EE23010 Assignment

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Question 12.13.3.55
There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X.

Solution:

parameters	description
A	number on the first card
B	number on the second card

$$p_A(k) = \begin{cases} \frac{1}{5}, & 1 \le k \le 5 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\tag{2}$$

X can take values ranging from 3 to 9.

$$p_X(k) = \sum_{i=1}^{5} \Pr\left(X = k \mid A = i\right) \cdot p_A(i) \tag{3}$$

Finding the expectation:

$$E[X] = \sum_{k=3}^{9} k \cdot p_X(k) \tag{4}$$

$$= \sum_{k=3}^{9} k \sum_{i=1}^{5} \Pr\left(X = k \mid A = i\right) \cdot p_A(i) \tag{5}$$

$$= \frac{1}{5} \sum_{k=3}^{9} k \sum_{i=1}^{5} \Pr\left(X = k \mid A = i\right) \tag{6}$$

Using a parameter t to establish a relation between k and i.

$$= \frac{1}{5} \cdot \frac{1}{4} \sum_{k=3}^{9} \sum_{i=1}^{5} k \left(\frac{1}{|k-i|!} \frac{d^{|k-i|}}{dt^{|k-i|}} t^{k-i} - t^{k-i} - t^{\frac{k}{2}-i} \right) \Bigg|_{t=0} \tag{7}$$

$$= \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} \left(\frac{k}{|k-i|!} \frac{d^{|k-i|}}{dt^{|k-i|}} t^{k-i} \right) \Bigg|_{t=0} - \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} k t^{k-i} \Bigg|_{t=0} - \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} k t^{\frac{k}{2}-i} \Bigg|_{t=0} \tag{8}$$

$$= \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} \left(\frac{k}{|k-i|!} \frac{d^{|k-i|}}{dt^{|k-i|}} t^{k-i} \right) \Bigg|_{t=0} - \frac{(3+4+5)}{20} - \frac{(4+6+8)}{20} \tag{9}$$

$$= \frac{1}{20} \left(3(3) + 4(4) + 5(5) + 6(5) + 7(4) + 8(3) + 9(2)\right) - \frac{12}{20} - \frac{18}{20} \tag{10}$$

$$= \frac{150}{20} - \frac{30}{20} \tag{11}$$

$$= 6 \tag{12}$$

$$Var(X) = E[X^2] - (E[X])^2 \tag{13}$$

$$= \sum_{k=3}^{9} k^2 \cdot p_X(k) - \left(\sum_{k=3}^{9} k \cdot p_X(k) \right)^2 \tag{14}$$

$$= \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} \left(\frac{k^2}{|k-i|!} \frac{d^{|k-i|}}{dt^{|k-i|}} t^{k-i} \right) \Bigg|_{t=0} - \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} k^2 t^{k-i} \Bigg|_{t=0} - \frac{1}{20} \sum_{k=3}^{9} \sum_{i=1}^{5} k^2 t^{\frac{k}{2}-i} \Bigg|_{t=0} - 6^2 \tag{15}$$

$$= \frac{1}{20} (3(9) + 4(16) + 5(25) + 5(36) + 4(49) + 3(64) + 2(81)) - \frac{9 + 16 + 25}{20} - \frac{16 + 36 + 64}{20} - 36 \tag{16}$$

$$= \frac{946}{20} - \frac{50}{20} - \frac{116}{20} - 36 \tag{17}$$

$$= 39 - 36 \tag{18}$$

$$= 3 \tag{19}$$