol^2))/(vol*sqrt(Tau-t))
d2=d1-vol*sqrt(Tau-t)

Then the **price of a call option at time 0<=t<=Tau** is given by the formula:

price(t)=S*N(d1)-Ke^(-r*(Tau-t))*N(d2) where **N(x)=Prob(N(0.1)<=x)**

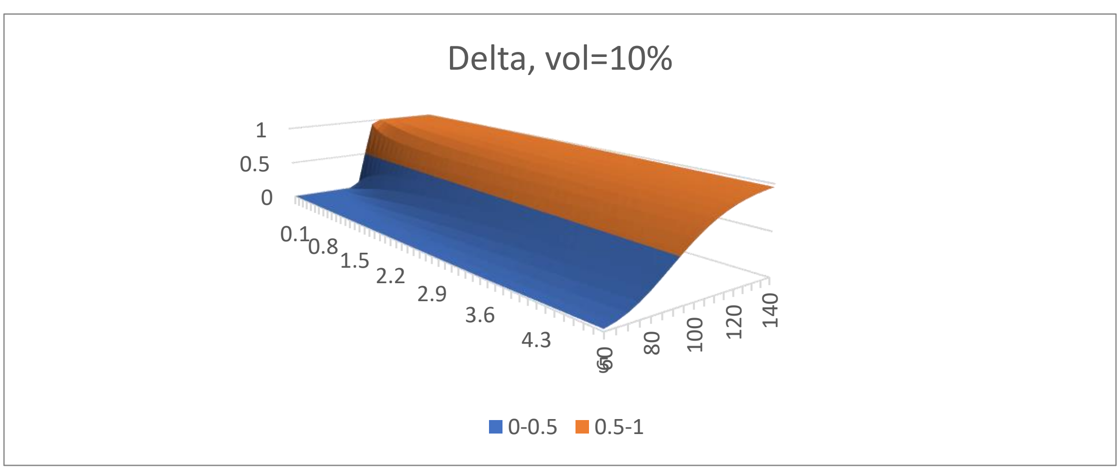
Greeks

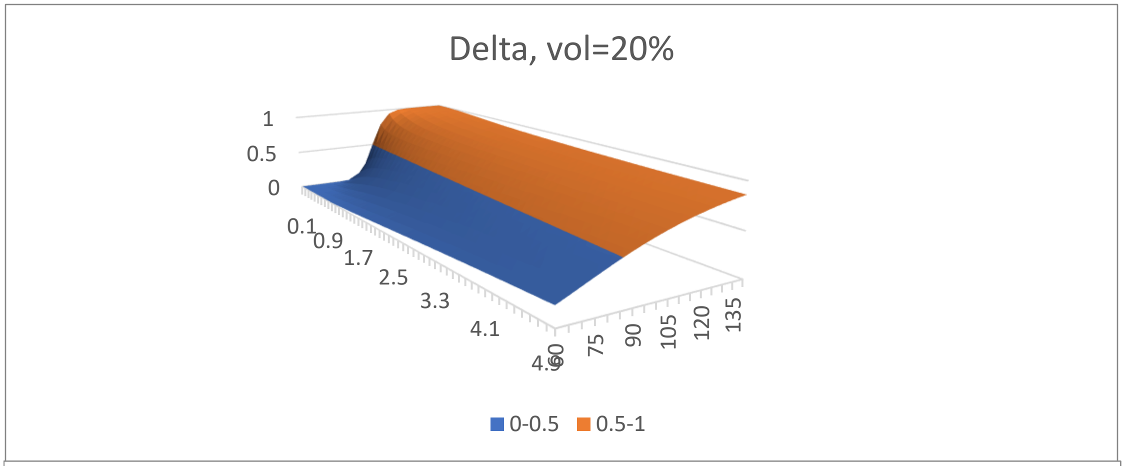
Delta

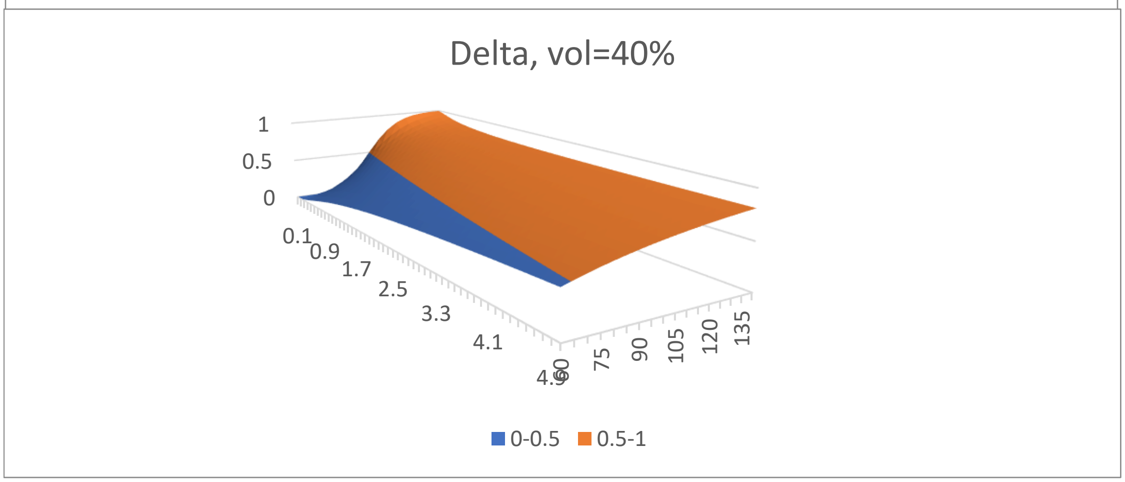
Delta is the **partial derivative** of the **price** with respect to **S**

Delta=N(d1)

These are the **3D plots** that I obtained for **vol=10%,20%,40%** respectively:







I can **observe** the following things:

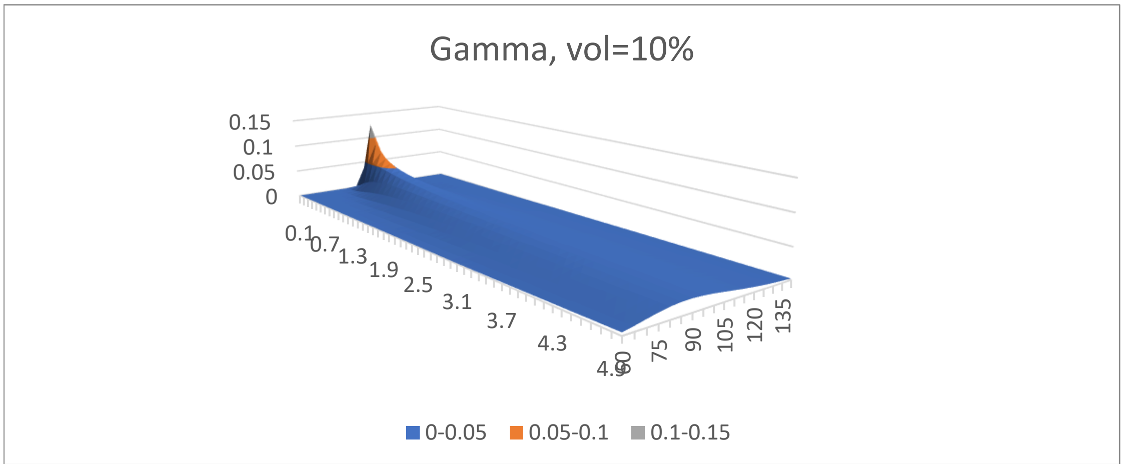
1)If I **fix** the initial price **S**, I can see that **Delta grows with the maturity time Tau if and only if S<=K**, **otherwise** it **decreases as Tau grows**.
2) If I **fix** the maturity time **Tau**, **Delta grows as the initial price grows**. We can also see that for **lower** values of **Tau**, **Delta** has a very **big jump** in values when we are **around the strike price**. **Otherwise** when **Tau** is **large** we have a more **linear growth**.
3) An **increase in the volatility** has **2 effects**: it makes **1) more prominent and 2) less prominent**.

