EE23010 Assignment

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Question 12.13.3.11
Prove that

(i) $\Pr(A) = \Pr(AB) + \Pr(AB')$

(ii) $\Pr(A+B) = \Pr(AB) + \Pr(AB') + \Pr(A'B)$

Solution:

(i) consider RHS:

$$A = A(B+B') \tag{1}$$
$$\Pr(A) = \Pr(A(B+B')) \tag{2}$$
$$= \Pr(AB+AB') \tag{3}$$
$$= \Pr(AB) + \Pr(AB') - \Pr((AB)(AB')) \tag{4}$$
$$= \Pr(AB) + \Pr(AB') - \Pr(ABB') \tag{5}$$
$$= \Pr(AB) + \Pr(AB') \tag{6}$$

(ii) consider RHS:

$$A+B = A(B+B') + B(A+A') \tag{7}$$
$$\Pr(A+B) = \Pr(A(B+B') + B(A+A')) \tag{8}$$
$$= \Pr(AB+AB+AB'+BA') \tag{9}$$
$$= \Pr(AB+AB'+BA') \tag{10}$$

But,

$$AB(AB') = 0 \tag{11}$$
$$AB(A'B) = 0 \tag{12}$$
$$AB'(A'B) = 0 \tag{13}$$

$\implies AB, AB', A'B$ are mutually exclusive as their pairwise product is zero.

$$= \Pr(AB) + \Pr(AB') + \Pr(A'B) - \Pr(AB(AB')) - \Pr(AB'(A'B)) - \Pr(A'B(AB)) + \Pr((AB)(AB')(A'B)) \tag{14}$$

From (11), (12) and (13), we get:

$$= \Pr(AB) + \Pr(AB') + \Pr(A'B) - 0 - 0 - 0 + 0 \tag{15}$$
$$= \Pr(AB) + \Pr(AB') + \Pr(A'B) \tag{16}$$