EE23010 Assignment

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Question 10.13.3.19
Two dice are thrown at the same time. Find the probability of getting
(i) same number on both dice.
(ii) different numbers on both dice.

Solution: Let the random variables:

parameters	value	description
X	$1 \leq X \leq 6$	outcome of the first die
Y	$1 \leq Y \leq 6$	outcome of the second die

Consider a random variable Z:

$$Z = X - Y \tag{1}$$

Z can take values ranging from {-5 to 5}.
We need to find the PMF of Z
We know that,

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$p_Y(k)$ is same as $p_X(k)$.

1. Finding the solution using convolution:

$$p_Z(k) = \Pr(k = X - Y) \tag{3}$$
$$= \Pr(X = k + Y) \tag{4}$$

We arrive at the expectation of Z:

$$E\left(p_X(k+Y)\right) = \sum_{m=1}^{6}[p_X(k+m) * p_Y(m)] \tag{5}$$
$$= \frac{1}{6}\sum_{m=1}^{6} p_X(k+m) \tag{6}$$
$$= \frac{1}{6}(p_x(k+1) + p_x(k+2) + p_x(k+3) + p_x(k+4) + p_x(k+5) + p_x(k+6)) \tag{7}$$

(i) Finding the probability for $Z = 0$
We need to find PMF(Z) at $Z = 0$
{From the result (7)}

$$\Pr(Z = 0) = \frac{1}{6}\left(p_x(1) + p_x(2) + p_x(3) + p_x(4) + p_x(5) + p_x(6)\right) \tag{8}$$
$$= \frac{1}{6}\left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right) \tag{9}$$
$$= \frac{1}{6} \tag{10}$$

(ii) Finding the probability for $Z \neq 0$

$$\Pr(Z \neq 0) = 1 - \Pr(Z = 0) \tag{11}$$
$$= 1 - \frac{1}{6} \tag{12}$$
$$= \frac{5}{6} \tag{13}$$

2. Finding the solution using z-transform:

PMF of Z using z-transform:

$$\Pr(k = Z) = \Pr(k = X - Y) \tag{14}$$

applying the z-transform on both the sides

$$z\{\Pr(k = Z)\} = z\{\Pr(k = X - Y)\} \tag{15}$$
$$p_Z(k) = z\{p_X(k)\} * z\{p_{-Y}(k)\} \tag{16}$$

Let $X(z)$ and $Y(-z)$ represent $z\{p_X(k)\}$ and $z\{p_{-Y}(k)\}$ respectively,

$$= X(z) \cdot Y(-z) \tag{17}$$
$$= \left(\sum_{i=1}^{6} p_X(i) \cdot z^i\right) * \left(\sum_{j=1}^{6} p_Y(j) \cdot z^{-j}\right) \tag{18}$$
$$= \left(\frac{1}{6}\sum_{i=1}^{6} z^i\right)\left(\frac{1}{6}\sum_{j=1}^{6} z^{-j}\right) \tag{19}$$
$$= \frac{1}{36}\sum_{i=1}^{6}\sum_{j=1}^{1}\left(z^i \cdot z^{-j}\right) \tag{20}$$
$$= \frac{1}{36}\sum_{i=1}^{6}\sum_{j=1}^{6} z^{i-j} \tag{21}$$

Here i and j represents outcomes on the dice X and dice Y respectively, so $i-j$ represents the value k.

$$\implies p_Z(k) = \frac{1}{36}\sum_{i=1}^{6}\sum_{j=1}^{6} z^{i-j}\Big|_{i-j=k} \tag{22}$$

Here i and j should satisfy the condition: $i-j=k$

$$= \frac{1}{36}\sum_{j=1,i=1+k}^{6,6+k} z^{k}\Big|_{i-j=k} \tag{23}$$

(i) Finding the probability for $Z = 0$

From the result (23)

$$p_Z(0) = \frac{1}{36}\sum_{j=1,i=1}^{6,6} z^{0}\Big|_{i=j} \tag{24}$$

The coefficient of $z^k(z^0$ here), is the probability $P(Z=0)$

$$\implies \Pr(Z=0) = \frac{1}{36}(6) \tag{25}$$

$$= \frac{1}{6} \tag{26}$$

(ii) Finding the probability for $Z \neq 0$

$$\Pr(Z \neq 0) = 1 - \Pr(Z=0) \tag{27}$$

$$= 1 - \frac{1}{6} \tag{28}$$

$$= \frac{5}{6} \tag{29}$$

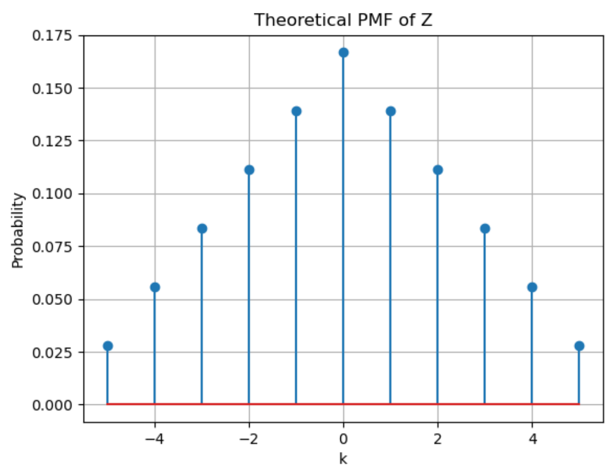

Fig. 0. Theorectical PMF of Z ($p_Z(k)$)