Q: Consider the wave elevation spectrum $S_{\eta\eta}(\omega)$ as shown in the figure. Then, the significant wave height is _____ m.

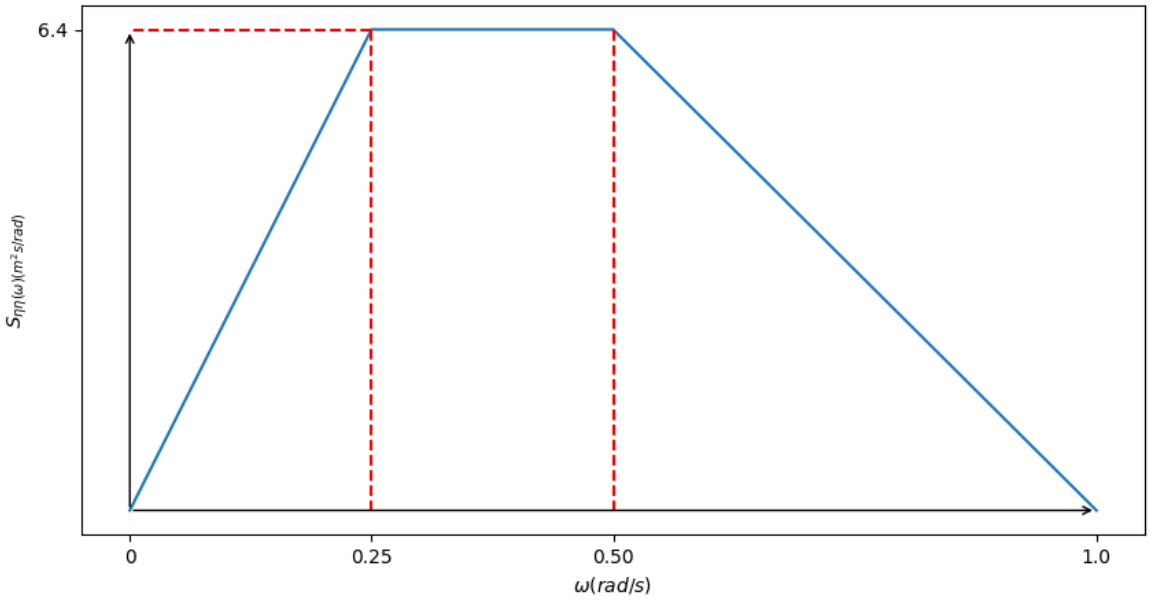

Fig. 1. Wave Elevation Spectrum

(A) 2
(B) 4
(C) 6
(D) 8

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Solution: Given:

$$S_{\eta\eta}(\omega)(m^2s/rad) = \begin{cases} 25.6\omega & \text{if } \omega \in [0, 0.25] \\ 6.4 & \text{if } \omega \in (0.25, 0.50] \\ 12.8\omega - 12.8 & \text{if } \omega \in (0.50, 1.0] \\ 0 & o.w \end{cases} \tag{1}$$

In terms of f:

$$S_{\eta\eta}(f)(m^2s) = \begin{cases} 51.2\pi f & \text{if } f \in [0, \frac{\pi}{2}] \\ 6.4 & \text{if } f \in (\frac{\pi}{2}, \pi] \\ 25.6\pi f - 12.8 & \text{if } f \in (\pi, 2\pi] \\ 0 & o.w \end{cases} \tag{2}$$

Significant Wave Height:

$$H_s = 4\sqrt{\int_0^\infty S(f)df} \tag{3}$$

From (2)

$$H_s = 4\sqrt{\int_0^{\frac{\pi}{2}} 51.2\pi f df + \int_{\frac{\pi}{2}}^{\pi} 6.4 df + \int_{\pi}^{2\pi} (25.6\pi f - 12.8) df} \tag{4}$$

$$= 4\sqrt{0.8 + 1.6 + 1.6} \tag{5}$$

$$\therefore H_s = 8 \tag{6}$$

Hence the answer is option (D).