EE23010 Assignment

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Question 1.5.3

Using (1.1.7.1) verify that

$$\angle BAI = \angle CAI.$$

Solution:

Given: The intersection $\mathbf{I}$ of the angle bisectors of B and C:

$$\mathbf{I} = \frac{1}{\sqrt{37} + 4 + \sqrt{61}} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \tag{1}$$

$$\cos A = \frac{(\mathbf{B} - \mathbf{A})^\top \mathbf{C} - \mathbf{A}}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \qquad (1.1.7.1) \tag{2}$$

$$\tag{3}$$

Let $\angle BAI$ be equal to θ_1 and $\angle CAI$ be θ_2.
We need to verify $\cos\theta_1 = \cos\theta_2$.
consider LHS:

$$\cos\theta_1 = \frac{(\mathbf{B} - \mathbf{A})^\top \mathbf{I} - \mathbf{A}}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{I} - \mathbf{A}\|} \tag{4}$$

$$= \frac{\left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)^\top \begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left\|\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\| \left\|\begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\|} \tag{5}$$

$$= \frac{\begin{pmatrix} -5 \\ 7 \end{pmatrix}^\top \begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1 \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1 \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1 \end{pmatrix}\right\|} \tag{6}$$

on simplifying $\mathbf{I}$, we get $\mathbf{I} = \begin{pmatrix} -1.48 & -0.79 \end{pmatrix}$

$$= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -1.48 - 1 \\ -0.79 + 1 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -1.48 - 1 \\ -0.79 + 1 \end{pmatrix}\right\|} \tag{7}$$

$$= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -2.48 \\ 0.21 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -2.48 \\ 0.21 \end{pmatrix}\right\|} \tag{8}$$

$$= \frac{13.87}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -2.48 \\ 0.21 \end{pmatrix}\right\|} \tag{9}$$

$$\tag{10}$$

from (1.1.2.1) length of the side AB

$$\implies \|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{B} - \mathbf{A})^T \mathbf{B} - \mathbf{A}} \tag{11}$$

$$= \frac{13.87}{\sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \sqrt{\begin{pmatrix} -2.48 & 0.21 \end{pmatrix} \begin{pmatrix} -2.48 \\ 0.21 \end{pmatrix}}} \tag{12}$$

$$= \frac{13.87}{\sqrt{74}\sqrt{6.19}} \tag{13}$$

$$= 0.64 \tag{14}$$

consider RHS:

$$\cos\theta_2 = \frac{(\mathbf{I}-\mathbf{A})^\top \mathbf{C}-\mathbf{A}}{\|\mathbf{I}-\mathbf{A}\|\|\mathbf{C}-\mathbf{A}\|} \tag{15}$$

$$= \frac{\left(\begin{pmatrix}\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right)^\top \begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}}{\left\|\begin{pmatrix}\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|} \tag{16}$$

$$= \frac{\left(\begin{pmatrix}-1.48\\-0.79\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right)^\top \begin{pmatrix}-4\\-4\end{pmatrix}}{\left\|\begin{pmatrix}-1.48\\-0.79\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}-4\\-4\end{pmatrix}\right\|} \tag{17}$$

$$= \frac{-4\begin{pmatrix}-2.48\\0.21\end{pmatrix}^T \begin{pmatrix}1\\1\end{pmatrix}}{4\left\|\begin{pmatrix}-2.48\\0.21\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\1\end{pmatrix}\right\|} \tag{18}$$

$$= -\frac{\begin{pmatrix}-2.48 & 0.21\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{\begin{pmatrix}-2.48 & 0.21\end{pmatrix}\begin{pmatrix}-2.48\\0.21\end{pmatrix}}\sqrt{\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}}} \tag{19}$$

$$= \frac{2.27}{\sqrt{6.19}\sqrt{2}} \tag{20}$$

$$= 0.64 \tag{21}$$

Therefore from the Equations (14) and (21), we get:

$$\cos\theta_1 = \cos\theta_2$$

$$LHS = RHS$$

Hence we have verified that $\angle BAI = \angle CAI$.