GATE 2022 -AE 63

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Question: For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are:

(A) $R_1 = 2\ \text{k}\Omega, R_2 = 3\ \text{k}\Omega$
(B) $R_1 = 5\ \text{k}\Omega, R_2 = 2\ \text{k}\Omega$
(C) $R_1 = 5\ \text{k}\Omega, R_2 = 2.5\ \text{k}\Omega$
(D) $R_1 = 2\ \text{k}\Omega, R_2 = 5\ \text{k}\Omega$

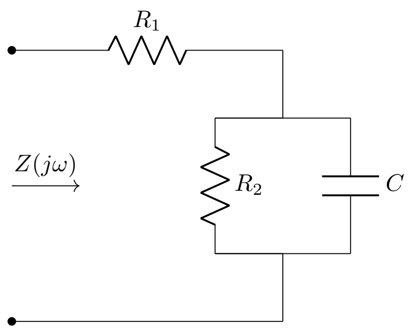

Fig. 1. Figure of circuit

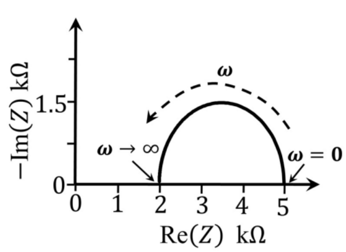

Fig. 2.

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Solution:

Parameter	Description	Value
$Z(j\omega)$	Impedance of circuit	?
R_1	Resistor 1	?
R_1	Resistor 2	?
C	Capacitor	?
ω	angular frequency of input voltage	ω

TABLE I
INPUT VALUES

In ω domain (i.e. after Laplace transform) Fig. 1 can be represented as Fig. 3 So, the impedance for

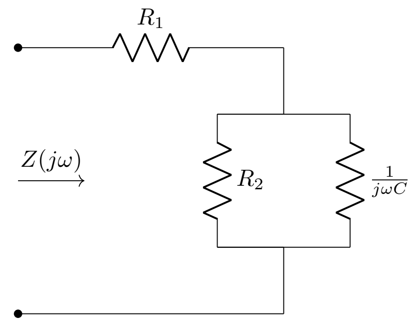

Fig. 3.

the circuit in ω domain is:

$$Z(j\omega) = R_1 + \frac{1}{\frac{1}{R_2} + j\omega C} \tag{1}$$

From Fig. 2, $Z(j\omega) = 2$ as $\omega \to \infty$ and $Z(j\omega) = 5$ as $\omega \to 0$

$$2 = R_1 + \lim_{\omega\to\infty} \frac{1}{\frac{1}{R_2} + j\omega C} \tag{2}$$

$$\implies 2 = R_1 + \lim_{\omega\to\infty} \frac{\frac{1}{R_2} - j\omega C}{\left(\frac{1}{R_2}\right)^2 + (\omega C)^2} \tag{3}$$

$$\implies 2 = R_1 + \lim_{\omega\to\infty} \frac{\frac{1}{R_2\omega^2} - j\frac{C}{\omega}}{\left(\frac{1}{R_2\omega}\right)^2 + C^2} \tag{4}$$

$$\therefore 2\Omega = R_1 \tag{5}$$

$$5 = R_1 + \frac{1}{\frac{1}{R_2} + j(0)} \tag{6}$$

$$\implies 5 = R_1 + R_2 \tag{7}$$

$$\therefore 3\Omega = R_2 \tag{8}$$

Hence, option (A) is correct.