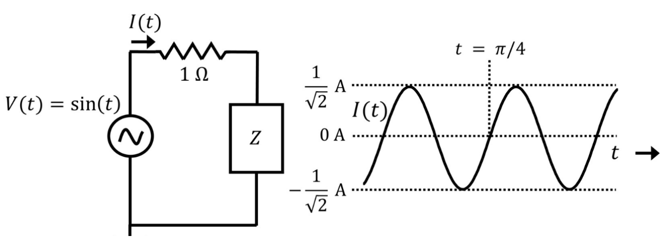
Assignment
GATE-EC-39

EE23BTECH11034 - Prabhat Kukunuri

I. Question

Consider the circuit shown in the figure with input V(t) in volts.The sinusoidal steady state current I(t) flowing through the circuit is shown graphically(where t is in seconds). The circuit element Z can be________.

1) a capacitor of 1 F
2) an inductor of 1 H
3) a capacitor of $\sqrt{3}$ H
4) an inductor of $\sqrt{3}$ H



Solution:

Symbol	Value	Description
$V(t)$	$\sin t$	Time varying voltage source
$I(()t)$	$\sin t - \frac{\pi}{4}$	Current flowing in the circuit
R	1Ω	Resistor in series to Z
Z	Z	Circuit element

TABLE 4
Variable description

The current through the circuit can be expressed as

$$I(t) = \sin\left(t - \frac{\pi}{4}\right) \tag{1}$$

Since, the voltage seems to be leading the current the circuit element z is an inductor with inductance L.
Applying KVL in the circuit,

$$R.I(t) + L\frac{dI(t)}{dt} = sin(t) \tag{2}$$

Applying Fourier transform to the differential equation,

$$R.I(s) + sL.I(s) - \frac{1}{s^2+1} = 0 \tag{3}$$

$$I(s)(R + sL) = \frac{1}{s^2+1} \tag{4}$$

$$\sin(at + b) \overset{\mathcal{L}}{\longleftrightarrow} \frac{a\cos(b) + s\sin(b)}{a^2 + s^2} \tag{5}$$

$$\sin\left(t - \frac{\pi}{4}\right) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1-s}{2(s^2+1)} \tag{6}$$

$$\frac{1-s}{2(s^2+1)}(R + sL) = \frac{1}{s^2+1} \tag{7}$$

Upon plugging in R=1Ω,

$$L = \frac{1}{s} \tag{8}$$

Applying inverse Laplace,

$$L = 1H \tag{9}$$

Appendix

Laplace transform of $\sin(at+b)$ is as follows,

$$\sin(at+b) \xleftrightarrow{\mathcal{L}} \int_0^\infty \sin(at+b)\, e^{-st} dt \tag{10}$$

$$\int_0^\infty \sin(at+b)\, e^{-st} dt = \cos b \int_0^\infty \sin(at)\, e^{-st} dt + \sin b \int_0^\infty \cos(at)\, e^{-st} dt \tag{11}$$

$$\int_0^\infty \cos(at)\, e^{-st} dt = \frac{e^{-st}}{a} sinat\Big|_0^\infty + \frac{s}{a}\int_0^\infty \sin(at)\, e^{-st} dt \tag{12}$$

$$\int_0^\infty \cos(at)\, e^{-st} dt = \frac{s}{a}\int_0^\infty \sin(at)\, e^{-st} dt \tag{13}$$

$$\int_0^\infty \cos(at)\, e^{-st} dt = \frac{s}{a}\left(\frac{-e^{-st}}{a} cosat\Big|_0^\infty + \frac{s}{a}\int_0^\infty \cos(at)\, e^{-st} dt\right) \tag{14}$$

$$\int_0^\infty \cos(at)\, e^{-st} dt = \frac{s}{a^2} + \frac{s^2}{a^2}\int_0^\infty \cos(at)\, e^{-st} dt \tag{15}$$

$$\int_0^\infty \cos(at)\, e^{-st} dt = \frac{s}{s^2+a^2}, s > 0 \tag{16}$$

From (13) we can say,

$$\int_0^\infty \sin(at)\, e^{-st} dt = \frac{a}{s^2+a^2}, s > 0 \tag{17}$$

$$\therefore \sin(at+b) \xleftrightarrow{\mathcal{L}} \frac{s\sin b + a\cos b}{s^2+a^2} \tag{18}$$