Wind Bridges project

The aim of this project is to assess the aeroelastic stability of a bridge deck. Using course contents and personal knowledge, this project aims to find the particular conditions for which the deck exhibits instabilities. This project has been realized by a group of four students.

Calculations have been carried out under Matlab. The codes and the data are available on my GitHub : https://github.com/PalomaPerrin/Wind-engineering.git

The analysis of the bridge is divided into two main parts. In the first part, the method will be based on the subdivision of the bridge deck into sections on which we will study aerodynamic forces. Then, we will analyze the full bridge response using a modal approach.

A nonlinear approach is used to assess the static position of the bridge for static load (traffic, loads of permanent wind). We use a linear approach for the assessment of the dynamic behavior of the bridge, linearizing around the static equilibrium configuration. Finite elements are used to schematize the bridge (beam elements for deck and tower, taut string for cables) and to compute its global behavior.

Summary

I - Application of the linearized Quasi-Steady Theory on a section model

The average wind produces a static load applied to all the bridge parts: deck, towers and cables. Since the bridge is a structure which is mainly extended in one direction, we can simplify the problem by studying only the bridge response when the wind arrives perpendicularly to its longitudinal axis. Applying this simplification, we are able to consider only three components: the lift force, the drag force and the pitching moment.

Moreover, the QST (Quasi-Steady Theory) is the most suitable approach to study problems related to stability. It reproduces well the aerodynamics forces on a deck, function of motion and wind turbulence, if the reduced velocity is greater than 10 (reduced velocity has no dimension). The deck is the most sensitive part of the bridge to the wind action. Thus, our analysis will be focused on its behavior.

1) Steady Response, nonlinear static problem

To begin the study of the bridge response to the wind, we first set the values of the parameters that will be used in the calculations (see Annex n°1). Then, with these parameters, we define the structural matrices being: the structural mass, damping and stiffness matrices:

$$M_{struct} = \begin{bmatrix} m_y & 0 & 0 \\ 0 & m_z & 0 \\ 0 & 0 & J \end{bmatrix} \quad K_{struct} = \begin{bmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & k_t \end{bmatrix} \quad R_{struct} = \begin{bmatrix} r_y & 0 & 0 \\ 0 & r_z & 0 \\ 0 & 0 & r_t \end{bmatrix}$$

The drag, lift and momentum coefficients (Cd, Cl, Cm) in function of the attack angle alpha are already given in the data for us to work with. We just extract them from the right column of the data sheet to define them separately. This allows us to plot them in function of the alpha.

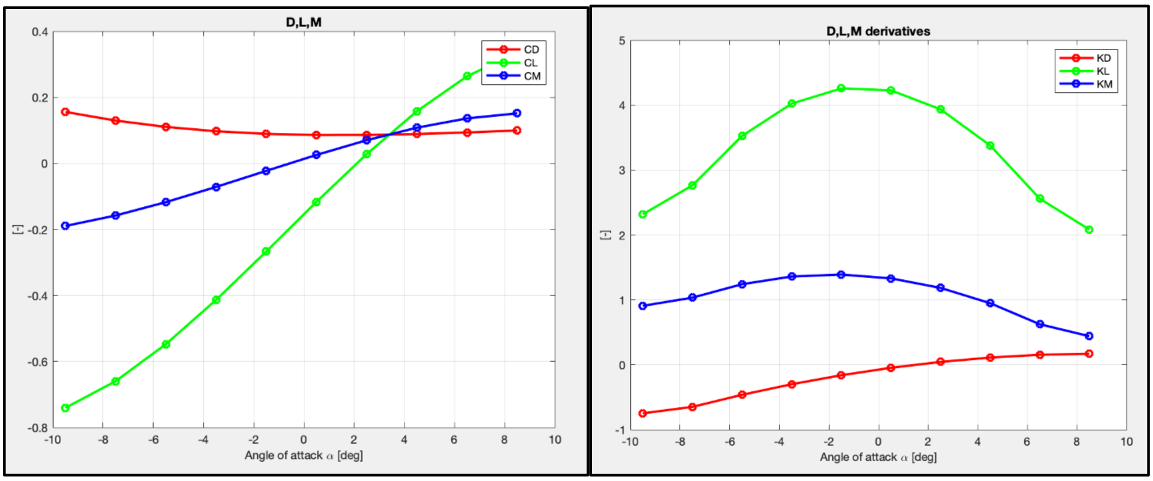


Figure 1: Drag, lift and torsional coefficients (left) and slopes (right)

2) Static Solution

Using the **linearized quasi-steady theory**, we can compute the static solution. The steady response is written in these terms (1) :

$$[K_{diag}].\underline{x_0} = \underline{F_{aero}} \iff \begin{bmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & k_t \end{bmatrix} . \begin{bmatrix} y_0 \\ z_0 \\ \theta_0 \end{bmatrix} = \frac{1}{2}\rho U^2 B \begin{bmatrix} C_D(\theta_0) \\ C_L(\theta_0) \\ B.C_M(\theta_0) \end{bmatrix} \quad (1)$$

Then we will focus on the computation of a non-linear solution and we will compare the results obtained using linearization. In this way, we can evaluate the error considering that the nonlinear solution is mathematically complex but generally more accurate.

Since the system is structurally uncoupled, we first solve the non-linear torsional equilibrium in θ (2).

$$k_\theta.\theta_0 = \frac{1}{2}.\rho.U^2.B^2.C_M(\theta_0) \quad (2)$$

We start by defining in the Matlab code a wind speed vector (U) which values go from 0 to 80 m/s. Then, a vector theta containing angles values from -10° to 10° converted into radians is created. It allows us to compute the moment aerodynamic coefficient. The goal of this first analysis is to find the angle theta for which the function "statica" (figure 2) is null (e.g. the angle at which the system is at equilibrium for every value of the wind velocity).

```
function f = statica(K_stru,theta,rho,U,B,L,alpha,Cm)
% theta is in [deg], but should be converted in [rad] when you compute K_theta*theta
% to find Cm(theta) use the 'interp1' function
% fsolve finds theta that makes f=0, so write the stati equilibrium accordingly
Cm_i = interp1(alpha,Cm,theta);
kt = L*K_stru(3,3);

f = kt*(theta*pi/180) - 0.5*rho*(U.^2)*(B^2)*Cm_i;
```

Figure 2: *Matlab function used to compute the static angle of attack with "fsolve"*

Using the values of the torsional equilibrium angles for every wind velocity, we can compute the horizontal and vertical components in order to get the values of z_0 and y_0.

