EE23010 Assignment

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Question 12.13.3.55

There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X.

Solution:

parameters	description
A	number on the first card
B	number on the second card

$$p_A(i) = \begin{cases} \frac{1}{5}, & 1 \le i \le 5 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$p_{AB}(k, i) = \Pr(B = k \mid A = i)\, p_A(i) \quad (2)$$

$$= \begin{cases} \frac{1}{20}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$p_B(k) = \sum_{i=-\infty}^{\infty} p_{AB}(k, i) \quad (4)$$

$$= \sum_{i=1}^{5} p_{AB}(k, i) \quad (5)$$

$$\implies p_B(k) = \begin{cases} \frac{1}{5}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Since $p_A(k) = p_B(k)$, A and B are identical.

$$X = A + B \quad (7)$$

$$\mathbb{E}[X] = \mathbb{E}[A + B] \quad (8)$$

$$= \mathbb{E}[A] + \mathbb{E}[B] \quad (9)$$

$$= \mathbb{E}[A] + \mathbb{E}[A] \quad (10)$$

$$= 2\mathbb{E}[A] \quad (11)$$

$$= 2\sum_{i=1}^{5} i\, p_A(i) \quad (12)$$

$$= 6 \quad (13)$$

$$\text{Var}\{X\} = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \quad (14)$$

$$= \mathbb{E}[(A + B)^2] - 6^2 \quad (15)$$

$$= \mathbb{E}[A^2 + B^2 + 2AB] - (2\mathbb{E}[A])^2 \quad (16)$$

$$= \mathbb{E}[A^2] + \mathbb{E}[B^2] + 2\mathbb{E}[AB] - 36 \quad (17)$$

$$= \mathbb{E}[A^2] + \mathbb{E}[A^2] + 2\sum_{i=1}^{5}\sum_{k=1}^{5} ik\, p_{AB}(k, i) - 36 \quad (18)$$

$$= 2\sum_{i=1}^{5} i^2 p_A(i) + \frac{2}{20}(170) - 36 \quad (19)$$

$$= 22 + 17 - 36 \quad (20)$$

$$= 3 \quad (21)$$