GATE 2021 EC 5

EE23BTECH11032 - Kaustubh Parag Khachane *

Question GATE 21 EC 5 :
Consider two 16-point sequences x $[n]$ and h $[n]$. Let the linear convolution of x $[n]$ and h $[n]$ be denoted by y $[n]$, while z $[n]$ denotes the 16-point inverse discrete Fourier transform ($IDFT$) of the product of the 16-point DFTs of x(n) and h $[n]$. The values of k for which z $[k]$ = y $[k]$ are

1) $k = 0, 1, 2, 3, ..., 15$
2) $k = 0$
3) $k = 15$
4) $k = 0$ and $k = 15$

(GATE EC 2021)

Solution:

We can write z $[n]$ as ,

Parameter	Description
x $[n]$	Given 16 point sequence
h $[n]$	Given 16 point sequence
y $[n]$	Linear convolution of x $[n]$ and h $[n]$
z $[n]$	IDFT of products of DFTs of h $[n]$ and x $[n]$
N	number of terms (16)

TABLE 4
Parameter Table

$$z[n] = IDFT[X[f]H[f]] \tag{1}$$

We know that product in frequency domain is convolution in time domain. However, z $[n]$ is not linear convolution of x $[n]$ and h $[n]$ in the time domain due to periodicity of DFT. Point-wise multiplication in the frequency domain (product of DFTs) doesn't translate directly to convolution in the time domain due to periodicity and potential aliasing.

z $[n]$ is the circular convolution of x $[n]$ and h $[n]$. Using the formula for circular convolution,

$$z[n] = \sum_{m=-\infty}^{\infty} \left(h[m] \sum_{p=-\infty}^{\infty} x[n-m-pN] \right) \tag{2}$$

y $[n]$ is a linear convolution of x $[n]$ and h $[n]$.

$$y[n] = \sum_{m=-\infty}^{\infty} x(m) h[n-m] \tag{3}$$

Each term of y $[n]$ will be sum of products of terms of x $[n]$ and h $[n]$. The number of terms in each summation will go from 1 for $n = 1$ to 16 for $n = 15$.

z $[n]$ is expressed as sum of 15 terms for all permissible values of n using $p = 0$ or $p = 1$ in (2).

Thus,$z[k] = y[k]$ can be possible for only $k = 15$.

For $k = 15$,

$$y[15] = x[0]h[15] + x[1]h[14] + ...x[15]h[0] \tag{4}$$

Using p = 0 in (2),

$$z[15] = h[0]x[15] + h[1]x[14] + ...h[15]x[0] \tag{5}$$

$$= x[0]h[15] + x[1]h[14] + ...x[15]h[0] \tag{6}$$

Thus , z $[15]$ = y $[15]$.

Graphically, let

$$x = [1, 2, 3, 4, ..., 16] \tag{7}$$

$$h = [1, 1, 1, 1, ...1] \tag{8}$$

Then the plot for z and y is as shown bellow.

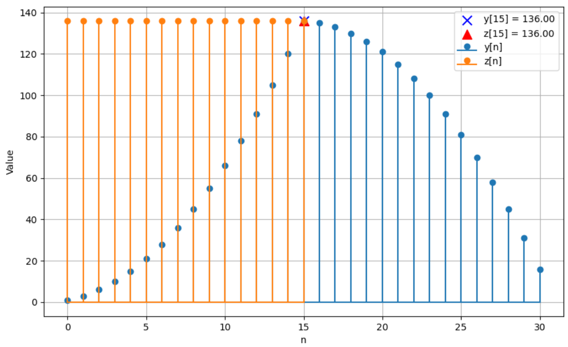

Fig. 4. Plot of y $[n]$ and z $[n]$