

# EE23010 Assignment

Sayyam Palrecha\* EE22BTECH11047

Question 10.13.3.19

Two dice are thrown at the same time. Find the probability of getting

- (i) same number on both dice.
- (ii) different numbers on both dice.

**Solution:** Let the random variables:

parameters	value	description
$X$	$1 \leq X \leq 6$	outcome of the first die
$Y$	$1 \leq Y \leq 6$	outcome of the second die

Consider a random variable  $W$ :

$$W = X - Y \quad (1)$$

$W$  can take values ranging from  $\{-5 \text{ to } 5\}$ .

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$p_Y(k)$  is same as  $p_X(k)$ .

PMF of  $W$  using  $z$ -transform:

applying the  $z$ -transform on both the sides

$$z\{W\} = z\{X - Y\} \quad (3)$$

$$M_W(z) = M_{X-Y}(z) \quad (4)$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X+Y}] \quad (5)$$

$$= E[z^{-X}] \cdot E[z^Y] \quad (6)$$

$$= \left( \sum_{i=1}^6 p_X(i) \cdot z^{-i} \right) \cdot \left( \sum_{j=1}^6 p_Y(j) \cdot z^j \right) \quad (7)$$

Extracting the PMF by considering the definition of  $z$ -transform

$$M_W(z) = p_W(0) + p_W(1)z + \dots + p_W(k)z^k + \dots \quad (8)$$

$$= \frac{1}{36} (z^{-1} + \dots + z^{-6}) \cdot (z^1 + \dots + z^6) \quad (9)$$

$$= \frac{1}{36} (z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \quad (10)$$

$$p_W(k) = \left( \frac{d^k}{dz^k} M_W(z) \right)_{z=0} \quad (11)$$

$$= \frac{1}{36} \left( \frac{d^k}{dz^k} (z^{-5} + \dots + z^5) \right)_{z=0} \quad (12)$$

(i) Finding the probability for  $W = 0$

From the result (12)

$$p_W(0) = \frac{1}{36} \left( \frac{d^0}{dz^0} (z^{-5} + \dots + z^5) \right)_{z=0} \quad (13)$$

$$\Rightarrow \Pr(W = 0) = \frac{1}{36}(6) \quad (14)$$

$$= \frac{1}{6} \quad (15)$$

(ii) Finding the probability for  $W \neq 0$

$$\Pr(W \neq 0) = 1 - \Pr(W = 0) \quad (16)$$

$$= 1 - \frac{1}{6} \quad (17)$$

$$= \frac{5}{6} \quad (18)$$

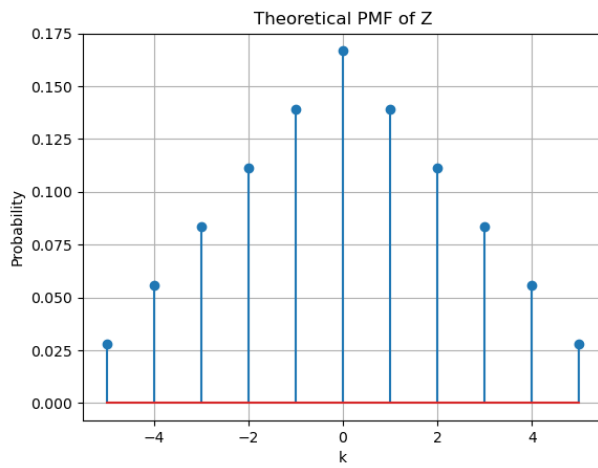


Fig. 0. Theoretical PMF of W ( $p_W(k)$ )