Assignment

GATE-IN-46

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I. Question

Consider a system with transfer-function $G(s) = \frac{2}{s+1}$. A unit-step function $\mu(t)$ is applied to the system, which results in an output y(t).

If $e(t) = y(t) - \mu(t)$ then $\lim_{t\to\infty} e(t)$ is_____.

Solution:

Symbol	Value	Description
$G(s)$	$\frac{2}{s+1}$	Transfer function
$e(t)$	$y(t) - \mu(t)$	Function of y(t) and $\mu(t)$
$Y(s)$	$G(s) \times U(s)$	Convolution in t domain is multiplication in s domain.
$\mu(t)$	$\begin{cases} 0 & \text{if t<0} \\ 1 & \text{if t>0} \end{cases}$	Unit step function

TABLE 0
Variable description

Applying Laplace transform on $\mu(t)$

$$\mu(t) \xleftrightarrow{\mathcal{L}} U(s) \tag{1}$$

$$U(s) = \frac{1}{s} \tag{2}$$

$$Y(s) = \left(\frac{2}{s+1}\right)\left(\frac{1}{s}\right) \tag{3}$$

$$Y(s) = \frac{2}{s} - \frac{2}{s+1} \tag{4}$$

The inverse Laplace transform of $\frac{a}{s+b}$ is $ae^{-bt}\mu(t)$

$$y(t) = 2\mu(t) - 2e^{-t}\mu(t) \tag{5}$$

$$e(t) = \mu(t)(1 - 2e^{-t}) \tag{6}$$

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \mu(t)(1 - 2e^{-t}) \tag{7}$$

$$\lim_{t\to\infty} e(t) = 1 \tag{8}$$