$$k_z.z_0 = \frac{1}{2}.\rho.U^2.B^2.C_L(\theta_0) \quad (3)$$

$$k_y.y_0 = \frac{1}{2}.\rho.U^2.B^2.C_D(\theta_0) \quad (4)$$

The linearization is performed under the hypothesis of small displacements, small velocities and small turbulent wind fluctuations. Now using linearization, we have to define the torsional coefficient and torsional stiffness for the null value of the angle of attack (respectively CM0 and KM0) by interpolation (interpolation of Cm and Km around 0). This allows us to compute the linearized torsional coefficient as:

$$C_M(\theta_0) = C_{M0} + K_{M0}.\theta_0 \quad (3)$$

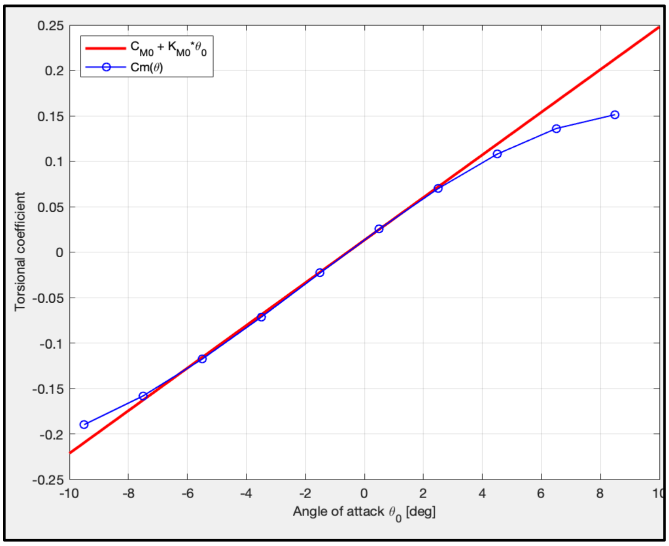


Figure 3: Torsional coefficients as a function of the angle of attack

Figure 3 allows us to compare the values of the torsional coefficient obtained by linearization (red line) with the value given in the data file. The linearization approach defines the torsional coefficient using an affine function, we thus get a line whose origin is the value of the interpolation computed earlier ($C_{M_0} = 0{,}0132$).

Then, we linearize the expression (2) by using the linearized torsional coefficient, which leads us to the linearized expression of the static torsional angle (figure 4):

$$\theta_0 = \frac{\frac{1}{2}\rho U^2 B^2 C_{M_0}}{k_\theta - \frac{1}{2}\rho U^2 B^2 K_{M_0}} \quad (4)$$

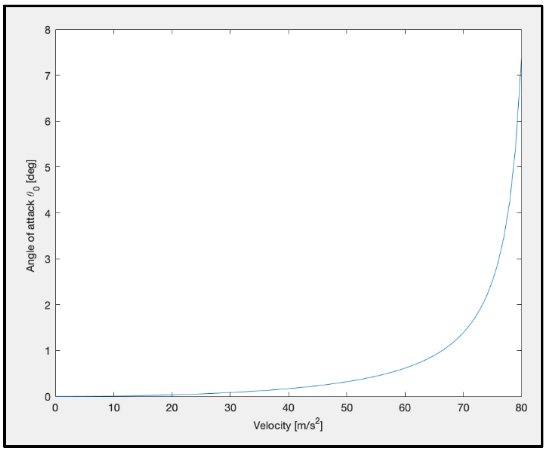


Figure 4: Linearized angle of attack as a function of the velocity

Having computed both linear and nonlinear solutions, we can graph (figure 5) the nonlinear horizontal and vertical static displacements, respectively y (red line) and z (green line). On the last plot of the figure (blue line), we plotted together the nonlinear and linear static torsional angle. We can see that the two approaches give very similar curves. Starting from the wind speed around 70 m/s, the linear curve increases more than the nonlinear approach giving a higher value for the 80 m/s wind velocity. (Around 7° against around 3,7°).

The static response is nonlinear. Thus, for the nonlinear procedure to have the theta, we take the static response and we use nonlinear coefficients (excel file). The equation to seek for the nonlinear equation is implicit. This is the reason why we have to use the function 'fsolve' to solve it. In the end, we need some iteration to have a great value of theta and the nonlinear function that we have set in fsolve gives us the value of theta for an almost null value of the equation. If we want to use a linear approach, it can only be correct if we are in presence of small displacements/ rotations.

At the end we can see that the linear curve goes higher than the nonlinear one because it doesn't take into account higher possible rotation.

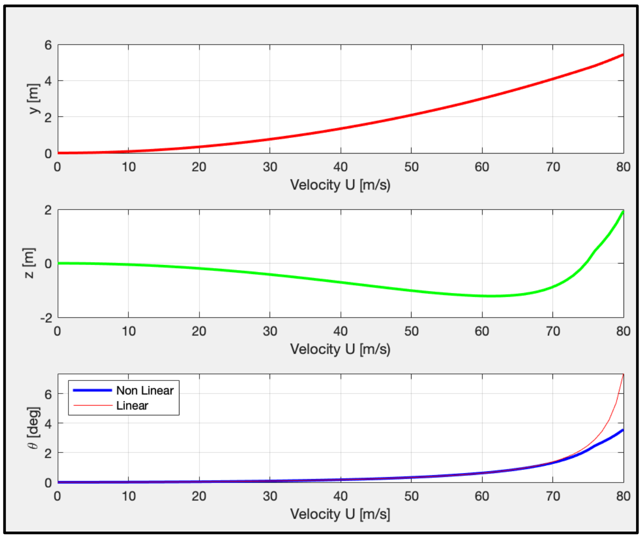


Figure 5: Static displacements and angle

Y and Z are nonlinear solutions using the theta, output of the nonlinear procedure.

II- Aeroelastic stability, eigenvalue problem, dynamic response

We are now interested in studying a two degrees of freedom instability, which is the flutter instability. The flutter instability is the combination of vertical and torsional motion producing instability effects over a specific wind speed (critical flutter wind speed). For this part, we use a reference wind speed equal to U=45m/s, independant from the height of the bridge. The turbulence components will be added to compute the buffeting part.

(Flutter instability happens when the damping matrix ($[K_A]$) is not symmetric.) Thus, the vertical and torsional frequencies tend to be equal, which leads to a synchronization of the two motions. The aerodynamic forces introduce energy in the system which leads to two degrees of instability. To have flutter, the torsional damping slope must be positive: $K_M > 0$.

1) <u>Parameters</u>

To assess the aeroelastic stability, we have to define the reference dimensions B for each direction. We also have to set the sizes of the frequency and damping vectors for each direction which are related to the length of the wind speed vector: all of them are row vectors of 81 columns.

Then, for each value of the wind velocity, we interpolate the drag, lift and torsional coefficients and stiffnesses at the query points theta equilibrium.