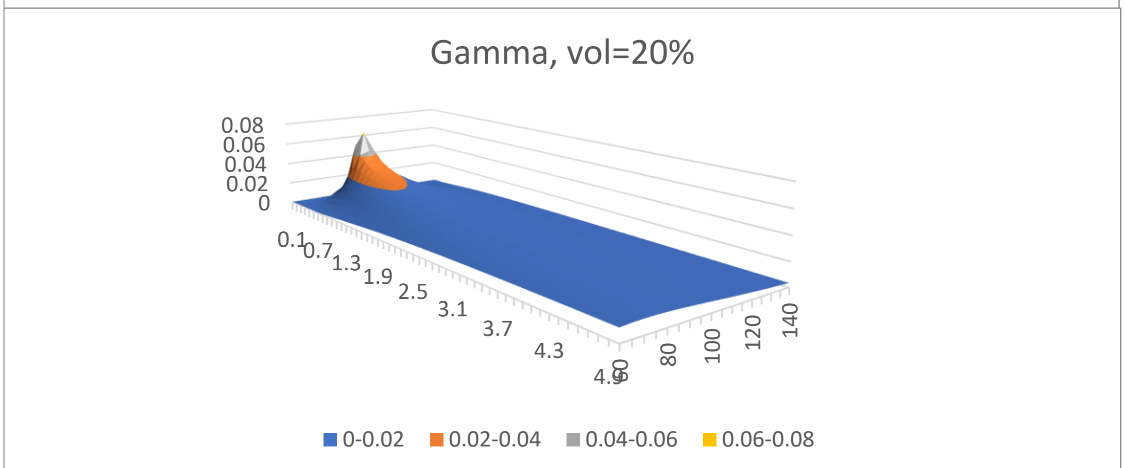
Gamma

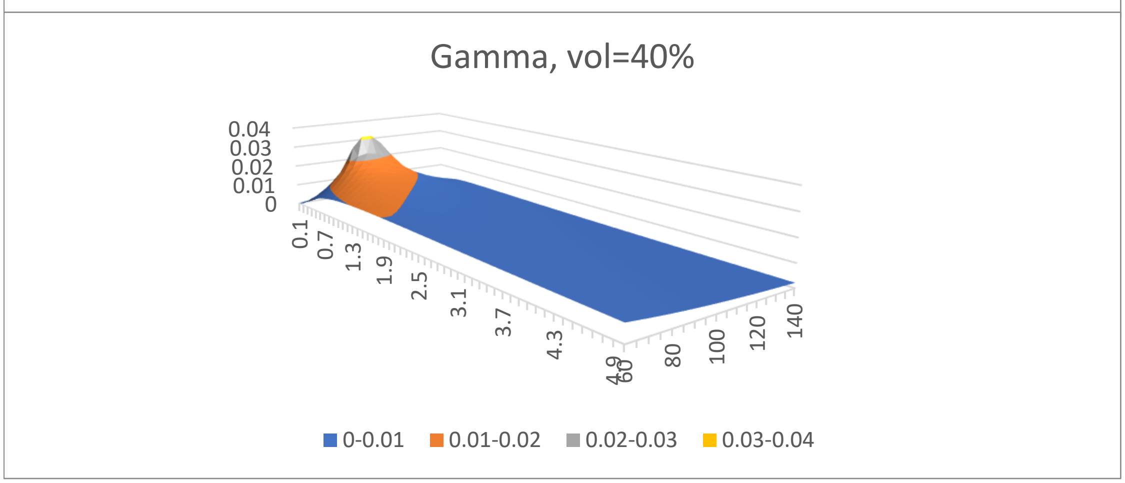
Gamma the **partial derivative of Delta** with respect to **S** (equivalently the second partial derivative of the price with respect to S)

