

# GATE 2021 Assignment

## EE1205 Signals and Systems

Kurre Vinay  
EE23BTECH11036

**Question:** The exponential Fourier series representation of a continuous-time periodic signal  $x(t)$  is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where  $\omega_0$  is the fundamental angular frequency of  $x(t)$  and the coefficients of the series are  $a_k$ . The following information is given about  $x(t)$  and  $a_k$

- I.  $x(t)$  is real and even, having a fundamental period of 6
- II. The average value of  $x(t)$  is 2.
- III.

$$a_k = \begin{cases} k, & 1 \leq k \leq 3 \\ 0, & k > 3 \end{cases} \quad (1)$$

(2)

The average power of the signal  $x(t)$  (rounded off to one decimal place) is \_\_\_\_\_.  
(GATE EC 2021)

**Solution:**

variable	value	description
$T_0$	6	Fundamental time period
$P_{avg}$	-	average power of the signal
$x(t)$	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	Input signal
$a_k$	$\begin{cases} k, & 1 \leq k \leq 3 \\ 0, & k > 3 \end{cases}$	coefficients of the series
$a_0$	2	average of $x(t)$

TABLE I

TABLE: INPUT PARAMETERS

$x(t)$  is even and real so,  $a_k = a_{-k}$

Parswal's Power Theorem

Proof

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt, \quad |x(t)|^2 = x(t) x^*(t) \quad (3)$$

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) x^*(t) dt \quad (4)$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad (5)$$

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left( \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \right) x^*(t) dt \quad (6)$$

$$P = \sum_{n=-\infty}^{\infty} C_n \left( \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^*(t) e^{jn\omega_0 t} dt \right) \quad (7)$$

$$= \sum_{n=-\infty}^{\infty} C_n C_n^* \quad (8)$$

$$\Rightarrow P = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad (9)$$

By using Parsval's Power Theorem

$$\frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \quad (10)$$

$$P_{avg} = \sum_{k=-\infty}^{\infty} |a_k|^2 \quad (11)$$

$$= 2 \sum_{k=1}^{\infty} |a_k|^2 + |a_0|^2 \quad (12)$$

$$= 2 \sum_{k=1}^3 |a_k|^2 + |a_0|^2 \quad (13)$$

$$= 2(1^2 + 2^2 + 3^2) + 2^2 \quad (14)$$

$$= 32 \quad (15)$$

$$x(n) = 2\text{Re}\left(e^{\frac{j\pi t}{3}} + 2e^{\frac{2j\pi t}{3}} + 3e^{j\pi t}\right) + 2 \quad (16)$$

$$\Rightarrow x(n) = 2\left(\cos\left(\frac{\pi t}{3}\right) + 2\cos\left(\frac{2\pi t}{3}\right) + 3\cos(\pi t)\right) + 2 \quad (17)$$

The average power of the signal  $x(t)$  (rounded off to one decimal place) is 32

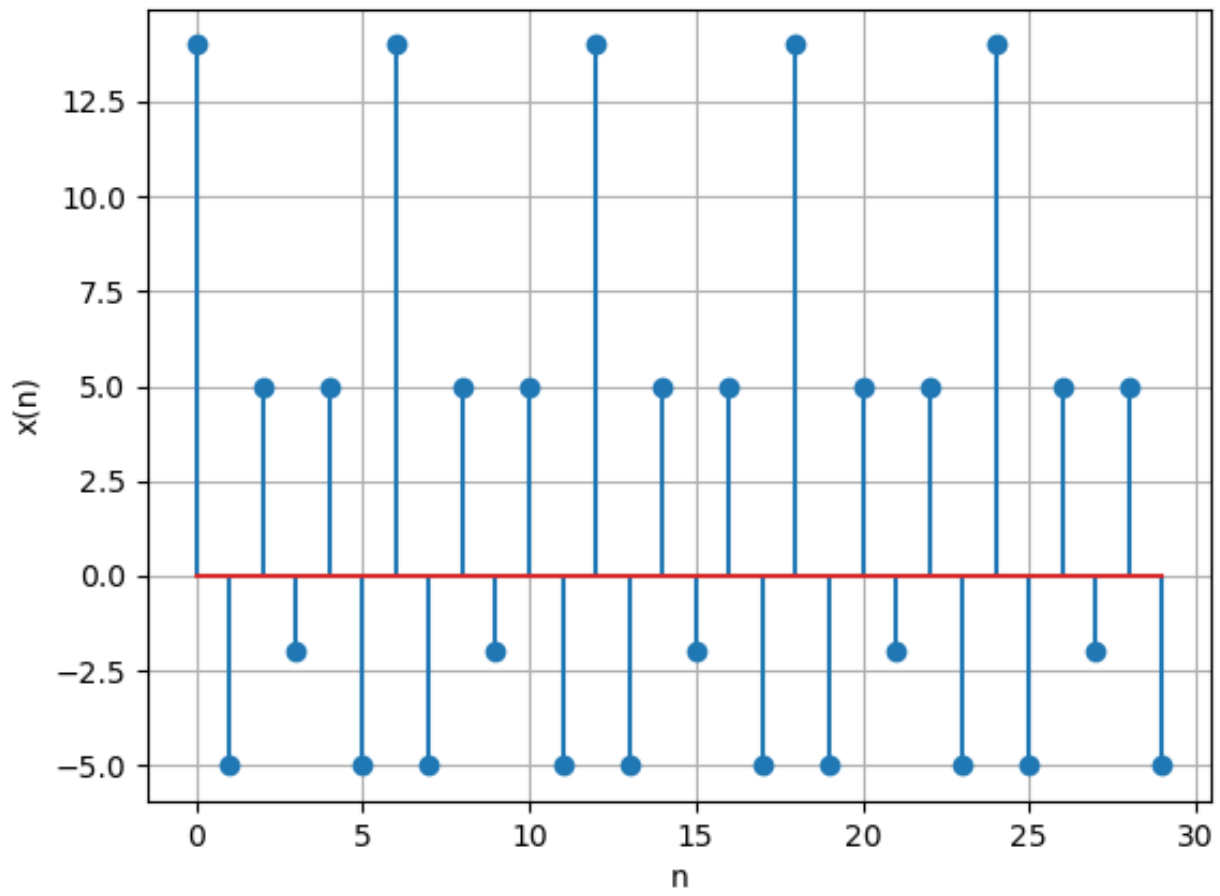


Fig. 1. STEM PLOT OF  $y(n]$