GATE 2021 EE 47

EE23BTECH11045 - Palavelli Srija*

Question: In the given figure, plant $G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K\left(\frac{1+T_1 s}{1+T_2 s}\right)$. The external disturbance input is D(s). It is desired that when the disturbance is a unit step, the steady-state error should not exceed 0.1 unit. The minimum value of K is

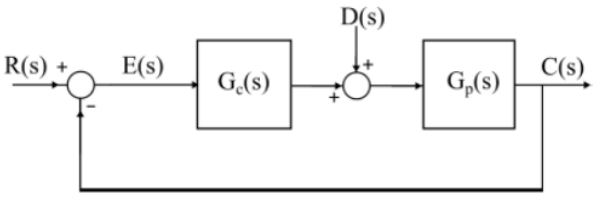

Fig. 0.

Solution:

Symbol	Value
$G_p(s)$	$\frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$
$G_c(s)$	$K\left(\frac{1+T_1 s}{1+T_2 s}\right)$
$\lvert e_{ss}\rvert$	≤ 0.1
K_{min}	??

TABLE 0
Input Parameters

From Fig. 0

$$E(s) = R(s) - C(s) \tag{1}$$

Assume R(s)=0

$$E(s) = -C(s) \tag{2}$$

$$C(s) = \left(E(s)G_c(s) + D(s)\right)G_p(s) \tag{3}$$

$$-E(s) = \left(E(s)G_c(s) + D(s)\right)G_p(s) \tag{4}$$

$$E(s) = \frac{-D(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{5}$$

Using final value theorem

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \tag{6}$$

Where $\mathcal{L}\{e(t)\} = E(s)$

$$e_{ss} = \lim_{s\to 0} sE(s) \tag{7}$$

$$= \lim_{s\to 0}\left(\frac{-sD(s)G_p(s)}{1 + G_c(s)G_p(s)}\right) \tag{8}$$

$$D(s) = \mathcal{L}\{u(t)\}$$

$$= \frac{1}{s} \tag{9}$$

$$e_{ss} = \lim_{s\to 0}\left(\frac{-s\frac{1}{s}G_p(s)}{1 + G_c(s)G_p(s)}\right) \tag{10}$$

$$= \lim_{s\to 0}\frac{\frac{-2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}}{1 + K\left(\frac{1+T_1 s}{1+T_2 s}\right)\frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}} \tag{11}$$

$$= \lim_{s\to 0}\frac{-2.2(1+T_2 s)}{(1+0.1s)(1+0.4s)(1+1.2s)(1+T_2 s) + 2.2K(1+T_1 s)} \tag{12}$$

$$|e_{ss}| = \frac{2.2}{1 + 2.2K} \tag{13}$$

given

$$|e_{ss}| \leq 0.1 \tag{14}$$

$$\frac{2.2}{1 + 2.2K} \leq 0.1 \tag{15}$$

$$K \geq 9.54 \tag{16}$$

$$K_{\min} = 9.54 \tag{17}$$