Gate Assignment

EE:1205 Signals and Systems
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Question: A Spectrometer is used to detect plasma oscillations in a sample. The spectrometer can work in the range of 3×10^{12} rad s^{-1} to 30×10^{12} rad s^{-1}. The minimum carrier concentration that can be detected by using this spectrometer is $n \times 10^{21}$ m^{-3}. The value of n is _________. (Round off to two places) (Charge on electron = -1.6×10^{-19} C^{-1}, mass of electron = 9.1×10^{-31} kg and $\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}) (GATE PH 35 2022)

Solution:

Parameter	Value	Description
ω_{p1}	3×10^{12} rad s^{-1}	Lower bound of plasma frequency
ω_{p2}	30×10^{12} rad s^{-1}	Upper bound of plasma frequency
$\Delta\omega_p = \omega_{p2} - \omega_{p1}$	27×10^{12} rad s^{-1}	Plasma Frequency
n_0	$n \times 10^{21}$	Minimum carrier concentration
e	-1.6×10^{-19}	Charge on electron
m	9.1×10^{-31}	Mass of electron

TABLE 1
Parameter Table

$$\Delta\omega_p = \sqrt{\frac{n_0 e^2}{m\epsilon_0}} \tag{1}$$

$$\implies n_0 = \frac{\left(\Delta\omega_p\right)^2 m\epsilon_0}{e^2} \tag{2}$$

$$n_0 = \frac{\left(27 \times 10^{12}\right)^2 \times \left(9.1 \times 10^{31}\right) \times \left(8.85 \times 10^{-12}\right)}{(-1.6 \times 10^{-19})^2} \tag{3}$$

$$\therefore n_0 = 2.83 \times 10^{21} \text{m}^{-3} \tag{4}$$

$$n = n_0 \times 10^{-21} \tag{5}$$

$$\therefore n = 2.83 \tag{6}$$

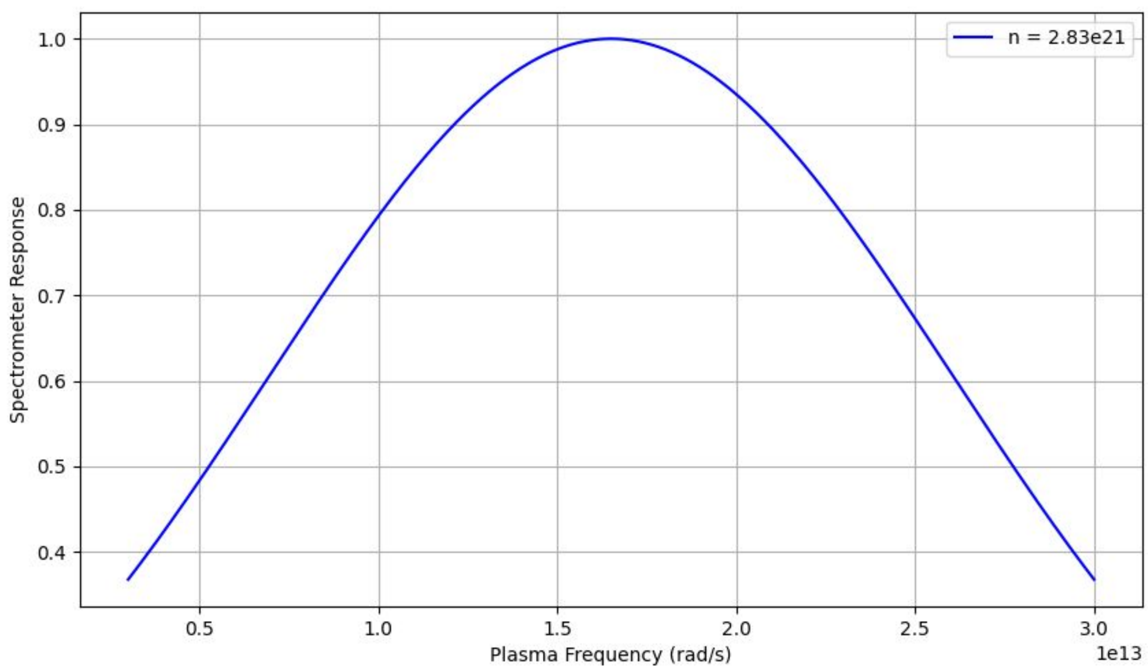

Fig. 1. Plot of Spectrometer response vs Plasma frequency