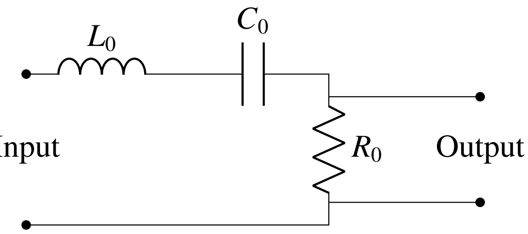
GATE 22 IN/33

EE23BTECH11040 - Manoj Kumar Ambatipudi*

QUESTION: In the bandpass filter circuit shown, $R_0 = 50\Omega$, $L_0 = 1mH$, $C_0 = 10nF$. The q factor of the filter is

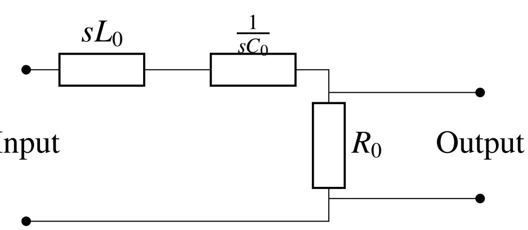


SOLUTION:

Variable	Description	Value
R_0	Resistance	50Ω
L_0	Inductance	$1mH$
C_0	Capacitance	$10nF$
ω_0	Resonant Angular Frequency	$\frac{1}{\sqrt{L_0C_0}}$

TABLE 1
VARIABLES AND THEIR DESCRIPTION

The corresponding Laplace domain circuit is



Input $X(s)$ can be written as

$$X(s) = I(s)\left(sL_0 + \frac{1}{sC_0} + R_0\right) \tag{1}$$

Output $Y(s)$ can be written as

$$Y(s) = I(s)R_0 \tag{2}$$

Transfer function $H(s)$ can be written as

$$H(s) = \frac{Y(s)}{X(s)} \tag{3}$$

$$= \frac{sC_0R_0}{s^2C_0L_0 + C_0R_0s + 1} \tag{4}$$

substituting $s = j\omega$

$$H(j,\omega) = \frac{j\omega C_0R_0}{-\omega^2C_0L_0 + jC_0R_0\omega + 1} \tag{5}$$

$$\implies |H(j,\omega)| = \frac{\omega C_0R_0}{\sqrt{\left(1-\omega^2C_0L_0\right)^2 + (C_0R_0\omega)^2}} \tag{6}$$

Differentiating w.r.t ω and equating to 0, we get

$$\frac{d\,|H(j,\omega)|}{d\omega} = \frac{C_0R_0}{\sqrt{\left(1-\omega^2C_0L_0\right)^2 + (C_0R_0\omega)^2}} + \frac{\omega C_0R_0}{2\left(\left(1-\omega^2C_0L_0\right)^2 + (C_0R_0\omega)^2\right)^{\frac{3}{2}}} \left(2\omega\,(C_0R_0)^2 - 2\left(1-\omega^2C_0L_0\right)2\omega\right) = 0 \tag{7}$$

$$\implies \omega_0 = \frac{1}{\sqrt{L_0C_0}} \tag{8}$$

from Table 1,

$$\omega_0 = 316227.76 \tag{9}$$

$Q-factor$ defined with reference to inductor

$$Q = \left|\frac{V_L}{V_R}\right|_{\omega_0} \tag{10}$$

$$= \frac{L_0\omega_0}{R_0} \tag{11}$$

$$= \frac{1}{R_0}\sqrt{\frac{L_0}{C_0}} \quad \text{(from (8))} \tag{12}$$

$Q-factor$ defined with reference to capacitor

$$Q = \left|\frac{V_C}{V_R}\right|_{\omega_0} \tag{13}$$

$$= \frac{1}{C_0\omega_0R_0} \tag{14}$$

$$= \frac{1}{R_0}\sqrt{\frac{L_0}{C_0}} \quad \text{(from (8))} \tag{15}$$

Substituting the values from Table 1, we get

$$Q = 200 \tag{16}$$

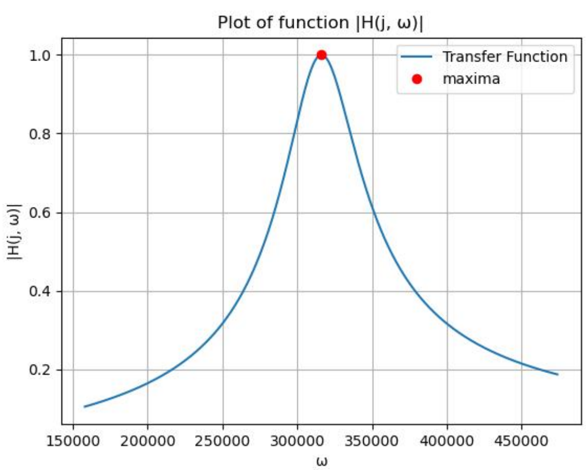


Fig. 1. Transfer function $|H(j, \omega)|$ taken from python3