This allows us to construct the mass, damping and stiffness aerodynamic matrices for each value of U. The mass matrix is 0 because the aerodynamic forces don't influence the second derivative. The matrices are defined as :

$$R_{aero} = \frac{1}{2}\rho U B \begin{bmatrix} 2.C_{D_0} & K_{D_0} - C_{L_0} & 0 \\ 2.C_{L_0} & K_{L_0} + C_{D_0} & 0 \\ B.2.\, C_{M_0} & B.\, K_{M_0} & B.\, K_{M_0}.\, B_{1_\theta} \end{bmatrix}$$

$$K_{aero} = \frac{1}{2}\rho U^2 B \begin{bmatrix} 0 & 0 & -K_{D_0} \\ 0 & 0 & -K_{L_0} \\ 0 & 0 & -B.\, K_{M_0} \end{bmatrix}$$

Finally, the total matrices are the sum of the structural and aerodynamic matrices. As an example, for the stiffness matrix, it gives us:

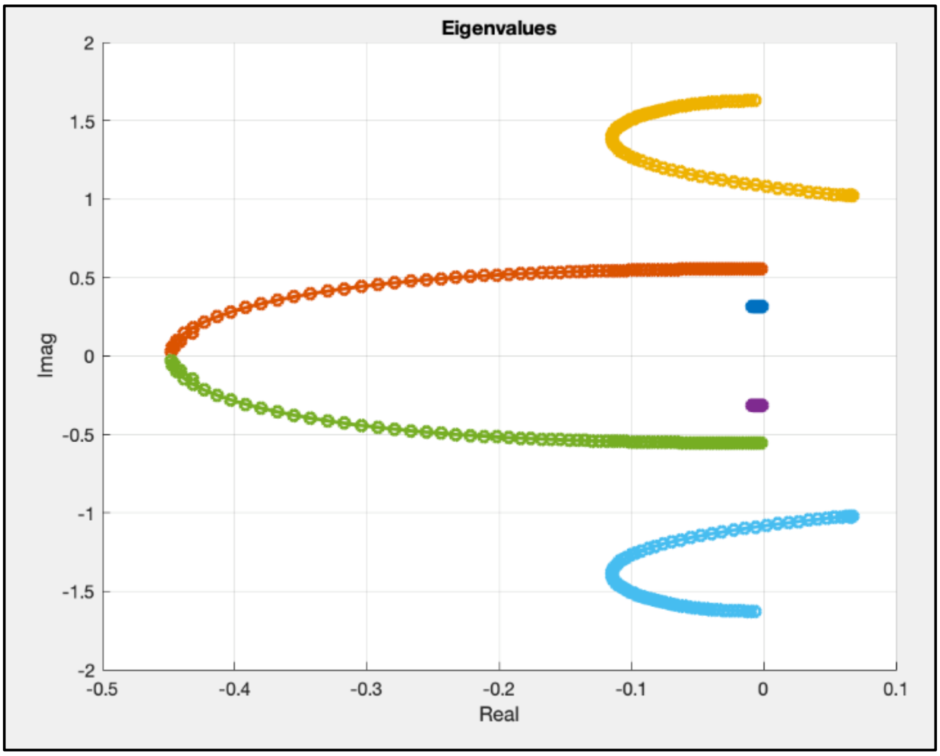
$$K = K_{stru} + K_{aero}$$

2) <u>Eigenvalue computation</u>

Using the function "polyeig" of Matlab allows us to directly solve the polynomial eigenvalue equation:

$$\left(\lambda^2[M_s] + \lambda[R_t] + [K_t]\right)\underline{\Phi} = [H(\lambda)]\Phi = 0 \ (7)$$

This is the equation of a 2 DOF dynamic system, specifically its homogeneous response which is the one without the force. All the aero coefs are put into the matrices (mass, damping, stiffness). The only matrix which is constant is the mass matrix. The function H is the transfer function that says what is the transfer between the position and the force. But here the force is zero, so we just have to evaluate the eigenvalue of the homogeneous equation.



<u>Figure 6:</u> Eigenvalues of the total matrices

The figure 6 represents the eigenvalues that we have for each velocity. Two eigenvalues are connected (coupled). All the eigenvalues have an imaginary part meaning that the system is underdamped. There is one exception at the first eigenvalue which is in critical damping (peak between red and orange). At a certain velocity, we have eigenvalues that are positives: instability occurs.

3) <u>Damping and frequency computation</u>

The formula of the eigenvalues of a damped system is:

$$\lambda_n = -\zeta.\,\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Having the eigenvalues, we can now compute the damping and frequencies for each direction, seaking for the velocity at which the flutter instability happens.

$$\zeta = \frac{-Re(\lambda)}{|\lambda|} \qquad\qquad f_d = \frac{|Im(\lambda)|}{2\pi} = \frac{2\,\pi f\sqrt{1-\zeta^2}}{2\,\pi}$$

We can now graph the damping and frequency values for the whole velocity vector.

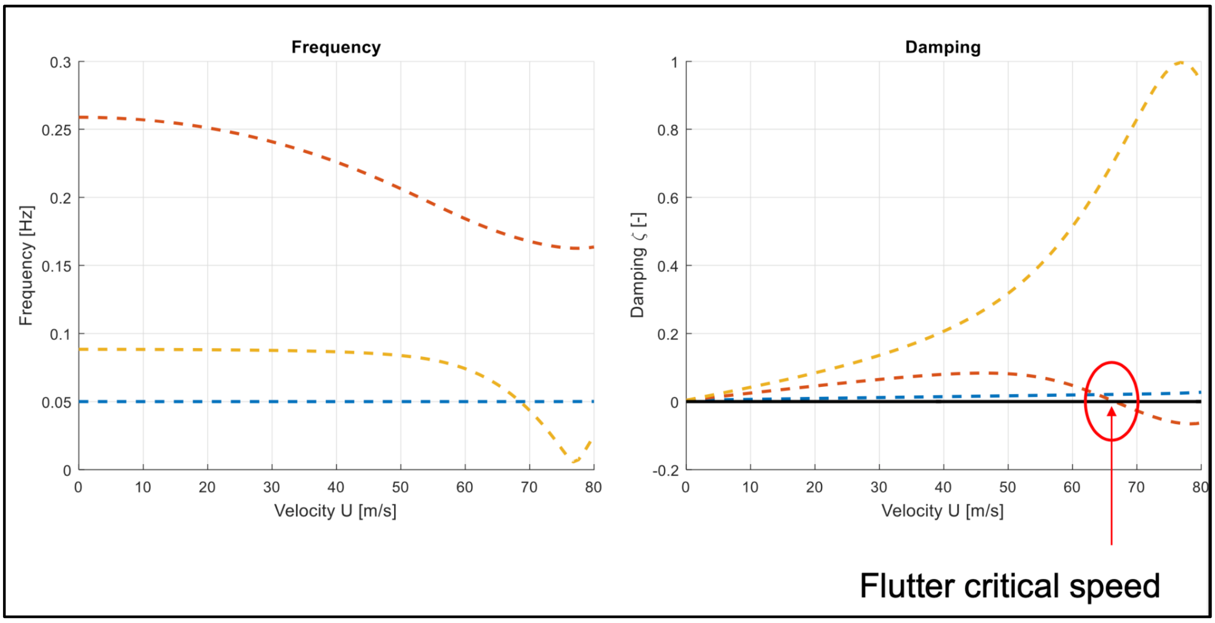


Figure 7: Damping and frequency curves function of the wind velocity

Knowing that flutter instability happens when the vertical and horizontal frequencies become equal and when the torsional damping becomes negative, we can graphically read the critical velocity V_c.

