

Assignment

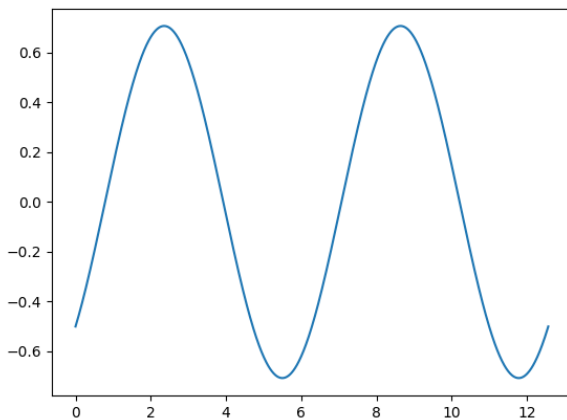
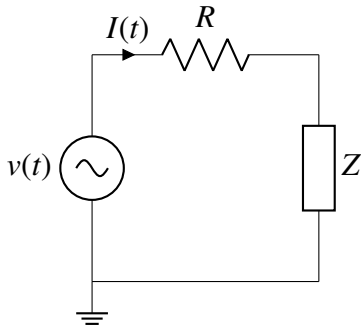
GATE-EC-39

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I. QUESTION

Consider the circuit shown in the figure with input $V(t)$ in volts. The sinusoidal steady state current $I(t)$ flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be_____.

- 1) a capacitor of 1 F
- 2) an inductor of 1 H
- 3) a capacitor of $\sqrt{3}$ H
- 4) an inductor of $\sqrt{3}$ H



Solution:

Symbol	Value	Description
$V(t)$	$\sin t$	Time varying voltage source
$I(t)$	$\sin t - \frac{\pi}{4}$	Current flowing in the circuit
R	1Ω	Resistor in series to Z
Z	Z	Circuit element

TABLE 4
VARIABLE DESCRIPTION

The current through the circuit can be expressed as

$$I(t) = \sin\left(t - \frac{\pi}{4}\right) \quad (1)$$

Since, the voltage seems to be leading the current the circuit element z is an inductor with inductance L .

Applying KVL in the circuit,

$$R.I(t) + L\frac{dI(t)}{dt} = \sin(t) \quad (2)$$

Applying Fourier transform to the differential equation,

$$R.I(s) + sL.I(s) - \frac{1}{s^2 + 1} = 0 \quad (3)$$

$$I(s)(R + sL) = \frac{1}{s^2 + 1} \quad (4)$$

$$\sin(at + b) \xleftrightarrow{\mathcal{L}} \frac{a \cos(b) + s \sin(b)}{a^2 + s^2} \quad (5)$$

$$\sin\left(t - \frac{\pi}{4}\right) \xleftrightarrow{\mathcal{L}} \frac{1 - s}{2(s^2 + 1)} \quad (6)$$

$$\frac{1 - s}{2(s^2 + 1)} (R + sL) = \frac{1}{s^2 + 1} \quad (7)$$

Upon plugging in $R=1\Omega$,

$$L = \frac{1}{s} \quad (8)$$

Applying inverse Laplace,

$$L = 1H \quad (9)$$

Appendix

Laplace transform of $\sin(at + b)$ is as follows,

$$\sin(at + b) \xleftrightarrow{\mathcal{L}} \int_0^{\infty} \sin(at + b) e^{-st} dt \quad (10)$$

$$\int_0^{\infty} \sin(at + b) e^{-st} dt = \cos b \int_0^{\infty} \sin(at) e^{-st} dt + \sin b \int_0^{\infty} \cos(at) e^{-st} dt \quad (11)$$

$$\int_0^{\infty} \cos(at) e^{-st} dt = \frac{e^{-st}}{a} \sin at \Big|_0^{\infty} + \frac{s}{a} \int_0^{\infty} \sin(at) e^{-st} dt \quad (12)$$

$$\int_0^{\infty} \cos(at) e^{-st} dt = \frac{s}{a} \int_0^{\infty} \sin(at) e^{-st} dt \quad (13)$$

$$\int_0^{\infty} \cos(at) e^{-st} dt = \frac{s}{a} \left(\frac{-e^{-st}}{a} \cos at \Big|_0^{\infty} + \frac{s}{a} \int_0^{\infty} \cos(at) e^{-st} dt \right) \quad (14)$$

$$\int_0^{\infty} \cos(at) e^{-st} dt = \frac{s}{a^2} + \frac{s^2}{a^2} \int_0^{\infty} \cos(at) e^{-st} dt \quad (15)$$

$$\int_0^{\infty} \cos(at) e^{-st} dt = \frac{s}{s^2 + a^2}, s > 0 \quad (16)$$

From (13) we can say,

$$\int_0^{\infty} \sin(at) e^{-st} dt = \frac{a}{s^2 + a^2}, s > 0 \quad (17)$$

$$\therefore \sin(at + b) \xleftrightarrow{\mathcal{L}} \frac{s \sin b + a \cos b}{s^2 + a^2} \quad (18)$$