Heat Simulation

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1 Software

This project requires no software or packages to download that do not come with the raspberry pi 4.

2 Hardware

This project requires no external hardware.

3 Project Description

My project simulates the heat equation for an anisotropic homogeneous medium in two dimensions. It simulates a rectangular object surrounded by a medium, which can have a different thermal diffusivity constant. The medium is surrounded by a thermal reservoir. The thermal diffusivity of the object and the medium, the temperature of the object, medium, and reservoir, as well as the location and dimensions of the object can all be determined by the user. The reservoir will have the same thermal diffusivity constant as the medium. I used the more generalized heat equation for anisotropic homogeneous mediums, which is as follows:

$$\begin{aligned} \frac{du(x,y,t)}{dt} &= \nabla \cdot (k(x,y)\nabla u(x,y,t)) \\ &= \frac{du}{dx}\frac{dk}{dx} + \frac{du}{dy}\frac{dk}{dy} + k(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}) \end{aligned} \tag{1}$$

Where u is the heat at the point x, y and k is the thermal diffusivity. Using the finite difference method, this equation is approximately:

$$
\begin{aligned}
\frac{u(t+\Delta t)-u}{\Delta t} &= \frac{u(x+\Delta x)-u}{\Delta x}\frac{k(x+\Delta x)-k}{\Delta x}+\frac{u(y+\Delta y)-u}{\Delta y}\frac{k(y+\Delta y)-k}{\Delta y} \\
&+k(\frac{u(x+\Delta x)+u(y+\Delta y)+u(x-\Delta x)+u(y-\Delta y)-4u}{\Delta x\Delta y}) \\
&= \frac{(u(x+\Delta x)-u(x-\Delta x))}{2\Delta x}\frac{(k(x+\Delta x)-k(x-\Delta x))}{2\Delta x} \\
&+\frac{(u(y+\Delta y)-u(y-\Delta y))}{2\Delta y}\frac{(k(y+\Delta y)-k(y-\Delta y))}{2\Delta y} \\
&+k(\frac{u(x+\Delta x)+u(y+\Delta y)+u(x-\Delta x)+u(y-\Delta y)-4u}{\Delta x\Delta y})
\end{aligned} \tag{2}
$$

Where the first two terms were changed to make the derivative symmetric about a point.

Then, assuming it is a square grid, $\Delta x = \Delta y = \Delta h$, and

$$
\begin{aligned}
u(t+\Delta t) = \frac{\Delta t}{\Delta h^2}\Bigg[&\frac{(u(x+\Delta x)-u(x-\Delta x))(k(x+\Delta x)-k(x-\Delta x))}{4} \\
&+\frac{(u(y+\Delta y)-u(y-\Delta y))(k(y+\Delta y)-k(y-\Delta y))}{4} \\
&+u(x+\Delta x)+u(y+\Delta y)+u(x-\Delta x)+u(y-\Delta y)-4u+u
\end{aligned} \tag{3}
$$

Note that the constant $\frac{\Delta t}{\Delta h^2}$ needs to be sufficiently small, otherwise the simulation will exhibit weird behavior. Necessarily, the difference in k will be smaller than the value of k itself (it is always positive), so the constant is changed based on the value of k that is the largest between the two media.

4 Results

After running the code, a menu will print showing the various options to change, such as the graph length and width, object length and width, k values for both mediums, and initial heat values for the object, surrounding media, and boundaries. Typing the number and pressing enter will allow you to change each of the values. After changing a value or, to leave it in the preset values, type 'c' and enter to start the animation.

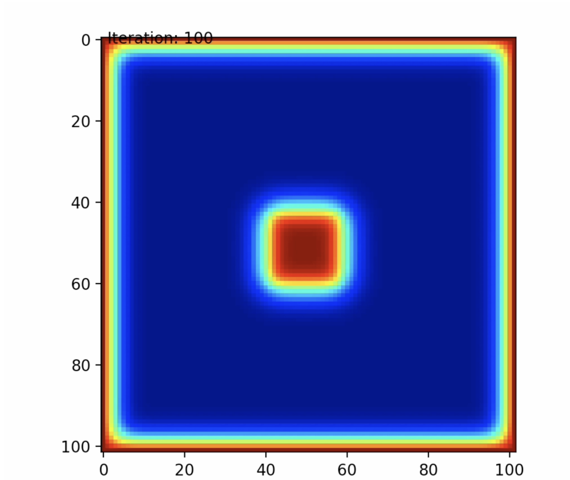

Figure 1: *Graph after 100 iterations, with the thermal diffusivity of object at* $k = 20$

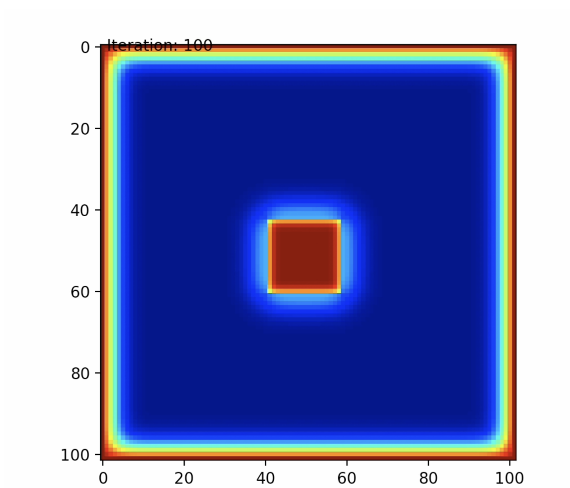

Figure 2: *Graph after 100 iterations, with the thermal diffusivity of object at* $k = 80$