GATE 2021 BM 46

EE23BTECH11007 - Aneesh Kadiyala*

Question: Consider a unity feedback system with closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{s+90}{s^2+10s+90}$$

The steady state error with respect to a unit ramp input is ______ . (rounded off to one decimal)

(GATE 2021 BM)

Solution:

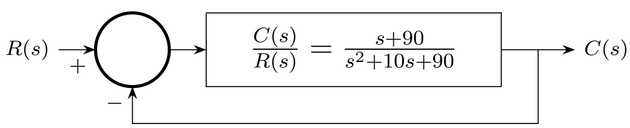

Fig. 0. Block Diagram of the System

$$\frac{C(s)}{R(s)} = \frac{s+90}{s^2+10s+90} \tag{1}$$

where $C(s)$ is the output and $R(s)$ is the input. Given that input is unit ramp function:

$$r(t) = tu(t) \tag{2}$$

$$\implies R(s) = \frac{1}{s^2} \tag{3}$$

$$\implies C(s) = \frac{s+90}{s^2(s^2+10s+90)} \tag{4}$$

$$E(s) = R(s) - C(s) \tag{5}$$

$$= \frac{s^2+9s}{s^2(s^2+10s+90)} \tag{6}$$

Steady state error is:

$$\lim_{s\to0} sE(s) = \frac{s+9}{s^2+10s+90} \tag{7}$$

$$= \frac{1}{10} \tag{8}$$

$\therefore$ steady state error for unit ramp input is 0.1.

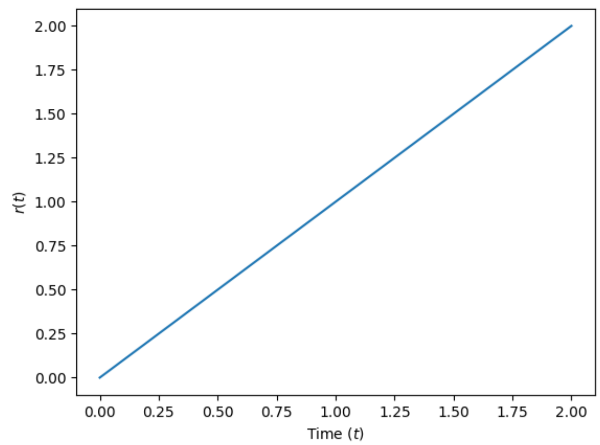

Fig. 0. Plot of $r(t)$ vs t

$$C(s) = \frac{s+90}{s^2(s^2+10s+90)} \tag{9}$$

$$= -\frac{1}{10s} + \frac{1}{s^2} + \frac{s}{10(s^2+10s+90)} \tag{10}$$

$$= -\frac{1}{10s} + \frac{1}{s^2} + \frac{s+5}{(s+5)^2+65} - \frac{1}{2}\left(\frac{1}{(s+5)^2+65}\right) \tag{11}$$

$$c(t) = u(t)\left(-\frac{1}{10} + t + \frac{e^{-5t}}{10}\cos\left(\sqrt{65}t\right) - \frac{e^{-5t}}{2\sqrt{65}}\sin\left(\sqrt{65}t\right)\right) \tag{12}$$

$$E(s) = R(s) - C(s) \tag{13}$$

$$\implies e(t) = r(t) - c(t) \tag{14}$$

$$= u(t)\left(\frac{1}{10} - \frac{e^{-5t}}{10}\cos\left(\sqrt{65}t\right) + \frac{e^{-5t}}{2\sqrt{65}}\sin\left(\sqrt{65}t\right)\right) \tag{15}$$

$$\text{Feedback Gain} = \frac{\frac{C(s)}{R(s)}}{1+\frac{C(s)}{R(s)}} \tag{16}$$

$$= \frac{s+90}{s^2+11s+180} \tag{17}$$

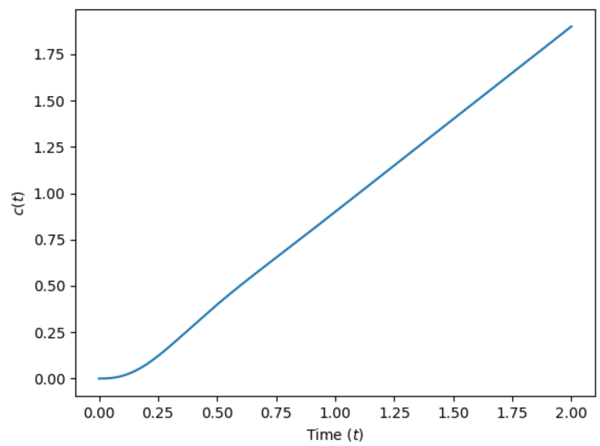

Fig. 0. Plot of $c\,(t)$ vs t

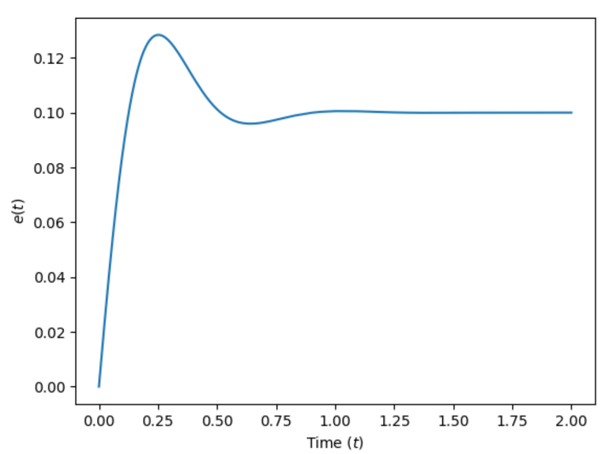

Fig. 0. Plot of $e\,(t)$ vs t