GATE 2021 1.CH

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Question: An ordinary differential equation (ODE) $\frac{dy}{dx} = 2y$ with an initial condition $y(0) = 1$ has the analytical solution $y = e^{2x}$ Using Runge-Kutta second order method, numerically integrate the ODE to calculate y at $x = 0.5$ using a step size of $h = 0.5$. If the relative percentage error is defined as

$$\epsilon = \left| \frac{y_{analytical} - y_{numerical}}{y_{analytical}} \right| \times 100$$

then the value of ϵ at $x = 0.5$ is

(GATE 1 CH 2021)

Solution:

Constant	Description
$y(0.5)$	$y_{analytical}$
y_1	$y_{numerical}$
$\frac{dy}{dx} = f(x, y)$	Function representing the ODE
h	Step size
K_1	First slope estimate in the Runge-Kutta
K_2	Second slope estimate in the Runge-Kutta
K	Weighted average of K_1 and K_2

TABLE 1

Analytical solution is given by:

$$y = e^{2x} \tag{1}$$

At $x = 0.5$, analytical solution is

$$y(0.5) = e^{2\times 0.5} = 2.718 \tag{2}$$

According to question:

$$f(x, y) = \frac{dy}{dx} = 2y \tag{3}$$

By Runge-kutta 2^{nd} order method,

$$K_1 = hf(x_o, y_o) = h(2y_o) \tag{4}$$

$$K_1 = 0.5(2 \times 1) = 1 \tag{5}$$

$$K_2 = h[f(x_o + h, y_o + K_1)] \tag{6}$$

$$K_2 = h[2(1 + 1)] \tag{7}$$

$$K_2 = 0.5 \times 4 = 2 \tag{8}$$

$$K = \frac{K_1 + K_2}{2} = \frac{1 + 2}{2} = 1.5 \tag{9}$$

$$y_1 = y_{numerical} = y_o + K = 1 + 1.5 = 2.5 \tag{10}$$

Hence,

$$\epsilon = \left| \frac{2.718 - 2.5}{2.718} \right| \times 100 \tag{11}$$

$$\epsilon = \frac{0.218}{2.718} \times 100 = 8\% \tag{12}$$