SASP-DAAP Homework 2

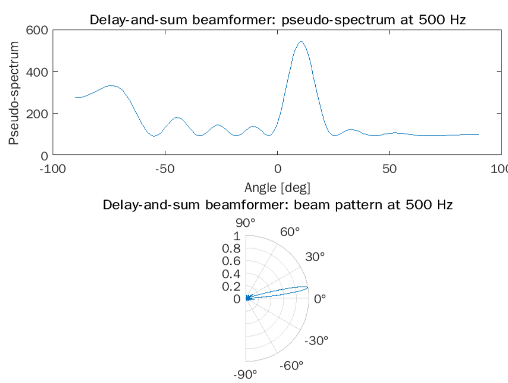
PERRIN Paloma (ID 10873558) and MORO Alessandra (ID 10620673)

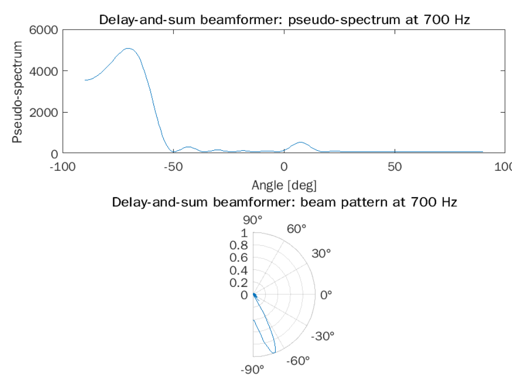
7 April 2022

This report is aimed to explain the main reasoning behind the implementation of the matlab code that estimates the Direction of Arrival (DOA) of two sources placed in an anechoic room and sampled by a microphone array of 64 microphones.

1 Delay-And-Sum beamformer (DAS)

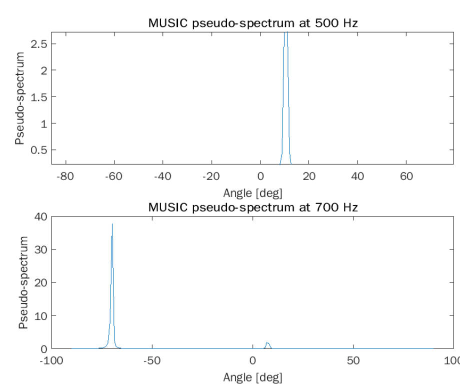
First of all, we defined in the Matlab code all the elements needed to compute properly the theory. We are working with a sampling frequency of 8kHz, 2 sources and 64 microphones. Each micro signal recorded 10000 values. The choice of d is detailed in question 2. To have an idea of the shape of the y signal, we plotted its spectrum and we found out that it was composed of two tones which frequencies where 500Hz and 700Hz.
We then arrive to the Delay and Sum Beamforming method. The purpose of this method is defined in question 1. In the code, we primarily defined the angles that we wanted to investigate : -90° to 90° with a 1° step (181 angles to analyze), converted in radians. Then, we had to define all the components useful to compute the pseudo-spectrum : the covariance matrix(R), the spatial frequency vectors(ws) and the propagation vectors (a) for each source. We plotted the pseudo spectrum for each source in which we see a peek for the corresponding DoAs. After computing the pseudo spectrum (PS), we took the two values of the angles (because we seek for 2 sources) for which the PS is maximum. Now that we have the DoAs of each source, we can take the corresponding values of the previous components already computed for each source to create the spatial filter (h) and the spatial response. Plotting the spatial response, we can define two beams having bigger width at the DoAs founded.
Finally, applying spatial filtering (h) to the microphones signals and putting it in the time domain, we obtain the two tones.





2 MUSIC algorithm

The purpose of the MUSIC algorithm is also described in question 1. To begin this part of the code, we computed the eigenvalue and eigenvectors of the covariance matrix and sorted the eigenvalue in descendent order. We then created accordingly a sorted diagonal eigenvalue matrix and the sorted eigenvector matrix. In this eigenvector matrix, we just took the component corresponding to the noise. As for the DS method, we computed the MUSIC pseudo-spectrum and found the angles for which it is maximized. The angles found being the same as the ones we found with DS method, we validated our computation.



3 Questions

Question 1 :

Delay-and-sum (DS) is a technique which calculates the DoA by measuring the signal strength at each possible arrival angle and selecting the arrival angles at power peaks : the highest power point corresponds to the estimated angle of arrival. Thus, given a known direction of arrival (DoA) and time delays, it aims to sum the delayed signals depending on the direction of arrival of the sound waves. This produces destructive interference in all directions but the direction of arrival. It is truly different from the MUSIC parametric method because MUSIC is a high-resolution direction-finding algorithm based on the eigenvalue decomposition of the sensor covariance matrix observed at an array. It works by searching for all arrival vectors that are orthogonal to the noise subspace.
In terms of pseudo spectra, while DS algorithm seeks for the higher power point, MUSIC constructs a pseudo-spectrum which peaks are infinite (never really infinite because of the unavoidable presence of noise) when an arrival vector is orthogonal to the noise subspace. The angles at which the spectrum has finite peaks are the desired directions of arrival. Despite being computationally simpler, the width and height of the side lobes limit the performance (discrimination capability) and effectiveness of the DS method when signals from multiple directions / sources are involved, implying in poor resolution. One way to improve it consists of increasing the number of sensors, which increases the delay-sum signal processing and complexity. Large numbers of computations are needed to search for the spectral angle when using the MUSIC algorithm, so in real applications its implementation can be difficult. In general, it is better to use MUSIC if there are several sources to locate because the results will be more accurate than with DS. However, the computation is much less complex with DS, which means that it's better to use it for the location of single source signal.

Question 2 :

In order to respect the anti-aliasing condition, the distance d between microphones in a microphone array should be at least less than half of the wavelength of the signal that is being detected by the microphone. Therefore, the maximum frequency this linear array can handle without ambiguity in direction finding is:

$$Fc \leq \frac{c}{2d} \tag{1}$$

Fs is the Nyquist frequency and, by respecting the sampling theorem, we obtain that the distance d is at least less than c/Fs.
Due to the fact that the signal from a source arrives at different microphones at times proportional to their distance, another way to estimate d is to measure the time differences of arrival (TDOA).

$$\tau = \frac{dsin(\theta)}{c} \Rightarrow d = \frac{c\tau}{sin(\theta)} \tag{2}$$

In conclusion, considering that d is proportional to the variation of the beam resolution, we need to calculate it using the expected frequency in order to have the maximum resolution. In fact, although using the smallest d possible is an assurance for the detection of all the frequencies, it reduces the beam resolution.

Question 3 :

To synthesize the array vector y(w), we need to recall the formula of the array model:

$$y(\omega) = AS(\omega) + e(\omega) \tag{3}$$

The point is to create two sine signals having the same frequency. We chose one with a zero-phase and the other one with a 10 radian phase. Then, by applying the dft on them, we get S(w). As we already know the DoAs of these two signals, we just need to compute their spatial frequency for these angles. Now, having the spatial frequencies and knowing that there are 3 microphones, we can compute the propagation matrix A. To create the noise, we just use the "random" formula from matlab. By applying these steps, we finally have all the components to synthesize the signal y. Everything is made in the matab code.