# EE23010 Assignment

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Question 10.13.3.19
Two dice are thrown at the same time. Find the probability of getting

(i) same number on both dice.
(ii) different numbers on both dice.

**Solution:** Let the random variables:

| parameters | value | description |
|---|---|---|
| $X$ | $1 \le X \le 6$ | outcome of the first die |
| $Y$ | $1 \le Y \le 6$ | outcome of the second die |

Consider a random variable Z:

$$Z = X - Y \tag{1}$$

$Z$ can take values ranging from {-5 to 5}.
We need to find the PMF of $Z$
We know that,

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \le k \le 6 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$p_Y(k)$ is same as $p_X(k)$.

(1) Finding the solution using convolution:

$$p_Z(k) = P(k = X - Y) \tag{3}$$
$$= P(X = k + Y) \tag{4}$$

We arrive at the expectation of $Z$:

$$E\left(p_X(k+Y)\right) = \sum_{m=1}^{6} [p_X(k+m) * p_Y(m)] \tag{5}$$
$$= \frac{1}{6}\sum_{m=1}^{6} p_X(k+m) \tag{6}$$

(i) Finding the probability for $Z = 0$

We need to find PMF($Z$) at $Z = 0$

$$P(Z=0) = \frac{1}{6}\sum_{m=1}^{6} p_X(0+m) \tag{7}$$
$$= \frac{1}{6}\sum_{m=1}^{6} p_X(m) \tag{8}$$
$$= \frac{1}{6}\sum_{m=1}^{6} \frac{1}{6} \tag{9}$$
$$= \frac{1}{6}(1) \tag{10}$$
$$= \frac{1}{6} \tag{11}$$

(ii) Finding the probability for $Z \neq 0$

$$P(Z \neq 0) = 1 - P(Z=0) \tag{12}$$
$$= 1 - \frac{1}{6} \tag{13}$$
$$= \frac{5}{6} \tag{14}$$

(2) Finding the solution using z-transform:

(i) PMF of $Z$ using $z$-transform:

$$P(k = Z) = P(k = X - Y) \tag{15}$$

applying the z-transform on bothe the sides

$$z\{P(k=Z)\} = z\{P(k=X-Y)\} \tag{16}$$
$$p_Z(k) = z\{p_X(k) * p_{-Y}(k)\} \tag{17}$$
$$= \left(\sum_{m=1}^{6} p_X(m)\cdot z^m\right) * \left(\sum_{m=1}^{6} p_Y(m)\cdot z^{-m}\right) \tag{18}$$
$$= \left(\frac{1}{6}\sum_{m=1}^{6} z^m\right)\left(\frac{1}{6}\sum_{m=1}^{6} k^{-m}\right) \tag{19}$$
$$= \frac{1}{36}\left(z^1+z^2+z^3+z^4+z^5+z^6\right)\cdot \left(z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}\right) \tag{20}$$

The coefficient of $z^k$ is the probability $P(k = Z)$

$$p_Z(0) = \frac{1}{36}\left(z^1 z^{-1} + z^2 z^{-2} + z^3 z^{-3} + z^4 z^{-4} + z^5 z^{-5} + z^6 z^{-6}\right) \tag{21}$$

$$= \frac{1}{36}(6) \tag{22}$$

$$= \frac{1}{6} \tag{23}$$

(ii) Finding the probability for $Z \neq 0$

$$P(Z \neq 0) = 1 - P(Z = 0) \tag{24}$$

$$= 1 - \frac{1}{6} \tag{25}$$

$$= \frac{5}{6} \tag{26}$$