

# GATE ES22 13

EE23BTECH11043 - BHUVANESH SUNIL NEHETE\*

**Question:** Assuming  $s > 0$ ; Laplace transform for  $f(x) = \sin(ax)$  is

- (A)  $\frac{a}{s^2+a^2}$
- (B)  $\frac{s}{s^2+a^2}$
- (C)  $\frac{a}{s^2-a^2}$
- (D)  $\frac{s}{s^2-a^2}$

**Solution:**

$$\mathcal{L}(f(x)) = \int_{-\infty}^{\infty} e^{-sx} f(x) dx \quad (1)$$

$$\text{We can write } \sin(ax) = \frac{e^{ax} - e^{-ax}}{2i} \quad (2)$$

From (2)

$$\mathcal{L}(\sin(ax)) = \int_0^{\infty} e^{-sx} \left( \frac{e^{iax} - e^{-iax}}{2i} \right) dx \quad (3)$$

$$= \frac{1}{2i} \int_0^{\infty} e^{-x(s-ia)} - e^{-x(s+ia)} dx \quad (4)$$

$$= \frac{1}{2i} \left( \frac{e^{-x(s-ia)}}{-(s-ia)} + \frac{e^{-x(s+ia)}}{-(s+ia)} \right)_0^{\infty} \quad (5)$$

$$= \frac{1}{2i} \left( \frac{1}{s-ia} - \frac{1}{s+ia} \right) \quad (6)$$

$$= \frac{a}{s^2 + a^2} \quad (7)$$

So, option (A) is correct.