EE23010 Assignment

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Question 12.13.3.11
How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?
Solution:

Parameter	Value	Description
n	n	number of coin tosses
p	$\frac{1}{2}$	getting a head on a coin toss
q	$\frac{1}{2}$	getting a tail on a coin toss
$\mu = np$	$\frac{n}{2}$	mean of the distribution
$\sigma^2 = npq$	$\frac{n}{4}$	variance of the distribution
Y	≥ 1	Number of heads

1) Gaussian:

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

The CDF of Y:

$$F_Y(y) = 1 - \Pr\left(Y > y\right) \tag{2}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{y-\mu}{\sigma}\right) \tag{3}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}\left(0,1\right) \tag{4}$$
$$\implies F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{5}$$

a) Without correction

$$\Pr\left(Y \geq 1\right) = 1 - F_Y(1) \tag{6}$$

From the result (5)

$$Q\left(\frac{2-n}{\sqrt{n}}\right) > 0.9 \tag{7}$$
$$\frac{2-n}{\sqrt{n}} < Q^{-1}(0.9) \tag{8}$$
$$\frac{2-n}{\sqrt{n}} < -1.28 \tag{9}$$

Squaring on both the sides

$$(n-2)^2 > (1.28\sqrt{n})^2 \tag{10}$$
$$n^2 - 5.6384n + 4 > 0 \tag{11}$$
$$n > 4.86, n < 0.8 \tag{12}$$
$$\implies n = 5 \tag{13}$$

b) With correction: 0.5 as correction term

$$\Pr\left(Y > 0.5\right) = 1 - F_Y(0.5) \tag{14}$$

From the result (5)

$$Q\left(\frac{1-n}{\sqrt{n}}\right) > 0.9 \tag{15}$$
$$\frac{1-n}{\sqrt{n}} < Q^{-1}(0.9) \tag{16}$$
$$\frac{1-n}{\sqrt{n}} < -1.28 \tag{17}$$

Squaring on both the sides

$$(n-1)^2 > (1.28\sqrt{n})^2 \tag{18}$$
$$n^2 - 3.6384n + 1 > 0 \tag{19}$$
$$n > 3.38, n < 0.29 \tag{20}$$
$$\implies n = 4 \tag{21}$$

2) Binomial:

$$X \sim \text{Bin}\left(n, p\right) \tag{22}$$

The PMF of X is given by:

$$p_X\left(k\right) = {}^{n}C_k\left(0.5\right)^k\left(0.5\right)^{n-k} \tag{23}$$

The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{24}$$
$$= \sum_{i=0}^{k} {}^{n}C_i\left(0.5\right)^{n-i}\left(0.5\right)^i \tag{25}$$

We have

$$\Pr\left(X \geq 1\right) > 0.9 \tag{26}$$
$$1 - p_X(0) > 0.9 \tag{27}$$
$$(2)^n > 10 \tag{28}$$
$$n > \log_2(10) \tag{29}$$
$$\implies n = 4 \tag{30}$$

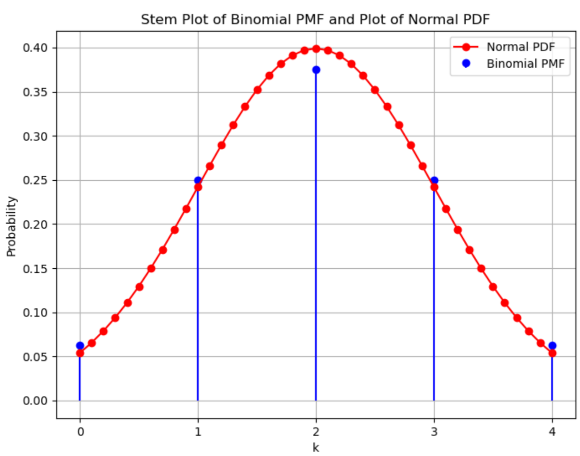

Fig. 2. Binomial PMF of X vs Normal PDF of Y