GATE 2021 EC 23

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Question: A speech signal, band limited to 4 kHz, is sampled at 1.25 times the Nyquist rate. The speech samples, assumed to be statistically independent and uniformly distributed in the range -5 V to +5 V, are subsequently quantized in an 8-bit uniform quantizer and then over a voice-grade AWGN telephone channel. If the ratio of transmitted signal power to channel noise power is 26 dB, the minimum channel bandwidth required to ensure reliable transmission of the signal with arbitrarily small probability of transmission error (*rounded off to one decimal place*) is _____ kHz.

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Solution:

TABLE 0
Input Parameters

Parameter	Value	Description
B_0	4 kHz	Bandwidth of signal
R_N	$2B_0$	Nyquist Rate
f_s	$1.25R_N$	Sampling Frequency
R	nf_s	Data Rate
C	$B\log_2\left(1+\frac{P}{N}\right)$	Capacity of AWGN Channel with bandwidth B
$10\log_{10}\frac{P}{N}$	26 dB	Signal to Noise Ratio

The signal is band limited to 4 kHz.

$$B_0 = 4\text{kHz} \tag{1}$$
$$\implies R_N = 8\text{kHz} \tag{2}$$
$$\implies f_s = 10\text{kHz} \tag{3}$$
$$R = (8)\,(10\text{kHz}) \tag{4}$$
$$\implies R = (8)\left(10^4\right)\ \text{bits/second} \tag{5}$$

Channel capacity for an Additive White Gaussian Noise channel is

$$C = B\log_2\left(1+\frac{P}{N}\right)\ \text{bits/second} \tag{6}$$

where P is the maximum channel power and N is the noise power and B is the channel bandwidth.

$$10\log_{10}\frac{P}{N} = 26\text{dB} \tag{7}$$
$$\implies \frac{P}{N} = 10^{2.6} \tag{8}$$
$$\approx 398.107 \tag{9}$$

For reliable transmission:

$$R \le C \tag{10}$$
$$8\left(10^4\right) \le B\log_2 399.107 \tag{11}$$
$$B \ge \frac{8\left(10^4\right)}{\log_2 399.107} \tag{12}$$
$$\implies B \ge 9258.58\text{Hz} \tag{13}$$

$\therefore$ the minimum channel bandwidth required to ensure reliable transmission of the signal is ≈ 9.26 kHz.