Gamma=phi(d1)/(S*vol*sqrt(Tau-t)) where phi(x)=exp(-x*x/2)/sqrt(2*pi)

These are the **3D plots** that I obtained for **vol=10%,20%,40%** respectively:







I can **observe** the follwing things:

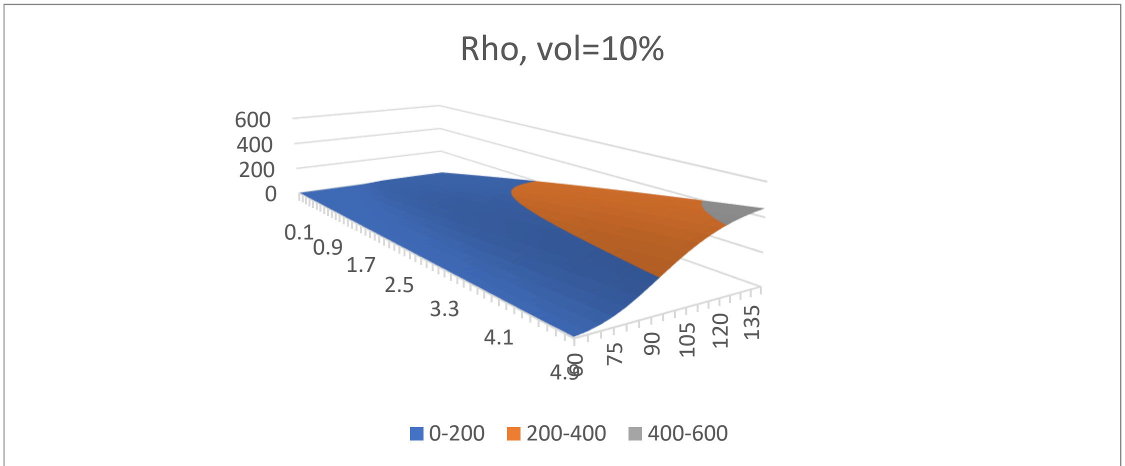
1) If I **fix** the initial price **S**, **Gamma grows with Tau if and only if S is far enough from the strike price K**. **Otherwise** it will **decrease as T increases**.
2) If I **fix** the time to maturity **Tau**, **Gamma follows a Bell curve with maximum value found at S=K**
3) **Increasing the volatility** has **2 effects**: it makes **1) more prominent** and makes the aformentioned **Bell curve** to have **higher variance**.

