EE23010 Assignment

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Question 12.13.3.55
There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X.

Solution:

parameters	description
A	number on the first card
B	number on the second card

$$p_A(i) = \begin{cases} \frac{1}{5}, & 1 \le i \le 5 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$p_{AB}(k,i) = \Pr(B = k \mid A = i)\, p_A(i) \tag{2}$$

$$= \begin{cases} \frac{1}{20}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

$$p_B(k) = \sum_{i=-\infty}^{\infty} p_{AB}(k,i) \tag{4}$$

$$= \sum_{i=1}^{5} p_{AB}(k,i) \tag{5}$$

$$\implies p_B(k) = \begin{cases} \frac{1}{5}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Since $p_A(k) = p_B(k)$, A and B are identical.

$$X = A + B \tag{7}$$

$$\mathbb{E}[X] = \mathbb{E}[A + B] \tag{8}$$

$$= \mathbb{E}[A] + \mathbb{E}[B] \tag{9}$$

$$= \mathbb{E}[A] + \mathbb{E}[A] \tag{10}$$

$$= 2\mathbb{E}[A] \tag{11}$$

$$= 2\sum_{i=1}^{5} i p_A(i) \tag{12}$$

$$= 6 \tag{13}$$

$$\mathrm{Var}\{X\} = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \tag{14}$$

$$= \mathbb{E}[(A + B)^2] - 6^2 \tag{15}$$

$$= \mathbb{E}[A^2 + B^2 + 2AB] - 36 \tag{16}$$

$$= \mathbb{E}[A^2] + \mathbb{E}[B^2] + 2\mathbb{E}[AB] - 36 \tag{17}$$

$$= \mathbb{E}[A^2] + \mathbb{E}[A^2] + 2\sum_{i=1}^{5}\sum_{k=1}^{5} ik\, p_{AB}(k,i) - 36 \tag{18}$$

$$= 2\sum_{i=1}^{5} i^2 p_A(i) + \frac{2}{20}(170) - 36 \tag{19}$$

$$= 22 + 17 - 36 \tag{20}$$

$$= 3 \tag{21}$$