

EE23010 Assignment

Sayyam Palrecha*

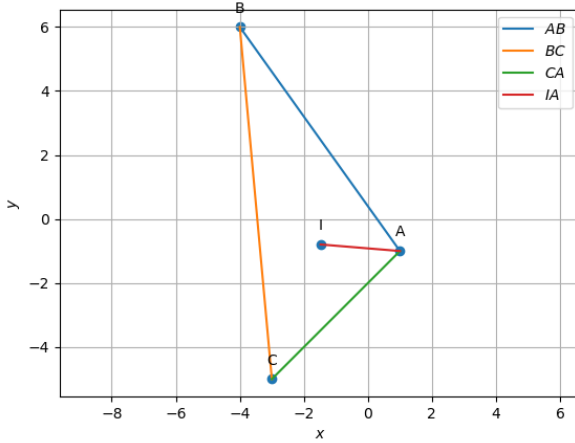


Fig. 0. Triangle generated using python

Question 1.5.3 Using (1.1.7.1) verify that

$$\angle BAI = \angle CAI. \quad (1)$$

Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (3)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (4)$$

The intersection \mathbf{I} of the angle bisectors of B and C :

$$\mathbf{I} = \frac{1}{\sqrt{37} + 4 + \sqrt{61}} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \quad (5)$$

$$\cos A = \frac{(\mathbf{B} - \mathbf{A})^\top \mathbf{C} - \mathbf{A}}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \quad (6)$$

Solution:

We need to verify

$$\angle BAI = \angle CAI. \quad (7)$$

consider LHS:

$$\cos \angle BAI = \frac{(\mathbf{B} - \mathbf{A})^\top \mathbf{I} - \mathbf{A}}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{I} - \mathbf{A}\|} \quad (8)$$

$$\begin{aligned} & \left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)^\top \left(\frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) \\ &= \frac{\left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)^\top \left(\frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)}{\left\| \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \left\| \frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\|} \end{aligned} \quad (9)$$

$$\begin{aligned} & \begin{pmatrix} -5 \\ 7 \end{pmatrix}^\top \left(\frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1} \right) \\ &= \frac{\begin{pmatrix} -5 \\ 7 \end{pmatrix}^\top \left(\frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1} \right)}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\| \left\| \frac{\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1}{-\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1} \right\|} \end{aligned} \quad (10)$$

on simplifying \mathbf{I} , we get

$$\mathbf{I} = \begin{pmatrix} -1.47756 \\ -0.79495 \end{pmatrix} \quad (11)$$

$$\begin{aligned} & \begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -1.47756 - 1 \\ -0.79495 + 1 \end{pmatrix} \\ &= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -1.47756 - 1 \\ -0.79495 + 1 \end{pmatrix}}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\| \left\| \begin{pmatrix} -1.47756 - 1 \\ -0.79495 + 1 \end{pmatrix} \right\|} \end{aligned} \quad (12)$$

$$\begin{aligned} & \begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix} \\ &= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\| \left\| \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix} \right\|} \end{aligned} \quad (13)$$

$$\begin{aligned} & \frac{13.82315}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\| \left\| \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix} \right\|} \\ &= \frac{13.82315}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\| \left\| \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix} \right\|} \end{aligned} \quad (14)$$

from (1.1.2.1) length of the side AB

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top \mathbf{B} - \mathbf{A}} \quad (15)$$

$$= \frac{13.82315}{\sqrt{74} \sqrt{6.2010538036}} \quad (16)$$

$$\Rightarrow \cos \angle BAI = 0.64529 \quad (17)$$

$$\Rightarrow \angle BAI = \cos^{-1} 0.64529 \quad (18)$$

$$= 49.7311 \quad (19)$$

consider RHS:

$$\cos \angle CAI = \frac{(\mathbf{I} - \mathbf{A})^\top \mathbf{C} - \mathbf{A}}{\|\mathbf{I} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \quad (20)$$

$$= \frac{\left(\begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ -\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)^\top \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left\| \begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ -\frac{\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \left\| \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\|} \quad (21)$$

$$= \frac{\left(\begin{pmatrix} -1.47756 \\ -0.79495 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)^\top \begin{pmatrix} -4 \\ -4 \end{pmatrix}}{\left\| \begin{pmatrix} -1.47756 \\ -0.79495 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \left\| \begin{pmatrix} -4 \\ -4 \end{pmatrix} \right\|} \quad (22)$$

$$= \frac{-4 \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}^\top \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{4 \left\| \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|} \quad (23)$$

$$= - \frac{\begin{pmatrix} -2.47756 & 0.20505 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\sqrt{\begin{pmatrix} -2.47756 & 0.20505 \end{pmatrix} \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}} \sqrt{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}} \quad (24)$$

$$= \frac{2.27251}{\sqrt{6.2010538036} \sqrt{2}} \quad (25)$$

$$\Rightarrow \cos \angle CAI = 0.64529 \quad (26)$$

$$\Rightarrow \angle CAI = \cos^{-1} 0.64529 \quad (27)$$

$$= 49.7311 \quad (28)$$

Therefore from the equations (19) and (28), we get:

$$\angle BAI = \angle CAI \quad (29)$$

$$\therefore LHS = RHS \quad (30)$$

Hence we have verified that

$$\angle BAI = \angle CAI. \quad (31)$$