

EE23010 Assignment

Sayyam Palrecha* EE22BTECH11047

Question 12.13.3.11

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution:

Parameter	Value	Description
n	n	number of coin tosses
p	$\frac{1}{2}$	getting a head on a coin toss
q	$\frac{1}{2}$	getting a tail on a coin toss
$\mu = np$	$\frac{n}{2}$	mean of the distribution
$\sigma^2 = npq$	$\frac{n}{4}$	variance of the distribution
Y	≥ 1	Number of heads

1) Gaussian:

The gaussian-distribution of Y :

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

The Q-function from the gaussian distribution:

$$Q\left(\frac{x-\mu}{\sigma}\right) = \int_x^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad (2)$$

$$= \int_x^\infty \sqrt{\frac{2}{\pi n}} e^{-\frac{(2t-n)^2}{2n}} dt \quad (3)$$

a) Without correction

$$\Pr(Y \geq 1) = Q\left(\frac{2-n}{\sqrt{n}}\right) > 0.9 \quad (4)$$

$$\frac{2-n}{\sqrt{n}} < Q^{-1}(0.9) \quad (5)$$

$$\frac{2-n}{\sqrt{n}} < -1.28 \quad (6)$$

Squaring on both the sides

$$(n-2)^2 > (1.28\sqrt{n})^2 \quad (7)$$

$$n^2 - 5.6384n + 4 > 0 \quad (8)$$

$$n > 4.86, n < 0.8 \quad (9)$$

$$\Rightarrow n = 5 \quad (10)$$

b) With correction: 0.5 as correction term

$$\Pr(Y > 0.5) = Q\left(\frac{1-n}{\sqrt{n}}\right) > 0.9 \quad (11)$$

$$\frac{1-n}{\sqrt{n}} < Q^{-1}(0.9) \quad (12)$$

$$\frac{1-n}{\sqrt{n}} < -1.28 \quad (13)$$

Squaring on both the sides

$$(n-1)^2 > (1.28\sqrt{n})^2 \quad (14)$$

$$n^2 - 3.6384n + 1 > 0 \quad (15)$$

$$n > 3.38, n < 0.29 \quad (16)$$

$$\Rightarrow n = 4 \quad (17)$$

2) Binomial:

$$X \sim \text{Bin}(n, p) \quad (18)$$

The PMF of X is given by:

$$p_X(k) = {}^nC_k (0.5)^k (0.5)^{n-k} \quad (19)$$

The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^k p_X(i) \quad (20)$$

$$= \sum_{i=0}^k {}^nC_i (0.5)^{n-i} (0.5)^i \quad (21)$$

We have

$$\Pr(X \geq 1) > 0.9 \quad (22)$$

$$1 - p_X(0) > 0.9 \quad (23)$$

$$(2)^n > 10 \quad (24)$$

$$n > \log_2(10) \quad (25)$$

$$\Rightarrow n = 4 \quad (26)$$

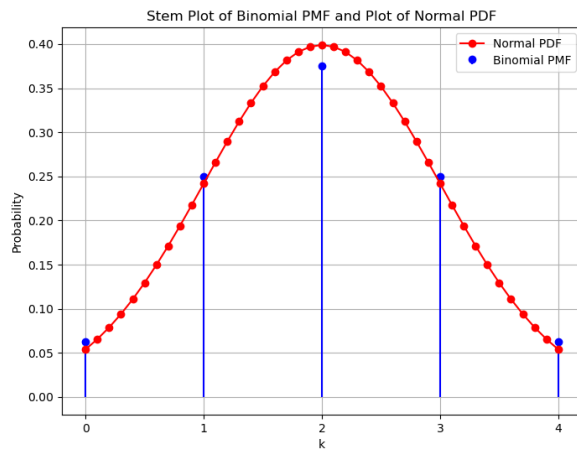


Fig. 2. Binomial PMF of X vs Normal PDF of Y