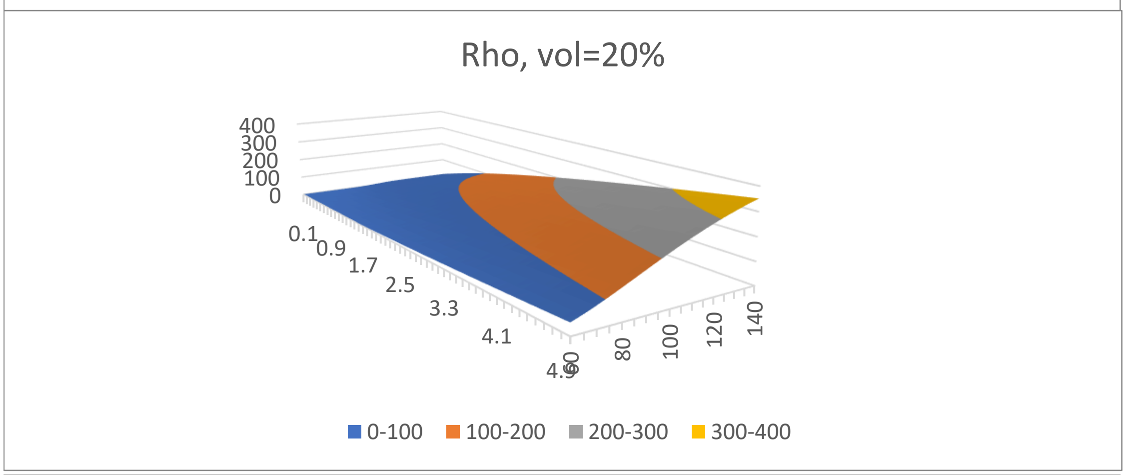
Rho

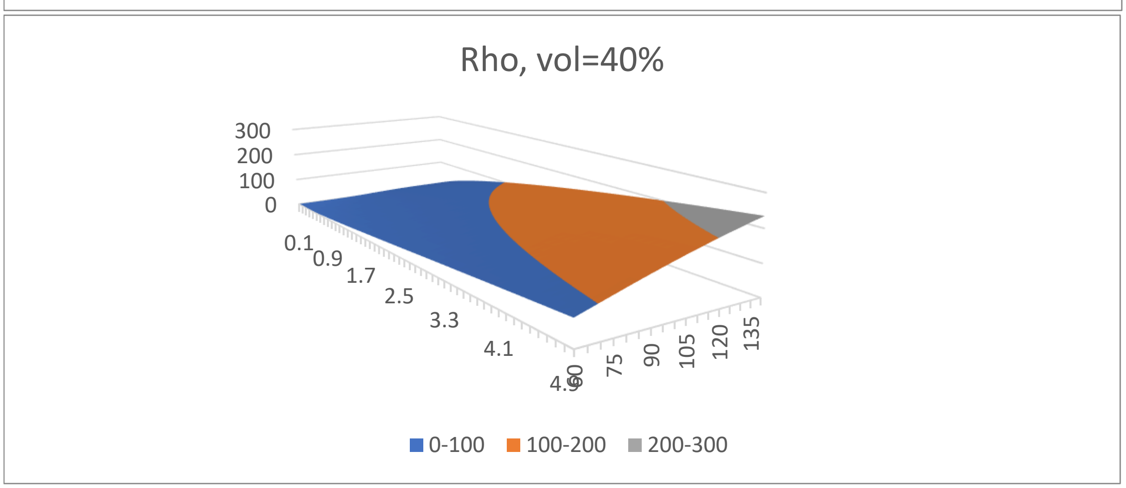
Rho is the **partial derivative of the price** with respect to the interest rate **r**.

Rho=K*(Tau-t)*exp(-r*(Tau-t))*N(d2)

These are the **3D plots** that I obtained for **vol=10%,20%,40%** respectively:







I can **observe** the following things:

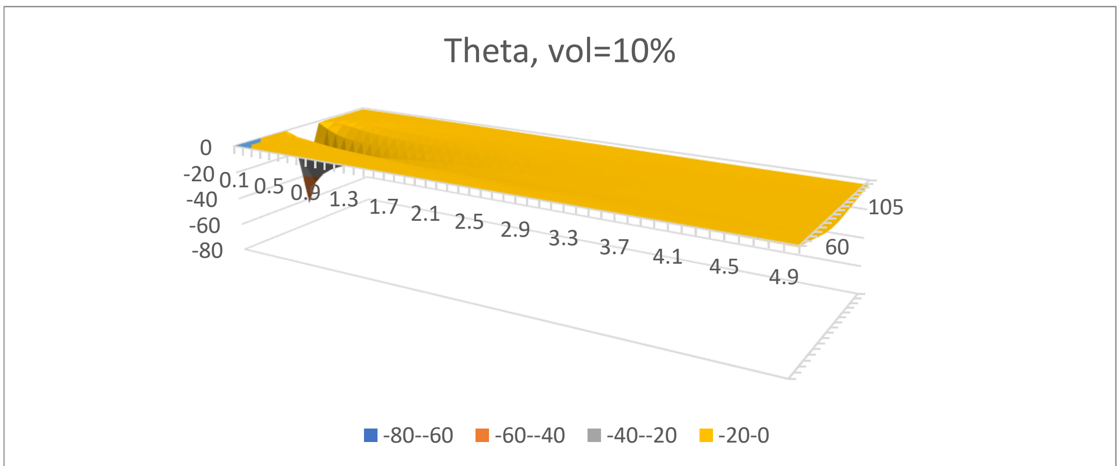
1) If I **fix** the initial price **S**, **Rho grows as the maturity time increases**. The **higher the initial price** the **more prominent** this **increase** is.
2) If I **fix** the maturity time **Tau**, **Rho grows as the initial price increases**.
3) Having **more volatility** causes **2 effects**: it makes **1) more prominent** and it **squeezes the effects of 2)** (i.e. increasing the volatility makes the **lower values higher** and the **higher values lower** when we fix Tau)

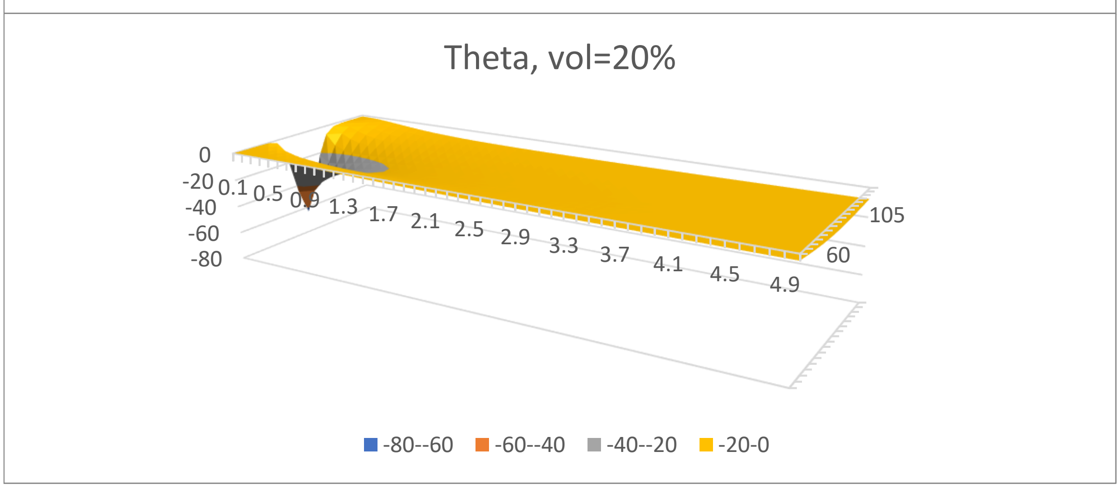
Theta

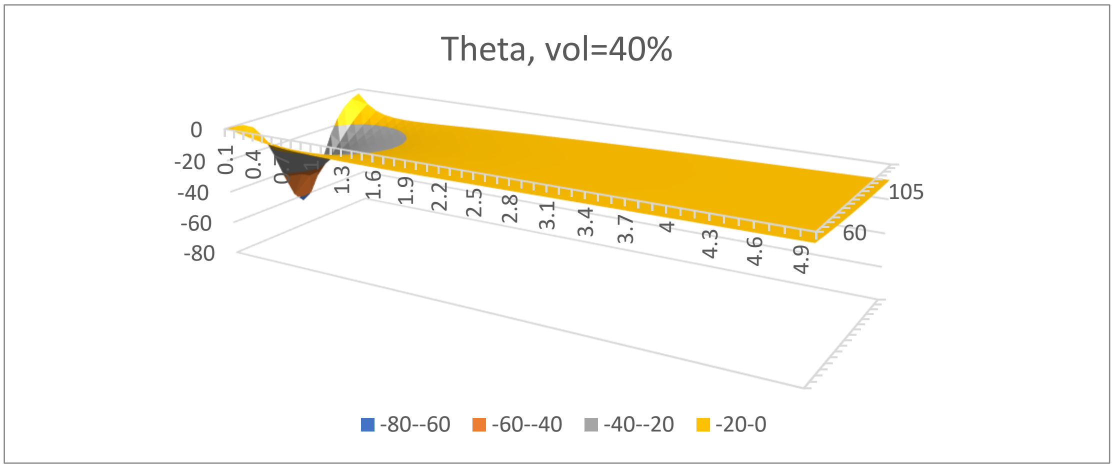
Theta is the **partial derivative of the price** with respect to the time **t**

