

EE23010 Assignment

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Question 12.13.3.11

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution:

Parameter	Value	Description
n	n	number of coin tosses
p	$\frac{1}{2}$	getting a head on a coin toss
q	$\frac{1}{2}$	getting a tail on a coin toss
$\mu = np$	$\frac{n}{2}$	mean of the distribution
$\sigma^2 = npq$	$\frac{n}{4}$	variance of the distribution
Y	≥ 1	Number of heads

1) Gaussian:

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

The CDF of Y :

$$F_Y(y) = 1 - \Pr(Y > y) \quad (2)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{y - \mu}{\sigma}\right) \quad (3)$$

But,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (4)$$

$$\Rightarrow F_Y(y) = 1 - Q\left(\frac{y - \mu}{\sigma}\right) \quad (5)$$

a) Without correction

$$\Pr(Y \geq 1) = 1 - F_Y(1) \quad (6)$$

From the result (5)

$$Q\left(\frac{2 - n}{\sqrt{n}}\right) > 0.9 \quad (7)$$

$$\frac{2 - n}{\sqrt{n}} < Q^{-1}(0.9) \quad (8)$$

$$\frac{2 - n}{\sqrt{n}} < -1.28 \quad (9)$$

Squaring on both the sides

$$\Rightarrow (n - 2)^2 > (1.28 \sqrt{n})^2 \quad (10)$$

$$\Rightarrow n^2 - 5.6384n + 4 > 0 \quad (11)$$

$$\Rightarrow n > 4.86, n < 0.8 \quad (12)$$

$$\Rightarrow n = 5 \quad (13)$$

b) With correction: 0.5 as correction term

$$\Pr(Y > 0.5) = 1 - F_Y(0.5) \quad (14)$$

From the result (5)

$$Q\left(\frac{1 - n}{\sqrt{n}}\right) > 0.9 \quad (15)$$

$$\frac{1 - n}{\sqrt{n}} < Q^{-1}(0.9) \quad (16)$$

$$\frac{1 - n}{\sqrt{n}} < -1.28 \quad (17)$$

Squaring on both the sides

$$\Rightarrow (n - 1)^2 > (1.28 \sqrt{n})^2 \quad (18)$$

$$\Rightarrow n^2 - 3.6384n + 1 > 0 \quad (19)$$

$$\Rightarrow n > 3.38, n < 0.29 \quad (20)$$

$$\Rightarrow n = 4 \quad (21)$$

2) Binomial:

$$X \sim \text{Bin}(n, p) \quad (22)$$

$$\Pr(X \geq 1) > 0.9 \quad (23)$$

$$\Rightarrow n = 4 \quad (24)$$

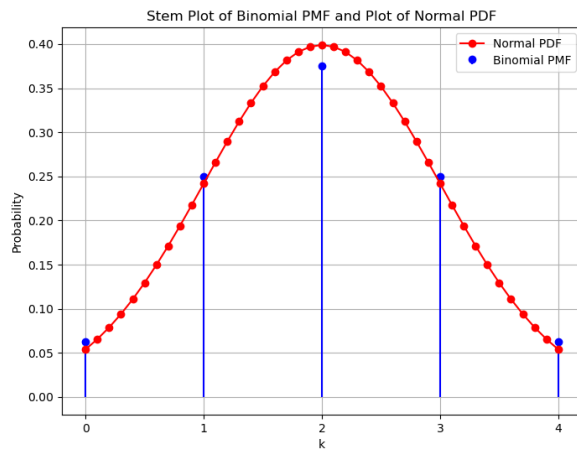


Fig. 2. Binomial PMF of X vs Normal PDF of Y