

EE23010 Assignment

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Question 46

Let X be a random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4}(x+1) & \text{if } -1 \leq x < 0 \\ \frac{1}{4}(x+3) & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases} \quad (1)$$

Which one of the following statements is true?

(A)

$$\lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) = \frac{5}{8} \quad (2)$$

(B)

$$\lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) = \frac{5}{8} \quad (3)$$

(C)

$$\lim_{n \rightarrow \infty} \Pr\left(X = \frac{1}{n}\right) = \frac{1}{2} \quad (4)$$

(D)

$$\Pr(X = 0) = \frac{1}{3} \quad (5)$$

(GATE ST 2023)

Solution:

(A)

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) &= \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) \\ &- \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} + \frac{1}{n}\right) - \lim_{n \rightarrow \infty} \Pr\left(X = -\frac{1}{n}\right) \end{aligned} \quad (6)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} + \frac{1}{n}\right) \\ &- \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) + \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) \end{aligned} \quad (7)$$

$$= \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} + \frac{1}{n}\right) \quad (8)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{n} + 1\right) - \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{2} + \frac{1}{n} + 1\right) \quad (9)$$

$$= \frac{1}{8} \quad (10)$$

\therefore (A) is not true.

(B)

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) &= \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) \\ &- \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} - \frac{1}{n}\right) - \lim_{n \rightarrow \infty} \Pr\left(X = \frac{1}{n}\right) \end{aligned} \quad (11)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} - \frac{1}{n}\right) \\ &- \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) + \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) \end{aligned} \quad (12)$$

$$= \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(-\frac{1}{2} - \frac{1}{n}\right) \quad (13)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4}\left(\frac{1}{n} + 3\right) - \lim_{n \rightarrow \infty} \frac{1}{4}\left(-\frac{1}{2} - \frac{1}{n} + 1\right) \quad (14)$$

$$= \frac{5}{8} \quad (15)$$

\therefore (B) is true.

(C)

$$\lim_{n \rightarrow \infty} \Pr\left(X = \frac{1}{n}\right) = \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) - \lim_{n \rightarrow \infty} F_X\left(\frac{1}{n}\right) \quad (16)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4}\left(\frac{1}{n} + 3\right) - \lim_{n \rightarrow \infty} \frac{1}{4}\left(\frac{1}{n} + 3\right) \quad (17)$$

$$= 0 \quad (18)$$

\therefore (C) is not true.

(D)

$$\Pr(X = 0) = F_X(0) - F_X(0^-) \quad (19)$$

$$= \frac{1}{4}(0 + 3) - \frac{1}{4}(0^- + 1) \quad (20)$$

$$= \frac{1}{2} \quad (21)$$

$\therefore (D)$ is not true.