EE23010 Assignment

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Question 1.5.3 Using (1.1.7.1) verify that

$$\angle BAI = \angle CAI. \tag{1}$$

Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{4}$$

The intersection $\mathbf{I}$ of the angle bisectors of B and C:

$$\mathbf{I} = \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} \tag{5}$$

$$\cos A = \frac{(\mathbf{B}-\mathbf{A})^\top \mathbf{C}-\mathbf{A}}{\|\mathbf{B}-\mathbf{A}\| \, \|\mathbf{C}-\mathbf{A}\|} \tag{6}$$

Solution:
Let $\angle BAI$ be equal to θ_1 and $\angle CAI$ be θ_2.
We need to verify

$$\theta_1 = \theta_2. \tag{7}$$

consider LHS:

$$\cos\theta_1 = \frac{(\mathbf{B}-\mathbf{A})^\top \mathbf{I}-\mathbf{A}}{\|\mathbf{B}-\mathbf{A}\| \, \|\mathbf{I}-\mathbf{A}\|} \tag{8}$$

$$= \frac{\left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)^\top \begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left\|\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\| \left\|\begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\|} \tag{9}$$

$$= \frac{\begin{pmatrix} -5 \\ 7 \end{pmatrix}^\top \begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1 \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} \frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} - 1 \\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}} + 1 \end{pmatrix}\right\|} \tag{10}$$

on simplifying $\mathbf{I}$, we get

$$\mathbf{I} = \begin{pmatrix} -1.47756 \\ -0.79495 \end{pmatrix} \tag{11}$$

$$= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -1.47756-1 \\ -0.79495+1 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -1.47756-1 \\ -0.79495+1 \end{pmatrix}\right\|} \tag{12}$$

$$= \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}\right\|} \tag{13}$$

$$= \frac{13.82315}{\left\|\begin{pmatrix} -5 \\ 7 \end{pmatrix}\right\| \left\|\begin{pmatrix} -2.47756 \\ 0.20505 \end{pmatrix}\right\|} \tag{14}$$

from (1.1.2.1) length of the side AB

$$\|\mathbf{A}-\mathbf{B}\| = \sqrt{(\mathbf{B}-\mathbf{A})^\top \mathbf{B}-\mathbf{A}} \tag{15}$$

$$= \frac{13.82315}{\sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \sqrt{\begin{pmatrix} -2.4775 & 0.2050 \end{pmatrix} \begin{pmatrix} -2.4775 \\ 0.2050 \end{pmatrix}}} \tag{16}$$

$$= \frac{13.82315}{\sqrt{74}\sqrt{6.2010538036}} \tag{17}$$

$$\implies \cos\theta_1 = 0.64529 \tag{18}$$

$$\implies \theta_1 = \cos^{-1} 0.64529 \tag{19}$$

$$\implies \theta_1 = 49.7311 \tag{20}$$

consider RHS:

$$\cos\theta_2 = \frac{(\mathbf{I}-\mathbf{A})^\top \mathbf{C}-\mathbf{A}}{\|\mathbf{I}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|} \tag{21}$$

$$= \frac{\left(\begin{pmatrix}\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right)^\top \begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}}{\left\|\begin{pmatrix}\frac{\sqrt{61}-16-3\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\\ \frac{-\sqrt{61}+24-5\sqrt{37}}{\sqrt{37}+4+\sqrt{61}}\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|} \tag{22}$$

$$= \frac{\left(\begin{pmatrix}-1.47756\\-0.79495\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right)^\top \begin{pmatrix}-4\\-4\end{pmatrix}}{\left\|\begin{pmatrix}-1.47756\\-0.79495\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}-4\\-4\end{pmatrix}\right\|} \tag{23}$$

$$= \frac{-4\begin{pmatrix}-2.47756\\0.20505\end{pmatrix}^\top \begin{pmatrix}1\\1\end{pmatrix}}{4\left\|\begin{pmatrix}-2.47756\\0.20505\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\1\end{pmatrix}\right\|} \tag{24}$$

$$= -\frac{\begin{pmatrix}-2.47756 & 0.20505\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{\begin{pmatrix}-2.47756 & 0.20505\end{pmatrix}\begin{pmatrix}-2.47756\\0.20505\end{pmatrix}}\sqrt{\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}}} \tag{25}$$

$$= \frac{2.27251}{\sqrt{6.2010538036}\sqrt{2}} \tag{26}$$

$$\implies \cos\theta_2 = 0.64529 \tag{27}$$

$$\implies \theta_2 = \cos^{-1} 0.64529 \tag{28}$$

$$\implies \theta_2 = 49.7311 \tag{29}$$

Therefore from the equations (20) and (29), we get:

$$\cos\theta_1 = \cos\theta_2 \tag{30}$$

$$\therefore LHS = RHS \tag{31}$$

Hence we have verified that

$$\angle BAI = \angle CAI. \tag{32}$$