EE1205 TA

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Question

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

parameters	value	description
a_0	2	first term of AP
d		common difference of AP

The Z-transform of a sequence $x(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \quad = \frac{a_0}{1 - z^{-1}} + \frac{dz^{-1}}{\left(1 - z^{-1}\right)^2} \tag{1}$$

Sum of first k terms S_k of the AP can be written as:

$$S_k = \sum_{m=0}^{k-1} \frac{1}{m!} \frac{d^m}{d\left(z^{-1}\right)^m} A(z) \bigg|_{z^{-1}=0} \tag{2}$$

$$= \sum_{m=0}^{k-1} \frac{1}{m!} \frac{d^m}{d\left(z^{-1}\right)^m} \left(\frac{a_0}{1 - z^{-1}} + \frac{dz^{-1}}{\left(1 - z^{-1}\right)^2} \right)_{z^{-1}=0} \tag{3}$$

$$= \sum_{m=0}^{k-1} \frac{a_0}{m!} \frac{d^m}{d\left(z^{-1}\right)^m} \left(\frac{1}{1 - z^{-1}} \right)_{z^{-1}=0} + \sum_{m=0}^{k-1} \frac{d}{m!} \frac{d^m}{d\left(z^{-1}\right)^m} \left(\frac{z^{-1}}{\left(1 - z^{-1}\right)^2} \right)_{z^{-1}=0} \tag{4}$$

$$= a_0 \sum_{m=0}^{k-1} \left(\frac{1}{1 - z^{-1}} \right)^{m+1}_{z^{-1}=0} + d \sum_{m=0}^{k-1} \left(\frac{m}{\left(1 - z^{-1}\right)^{m+1}} \right)_{z^{-1}=0} \tag{5}$$

After simplifying the above expression we get:

$$S_k = a_0 k + d \frac{(k)(k-1)}{2} \tag{6}$$

$$= 2k + d \frac{(k)(k-1)}{2} \tag{7}$$

From the given information:

$$S_5 = \frac{1}{4}\left(S_{10} - S_5\right) \tag{8}$$

$$5S_5 = S_{10} \tag{9}$$

$$5(10 + 10d) = 20 + 45d \tag{10}$$

$$\implies d = -6 \tag{11}$$

Any term of the AP can be given by:

$$a_k = \frac{1}{k!} \frac{d^k}{d\left(z^{-1}\right)^k} A(z) \bigg|_{z^{-1}=0} \tag{12}$$

Therefore the 20^{th} is given by:

$$a_{19} = \frac{1}{19!} \frac{d^{19}}{d\left(z^{-1}\right)^{19}} A(z) \bigg|_{z^{-1}=0} \tag{13}$$

$$= a_0 \left(\frac{1}{\left(1 - z^{-1}\right)^{20}} \right)_{z^{-1}=0} + d \left(\frac{19}{\left(1 - z^{-1}\right)^{20}} \right)_{z^{-1}=0} \tag{14}$$

$$\implies a_{19} = -112 \tag{15}$$