# Find Iteration2

6 กุมภาพันธ์ 2568 19:33

$f(x) = x^2 - 2x$ , $\varepsilon = 0.05$ , $x \in [3/2, 3]$, find iteration

Sol $f(x) = 0 \Leftrightarrow \underbrace{\alpha(x^2 - 2x) + x}_{g(x)} = x$

$L = ?$ : $|g'(x)| \leq L < 1$, $\forall x \in [3/2 \;\; 3]$

$g'(x) = \alpha[2x - 2] + 1$ ; $\Leftrightarrow$ $3/2 \leq x \leq 3$

$\Leftrightarrow 3 \leq 2x \leq 6$

$\Leftrightarrow 1 \leq 2x - 2 \leq 4$

We need to chose $\alpha$ such that $|g'(x)| < 1$

We know that $1 \leq 2x - 2 \leq 4$

Consider $2x - 2 = 1$

then, $g'(x) = \alpha[2x-2] + 1$

$= \alpha(1) + 1$

$g'(x) = \alpha + 1$

from; $|g'(x)| < 1$

we got; $|\alpha + 1| < 1$

$\Leftrightarrow -1 < \alpha + 1 < 1$

$\rightarrow -2 < \alpha < 0$ — (1)

Consider, $2x - 2 = 4$

then $g'(x) = \alpha[2x-2] + 1$

$= \alpha[4] + 1$

$g'(x) = 4\alpha + 1$

from; $|g'(x)| < 1$

we got $|4\alpha + 1| < 1$

$\rightarrow -1 < 4\alpha + 1 < 1$

$\rightarrow -2 < 4\alpha < 0$

$\rightarrow -\frac{1}{2} < \alpha < 0$ — (2)

from (1) and (2) we can chose $\alpha \in \left(\frac{-1}{2}, 0\right)$

So, we chose $\alpha = \frac{-2}{5}$

We got $|g'(x)| \leq L < 1$

So, $\left|\frac{-2}{5}(2x-2) + 1\right| \leq L < 1$

Since, $2x - 2 \leq 4$ then $\left|\frac{-2}{5}(4) + 1\right| \leq L < 1$

$\Leftrightarrow \left|\frac{-3}{5}\right| \leq L < 1$

$\Leftrightarrow \frac{3}{5} \leq L < 1$

from; $\frac{3}{5} \leq L < 1$ Consider, $L = \frac{3}{5}$

from $e_k \leq L^k(b - a)$

$e_k \leq \left(\frac{3}{5}\right)^k (1.5)$

from $e_k \leq \varepsilon$

$\Leftrightarrow \left(\frac{3}{5}\right)^k (1.5) \leq 0.05$

$\Leftrightarrow \left(\frac{3}{5}\right)^k \leq \frac{\cancel{5} \times 10}{100 \times \cancel{15}_{3}}$

$\Leftrightarrow \left(\frac{3}{5}\right)^k \leq \frac{1}{30}$

$\Leftrightarrow \ln\left(\frac{3}{5}\right)^k \leq \ln(1/30)$

$\Leftrightarrow k \ln\left(\frac{3}{5}\right) \leq \ln(1/30)$

$\rightarrow k \leq \dfrac{\ln(1/30)}{\ln\left(\frac{3}{5}\right)}$

$\Leftrightarrow k \leq \log_{3/5}\left(\frac{1}{30}\right)$

Since, $k \leq \log_{3/5}(1/30)$

Therefore, maximum iteration

$= \log_{3/5}\left(\frac{1}{30}\right) \approx \lceil 6.65 \rceil = 7$