GATE: EE - 11.2022

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Question: The transfer function of a real system $H(S)$ is given as:

$$H(s) = \frac{As + B}{s^2 + Cs + D}$$

where A, B, C and D are positive constants. This system cannot operate as

(A) Low pass filter
(B) High pass filter
(C) Band pass filter
(D) An Integrator

(GATE EE 2022)

Solution:

The transfer function $H(s)$ is given by:

$$H(s) = \frac{As + B}{s^2 + Cs + D} \tag{1}$$

Put $s = j\omega$ in (1):

$$H(j\omega) = \frac{A(j\omega) + B}{(j\omega)^2 + C(j\omega) + D} \tag{2}$$

$$|H(j\omega)| = \frac{\sqrt{(A\omega)^2 + B^2}}{\sqrt{(D - \omega^2)^2 + (\omega C)^2}} \tag{3}$$

Parameter	Description
Low Pass Filter	The gain should be finite at low frequency
High Pass Filter	The gain should be finite at high frequency
Band Pass Filter	Finite gain over frequency band
Integrator	Transfer function should have at least one pole at origin

TABLE 1: Conditions

a) Low Pass Filter:
At low frequency ($\omega = 0$):

$$|H(\omega = 0)| = \frac{B}{D} \tag{4}$$

$\therefore$ H(s) can operate as Low pass filter.

b) High Pass Filter:
At high frequency ($\omega = \infty$):

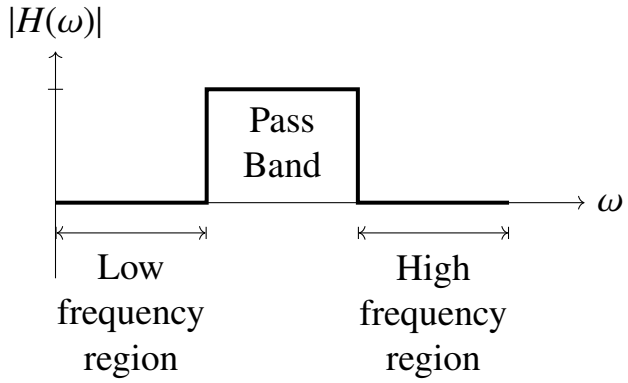
$$|H(\omega = \infty)| = 0 \tag{5}$$

$\therefore$ $H(s)$ cannot operate as High pass filter.

c) Band Pass Filter:
Assuming B is a very less positive valued constant as compared to others:

$$|H(j\omega)| = \frac{(A\omega)}{\sqrt{(D - \omega^2)^2 + (\omega C)^2}} \tag{6}$$

$$\implies |H(\omega = 0)| = 0 \text{ and } |H(\omega = \infty)| = 0 \tag{7}$$



$\because$ $H(s)$ passes frequency between low and high frequencies.
$\therefore$ $H(s)$ can operate as a band pass filter.

d) Integrator:
At very high value of frequency($\omega \to \infty$):

$$H(s) \approx \frac{As}{s^2} \approx \frac{A}{s} \tag{8}$$

From Table 1:
$\therefore$ $H(s)$ can operate as an Integrator.