GATE 2021 EE 43

EE23BTECH11022 - G DILIP REDDY

Question:

Consider a continuous-time signal $x(t)$ defined by $x(t) = 0$ for $|t| > 1$, and $x(t) = 1 - |t|$ for $|t| \leq 1$. Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$. The maximum magnitude of $X(\omega)$ is

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Solution:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \tag{1}$$

$$X(f) = \int_{-1}^{1} (1 - |t|) e^{-j2\pi ft} dt \tag{2}$$

$$X(f) = \int_{-1}^{1} e^{-j2\pi ft} dt - \int_{-1}^{1} |t| e^{-j2\pi ft} dt \tag{3}$$

$$X(f) = 2 \int_{0}^{1} \cos(2\pi ft) \, dt - 2 \int_{0}^{1} t \cos(2\pi ft) \, dt \tag{4}$$

$$X(f) = 2 \frac{\sin(2\pi f)}{2\pi f} - 2 \left[\frac{\sin(2\pi f)}{2\pi f} + \frac{\cos(2\pi f)}{(2\pi f)^2} - \frac{1}{(2\pi f)^2} \right] \tag{5}$$

$$X(f) = 2 \frac{1 - \cos(2\pi f)}{(2\pi f)^2} \tag{6}$$

$$X(f) = 2 \frac{2 \sin^2 \left(\frac{2\pi f}{2} \right)}{(2\pi f)^2} \tag{7}$$

$$X(f) = \frac{\sin^2(\pi f)}{(\pi f)^2} \tag{8}$$

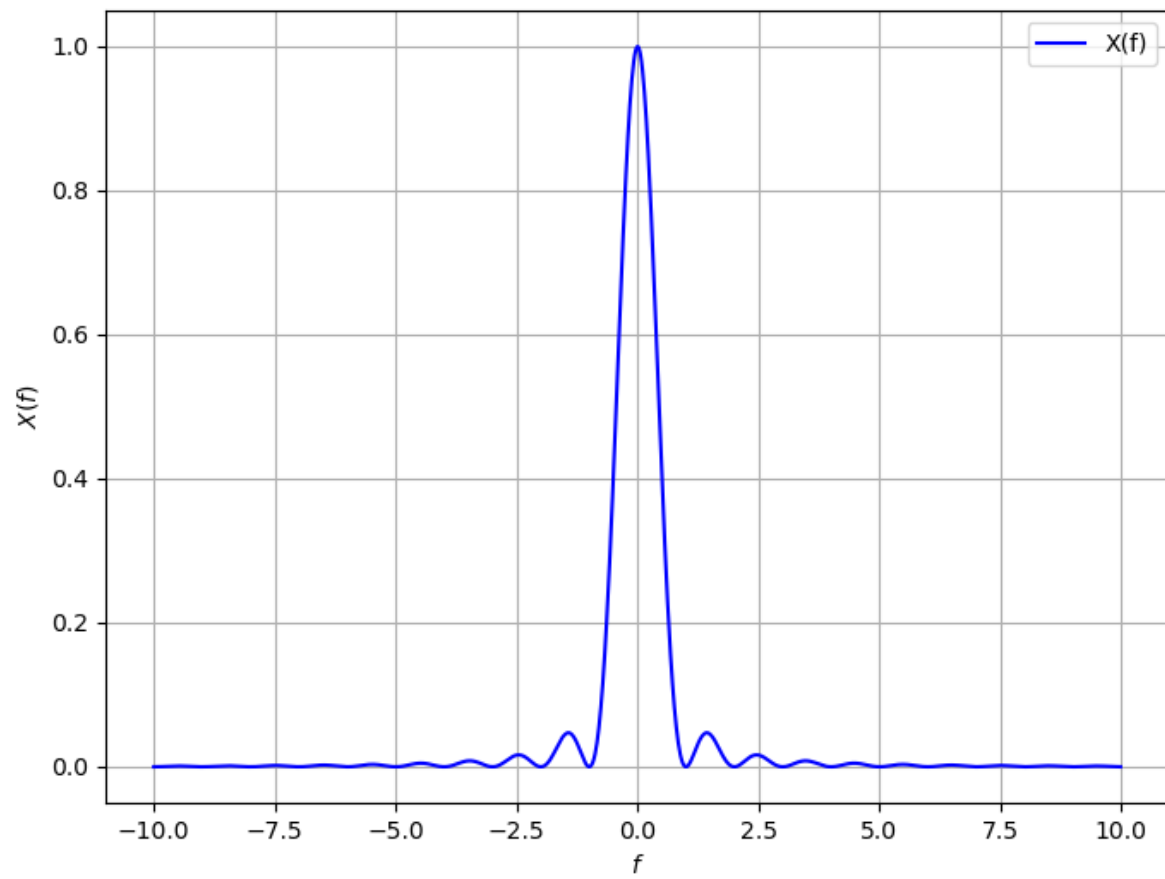$$f \to 0 \implies X(f) \to 1 \tag{9}$$

Maximum of magnitude of $X(f) = 1$

Fig. 1: plot of X(f)