GATE 2022 XE 80

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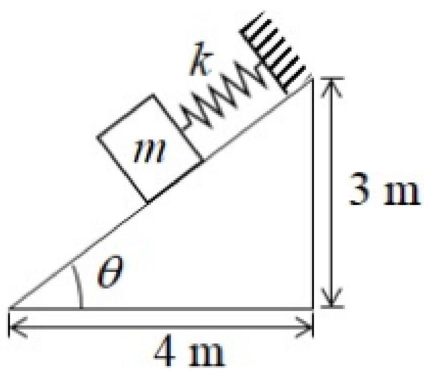
Question GATE 22 XE 80 :
A mass m = 10 kg is attached to a spring as shown in the figure. The coefficient of friction between the mass and the inclined plane is 0.25. Assume that the acceleration due to gravity is 10 m/s^2 and that static and kinematic friction coefficients are the same. Equilibrium of the mass is impossible if the spring force is



1) 30 N
2) 45 N
3) 60 N
4) 75 N

(GATE XE 2022)

Solution:

If the spring force is imum, frictional force is downwards and block is just about to move upwards and is at rest and equilibrium currently.

From Fig. 4 and Table 4, the force equation for the object is

$$\mu mg\cos\theta + mg\sin\theta - F_s = m\frac{d^2x}{dt^2} \tag{1}$$

Parameter	Description	Value
m	Mass of object	10 Kg
μ	Frictional coefficient (*static*)	0.25
x(t)	Displacement of block	
$x(0)$	Initial displacement	0 (*assumed*)
g	Gravitational acceleration	10 m/s^2
F_s	Spring force	
f	frictional force	μ N
N	Normal Force	mg $cos(\theta)$

TABLE 4
PARAMETER TABLE

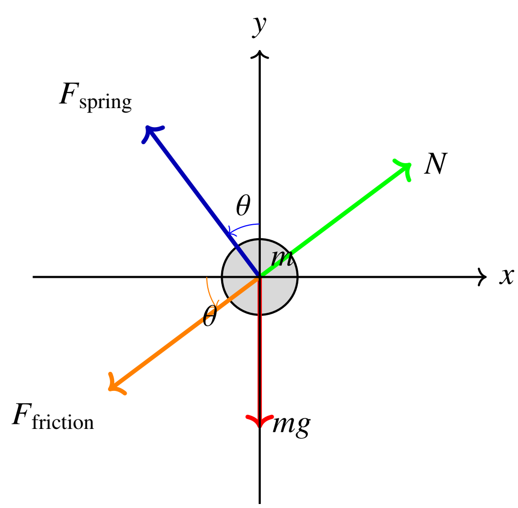


Fig. 4. Maximum spring force FBD

the Laplace transform of terms is

$$k \overset{\mathcal{L}}{\longleftrightarrow} \frac{k}{s} \tag{2}$$

$$\frac{d^2x}{dt^2} \overset{\mathcal{L}}{\longleftrightarrow} s^2X(s) - sx(0) - \dot{x}(0) \tag{3}$$

Applying Laplace transform to equation (1),

$$\frac{\mu mg\cos\theta + mg\sin\theta - F_s}{s} = s^2X(s) - sx(0) - \dot{x}(0) \quad (4)$$

$$\implies \frac{\mu mg\cos\theta + mg\sin\theta - F_s}{s^3} = X(s) \quad (5)$$

The inverse Laplace transform is

$$\frac{k}{s^3} \overset{\mathcal{L}^-}{\longleftrightarrow} \frac{k}{2}t^2 \quad (6)$$

The inverse Laplace of (5) is

$$(\mu mg\cos\theta + mg\sin\theta - F_s)\,t^2 = x(t) \quad (7)$$

As it is always at equilibrium, $\frac{dx}{dt}$ is 0

$$2t(\mu mg\cos\theta + mg\sin\theta - F_s) = 0 \quad (8)$$

$$\implies \mu mg\cos\theta + mg\sin\theta - F_s = 0 \quad (9)$$

$$\implies \mu mg\cos\theta + mg\sin\theta = F_s \quad (10)$$

Using Table 4 , the maximum value for equilibrium is

$$F_s = 80N \quad (11)$$

Now consider the other case. F_s is minimum possible for equilibrium. The block is about to move downwards.

From Fig. 4 and Table 4, the force equation

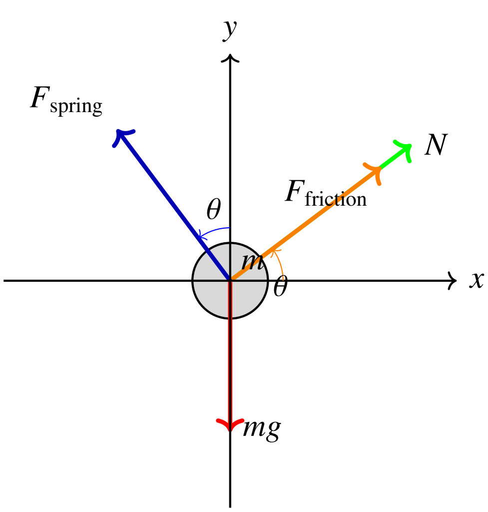


Fig. 4. Minimum spring force FBD

for the object is

$$mg\sin\theta - \mu mg\cos\theta - F_s = m\frac{d^2x}{dt^2} \quad (12)$$

Laplace transform is

$$\frac{mg\sin\theta - \mu mg\cos\theta - F_s}{s^3} = X(s) \quad (13)$$

The inverse Laplace of (5) is

$$(mg\sin\theta - \mu mg\cos\theta - F_s)\,t^2 = x(t) \quad (14)$$

Hence the minimum force for equilibrium is

$$F_s = mg\sin\theta - \mu mg\cos\theta \quad (15)$$

$$= 40N \quad (16)$$

Hence , block is in equilibrium for F_s between 40 and 80N. At 30 N, it is not at equilibrium.