EE23010 Assignment

Sayyam Palrecha* EE22BTECH11047

Question 12.13.3.11
Prove that

(i) $\Pr(A) = \Pr(AB) + \Pr\left(A\overline{B}\right)$
(ii) $\Pr(A + B) = \Pr(AB) + \Pr\left(A\overline{B}\right) + \Pr\left(\overline{A}B\right)$

Solution:

(i) consider RHS:

$$\Pr(A) = \Pr\left(A(B + \overline{B})\right) \tag{1}$$
$$= \Pr\left(AB + A\overline{B}\right) \tag{2}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) - \Pr\left((AB)(A\overline{B})\right) \tag{3}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) - \Pr\left(AB\overline{B}\right) \tag{4}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) - 0 \tag{5}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) \tag{6}$$

From (6)

$$\Pr(A) = \Pr(AB) + \Pr\left(A\overline{B}\right) \tag{7}$$
$$\therefore LHS = RHS \tag{8}$$

(ii) consider RHS:

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{9}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) + \Pr(BA) + \Pr\left(\overline{A}B\right) - \Pr(AB) \tag{10}$$
$$= \Pr(AB) + \Pr\left(A\overline{B}\right) + \Pr\left(\overline{A}B\right) \tag{11}$$

Using the result (7), we have proved:

$$\Pr(A + B) = \Pr(AB) + \Pr\left(A\overline{B}\right) + \Pr\left(\overline{A}B\right) \tag{12}$$
$$\therefore LHS = RHS \tag{13}$$