

EE23010 Assignment

Sayyam Palrecha* EE22BTECH11047

Question 46

Let X be a random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4}(x+1) & \text{if } -1 \leq x < 0 \\ \frac{1}{4}(x+3) & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases} \quad (1)$$

Which one of the following statements is true?

(A)

$$\lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) = \frac{5}{8} \quad (2)$$

(B)

$$\lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) = \frac{5}{8} \quad (3)$$

(C)

$$\lim_{n \rightarrow \infty} \Pr\left(X = \frac{1}{n}\right) = \frac{1}{2} \quad (4)$$

(D)

$$\Pr(X = 0) = \frac{1}{3} \quad (5)$$

(GATE ST 2023)

Solution:

$$f_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4} & \text{if } -1 \leq x < 0 \\ \frac{1}{2} & \text{if } x = 0 \\ \frac{1}{4} & \text{if } 0 < x < 1 \\ 0 & \text{if } x \geq 1 \end{cases} \quad (6)$$

(A)

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) \\ = \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} + \frac{1}{n}\right) \end{aligned} \quad (7)$$

$$= \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} + \frac{1}{n}\right) \quad (8)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{n} + 1\right) - \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{2} + \frac{1}{n} + 1\right) \quad (9)$$

$$= \frac{1}{8} \quad (10)$$

\therefore (A) is not true.

(B)

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) \\ = \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} - \frac{1}{n}\right) \end{aligned} \quad (11)$$

$$= \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} - \frac{1}{n}\right) \quad (12)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4}\left(\frac{1}{n} + 3\right) - \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{2} - \frac{1}{n} + 1\right) \quad (13)$$

$$= \frac{5}{8} \quad (14)$$

\therefore (B) is true.

(C) From (6)

$$\lim_{n \rightarrow \infty} \Pr\left(X = \frac{1}{n}\right) = 0 \quad (15)$$

\therefore (C) is not true.

(D) From (6)

$$\Pr(X = 0) = \frac{1}{2} \quad (16)$$

\therefore (D) is not true.

Steps for the simulation of r.v X :

- 1) Identify the point of discontinuity (0 here).
- 2) Define the simulation size for the simulation data set (num_sim).
- 3) Define the functions of CDF and PDF of X .
- 4) Find $\Pr(X = 0)$ from the PDF of X .
- 5) For this simulation, the remaining numbers in $[-1, 1)$ have probability of $1 - \Pr(X = 0)$.
- 6) Generate random sample in $[-1, 1) - \{0\}$ of the size = num_sim \times (1 - $\Pr(X = 0)$).

- 7) Generate sample containing only zeros of the size = $\text{num_sim} \times \Pr(X = 0)$.
- 8) Combine all the generated samples to make a single sample and we generate the required r.v X .

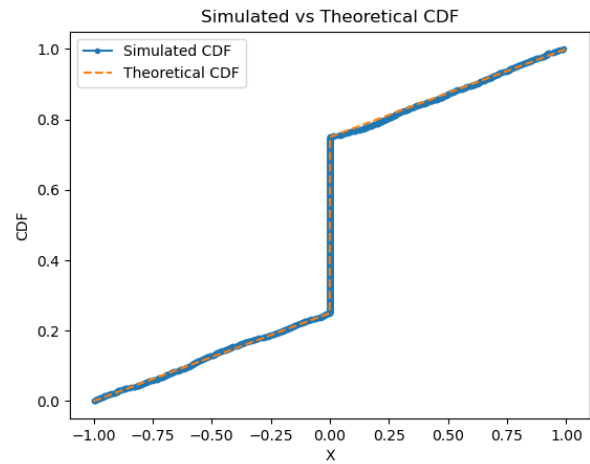


Fig. 8. CDF of X -(simulation vs actual)