On the damping graph (figure 7, left), we see that the torsional damping, which is the yellow dot line, becomes negative for a velocity which is around 66 m/s. On the frequency graph (figure 7, right), the vertical and horizontal lines are crossing at 70m/s. This means that flutter instability happens for a velocity at 66 m/s.

Km is always positive so 1 DOF instability does not occur in our case.

III - Buffeting calculation

1) Computation of the turbulent wind

The wind in the atmospheric boundary layer is always turbulent. The transition from laminar to turbulent flow occurs in the BL region. Shear stresses at the body surface increase when the flow becomes turbulent. Thus, in order to compute the buffeting forces, we need to define some atmospheric turbulence statistical quantities with a single point analysis (time) for both w and u components. Using the excel file given for the project, we extract different parameters, function of the height of the bridge from the ground:

- Wind speed profile $\frac{U}{U_{ref}}$
- Longitudinal turbulence intensity I_u
- Transversal turbulence intensity I_w
- Longitudinal integral length scale x_{Lu}
- Longitudinal integral length scale x_{Lw}

Note: Integral length scales are a measure of the size of the vortices in the wind.

The deck height being at 64 meters, we were missing the value of the previous parameters at this specific height. Thus, we created affine functions to interpolate their values (given for 60m and 70m) at the wanted height. We will define the procedure for the u component of the turbulent wind.

Having the values of the parameters previously defined for a 60 m and 70 m height, we seek for the value of the wind speed profile at 64m. The slope coefficient of the affine function is:

$$m_u = \frac{U(70) - U60)}{70-60} = \frac{0{,}9721 - 0{,}9893}{70-60} = 0.00143 \text{ [1/s]}$$

The origin value of the function is defined as:

$$q_u = U(70) - m_u * 70 = 0.889 \text{ [m/s]}$$

Thus, the formula to find the u component of the turbulent wind is:

$$y_{k_u}(x) = m_u . x + q_u$$

It allows us to compute the u component velocity of the turbulent wind at the height of the deck being 64 m, the reference velocity U = 45 m/s, computing: $y_{k_u}(64) = 0.981 * 45$ [m/s]

The obtained value is coherent with the ones contained in the excel file, which seems to confirm our calculations. We apply the same steps to define the value of the turbulence intensity and of the integral length scale at 64m for both u and w components.

With these values (wind speed, integral length scale and turbulence intensity at the deck height), we can now compute the Von Karman non-dimensional power spectral density for both components of the turbulent wind. We define a reference velocity U of 45 m/s. This non-dimensional spectrum is computed (for the u component) using the formula:

$$\frac{f.S_{uuN}}{{\sigma_u}^2} = \frac{4(\frac{f.L_u^x}{U})}{[1 + 70{,}8.(\frac{f.L_u^x}{U})^2]^{5/6}}$$

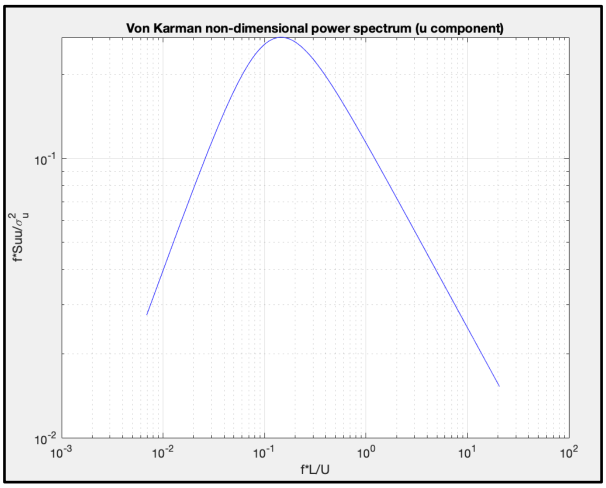


Figure 8: Von Karman non-dimensional power density spectrum

To compute the buffeting forces, we need the dimensional power spectrum given by:

$$S_{uu} = \frac{S_{uuN.\sigma_u}}{f}$$

The power spectral amplitude of the u component can be related to the amplitude of a sine wave by (df is the discretization interval of the spectrum):

$$A = \sqrt{2.G_{xx}} \text{ with } G_{xx} = S_{uu}.df$$

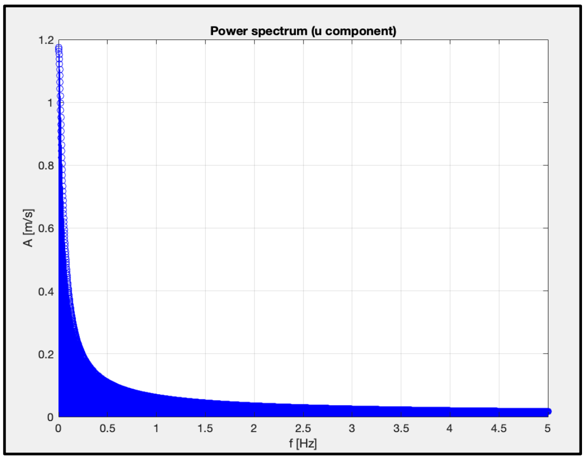


Figure 9: Power spectrum amplitude of the u component

The amplitude is maximal near 0 because the wind is powerful at lower frequencies.
We computed A only between 0-5 Hz because it's the most relevant part of the spectral density. The wind is powerful at lower frequencies.

To compute the time history of the wind velocity, which is a random signal, we can treat it as a periodic signal constructed through wave superposition:

$$u(t) = A_0 + \sum_{i=1}^{N} A_i \sin\left(2\pi f_i t + \phi_i\right)$$

with A the value of the amplitude of the power spectrum at every frequency between 0 and 600 Hz.

