GATE-2022, In-62

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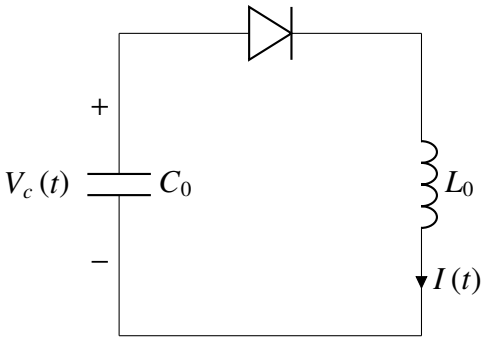
Question: In the circuit shown, the capacitance $C_0 = 10\mu F$ and inductance $L_0 = 1mH$ and the diode is ideal. The capacitor is initially charged to 10V and the current in the inductor is initially zero. If the switch is closed at t=0, the voltage $V_c(t)$(in volts) across the capacitor at t=0.5s is? (round off to one decimal place)



Solution:

$$V_c(0^-) = 10V \tag{1}$$

$$t > 0 \tag{2}$$

convert circuit into laplace form

parameter	laplace transform
C_0	$\frac{1}{sC_0} - V(0^-)C_0$
L_0	sL_0
$i(t)$	$I(s)$
$10cos\left(10^{-4}t\right)V$	$\frac{10V}{10^{-8}s^2-1}$
$sin\left(10^4t\right)A$	$\frac{10^{-7}sA}{10^{-8}s^2-1}$
$v(t)$	$V(s)$

apply KVL,

$$-V_c(s) + I(s)\,sL_0 - \frac{10}{s} = 0 \tag{3}$$

$$\frac{10}{s} - I(s)\,sL_0 = -V_c(s) \tag{4}$$

$$\frac{10}{s} - V_c(s)sC_0sL_0 = -V_c(s) \tag{5}$$

$$\frac{10V}{10^{-8}s^2 - 1} = V_c(s) \tag{6}$$

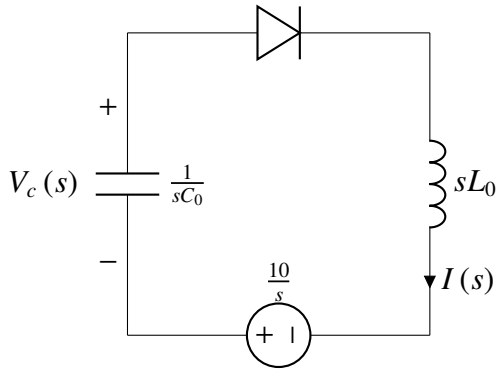


Fig. 0. s domain circuit

apply inverse laplace transform

$$V_c(t) = 10cos\left(10^{-4}t\right)V \tag{7}$$

for the inductor

$$V_L(s) = \frac{I_L(s)}{sL_0C_0} \tag{8}$$

$$I_L(s) = \frac{10^{-7}sA}{10^{-8}s^2 - 1} \tag{9}$$

apply inverse laplace transform

$$i_L(t) = sin\left(10^4t\right)A \tag{10}$$

At,

$$10^4t = \pi \tag{11}$$

$$i_L(\pi) = 0 \tag{12}$$

$$V_c(\pi) = 10cos(\pi) = -10V \tag{13}$$

So, this time capacitor plates are changed opposite to its initial,
so after

$$10^4t = \pi, \tag{14}$$

$$t = \frac{\pi}{10^4} \tag{15}$$

$$t = 10^{-4}\pi sec \tag{16}$$

capacitor voltage is always

$$-10V \tag{17}$$

as,

$$0.5s > 10^{-4}\pi \tag{18}$$

$$\implies V_c(0.5) = -10V \tag{19}$$

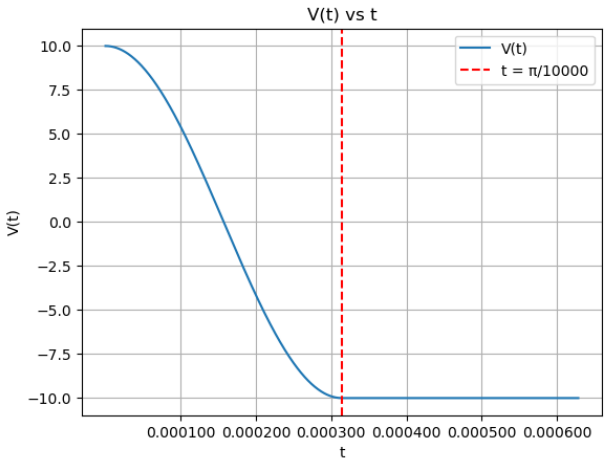


Fig. 0.