

# GATE-CE.26

EE23BTECH11046 - Poluri Hemanth\*

**Question:** The solution of second-order differential equation

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$  with boundary conditions  $y(0) = 1$  and  $y(1) = 3$ .

(GATE 2021 CE.26)

**Solution:**

Symbol	Values	Description
$Y(s)$	-	y in s domain
$y(x)$	-	y in x domain
$y(0)$	1	y at $x = 0$
$y(1)$	3	$y(x)$ at $x = 1$
$u(x)$	$= \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{o.w} \end{cases}$	unit step function

TABLE I  
PARAMETERS

From (7),

$$y(1) = (1 - 3 - y'(0))e^{-1} \quad (8)$$

$$3 = (1 - 3 - y'(0))e^{-1} \quad (9)$$

$$\Rightarrow y'(0) = -(2 + 3e) \quad (10)$$

From(10),(7)

$$y(x) = (e^x + (3e - 1)xe^{-x})u(x) \quad (11)$$

Applying Laplace transform

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) + 2sY(s) - 2y(0) + Y(s) \quad (1)$$

$$Y(s)(s^2 + 2s + 1) = s - 2 - y'(0) \quad (2)$$

$$\Rightarrow Y(s) = \frac{s - 2 - y'(0)}{s^2 + 2s + 1} \quad (3)$$

$$= \frac{1}{s + 1} - \frac{2 + y'(0)}{(s + 1)^2} \quad (4)$$

For inversion of  $Y(s)$  in partial fractions-

$$\frac{b}{(s + a)^n} \xleftrightarrow{\mathcal{L}} \frac{b}{(n - 1)!} \cdot x^{n-1} e^{-ax} \cdot u(x) \quad (5)$$

Applying Laplace inverse-

From (4),(5)

$$y(x) = \frac{1}{0!} e^{-x} \cdot u(x) - \frac{3 + y'(0)}{1!} x \cdot e^{-x} \cdot u(x) \quad (6)$$

$$= (1 - (3 + y'(0))x)e^{-x}u(x) \quad (7)$$

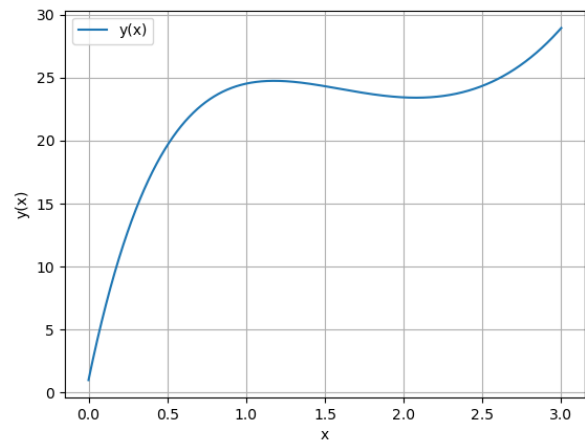


Fig. 1. Plot of  $y(x)$