Gate 2022_EE_17

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The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is $-180°$. This system has

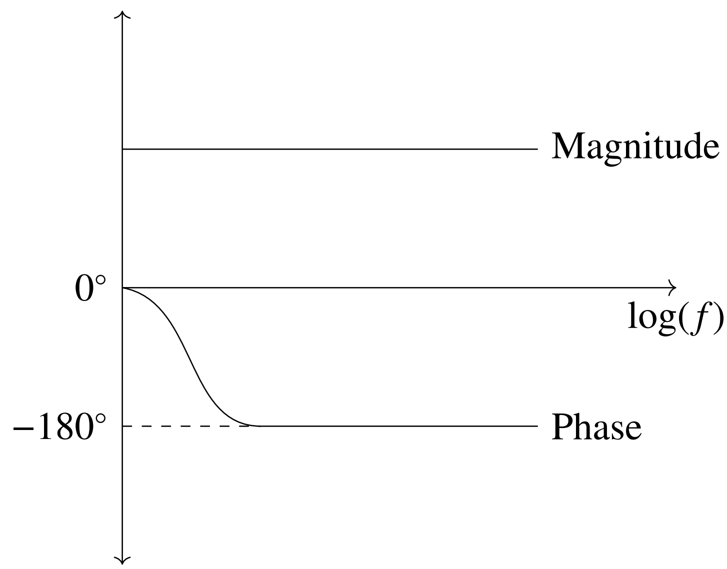

Fig. 1.

(A) one LHP pole and one RHP zero at the same frequency.
(B) one LHP pole and one LHP zero at the same frequency.
(C) two LHP poles and one RHP zero.
(D) two RHP poles and one LHP zero.
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Solution:
Flat constant magnitude response for all frequency of system shows that it is an all pass system.
In all pass system, poles and zeros are symmetrical about $j\omega$ axis.
Possible transfer functions are

$$T_1(s) = \frac{s-a}{s+a} \quad a > 0 \tag{1}$$
$$T_2(s) = \frac{a-s}{a+s} \quad a > 0 \tag{2}$$
$$s = j\omega \tag{3}$$

From the phase plot as $\omega \to \infty$ shows $\phi = -180°$.

1) For $T_1(s)$:
   Using equation (3)

$$T_1(j\omega) = \frac{j\omega - a}{j\omega + a} \quad a > 0 \tag{4}$$

$$\angle T_1(j\omega) = 180^\circ - \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) \tag{5}$$

$$= 180^\circ - 2\tan^{-1}\left(\frac{\omega}{a}\right) \tag{6}$$

   At $\omega = \infty$,

$$\angle T_1(j\omega) = 0^\circ \tag{7}$$

2) For $T_2(s)$:
   Using equation (3)

$$T_2(j\omega) = \frac{a - j\omega}{a + j\omega} \quad a > 0 \tag{8}$$

$$\angle T_2(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) \tag{9}$$

$$= -2\tan^{-1}\left(\frac{\omega}{a}\right) \tag{10}$$

   At $\omega = \infty$,

$$\angle T_2(j\omega) = -180^\circ \tag{11}$$

Hence, the transfer function of given all pass filter.

$$T(s) = \frac{a - s}{a + s} \quad a > 0 \tag{12}$$

Hence, the system has one LHP pole and one RHP zero at the same frequency.