GATE EC 41Q

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Question

Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$$

(rounded off to one decimal place) is.

Solution

Symbol	Value	description
$x[n]$	$2^{n-1}u[-n+2]$	Discrete time signal
$y[n]$	$2^{-n+2}u[n+1]$	Discrete time signal

TABLE 0

$$x[n] * y[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega})Y(e^{j\omega}) \tag{1}$$

$$x[n] \xleftrightarrow[transform]{Fourier} X(e^{j\omega}) \tag{2}$$

$$y[n] \xleftrightarrow[transform]{Fourier} Y(e^{j\omega}) \tag{3}$$

The

$$y(n) \xleftrightarrow[transform]{Fourier} y(e^{j\omega}) \tag{4}$$

By using the time reversal property:

$$y[-n] \xleftrightarrow[transform]{Fourier} y(e^{-j\omega}) \tag{5}$$

Let assume

$$z[n] = x[n] * y[-n] \tag{6}$$

$$Z\left(e^{j\omega}\right) = X(e^{j\omega})Y(e^{-j\omega}) \tag{7}$$

$$z[n] = \frac{1}{2\pi}\int_0^{2\pi} Z(e^{j\omega})e^{j\omega n}d\omega \tag{8}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega n}d\omega. \tag{9}$$

putting n=0, we get

$$z[0] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega \tag{10}$$

$$z[n] = x[n] * y[-n] \tag{11}$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1}u[-k+2]\cdot 2^{n-k+2}u[-n+k+1 \tag{12}$$

$$= \sum_{k=-\infty}^{2} 2^{k-1}\cdot 2^{n-k+2}u[-n+k+1] \tag{13}$$

$$= \sum_{k=-\infty}^{2} 2^{k-1+n-k+2}u[-n+k+1] \tag{14}$$

$$= \sum_{k=-\infty}^{2} 2^{n+1}u[-n+k+1] \tag{15}$$

Putting $n = 0$, we get:

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega = z[0] \tag{16}$$

$$= \sum_{k=-\infty}^{2} 2\cdot u[k+1] \tag{17}$$

$$= \sum_{k=-1}^{2} 2(1) = 2\times 4 \tag{18}$$

$$= 8 \tag{19}$$