EE23010 Assignment

Sayyam Palrecha* EE22BTECH11047

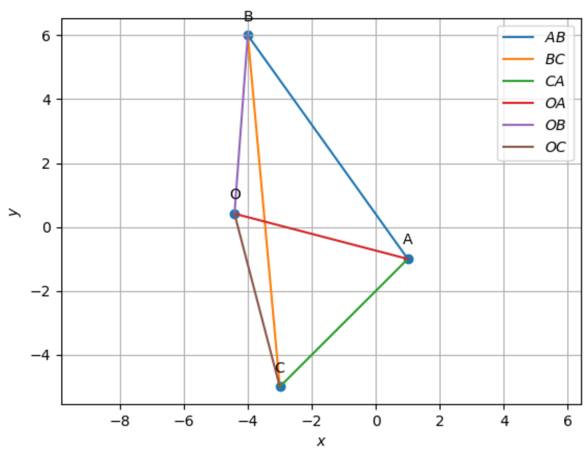

Fig. 0. Triangle generated using python

Question 1.5.3 verify that

$$OA = OB = OC \tag{1}$$

Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{4}$$

The intersection $\mathbf{O}$ of the perpendicular bisectors of AB and AC:

$$\mathbf{O} = \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{5}$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top \mathbf{B} - \mathbf{A}} \tag{6}$$

We need to verify

$$OA = OB = OC \tag{7}$$

consider OA:

$$OA = \|\mathbf{O} - \mathbf{A}\| \tag{8}$$
$$= \sqrt{(\mathbf{O} - \mathbf{A})^\top \mathbf{O} - \mathbf{A}} \tag{9}$$
$$= \sqrt{\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)^\top \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}} \tag{10}$$
$$= \sqrt{\begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}^\top \begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}} \tag{11}$$
$$= \sqrt{\begin{pmatrix} -\frac{65}{12} & \frac{17}{12} \end{pmatrix} \begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}} \tag{12}$$
$$= \sqrt{\left(\frac{65}{12}\right)^2 + \left(\frac{17}{12}\right)^2} \tag{13}$$
$$= 5.5988 \tag{14}$$

consider OB:

$$OB = \|\mathbf{O} - \mathbf{B}\| \tag{15}$$
$$= \sqrt{(\mathbf{O} - \mathbf{B})^\top \mathbf{O} - \mathbf{B}} \tag{16}$$
$$= \sqrt{\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}\right)^\top \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}} \tag{17}$$
$$= \sqrt{\begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}^\top \begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}} \tag{18}$$
$$= \sqrt{\begin{pmatrix} -\frac{5}{12} & -\frac{67}{12} \end{pmatrix} \begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}} \tag{19}$$
$$= \sqrt{\left(\frac{5}{12}\right)^2 + \left(\frac{67}{12}\right)^2} \tag{20}$$
$$= 5.5988 \tag{21}$$

consider OC:

$$OC = ||\mathbf{O} - \mathbf{C}|| \tag{22}$$
$$= \sqrt{(\mathbf{O} - \mathbf{C})^{\top}\mathbf{O} - \mathbf{C}} \tag{23}$$
$$= \sqrt{\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix}\right)^{\top} \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix}} \tag{24}$$
$$= \sqrt{\begin{pmatrix} -\frac{17}{12} \\ \frac{65}{12} \end{pmatrix}^{\top} \begin{pmatrix} -\frac{17}{12} \\ \frac{65}{12} \end{pmatrix}} \tag{25}$$
$$= \sqrt{\begin{pmatrix} -\frac{17}{12} & \frac{65}{12} \end{pmatrix}^{\top} \begin{pmatrix} -\frac{17}{12} \\ \frac{65}{12} \end{pmatrix}} \tag{26}$$
$$= \sqrt{\left(\frac{17}{12}\right)^2 + \left(\frac{65}{12}\right)^2} \tag{27}$$
$$= 5.5988 \tag{28}$$

Therefore from the equations (14), (21) and (28), we get:

$$OA = OB = OA \tag{29}$$

Hence we have verified that

$$OA = OB = OC \tag{30}$$