Gate 2021 BM Q8

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For a linear stable second order system, if the unit step response is such that peak time is twice the rise time, then the system is .

1) underdamped
2) undamped
3) overdamped
4) critically damped

Solution:

Parameter	Description
ω_n	natural frequency
ζ	damping ratio
θ	is the angle in the complex plane corresponding to the pole location

TABLE 4
Given Parameters

The rise time is given by:

$$t_r = \frac{\pi - \theta}{\omega_n \sqrt{1-\zeta^2}} \tag{1}$$

The peak time is given by:

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{2}$$

as, peak time is twice the rise time:

$$t_p = 2t_r \tag{3}$$

$$\frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 2\frac{\pi - \theta}{\omega_n \sqrt{1-\zeta^2}} \tag{4}$$

$$\theta = \frac{\pi}{2} \tag{5}$$

as, $\theta = \frac{\pi}{2}$, both roots of the system are imaginary, so

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{6}$$

So, for the denominator to have two imaginary roots

$$s = +\jmath\omega_n \tag{7}$$

$$s = -\jmath\omega_n \tag{8}$$

$2\zeta\omega_n$ should be zero.

$$\zeta = 0 \tag{9}$$

The Routh-Hurwitz criterion is a method used to determine the stability of a system based on the locations of the roots of the characteristic equation in the complex plane.

The coefficients of s, s^2 and 1, which are $2\zeta\omega_n$, 1 and ω_n^2 are non negative, hence the system is stable.So,the syatem is either undamped or overdamped.As, ζ is zero, system is undamped.