Theta=-(-S*phi(d1)/(2*sqrt(Tau-t))-r*K*exp(-r*(Tau-t))*N(d2)

These are the **3D plots** that I obtained for **vol=10%,20%,40%** respectively:







I can **observe** the following things:

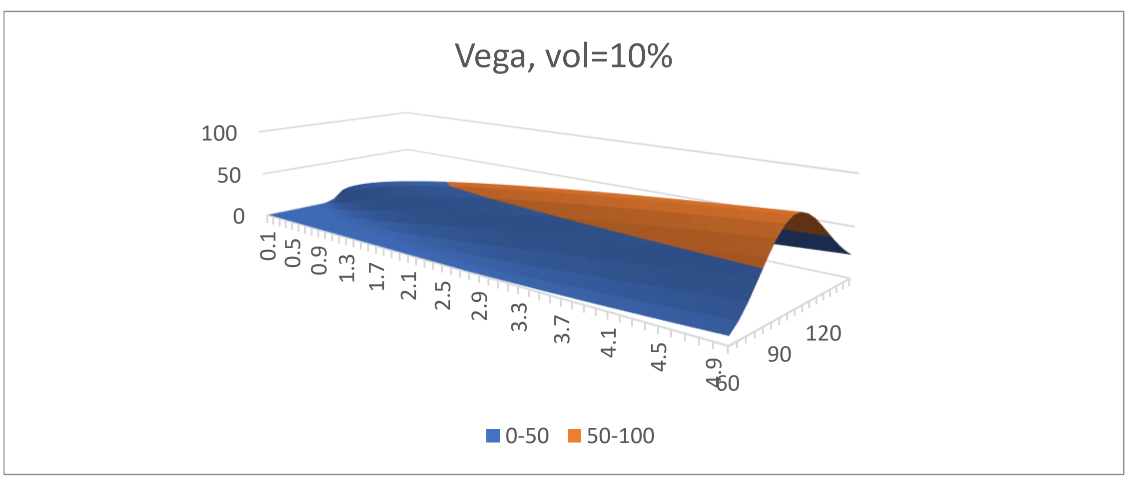
1)This is the **only Greek** that assumes values **lower than 0**.
2) If I **fix** the initial price **S**, **Theta** behaves in the **same** (but **mirrored** on the plane **z=0**) of **Gamma**
3)If I **fix** the maturity time **Tau**, **Theta assumes the shape of a Bell curve** where the **tail** composed by the **higher initial prices is lower** (thus higher in absolute value) than the **other** tail.
4)**Increasing the volatility** has 2 effects: **1) is more prominen**t and it **exacerbates the shape of our function** when Tau is fixed (i.e. makes the **higher values higher**, the **lower values lower** and **reducing the variance** of the Bell curve)

