EE23010 Assignment

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Question 12.13.3.11
How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution:

1) Binomial:
Let, X_i be the sequence of independent Bernoulli random varibles.

$$\implies X = \sum_{i=1}^{n} X_i \tag{1}$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{2}$$

$$p_X(k) = \begin{cases} 0.5 = p, & k = 0 \\ 0.5 = q, & k = 1 \end{cases} \tag{3}$$

Let, the total number of trials be n and the PMF of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^nC_k\,(p)^k\,(q)^{n-k} \tag{5}$$
$$= {}^nC_k\,(0.5)^k\,(0.5)^{n-k} \tag{6}$$

The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{7}$$
$$= \sum_{i=0}^{k} {}^nC_i\,(0.5)^{n-i}\,(0.5)^i \tag{8}$$

$$\Pr(X > 0) > 0.9 \tag{9}$$
$$1 - p_X(0) > 0.9 \tag{10}$$
$$(2)^n > 10 \tag{11}$$
$$n > \log_2(10) \tag{12}$$
$$n > 3.32 \tag{13}$$
$$\implies n = 4 \tag{14}$$

2) Gaussian:
Let Y be the gaussian variable,

$$\mu = np \tag{15}$$
$$= \frac{n}{2} \tag{16}$$
$$\sigma^2 = np(1-p) \tag{17}$$
$$= \frac{n}{4} \tag{18}$$

Using the normal-distribution of Y,

$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \tag{19}$$

$$\Pr(Y > 0) = 1 - \Pr(Y = 0) \tag{20}$$
$$= 1 - f_Y(0) \tag{21}$$
$$= 1 - \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(0-\mu)^2}{2\sigma^2}} \tag{22}$$
$$= 1 - \frac{2}{\sqrt{2\pi n}} e^{-\frac{n}{2}} \tag{23}$$

$$\Pr(Y > 0) > 0.9 \tag{24}$$
$$1 - \frac{2}{\sqrt{2\pi n}} e^{-\frac{n}{2}} > 0.9 \tag{25}$$
$$\sqrt{\frac{2}{\pi n}} e^{-\frac{n}{2}} < 0.1 \tag{26}$$
$$\frac{2}{\pi n} e^{-n} < 0.01 \tag{27}$$
$$e^{-n} < 0.005\pi n \tag{28}$$
$$-n < \ln 0.0157 + \ln n \tag{29}$$
$$n + \ln n > 4.15 \tag{30}$$
$$\implies n = 4 \tag{31}$$

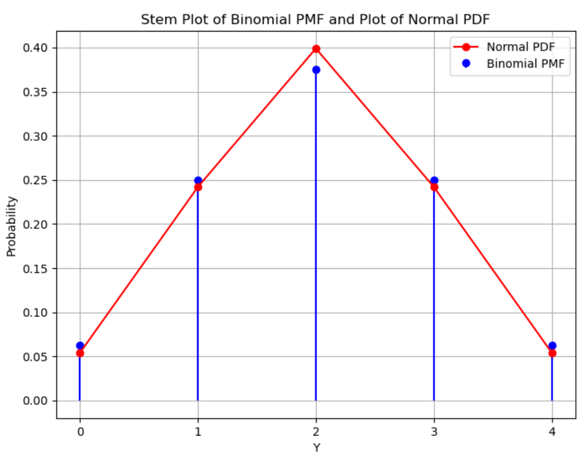

Fig. 2. Binomial PMF of X vs Normal PDF of Y