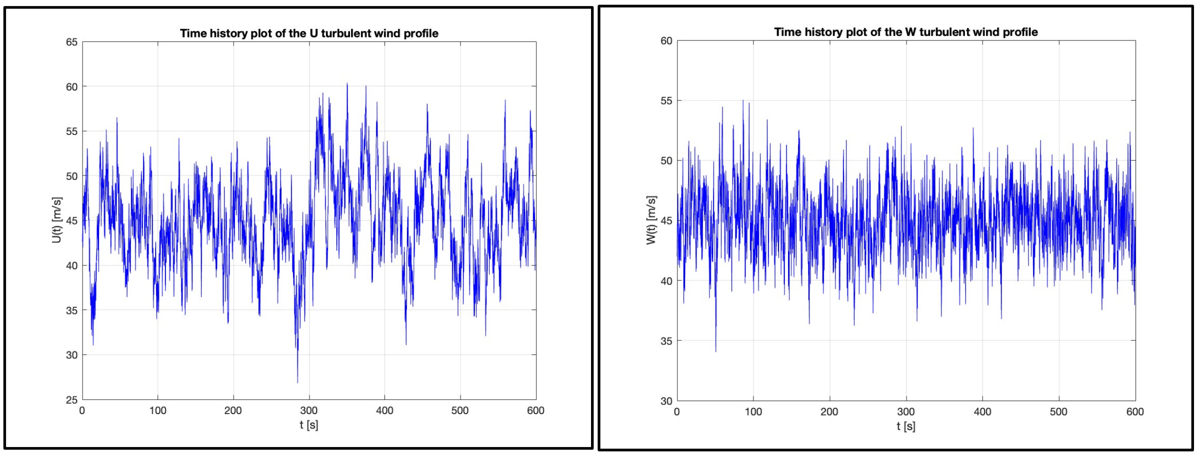


Figure 10: Time histories of the u and w turbulent wind profiles

2) Computation of the buffeting forces

The buffeting force is the aerodynamic part of the force induced by the presence of turbulent fluctuations in the incoming flow. The presence of time-varying fluctuating components in the wind velocity that impinges on the bridge results in a fluctuating aerodynamic force whose characteristics depend on the bridge (deck in our case) shape and from turbulence characteristics

Buffeting phenomenon cannot be avoided but it can be reduced or controlled by increasing the aerodynamic damping.

To compute the buffeting forces, we need to define the torsional angle of the bridge with which we will interpolate the aerodynamic coefficient for the reference wind speed U=45 m/s, to construct the buffeting aerodynamic matrices.

Having the aerodynamic coefficients interpolated with the torsional angle, we can express the aerodynamic matrices and the buffeting force accordingly:

$$R_{aero} = \frac{1}{2}\rho U B \begin{bmatrix} 2.C_{D_{buff}} & K_{D_{buff}} - C_{L_{buff}} & B_{1_y}\left(K_{D_{buff}} - C_{L_{buff}}\right) \\ 2.C_{buff} & K_{L_{buff}} + C_{D_{buff}} & B_{1_z}\left(K_{L_{buff}} + C_{D_{buff}}\right) \\ B.2.C_{buff} & B.K_{M_{buff}} & B.K_{M_{buff}}.B_{1_\theta} \end{bmatrix}$$

$$K_{aero} = \frac{1}{2}\rho U^2 B \begin{bmatrix} 0 & 0 & -K_{D_{buff}} \\ 0 & 0 & -K_{L_{buff}} \\ 0 & 0 & -B.\,K_{M_{buff}} \end{bmatrix}$$

$$M = M_{stru}$$

$$F_{buff} = \frac{1}{2}\rho U^2 B \begin{bmatrix} 2C_{D_{buff}} & K_{D_{buff}} - C_{L_{buff}} \\ 2C_{L_{buff}} & K_{L_{buff}} + C_{D_{buff}} \\ 2C_{M_{buff}}B & K_{M_{buff}}B \end{bmatrix} \begin{bmatrix} \frac{u}{U} \\ \frac{w}{U} \end{bmatrix}$$

With u and w respectively the amplitudes of the power spectrum for the corresponding components for every frequency value.

We defined all the components allowing us to compute the solution of the equation of motion being:

$$[M_{diag}]\ddot{x} + ([R_{diag}] + [R_{aero}])\dot{x} + ([K_{diag}] + [K_{aero}]) = F_{buff}$$

The results of the equation give us the following graphs:

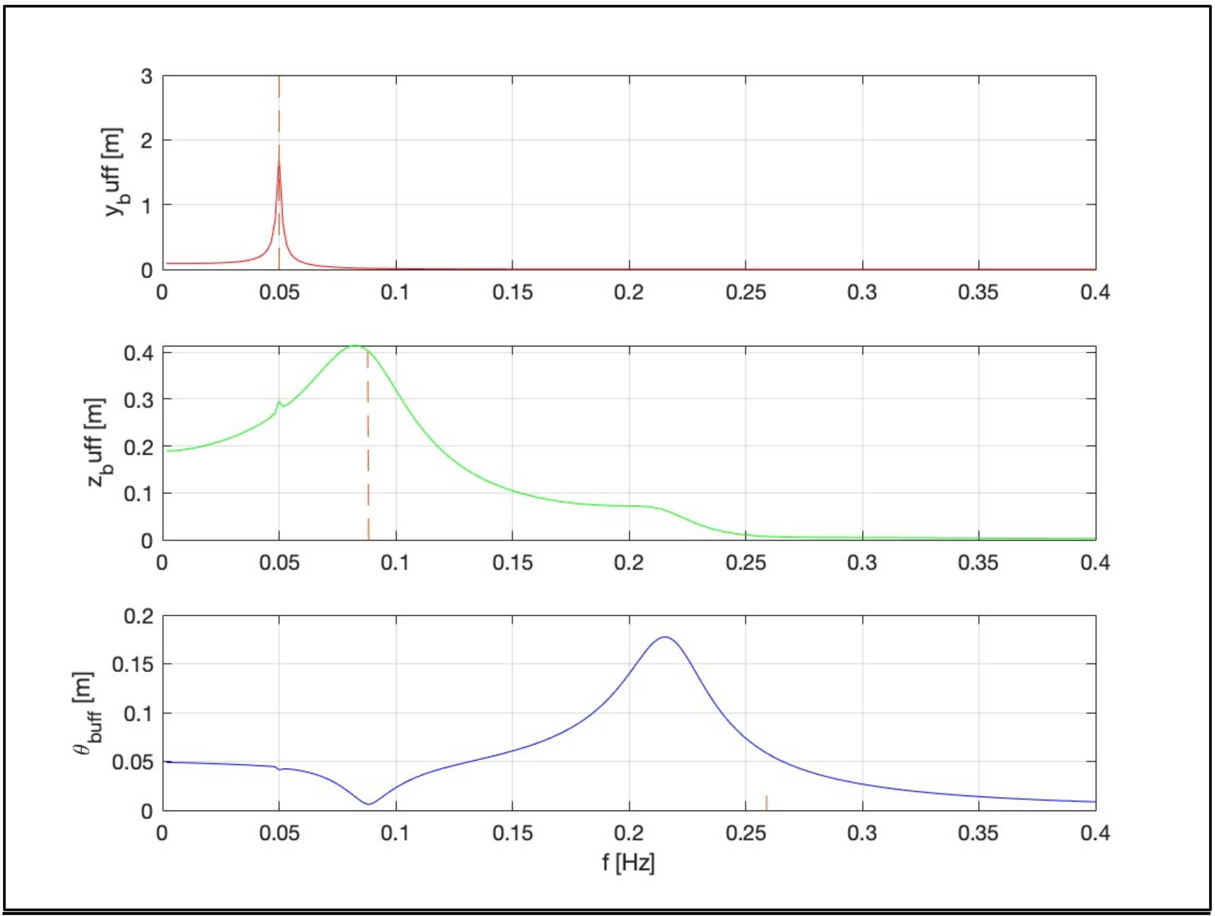


Figure 11: Time histories of the u and w turbulent wind profiles

In the Y direction, the bridge is quite still in the frequency range. It exhibits a very narrow peak of amplitude 1,7 meters at the frequency 0,05 Hz. This wind frequency might be the one with the higher energy. We recall that the wind energy is the highest in the lowest frequencies. Another important point is that the peak is located at the Y-axis natural frequency of the bridge which is 0,05 Hz. For this direction, we might say that we have a combination of the force frequency of the wind and the natural frequency of the bridge (Y-axis). The peak is very sharp because in the Y direction we have the lowest global damping.