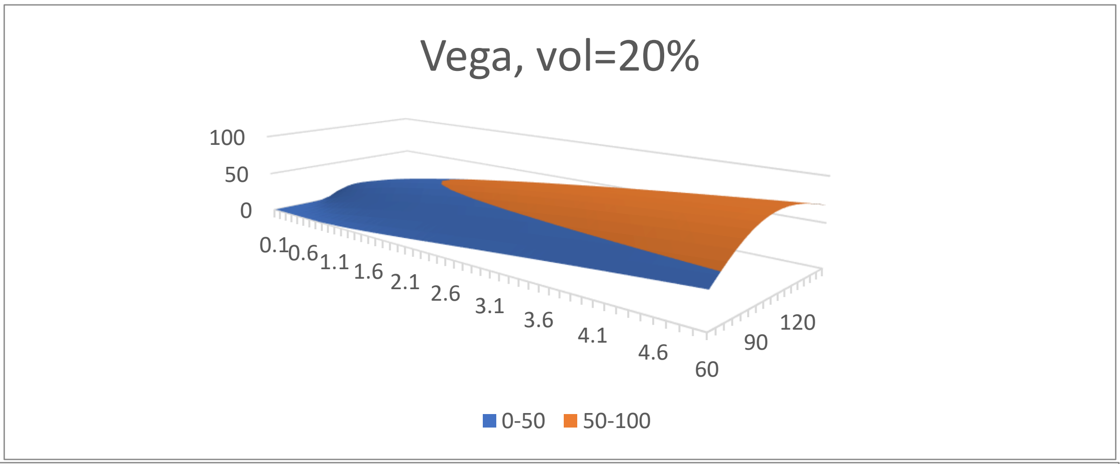
Vega

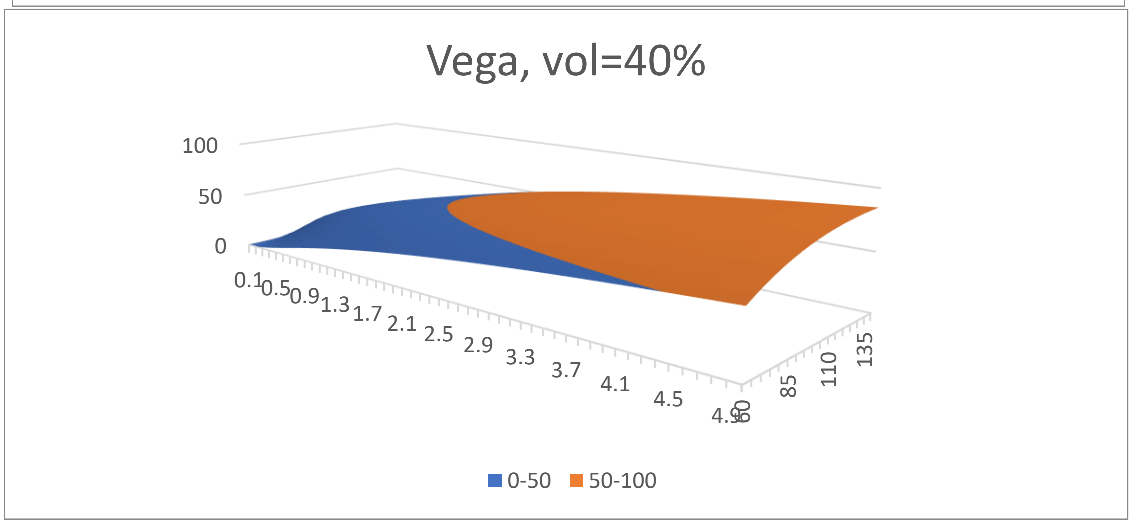
Vega is the **partial derivative of the price** with respect to the **volatility**.

Vega=S*phi(d1)*sqrt(Tau-t)

These are the **3D plots** that I obtained for **vol=10%,20%,40%** respectively:







I can **observe** the following things:

1) If I **fix** the initial price **S**, **Vega grows as the time to maturity Tau increases**.
2) If I **fix** the maturity **Tau**, then **Vega** has the **shape of a Bell curve** where the **tail composed by the higher values of S** is **higher than its counterpart**.
3)An **increase of the volatility** has **2 effects**: it makes **1) more prominent** and makes the **Bell curve** have a **higher variance**