EE23010 Assignment

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Question 12.13.3.11
How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?
Solution: Let, X_i be the sequence of independent Bernoulli random varibles.

$$\implies X = \sum_{i=1}^{n} X_i \tag{1}$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{2}$$

$$p_X(k) = \begin{cases} 0.5 = p, & k = 0 \\ 0.5 = q, & k = 1 \end{cases} \tag{3}$$

Let, the total number of trials be n and the PMF of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^nC_k (p)^k (q)^{n-k} \tag{5}$$
$$= {}^nC_k (0.5)^k (0.5)^{n-k} \tag{6}$$

The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{7}$$

$$= \sum_{i=0}^{k} {}^nC_i (0.5)^{n-i} (0.5)^i \tag{8}$$

$$\Pr(X \geq 1) > 0.9 \tag{9}$$
$$1 - p_X(0) > 0.9 \tag{10}$$
$$(2)^n > 10 \tag{11}$$
$$n > \log_2(10) \tag{12}$$
$$n > 3.32 \tag{13}$$
$$\implies n = 4 \tag{14}$$