EE23010 Assignment

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Question 12.13.3.11
How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?
Solution:

1) Binomial:
Let, X_i be the sequence of independent Bernoulli random varibles.

$$\implies X = \sum_{i=1}^{n} X_i \tag{1}$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{2}$$

$$p_X(k) = \begin{cases} 0.5 = p, & k = 0 \\ 0.5 = q, & k = 1 \end{cases} \tag{3}$$

Let, the total number of trials be n and the PMF of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^nC_k\,(p)^k\,(q)^{n-k} \tag{5}$$
$$= {}^nC_k\,(0.5)^k\,(0.5)^{n-k} \tag{6}$$

The CDF of X is defined as:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{7}$$
$$= \sum_{i=0}^{k} {}^nC_i\,(0.5)^{n-i}\,(0.5)^i \tag{8}$$

$$\Pr(X > 0) > 0.9 \tag{9}$$
$$1 - p_X(0) > 0.9 \tag{10}$$
$$(2)^n > 10 \tag{11}$$
$$n > \log_2(10) \tag{12}$$
$$n > 3.32 \tag{13}$$
$$\implies n = 4 \tag{14}$$

2) Gaussian:
Let Y be the gaussian variable,

$$\mu = np = \frac{n}{2} \tag{15}$$
$$\sigma^2 = np(1-p) = \frac{n}{4} \tag{16}$$

Let

$$Z \approx \frac{Y - \mu}{\sigma} \tag{17}$$

Here, Z is a random variable with $\mathcal{N}(0, 1)$,
The normal-Distribution of Z is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{18}$$

The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{19}$$
$$= \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\,dt \tag{20}$$

a) Without correction

$$Y \geq 1 \tag{21}$$
$$Z \geq \frac{1-\mu}{\sigma} \tag{22}$$
$$Z \geq \frac{2-n}{\sqrt{n}} \tag{23}$$

Since we know that,

$$\Pr(Z > x) > 0.9 \tag{24}$$
$$Q(x) > 0.9 \tag{25}$$
$$\implies Z > 1.28 \tag{26}$$

From (23)

$$\frac{2-n}{\sqrt{n}} < 1.28 \tag{27}$$
$$n + 1.28\sqrt{n} > 2 \tag{28}$$
$$\implies n = 1 \tag{29}$$

b) With correction

$$Y > 0.1 \tag{30}$$
$$Z > \frac{0.1 - n}{\sqrt{n}} \tag{31}$$

From (26)

$$\frac{0.1 - n}{\sqrt{n}} > 1.28 \qquad (32)$$

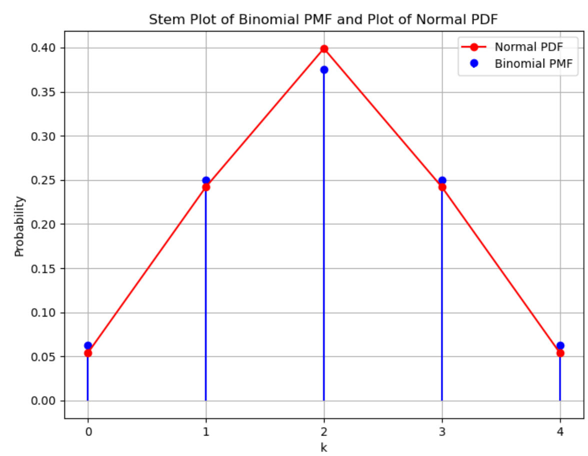

Fig. 2. Binomial PMF of X vs Normal PDF of Y