EE23010 Assignment

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Question 12.13.3.55

There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X.

Solution:

parameters	description
A	number on the first card
B	number on the second card

$$p_A(k) = \begin{cases} \frac{1}{5}, & 1 \le k \le 5 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$p_B(k) = \sum_{i=1}^{5} \Pr(B = k \mid A = i) \cdot p_A(i) \tag{2}$$

$$= \frac{1}{5} \sum_{i=1}^{5} \Pr(B = k \mid A = i) \tag{3}$$

$$\Pr(B = k \mid A = i) = \begin{cases} \frac{1}{4}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$\implies p_B(k) = \begin{cases} \frac{1}{20}, & 1 \le k \le 5, k \ne i \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

X can take values ranging from 3 to 9.

$$X = A + B \tag{6}$$

$$p_X(k) = \sum_{i=1}^{5} \Pr(X = k \mid A = i) \cdot p_A(i) \tag{7}$$

Finding mean or the expectation:

$$\mathbb{E}[X] = \sum_{k=3}^{9} k \cdot p_X(k) \tag{8}$$

$$= \sum_{k=3}^{9} k \sum_{i=1}^{5} \Pr(X = k \mid A = i) \cdot p_A(i) \tag{9}$$

$$= \sum_{k=3}^{9} k \sum_{i=1}^{5} \Pr(B = k - i \mid A = i) \cdot p_A(i) \tag{10}$$

$$= \sum_{k=3}^{9} k \cdot p_B(k - i) \tag{11}$$

$$p_B(k-i) = \begin{cases} \frac{1}{20}, & 1 \le k - i \le 5, k - i \ne i \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

$$= \begin{cases} \frac{1}{20}, & i + 1 \le k \le i + 5, k \ne 2i \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

$$\implies \mathbb{E}[X] = 3p_B(3 - i) + ... + 9p_B(9 - i) \tag{14}$$

$$= \frac{1}{20}(120) \tag{15}$$

$$= 6 \tag{16}$$

$$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \tag{17}$$

$$= 3^2 p_B(3 - i) + ... + 9^2 p_B(9 - i) - 6^2 \tag{18}$$

$$= 39 - 36 \tag{19}$$

$$= 3 \tag{20}$$

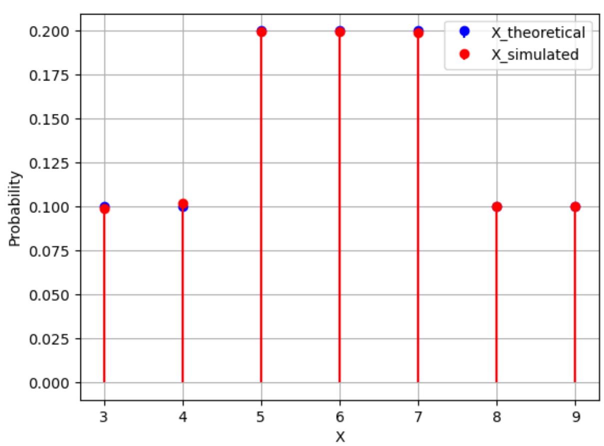

Fig. 0. PMF analysis of X ($p_X(k)$)