

# EE23010 Assignment

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## Question 12.13.3.55

There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let  $X$  denote the sum of the numbers on two cards drawn. Find the mean and variance of  $X$ .

### Solution:

parameters	description
$A$	number on the first card
$B$	number on the second card

$$p_A(i) = \begin{cases} \frac{1}{5}, & 1 \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$p_{AB}(i, k) = \Pr(B = k | A = i) p_A(i) \quad (2)$$

$$= \begin{cases} \frac{1}{20}, & 1 \leq k \leq 5, k \neq i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$p_B(k) = \sum_{i=-\infty}^{\infty} p_{AB}(i, k) \quad (4)$$

$$= \sum_{i=1}^5 p_{AB}(i, k) \quad (5)$$

Using the result from (3)

$$\Rightarrow p_B(k) = \begin{cases} \frac{1}{5}, & 1 \leq k \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Since  $p_A(k) = p_B(k)$ ,  $A$  and  $B$  are identical.

$$X = A + B \quad (7)$$

$$\mathbb{E}[X] = \mathbb{E}[A + B] \quad (8)$$

$$= \mathbb{E}[A] + \mathbb{E}[B] \quad (9)$$

$$= \mathbb{E}[A] + \mathbb{E}[A] \quad (10)$$

$$= 2\mathbb{E}[A] \quad (11)$$

$$= 2 \sum_{i=1}^5 i p_A(i) \quad (12)$$

$$= 6 \quad (13)$$

$$\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \quad (14)$$

$$= \mathbb{E}[(A + B)^2] - 6^2 \quad (15)$$

$$= \mathbb{E}[A^2 + B^2 + 2AB] - 36 \quad (16)$$

$$= \mathbb{E}[A^2] + \mathbb{E}[B^2] + 2\mathbb{E}[AB] - 36 \quad (17)$$

$$= \mathbb{E}[A^2] + \mathbb{E}[A^2] + 2 \sum_{i=1}^5 \sum_{k=1}^5 ik p_{AB}(i, k) - 36 \quad (18)$$

Using the result from (3)

$$\Rightarrow \text{Var}(X) = 2 \sum_{i=1}^5 i^2 p_A(i) + \frac{2}{20}(170) - 36 \quad (19)$$

$$= 3 \quad (20)$$