EE23010 Assignment

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Question 12.13.3.11
How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution:

1) Binomial:
   Let, X_i be the sequence of independent Bernoulli random varibles.

$$\implies X = \sum_{i=1}^{n} X_i \tag{1}$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{2}$$

$$X \sim \text{Bin}(n, p) \tag{3}$$

   Let, the total number of trials be n and the PMF of getting k heads is given by:

$$p_X(k) = {}^nC_k (0.5)^k (0.5)^{n-k} \tag{4}$$

   The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{5}$$

$$= \sum_{i=0}^{k} {}^nC_i (0.5)^{n-i} (0.5)^i \tag{6}$$

   We have

$$\Pr(X \geq 1) > 0.9 \tag{7}$$

$$1 - p_X(0) > 0.9 \tag{8}$$

$$(2)^n > 10 \tag{9}$$

$$n > \log_2(10) \tag{10}$$

$$\implies n = 4 \tag{11}$$

2) Gaussian:
   Let Y be the gaussian variable,

$$\mu = np = \frac{n}{2} \tag{12}$$

$$\sigma^2 = np(1-p) = \frac{n}{4} \tag{13}$$

   Let

$$Z \approx \frac{Y - \mu}{\sigma} \tag{14}$$

   Here, Z is a random variable with $\mathcal{N}(0, 1)$, The normal-Distribution of Z is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{15}$$

   The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{16}$$

$$= \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \tag{17}$$

a) Without correction

$$Y \geq 1 \tag{18}$$

$$Z \geq \frac{1-\mu}{\sigma} \tag{19}$$

$$Z \geq \frac{2-n}{\sqrt{n}} \tag{20}$$

   Since we know that,

$$\Pr(Z > x) > 0.9 \tag{21}$$

$$Q(x) > 0.9 \tag{22}$$

$$x > Q^{-1}(0.9) \tag{23}$$

$$\implies x > -1.28 \tag{24}$$

   From (20)

$$\frac{2-n}{\sqrt{n}} < -1.28 \tag{25}$$

$$n - 1.28\sqrt{n} > 2 \tag{26}$$

$$\implies n = 5 \tag{27}$$

b) With correction: 0.5 as correction term

$$Y > 0.5 \tag{28}$$

$$Z > \frac{1-n}{\sqrt{n}} \tag{29}$$

   From (24)

$$\frac{1-n}{\sqrt{n}} > -1.28 \tag{30}$$

$$n - 1.28\sqrt{n} > 1 \tag{31}$$

$$\implies n = 4 \tag{32}$$

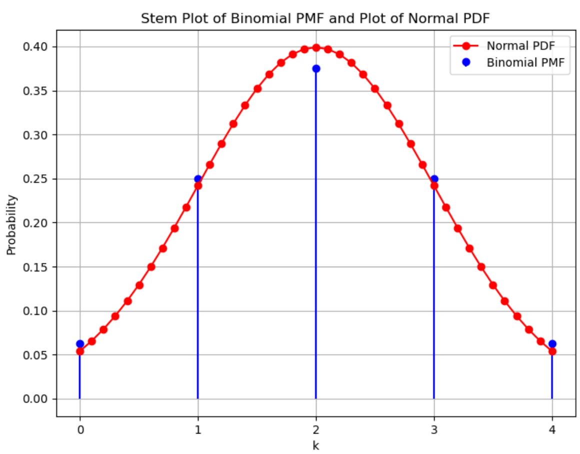

Fig. 2. Binomial PMF of X vs Normal PDF of Y