In the Z direction, the displacement starts from 0,2 meters and increases slowly up to 0,4 meters at 0,08 Hz. The peak isn't as sharp as the Y displacement one because we have an aerodynamic damping which is quite high compared to the structural one. The peak is happening at this frequency because it is close to the natural Z frequency of the bridge 0,0884 Hz. Thus, it is decreasing smoothly.

In the theta direction, again, we see that the peak is happening around the natural torsional frequency of the bridge which is 0,259 Hz (peak at 0,215 Hz amp 0,17 m). The damping is in between the two others. The counter peak might be due to the second derivative of Cm.

As we saw earlier, the flutter velocity is the wind speed at which the velocity becomes negative. It means that it is very important to have high critical flutter velocity to avoid the decrease of the damping coefficient (even better if we increase it), reducing turbulence induced motions.

3) <u>Power Spectral Density</u>

The buffeting forces can also be expressed in the frequency domain using Power Spectral Density or finite Fourier Transform. In our project, we only computed the PSD of the buffeting forces which is a real and symmetric function since it is defined as the square amplitude of the force.

The passage from the time domain to the frequency domain using PSD determines the amount of information loss contained in the signal phase. These informations are very important to describe the point and spatial coherence of wind turbulence.

If we are interested in the phase informations, it is better to use the FFT. But we prefer to work with PSD because it gives us an output which is independent from the frequency resolution.

$$A_y = \sqrt{\frac{2|G_{yy}|^2}{\omega_0}}$$

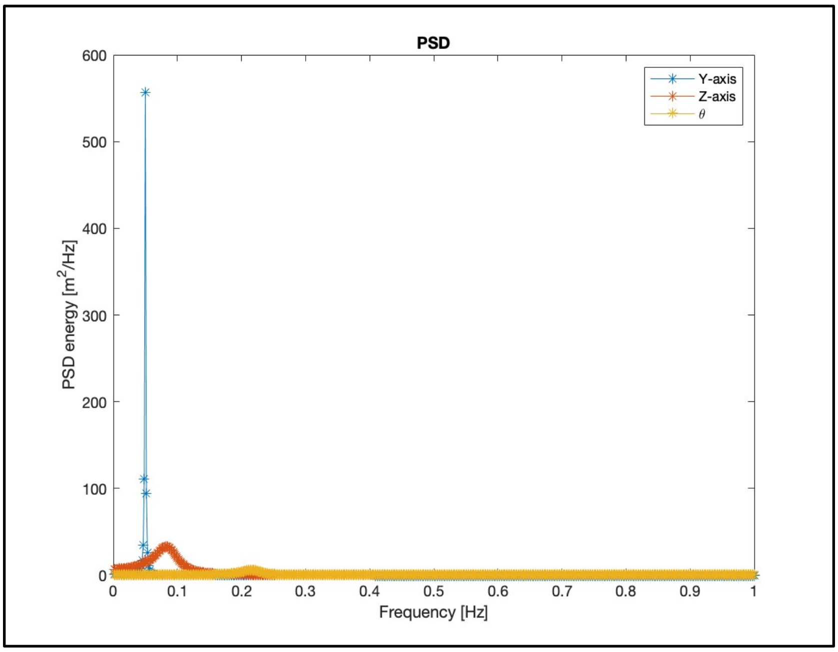


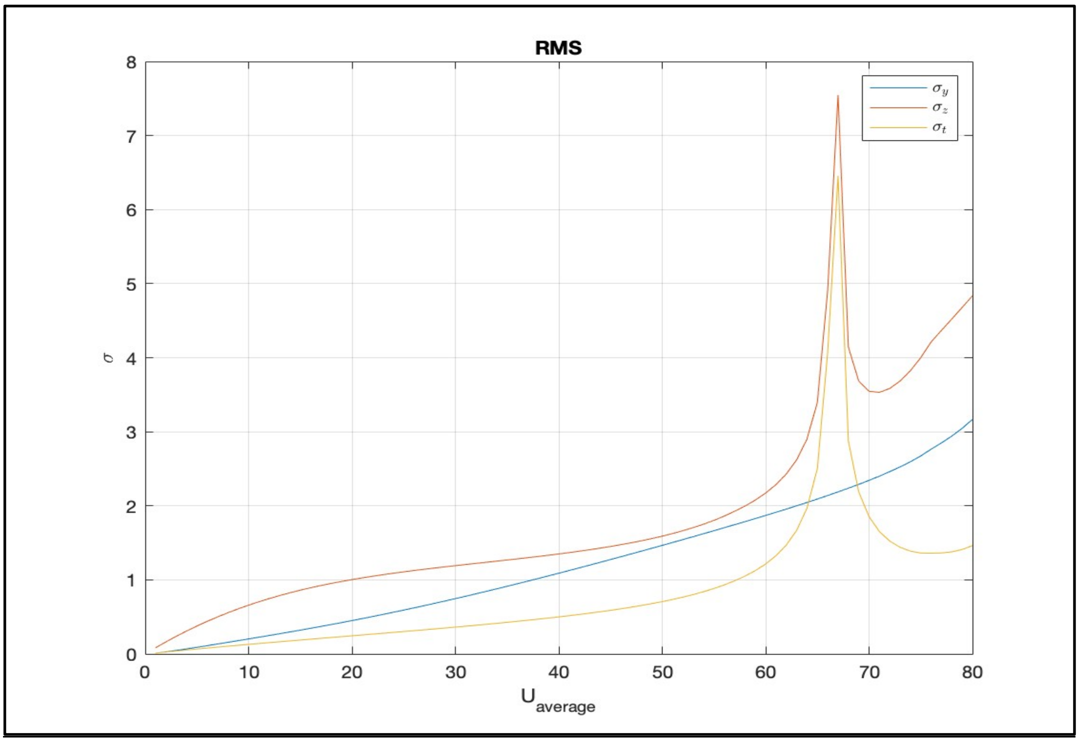
Figure 12: Power spectral density in each direction

A lot of energy is incoming in the Y motion because the damping is really low compared to the other directions. Peaks are near the resonance frequency of the bridge. Even if the Z displacement is the most damped one, we have a higher energy for it than for Y: it means that the wind is pushing a lot of the Z-axis direction.

