

EE23010 Assignment

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Question 12.13.3.55

There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X .

Solution:

parameters	description
A	number on the first card
B	number on the second card

$$p_A(k) = \begin{cases} \frac{1}{5}, & 1 \leq k \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$p_B(k) = \sum_{i=1}^5 \Pr(B = k | A = i) \cdot p_A(i) \quad (2)$$

$$= \frac{1}{5} \sum_{i=1}^5 \Pr(B = k | A = i) \quad (3)$$

$$\Pr(B = k | A = i) = \begin{cases} \frac{1}{4}, & 1 \leq k \leq 5, k \neq i \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$\Rightarrow p_B(k) = \begin{cases} \frac{1}{20}, & 1 \leq k \leq 5, k \neq i \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

X can take values ranging from 3 to 9.

$$X = A + B \quad (6)$$

$$p_X(k) = \sum_{i=1}^5 \Pr(X = k | A = i) \cdot p_A(i) \quad (7)$$

Finding mean or the expectation:

$$\mathbb{E}[X] = \sum_{k=3}^9 k \cdot p_X(k) \quad (8)$$

$$= \sum_{k=3}^9 k \sum_{i=1}^5 \Pr(X = k | A = i) \cdot p_A(i) \quad (9)$$

$$= \sum_{k=3}^9 k \sum_{i=1}^5 \Pr(B = k - i | A = i) \cdot p_A(i) \quad (10)$$

$$= \sum_{k=3}^9 k \cdot p_B(k - i) \quad (11)$$

$$p_B(k - i) = \begin{cases} \frac{1}{20}, & 1 \leq k - i \leq 5, k - i \neq i \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

$$= \begin{cases} \frac{1}{20}, & i + 1 \leq k \leq i + 5, k \neq 2i \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$\Rightarrow \mathbb{E}[X] = \frac{1}{20}(120) \quad (14)$$

$$= 6 \quad (15)$$

$$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \quad (16)$$

$$= \sum_{k=3}^9 k^2 p_B(k - i) - 6^2 \quad (17)$$

$$= 39 - 36 \quad (18)$$

$$= 3 \quad (19)$$

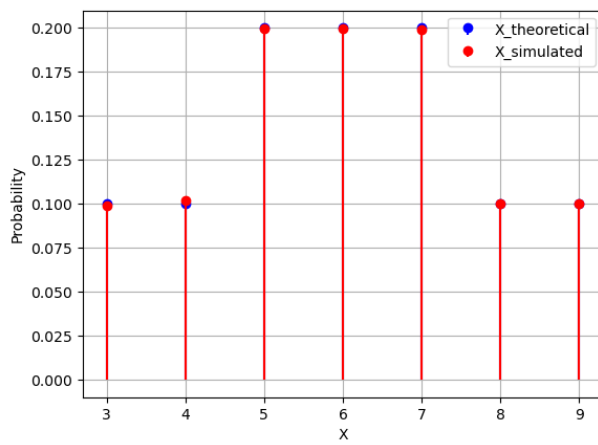


Fig. 0. PMF analysis of X ($p_X(k)$)