

GATE: AG-26 2021

EE23BTECH11038 - Rohith Madhani*

Question : Solution of differential equation $y'' + y' + 0.25y = 0$ with initial values $y(0) = 3$ and $y'(0) = -3.5$ is

- (A) $y = (3 - 2x)e^{0.5x}$
 (B) $y = (3 - 2x)e^{-0.25x}$
 (C) $y = (3 - 2x)e^{-0.5x}$
 (D) $y = (2 - 3x)e^{-0.5x}$

(GATE AG 2021)

Solution:

Parameter	Description	Value
$y(t)$	y in time domain	?
$y(0)$	y at $t = 0$	3
$y'(0)$	y' at $t = 0$	-3.5

TABLE 0: Given parameters

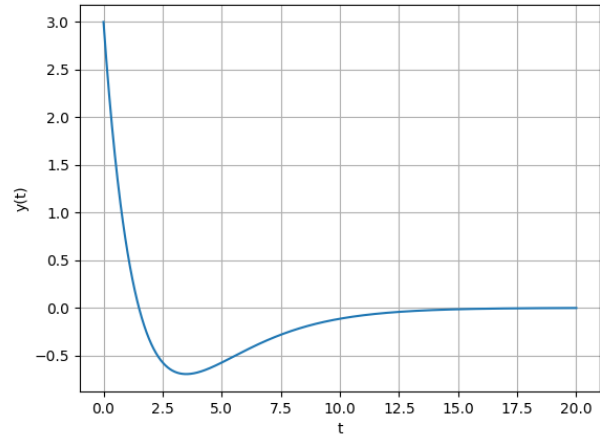


Fig. 0: $y(t) = [(3 - 2t)e^{-0.5t}]u(t)$

By applying laplace transform to the differential equation,

$$y'' + y' + 0.25y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) + sY(s) - y(0) + 0.25Y(s) \quad (1)$$

$$Y(s)(s^2 + s + 0.25) = 3s - 0.5 \quad (2)$$

$$\begin{aligned} \Rightarrow Y(s) &= \frac{3s - 0.5}{s^2 + s + 0.25} \quad (3) \\ &= \frac{3}{s + 0.5} - \frac{2}{(s + 0.5)^2}; \operatorname{Re}(s) > -0.5 \quad (4) \end{aligned}$$

As we know,

$$\frac{b}{(s + a)^n} \xleftrightarrow{\mathcal{L}^{-1}} \frac{b}{(n - 1)!} t^{n-1} e^{-at} u(t) \quad (5)$$

By taking inverse laplace of (4), we get

$$y(t) = \frac{3}{0!} e^{-0.5t} u(t) - \frac{2}{1!} t e^{-0.5t} u(t) \quad (6)$$

$$\Rightarrow y(t) = [(3 - 2t)e^{-0.5t}] u(t) \quad (7)$$

Hence the correct answer is option (C)