4) <u>Root Mean Square</u>

The RMS is computed using the integration of the PSD for each velocity. To do so, we used the rectangle integration method to integrate the PSD and then we put this result under a root to evaluate the sigma. This procedure is done for each velocity and each axis.

$$\sigma_y = \int \sqrt{\frac{|G_{yy}|^2}{2\omega_0}}$$



<u>Figure 13:</u> Root Mean Square for each direction

For each velocity, we have an index of the global displacement. For the theta and Z directions between 60 and 70 m/s, we have instability. This is 2 DOF because there's a coupling between both directions. Moreover, we detect that the peaks are located at the flutter velocity (e.g.66 m/s).

IV - Full bridge analysis

We now want to extend our deck section analysis to the multi-sectional full bridge model. To do so, we rely on a modal approach because it has a smaller computational cost. There is a large number of physical coordinates that describe the bridge response. Their number can be reduced to a compact set of modal coordinates.

Modal information can be obtained from a finite element formulation by modeling the continuous structure with beam elements. By exploiting equilibrium conditions, we get the homogeneous equation of motion as (X contains all the DOF of the structure):

$$[M_S]\ddot{X} + [R_S]\dot{X} + [K_S]X = 0$$

We can use a separation of variables, where phi is the modal matrix including the number of mode shapes (eigenvectors matrix) for every direction and q is the vector of the modal coordinates (vector of the dimensionless generalized displacements).

$$X(\epsilon, t) = [\Phi(\epsilon)].\, q(t)$$

The homogeneous equation should have been used to get the modal matrices and the mode shape matrix but since we already have them in the given data (fisez), we skipped this phase. Phi contains the mode shapes associated with the displacements y, z and the rotation theta.
We also used the database of the 14th natural mode shapes for all 3 directions for all the 243 nodes of the bridge. Thanks to this data, we were able to compute the full bridge analysis.

1) Computation of the static problem:

Neglecting acceleration and velocity terms, we compute the solution qn0 of the following equation:

$$k_n q_{n0} = (\underline{\phi})^T \underline{F}_{stat} = \sum_{i=1}^{N} (\phi_{yi}^n F_{yi}^{stat} + \phi_{zi}^n F_{zi}^{stat} + \phi_{\theta i}^n M_i^{stat})$$

with the force expressed as:

$$F_{yi}^{stat} = \frac{1}{2}\rho B L_i V^2 C_D(\theta_{0,i})$$
$$F_{zi}^{stat} = \frac{1}{2}\rho B L_i V^2 C_L(\theta_{0,i})$$
$$M_i^{stat} = \frac{1}{2}\rho B^2 L_i V^2 C_M(\theta_{0,i})$$

The aerodynamic forces have a coupling effect since the static coefficients Cd, Cl, Cm are function of the deck rotation:

$$\theta_{0,i} = \sum_{n=1}^{M} \phi_{\theta i}^n q_{n0}$$

As a result, we put in the MATLAB code the following equation, previously defining the forces of each axis separately, the previous angle of theta and interpolating the aerodynamic coefficient around this value of theta :

$$k_n q_{n0} - \frac{1}{2}\rho B \sum_{i=1}^{N} V_i^2 L_i \left[C_D \left(\sum_{n=1}^{M} \phi_{\theta i}^n q_{n0} \right) + \right.$$
$$\left. + C_L \left(\sum_{n=1}^{M} \phi_{\theta i}^n q_{n0} \right) + B C_M \left(\sum_{n=1}^{M} \phi_{\theta i}^n q_{n0} \right) \right] = 0$$

The equation and the interpolation of the aerodynamic coefficients are implicit because we are seeking for q0 which is present in the expression of the aerodynamic coefficients and on the second term of the differential equation.

We solve the nonlinear system of static equilibrium equations with iterative numerical methods. We express the displacements (y, z and theta static displacements using) the expression :

$$y = \phi_y . q_{n0}$$

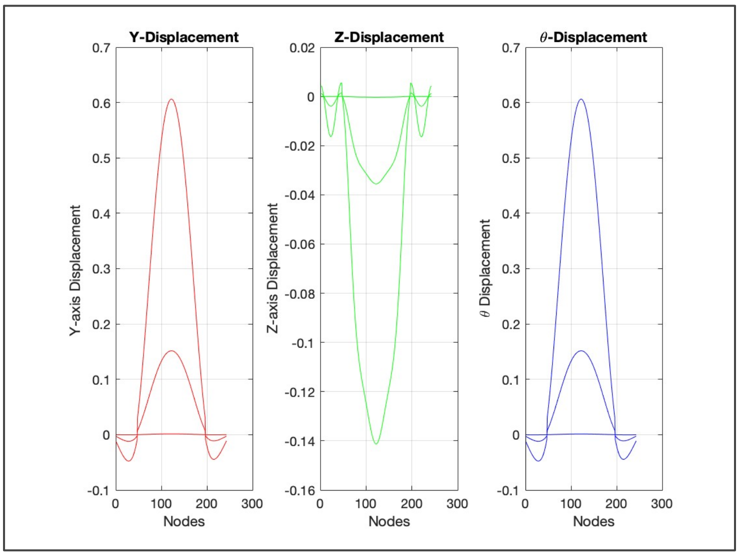


Figure 14: Displacements of the whole deck in each direction

Thanks to the iterative method, we computed the static displacements for every direction at every velocity (for every node). Compared to the sectional model, for the static displacement of the full bridge, we have for the same velocity, a smaller displacement for every direction. This is due to the fact that, in the full bridge analysis, the entire stiffness effect (the stiffness of all the sections combined) is taken into account.

The peak of the displacement for each direction is at the middle of the bridge. (It depends on the first mode shapes because they are the most influential on the behavior of the system.)

The three plots in each graph correspond to the displacements for 0, 40, 80 m/s.

