

EE23010 Assignment

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Question 10.13.3.19

Two dice are thrown at the same time. Find the probability of getting

- (i) same number on both dice.
- (ii) different numbers on both dice.

Solution: Let the random variables:

X	outcome of the first die
Y	outcome of the second die

Since each die has 6 sides, both X and Y can take on values from the set $\{1, 2, 3, 4, 5, 6\}$.

Consider a random variable Z :

$$Z = X - Y \quad (1)$$

Z can take values ranging from $\{-5$ to $5\}$.

We need to find the PMF of Z

We know that,

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$p_Y(k)$ is same as $p_X(k)$.

(1) Finding the solution using convolution:

$$p_Z(k) = P(k = X - Y) \quad (3)$$

$$= P(X = k + Y) \quad (4)$$

We arrive at the expectation of Z :

$$E(p_X(k + Y)) = \sum_{m=1}^6 [p_X(k + m) * p_Y(m)] \quad (5)$$

$$= \frac{1}{6} \sum_{m=1}^6 p_X(k + m) \quad (6)$$

(i) Finding the probability for $Z = 0$

We need to find PMF(Z) at $Z = 0$

$$P(Z = 0) = \frac{1}{6} \sum_{m=1}^6 p_X(0 + m) \quad (7)$$

$$= \frac{1}{6} \sum_{m=1}^6 p_X(m) \quad (8)$$

$$= \frac{1}{6} \sum_{m=1}^6 \frac{1}{6} \quad (9)$$

$$= \frac{1}{6} (1) \quad (10)$$

$$= \frac{1}{6} \quad (11)$$

(ii) Finding the probability for $Z \neq 0$

$$P(Z \neq 0) = 1 - P(Z = 0) \quad (12)$$

$$= 1 - \frac{1}{6} \quad (13)$$

$$= \frac{5}{6} \quad (14)$$

(2) Finding the solution using z-transform:

(i) PMF of Z using z -transform:

$$P(k = Z) = P(k = X - Y) \quad (15)$$

applying the z -transform on both the sides

$$z\{P(k = Z)\} = z\{P(k = X - Y)\} \quad (16)$$

$$p_Z(k) = z\{p_X(k) * p_{-Y}(k)\} \quad (17)$$

$$= \left(\sum_{m=1}^6 p_X(m) \cdot z^m \right) * \left(\sum_{m=1}^6 p_Y(m) \cdot z^{-m} \right) \quad (18)$$

$$= \left(\frac{1}{6} \sum_{m=1}^6 z^m \right) \left(\frac{1}{6} \sum_{m=1}^6 z^{-m} \right) \quad (19)$$

$$= \frac{1}{36} (z^1 + z^2 + z^3 + z^4 + z^5 + z^6) \cdot (z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}) \quad (20)$$

The coefficient of z^k is the probability $P(k = Z)$

$$p_Z(0) = \frac{1}{36} (z^1 z^{-1} + z^2 z^{-2} + z^3 z^{-3} + z^4 z^{-4} + z^5 z^{-5} + z^6 z^{-6}) \quad (21)$$

$$= \frac{1}{36}(6) \quad (22)$$

$$= \frac{1}{6} \quad (23)$$

(ii) Finding the probability for $Z \neq 0$

$$P(Z \neq 0) = 1 - P(Z = 0) \quad (24)$$

$$= 1 - \frac{1}{6} \quad (25)$$

$$= \frac{5}{6} \quad (26)$$