EE23010 Assignment

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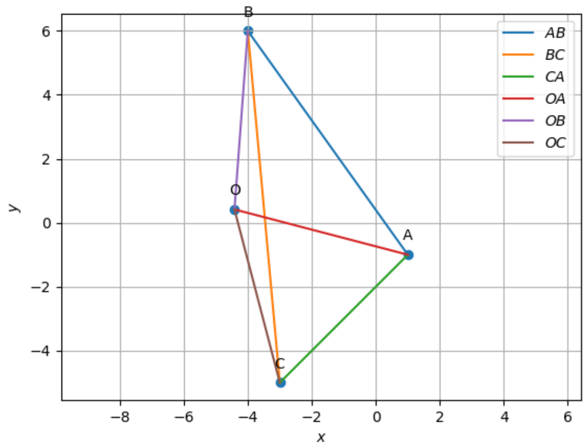

Fig. 0. Triangle generated using python

Question 1.5.3 verify that

$$OA = OB = OC \tag{1}$$

Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{4}$$

The intersection $\mathbf{O}$ of the perpendicular bisectors of AB and AC:

$$\mathbf{O} = \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{5}$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top \mathbf{B} - \mathbf{A}} \tag{6}$$

We need to verify

$$OA = OB = OC \tag{7}$$

consider OA:

$$\begin{aligned}
OA &= \|\mathbf{O} - \mathbf{A}\| && (8)\\
&= \sqrt{(\mathbf{O} - \mathbf{A})^\top \mathbf{O} - \mathbf{A}} && (9)\\
&= \sqrt{\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)^\top \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}} && (10)\\
&= \sqrt{\begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}^\top \begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}} && (11)\\
&= \sqrt{\begin{pmatrix} -\frac{65}{12} & \frac{17}{12} \end{pmatrix} \begin{pmatrix} -\frac{65}{12} \\ \frac{17}{12} \end{pmatrix}} && (12)\\
&= \sqrt{\left(\frac{65}{12}\right)^2 + \left(\frac{17}{12}\right)^2} && (13)\\
&= 5.5988 && (14)
\end{aligned}$$

consider OB:

$$\begin{aligned}
OB &= \|\mathbf{O} - \mathbf{B}\| && (15)\\
&= \sqrt{(\mathbf{O} - \mathbf{B})^\top \mathbf{O} - \mathbf{B}} && (16)\\
&= \sqrt{\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}\right)^\top \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}} && (17)\\
&= \sqrt{\begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}^\top \begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}} && (18)\\
&= \sqrt{\begin{pmatrix} -\frac{5}{12} & -\frac{67}{12} \end{pmatrix} \begin{pmatrix} -\frac{5}{12} \\ -\frac{67}{12} \end{pmatrix}} && (19)\\
&= \sqrt{\left(\frac{5}{12}\right)^2 + \left(\frac{67}{12}\right)^2} && (20)\\
&= 5.5988 && (21)
\end{aligned}$$

consider OC:

$$\begin{aligned}
OC &= ||\mathbf{O}-\mathbf{C}|| &(22)\\
&= \sqrt{(\mathbf{O}-\mathbf{C})^{\top}\mathbf{O}-\mathbf{C}} &(23)\\
&= \sqrt{\left(\begin{pmatrix}-\frac{53}{12}\\ \frac{5}{12}\end{pmatrix}-\begin{pmatrix}-3\\-5\end{pmatrix}\right)^{\top}\begin{pmatrix}-\frac{53}{12}\\ \frac{5}{12}\end{pmatrix}-\begin{pmatrix}-3\\-5\end{pmatrix}} &(24)\\
&= \sqrt{\begin{pmatrix}-\frac{17}{12}\\ \frac{65}{12}\end{pmatrix}^{\top}\begin{pmatrix}-\frac{17}{12}\\ \frac{65}{12}\end{pmatrix}} &(25)\\
&= \sqrt{\begin{pmatrix}-\frac{17}{12} & \frac{65}{12}\end{pmatrix}^{\top}\begin{pmatrix}-\frac{17}{12}\\ \frac{65}{12}\end{pmatrix}} &(26)\\
&= \sqrt{\left(\frac{17}{12}\right)^2+\left(\frac{65}{12}\right)^2} &(27)\\
&= 5.5988 &(28)
\end{aligned}$$

Therefore from the equations (14), (21) and (28), we get:

$$OA = OB = OA \qquad (29)$$

Hence we have verified that

$$OA = OB = OC \qquad (30)$$