2) <u>Self-excited forces</u>

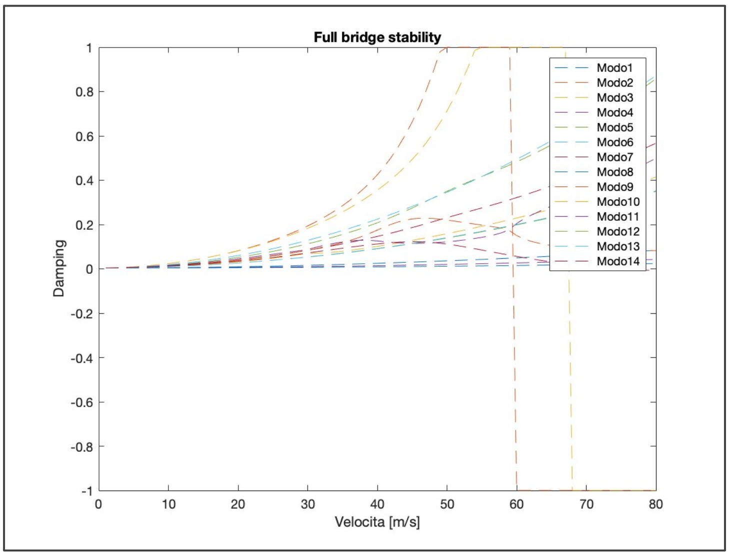
A part of the aerodynamic forces is generated by the motion of the body itself which is referred to as self-excited forces. Considering only this part of the aerodynamic forces, we have the equation of motion:

$$m_n\ddot{q_n} + r_n\dot{q_n} + k_n q_n = \sum_{i=1}^{N}(\phi_{yi}^n F_{yi}^{se} + \phi_{zi}^n F_{zi}^{se} + \phi_{\theta i}^n M_i^{se})$$

For each 243 node and each value of the wind speed (80m/s), we interpolated the value of the aerodynamic coefficient around the value of the static angle theta in order to construct the matrices used in the self-excited forces are expressed as:

$$\underline{F}_{se,i} = -([K_{aero,i}] \cdot \underline{x}_i + [R_{aero,i}] \cdot \dot{\underline{x}}_i)$$

Having the matrices Mn, Rn and Kn being the sum of the modal matrix with the aerodynamic matrix, we applied the eigenvalue method in order to get the damping values for each mode and each direction for all bridge.



<u>Figure 15:</u> Damping of the bridge for each mode

We got the damping for each velocity in Lagrangian coordinates. We were able to compute the damping value thanks to the dynamic response of the all bridge. This dynamic response is based on the solution of the dynamic system using the Quasi Steady Theory which is a linear method. The aerodynamic coefficients were taken from the static solution (interpolated around the nonlinear value of theta). It gave us the aerodynamic matrices. Using the eigenvalue method, we defined the damping and the frequency for each velocity and modal coordinates.

We directly identify two modes that exhibit huge instabilities (flutter instability): mode 2 which damping drastically falls down at 60 m/s and same phenomena with the mode 3 which almost fall at the flutter velocity found for the sectional mode 68 m/s (we found 66 m/s). They both are flexural Z modes and have a big increase of the damping before the instability occurs (such as in the sectional model).

The first mode doesn't give instability because of its bond to the Y direction (one of the blue dashed line close to 0).

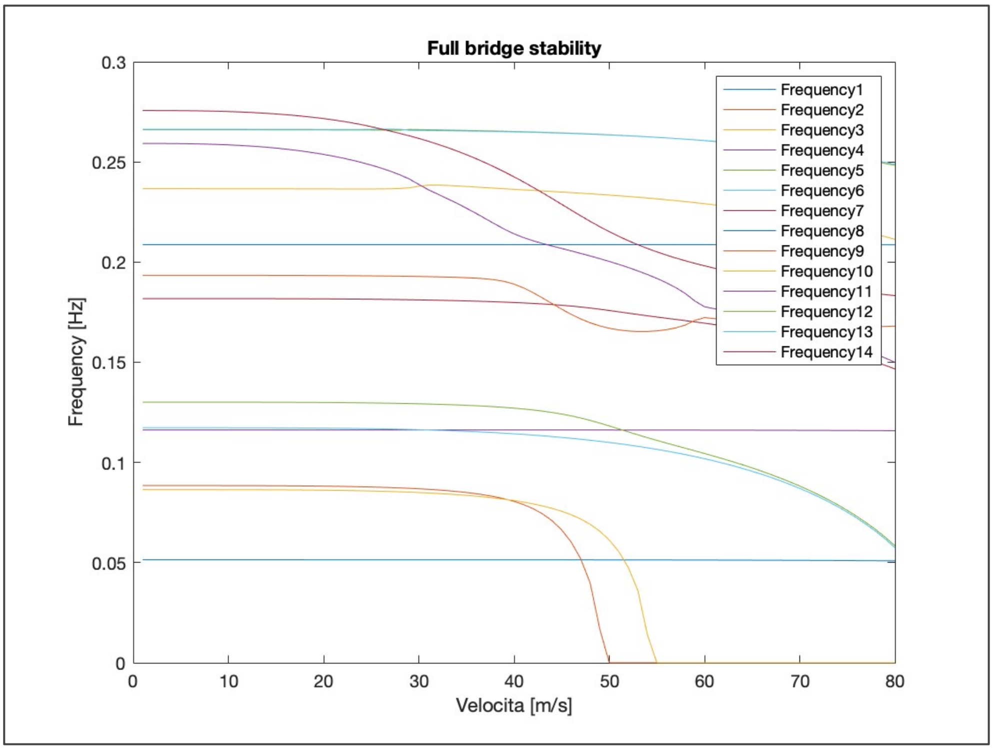


Figure 16: Frequencies of the whole deck

By looking at the graph, we saw that frequency 9 crosses the frequency 11 respectively and 69 m/s. Thanks to the damping graph, we can say that the instability is occurring between 60 and 70 m/s. There are also crossings before these values of velocity but we saw earlier on the damping graph that the instability was really high at 60 m/s and 70 m/s so we can neglect their impact on the instability behavior of the bridge.

Frequency 9 is a flexural mode and frequency 11 is a torsional mode. It respects the definition of the flutter instability being a coupling between a flexural and a torsional mode/frequency.

Annex

Air density	$\rho = 1{,}25\ kg/m3$
Deck chord	$B = 35{,}6\ m$
Deck length	$L = 1\ m$
Horizontal mass	$m_y = 23160\ kg/\ m3$
Vertical mass	$m_z = m_y = 23160\ kg/\ m3$
Inertia	$J = 2{,}77 * 10^{\wedge 6}\ kg/\ m2$
Damping coefficient	$\zeta = 4 * 10^{\wedge 3}$
Horizontal frequency	$f_y = 0{,}05\ Hz$
Vertical frequency	$f_z = 0{,}0884\ Hz$
Torsional frequency	$f_t = 0{,}0259\ Hz$
Horizontal stiffness	$k_y = (2\,\pi\, f_y)^{\wedge 2}.\, m_y$
Vertical stiffness	$k_z = (2\,\pi\, f_z)^{\wedge 2}.\, m_z$
Torsional stiffness	$k_t = (2\,\pi\, f_t)^{\wedge 2}.\, m_t$
Horizontal damping	$r_y = \zeta.\, 2.\, m_y\, .2\pi\, f_y\ .$

Vertical damping	$r_Z = \zeta . 2 . m_Z .2\pi \; f_Z$.
Torsional damping	$r_t = \zeta . 2 . m_t .2\pi \; f_t$.