

GATE 22 EE/46

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QUESTION: Let a causal LTI system be governed by the following differential equation,

$$y(t) + \frac{1}{4} \frac{dy}{dt} = 2x(t) \quad (1)$$

where $x(t)$ and $y(t)$ are the input and output respectively. Its impulse response is (GATE EE-2022)

Solution:

From (1), corresponding Laplace transform,

$$Y(s) + \frac{1}{4}(sY(s) - y(0)) = 2X(s) \quad (2)$$

Since it is causal LTI system,

$$y(0) = 0 \quad (3)$$

$$\Rightarrow Y(s) + \frac{1}{4}sY(s) = 2X(s) \quad (4)$$

$$\Rightarrow Y(s) = X(s) \frac{8}{4+s} \quad (5)$$

$$\Rightarrow H(s) = \frac{8}{4+s} \quad ROC : Re(s) > -4 \quad (6)$$

Taking inverse laplace transform and applying causality conditions

$$h(t) = 8e^{-4t}u(t) \quad (7)$$

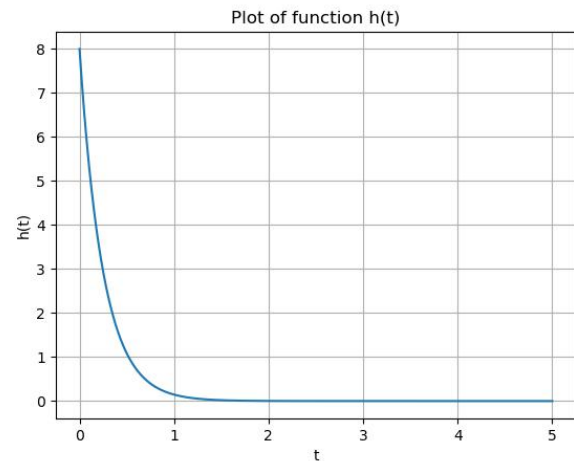


Fig. 1. Plot of $h(n)$, taken from python3