

EE23010 Assignment

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Question 12.13.3.11

Prove that

- 1) $\Pr(A) = \Pr(AB) + \Pr(AB')$
- 2) $\Pr(A + B) = \Pr(AB) + \Pr(AB') + \Pr(A'B)$

Solution:

- 1) consider *RHS*:

$$A = A(B + B') \quad (1)$$

$$\Pr(A) = \Pr(A(B + B')) \quad (2)$$

$$= \Pr(AB + AB') \quad (3)$$

$$= \Pr(AB) + \Pr(AB') - \Pr((AB)(AB')) \quad (4)$$

$$= \Pr(AB) + \Pr(AB') - \Pr(ABB') \quad (5)$$

$$= \Pr(AB) + \Pr(AB') \quad (6)$$

- 2) consider *RHS*:

$$A + B = A(B + B') + B(A + A') \quad (7)$$

$$\Pr(A + B) = \Pr(A(B + B') + B(A + A')) \quad (8)$$

$$= \Pr(AB + AB' + BA' + BA) \quad (9)$$

$$= \Pr(AB + AB' + BA') \quad (10)$$

But,

$$AB(AB') = 0 \quad (11)$$

$$AB(A'B) = 0 \quad (12)$$

$$AB'(A'B) = 0 \quad (13)$$

$\implies AB, AB', A'B$ are mutually exclusive as their pairwise product is zero.

$$\begin{aligned} \Pr(A + B) &= \Pr(AB) + \Pr(AB') + \Pr(A'B) - \Pr(AB(AB')) \\ &\quad - \Pr(AB'(A'B)) - \Pr(A'B(AB)) + \Pr((AB)(AB')(A'B)) \end{aligned} \quad (14)$$

From (11), (12) and (13), we get:

$$= \Pr(AB) + \Pr(AB') + \Pr(A'B) - 0 - 0 - 0 + 0 \quad (15)$$

$$= \Pr(AB) + \Pr(AB') + \Pr(A'B) \quad (16)$$