

Gate 2021- Instrumentation Engineering

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Question 43: Given $y(t) = e^{-3t}u(t) * u(t+3)$, where * denotes convolution operation. The value of $y(t)$ as $t \rightarrow \infty$ is (GATE IN 2021)

Solution:

$$y(t) = e^{-3t}u(t) * u(t+3) \quad (1)$$

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \quad (2)$$

$$x(t - t_o) \xleftrightarrow{\mathcal{L}} e^{-st_o} X(s) \quad (3)$$

$$x_1(t) * x_2(t) \xleftrightarrow{\mathcal{L}} X_1(s)X_2(s) \quad (4)$$

$$e^{-at}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+a} \quad (ROC : Re(s) > -a) \quad (5)$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (ROC : Re(s) > 0) \quad (6)$$

$$u(t+3) \xleftrightarrow{\mathcal{L}} \frac{e^{3s}}{s} \quad (ROC : Re(s) > 0) \quad (7)$$

$$Y(s) = \left(\frac{1}{s+3} \right) \left(\frac{e^{3s}}{s} \right) \quad (ROC : Re(s) > 0) \quad (8)$$

By using Final Value Theorem,

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (9)$$

$$= \lim_{s \rightarrow 0} s \left(\frac{1}{s+3} \right) \left(\frac{e^{3s}}{s} \right) \quad (10)$$

$$= \frac{1}{3} \quad (11)$$

By solving the equation (8) through partial fractions,

$$Y(s) = \frac{e^{3s}}{3s} - \frac{e^{3s}}{3(s+3)} \quad (12)$$

By applying inverse laplace transform,

$$y(t) = \frac{u(t+3)}{3} - \frac{e^{-3(t+3)}u(t+3)}{3} \quad (13)$$

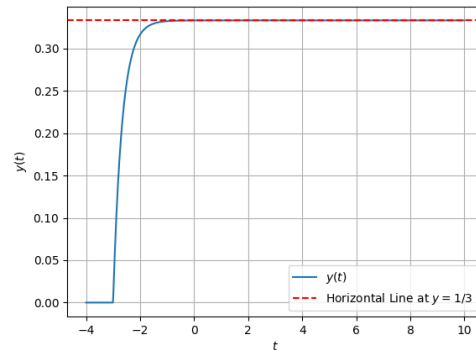


Fig. 0. plot of $y(t)$