CBSE Class 12, 2016, 065/SET2/N

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1 Matrices

1. If A is 3×3 matrix and $|3A| = k|A|$, then write the value of k.

2. If $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$, find $alpha$ satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}I_2$; where A^T is transpose of A.

3. Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

4. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$ find k.

2 Probability

5. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

6. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

7. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

3 Vector

8. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

9. Find λ and μ if
$$\left(\hat{i} + 3\hat{j} + 9\hat{k}\right) \times \left(3\hat{i} - \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$$

10. Write the sum of intercepts cut by the plane $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axes.

11. Find the equations of the plane which contains the line of intersection of the planes

$$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0$$
$$\vec{r} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$$

and whose intercept on x-axis is equal to that of y-axis.

4 3D-Geometry

12. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $A(2, 3, -1)$. Hence find the image of the point A in the line BC.

13. Show that the four point $A(4, 5, 1)$, $B(0, -1, -1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

5 Differention

14. Find the particular solution of the differential equation

$$2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$$

given that $x = 0$ when $y = 1$.

15. If $x\cos(a+y) = \cos y$ then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin^2(a+y)\frac{dy}{dx} = 0$.

16. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{6x - 4\sqrt{1-4x^2}}{5}\right)$

17. If

$$f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of a and b.

18. For what values of k, the system of linear equations

$$x + y + z = 2$$
$$2x + y + z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution?

6 Integration

19. Find: $\int (x+3) \sqrt{3 - 4x - x^2}dx$.

20. Find: $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

21. Find: $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$

22. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$.

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23. Find the particular solution of differential equation: $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$ given that $y = 1$ when $x = 0$.

24. Using the method of integration, find the area of the triangular region whose vertices are $(2, -2), (4, -3)$ and $(1, 2)$.

7 Trigonometry

25. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpenicular to line $x + 14y + 3 = 0$.

26. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$.

27. Show that semi-vertical angle of a cone of a maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

28. Solve for x: $\tan^{-1}(x - 1) + \tan^{-1}(x + 1) = \tan^{-1}(3x)$.

29. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) + \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1} 2x; |2x| < \frac{1}{\sqrt{3}}$.

8 Functions

30. Let $A = R \times R$ and * be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Find the identity elemeny for * on A. Also find the inverse of every element $(a, b) \in A$.

9 Optimization

31. A typist charges |145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are |180. Using matrices, find the charges of typing one English and one English page separately. However typist charged only |2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this peeo boy? Which values are reflected in this problem?

32. A retired person wants to invest an amount of |50,000. His broker recommends investing in the type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least |20,000 in bond 'A' and at least |10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns.