# EE23BTECH11054 - Sai Krishna Shanigarapu*

## Gate ES 2022

13. Assuming $s > 0$; Laplace transform for $f(x) = \sin(ax)$ is

(A) $\frac{a}{s^2+a^2}$

(B) $\frac{s}{s^2+a^2}$

(C) $\frac{a}{s^2-a^2}$

(D) $\frac{s}{s^2-a^2}$

(GATE 2022 ES)

**Solution:**
Assuming $Re(s) > 0$
Using Table I

1) Proof 1: Using Definition

$$\mathcal{L}(\sin(ax)) = \int_0^\infty e^{-sx}\sin(ax)\ dx \tag{1}$$
$$= \lim_{L\to\infty}\int_0^L e^{-sx}\sin(ax)\ dx \tag{2}$$
$$= \frac{s\sin(0)+a\cos(0)}{s^2-a^2} - 0 \tag{3}$$
$$= \frac{a}{s^2+a^2} \tag{4}$$

2) Proof 2: Using Euler's identity

$$\mathcal{L}(\sin(ax)) = \mathcal{L}\left(\frac{e^{jax}-e^{-jax}}{2j}\right) \tag{5}$$
$$= \frac{\left(\mathcal{L}\left(e^{jax}\right)-\mathcal{L}\left(e^{-jax}\right)\right)}{2j} \tag{6}$$
$$= \frac{2}{2j}\left(\frac{1}{s-ja}-\frac{1}{s+ja}\right) \tag{7}$$
$$= \frac{1}{2j}\left(\frac{2ja}{s^2+a^2}\right) \tag{8}$$
$$= \frac{a}{s^2+a^2} \tag{9}$$

3) Proof 3: Using Laplace Transform of Second Derivative

$$\mathcal{L}(f''(x)) = s^2\mathcal{L}(f(x)) - sf(0) - f'(0) \tag{10}$$

Then ,

$$f(x) = \sin(ax) \tag{11}$$
$$\implies f'(x) = a\cos(ax) \tag{12}$$
$$f''(x) = -a^2\ \sin(ax) \tag{13}$$
$$f(0) = 0 \tag{14}$$
$$f'(0) = 0 \tag{15}$$
$$\implies \mathcal{L}\left(-a^2\ \sin(ax)\right) = s^2\mathcal{L}(\sin(ax)) - a \tag{16}$$
$$\implies -a^2\mathcal{L}(\sin(ax)) = s^2\mathcal{L}(\sin(ax)) - a \tag{17}$$
$$\implies \mathcal{L}(\sin(ax)) = \frac{a}{s^2+a^2} \tag{18}$$

4) Proof 4: Using Laplace Transform of exponential

$$\mathcal{L}\left(e^{jax}\right) = \mathcal{L}(\cos(ax) + j\sin(ax)) \tag{19}$$
$$= \mathcal{L}(\cos(ax)) + j\mathcal{L}(\sin(ax)) \tag{20}$$
$$\implies \mathcal{L}(\sin(ax)) = \mathbf{Im}\left(\mathcal{L}\left(e^{jax}\right)\right) \tag{21}$$
$$= \mathbf{Im}\left(\frac{s+ja}{s^2+a^2}\right) \tag{22}$$
$$= \frac{a}{s^2+a^2} \tag{23}$$

$\therefore$ Option (A) is correct.

| $y(x)$ | $\mathcal{L}(y(x))$ |
|---|---|
| $e^{ax}$ | $\frac{1}{s-a}$ |
| $k\,y(x)$ | $k\mathcal{L}(y(x))$, where k is constant |
| $y''(x)$ | $s^2\mathcal{L}(y(x)) - sy(0) - y'(0)$ |

TABLE I
LAPLACE TRANSFORMS

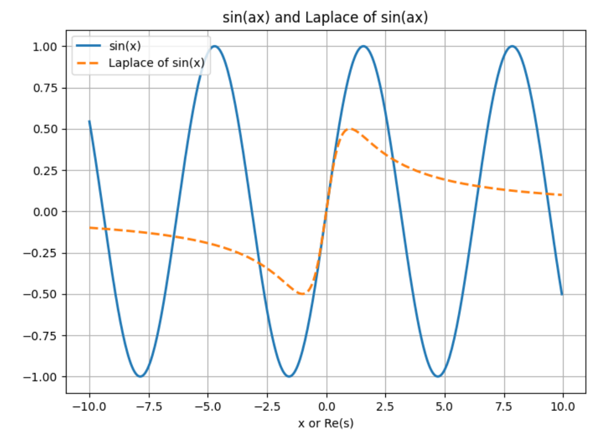


Fig. 1. plot of sin(ax) and it’s laplace transform

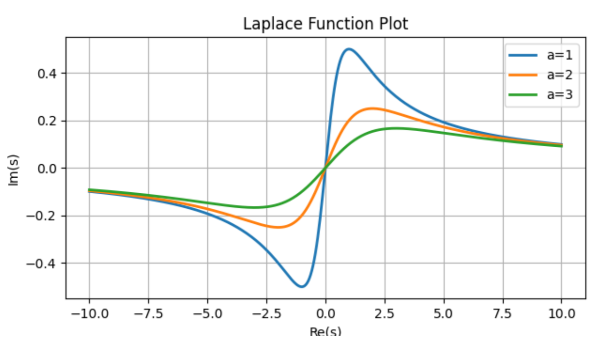


Fig. 2. plots of laplace forms of sin(ax)