

EE23010 Assignment

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Question 10.13.3.19

Two dice are thrown at the same time. Find the probability of getting

- (i) same number on both dice.
- (ii) different numbers on both dice.

Solution: Let the random variables:

parameters	value	description
X	$1 \leq X \leq 6$	outcome of the first die
Y	$1 \leq Y \leq 6$	outcome of the second die

Consider a random variable W :

$$W = X - Y \quad (1)$$

W can take values ranging from $\{-5 \text{ to } 5\}$.

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$p_X(k) = p_Y(k) \quad (3)$$

PMF of W using z -transform:

applying the z -transform on both the sides

$$z\{W\} = z\{X - Y\} \quad (4)$$

$$M_W(z) = M_{X-Y}(z) \quad (5)$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X+Y}] \quad (6)$$

$$= E[z^{-X}] \cdot E[z^Y] \quad (7)$$

$$= \left(\sum_{i=1}^6 p_X(i) \cdot z^{-i} \right) \cdot \left(\sum_{j=1}^6 p_Y(j) \cdot z^j \right) \quad (8)$$

Extracting the PMF by considering the definition of z -transform

$$M_W(z) = p_W(0) + p_W(1)z + \dots + p_W(k)z^k + \dots \quad (9)$$

$$= \frac{1}{36} (z^{-1} + \dots + z^{-6}) \cdot (z^1 + \dots + z^6) \quad (10)$$

$$= \frac{1}{36} (z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \quad (11)$$

defined for all the values of $-5 \leq k \leq 5$

$$p_W(k) = \frac{1}{k!} \left(\frac{d^{|k|}}{dz^{|k|}} M_W(z) \right)_{z=0} \quad (12)$$

$$= \frac{1}{36k!} \left(\frac{d^{|k|}}{dz^{|k|}} (z^{-5} + \dots + z^5) \right)_{z=0} \quad (13)$$

$$= \frac{1}{36k!} \left(\frac{d^{|k|}}{dz^{|k|}} \left(\sum_{m=-5}^5 (6 - |m|) z^m \right) \right)_{z=0} \quad (14)$$

$$= \frac{1}{36k!} (6 - |k|) \frac{d^{|k|}}{dz^{|k|}} z^k \quad (15)$$

$$= \frac{1}{36k!} (6 - |k|) k! \quad (16)$$

$$= \frac{1}{36} (6 - |k|) \quad (17)$$

(i) Finding the probability for $W = 0$

From the result (17)

$$p_W(0) = \frac{1}{36} (6 - |0|) \quad (18)$$

$$\Rightarrow \Pr(W = 0) = \frac{1}{36} (6) \quad (19)$$

$$= \frac{1}{6} \quad (20)$$

(ii) Finding the probability for $W \neq 0$

$$\Pr(W \neq 0) = 1 - \Pr(W = 0) \quad (21)$$

$$= 1 - \frac{1}{6} \quad (22)$$

$$= \frac{5}{6} \quad (23)$$

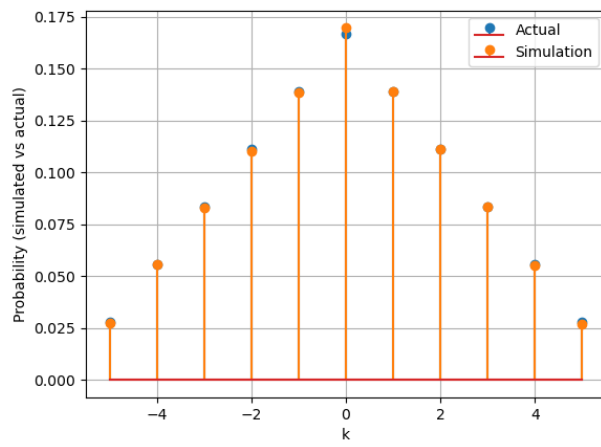


Fig. 0. PMF analysis of W ($p_W(k)$)