EE23010 Assignment

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Question 10.13.3.19
Two dice are thrown at the same time. Find the probability of getting
(i) same number on both dice.
(ii) different numbers on both dice.

Solution: Let the random variables:

parameters	value	description
X	$1 \le X \le 6$	outcome of the first die
Y	$1 \le Y \le 6$	outcome of the second die

Consider a random variable W:

$$W = X - Y \tag{1}$$

W can take values ranging from {-5 to 5}.

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \le k \le 6 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$p_Y(k)$ is same as $p_X(k)$.

PMF of W using z-transform:

applying the z-transform on both the sides

$$z\{W\} = z\{X - Y\} \tag{3}$$
$$M_W(z) = M_{X-Y}(z) \tag{4}$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X+Y}] \tag{5}$$
$$= E[z^{-X}] \cdot E[z^{Y}] \tag{6}$$
$$= \left(\sum_{i=1}^{6} p_X(i) \cdot z^{-i}\right) \cdot \left(\sum_{j=1}^{6} p_Y(j) \cdot z^{j}\right) \tag{7}$$

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = p_W(0) + p_W(1)z + ... + p_W(k)z^k + ... \tag{8}$$
$$= \frac{1}{36}\left(z^{-1} + ... + z^{-6}\right) \cdot \left(z^{1} + ... + z^{6}\right) \tag{9}$$
$$= \frac{1}{36}(z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6 + 5z^{1} + 4z^{2} + 3z^{3} + 2z^{4} + z^{5}) \tag{10}$$

$$p_W(k) = \left(\frac{d^{|k|}}{dz^{|k|}} M_W(z)\right)_{z=0} \tag{11}$$

defined for all the values of $-5 \le k \le 5$

$$= \frac{1}{36}\left(\frac{d^{|k|}}{dz^{|k|}}\left(z^{-5} + ... + z^{5}\right)\right)_{z=0} \tag{12}$$

the above expression extracts the coefficient of z^k for the desired value of k and w.k.t the coefficients of z^k and z^{-k} are the same.

(i) Finding the probability for $W = 0$

From the result (12)

$$p_W(0) = \frac{1}{36}\left(\frac{d^{0}}{dz^{0}}\left(z^{-5} + ... + z^{5}\right)\right)_{z=0} \tag{13}$$
$$\implies \Pr(W = 0) = \frac{1}{36}(6) \tag{14}$$
$$= \frac{1}{6} \tag{15}$$

(ii) Finding the probability for $W \neq 0$

$$\Pr(W \neq 0) = 1 - \Pr(W = 0) \tag{16}$$
$$= 1 - \frac{1}{6} \tag{17}$$
$$= \frac{5}{6} \tag{18}$$

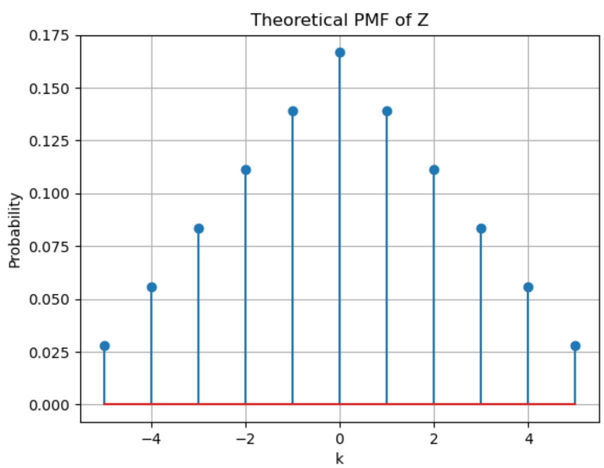

Fig. 0. Theorectical PMF of W ($p_W(k)$)