GATE 2022 BM.38

EE23BTECH11010 - VENKATESH BANDAWAR*

Question: An input $x(t)$ is applied to a system with a frequency transfer function given by $H(j\omega)$ as shown below. The magnitude and phase response of the transfer function are shown below. If $y(t_d) = 0$ for $x(t) = u(t)$, the time $t_d(> 0)$ is.
(Gate 2022 BM.38)

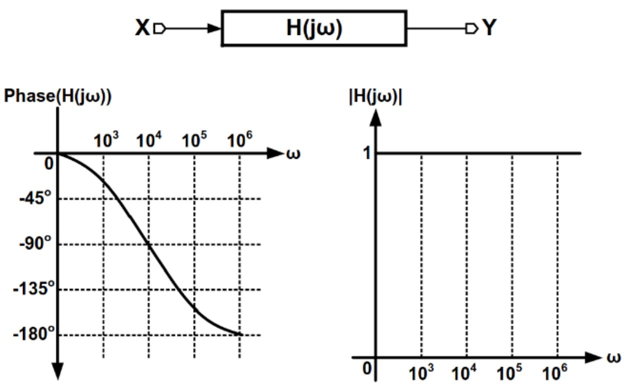

Fig. 1: Graph of y(t)

Solution:

Parameter	Description
$x(t) = u(t)$	Input signal
$y(t)$	Output signal
$X(j\omega)$	Fourier Transform of $x(t)$
$Y(j\omega)$	Fourier Transform of $y(t)$
$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$	Transfer function

TABLE I: Input Parameters Table

from graph 2

$$\angle H(j\omega) = -2\tan^{-1}\left(\frac{\omega}{a}\right) \tag{1}$$

At $\omega = 10^4, \angle H(j\omega) = -\frac{\pi}{2}$

$$\implies a = 10^4 \tag{2}$$

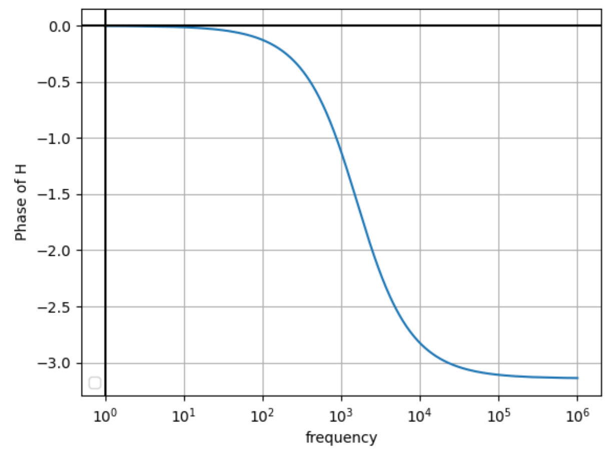

Fig. 2: Phase of H(f)

$$\angle H(j\omega) = \tan^{-1}\left(\frac{-\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) \tag{3}$$

$$H(j\omega) = \frac{e^{j\tan^{-1}\left(\frac{-\omega}{a}\right)}}{e^{j\tan^{-1}\left(\frac{\omega}{a}\right)}} \tag{4}$$

$$= \frac{\frac{a-j\omega}{\sqrt{a^2+\omega^2}}}{\frac{a+j\omega}{\sqrt{a^2+\omega^2}}} \tag{5}$$

$$= \frac{a - j\omega}{a + j\omega} \tag{6}$$

Substitute $s = j\omega$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \tag{7}$$

$$Y(s) = \frac{1}{s}\frac{a-s}{a+s} \tag{8}$$

$$= \frac{1}{s} - \frac{2}{a+s} \tag{9}$$

$$\frac{1}{s} \xleftrightarrow{\mathcal{L}^{-1}} u(t) \tag{10}$$

$$\frac{1}{a+s} \xleftrightarrow{\mathcal{L}^{-1}} e^{-at}u(t) \tag{11}$$

$$y(t) = (1 - 2e^{-at})u(t) \tag{12}$$

Verification of laplace transform:

$$\because y(t_d) = 0 \tag{13}$$

$$t_d = 100\ln 2\mu s \tag{14}$$

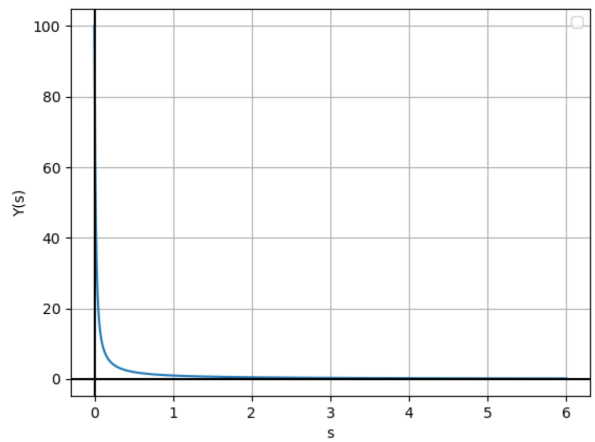

Fig. 3: Laplace Transform of $u(t)$

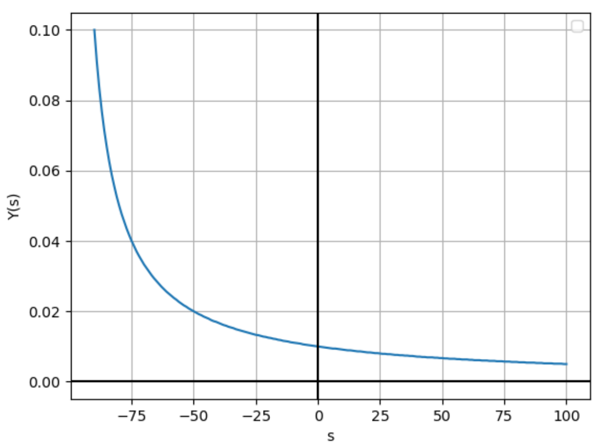

Fig. 4: Laplace Transform $of e^{-at}u(t)$