GATE: CH - 60.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: Consider a single-input-single-output (SISO) system with the transfer function

$$G_p(s) = \frac{2(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)}$$

where the time constants are in minutes. The system is forced by a unit step input at time $t = 0$. The time at which the output response reaches the maximum is ____ minutes (rounded off to two decimal places). (GATE CH 2022)

Solution:

Parameters	Description	Value
$y(t)$	Output response	
$G_p(s)$	Transfer function	$\frac{2(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)}$
$x(t)$	Input	$u(t)$
$X(s)$	Laplace transform of x(t)	$\frac{1}{s}$
$y'(t)$	$\frac{dy}{dt}$	

TABLE 1
PARAMETERS

$$Y(s) = G_p(s)X(s) \tag{1}$$

$$= \frac{16(s+1)}{s(s+2)(s+4)} \tag{2}$$

$$= \frac{2}{s} + \frac{4}{s+2} - \frac{6}{s+4} \tag{3}$$

$$u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \tag{4}$$

$$e^{-at}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \tag{5}$$

From Laplace transforms (4) and (5), we get

$$y(t) = \left(2 + 4e^{-2t} - 6e^{-4t}\right)u(t) \tag{6}$$

For maximum value of $y(t)$,

$$y'(t) = 0 \tag{7}$$

$$\implies -8e^{-2t} + 24e^{-4t} = 0 \tag{8}$$

$$e^{2t} = 3 \tag{9}$$

$$\implies t = \frac{\ln 3}{2} \tag{10}$$

$$\approx 0.55 \tag{11}$$

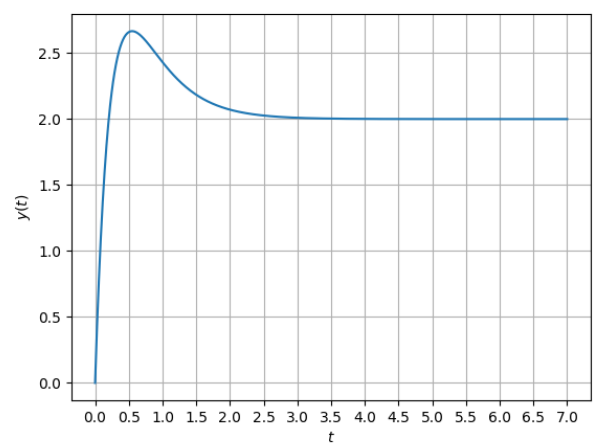

Fig. 1. Plot of $y(t)$