

ME 36

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QUESTION: Given that S is the unit circle in the counter clock-wise direction with its centre at origin, the integral $\oint \left(\frac{z^3}{4z-j} \right) dz = \text{_____}$ (round off to three decimal places) (GATE 2022 ME)

Solution:

For pole

$$4z - j = 0 \quad (1)$$

$$z = \frac{j}{4} \text{ order of pole is } 1 \quad (2)$$

Pole inside unit circle is $\frac{j}{4}$

$$\oint \left(\frac{z^3}{4z-j} \right) dz = \oint \left(\frac{\frac{z^3}{4}}{z - \frac{j}{4}} \right) dz \quad (3)$$

$$= 2\pi j \left(\frac{z^3}{4} \right) \text{ at } z = \frac{j}{4} \text{ (using Cauchy integral)} \quad (4)$$

$$= 2\pi j \left(\frac{-j}{256} \right) \quad (5)$$

$$= \frac{\pi}{128} \quad (6)$$

$$= 0.02 \quad (7)$$