GATE-2022, BM-37

EE23BTECH11033- JASWANTH KILLANA

Question: Solution of the differential equation $\frac{dy}{dx} - y = cos(x)$ is

$$(A) \quad y = \frac{sin(x) - cos(x)}{2} + ce^{x} \tag{1}$$

$$(B) \quad y = \frac{sin(x) + cos(x)}{2} + ce^{x} \tag{2}$$

$$(C) \quad y = \frac{sin(x) + cos(x)}{2} + ce^{-x} \tag{3}$$

$$(D) \quad y = \frac{sin(x) - cos(x)}{2} + ce^{-x} \tag{4}$$

Solution:

$$\frac{dy}{dx} - y = cos(x) \tag{5}$$

Apply laplace transform

$$\mathcal{L}\left(\frac{dy}{dx}\right) - \mathcal{L}(y) = \mathcal{L}(cos(x)) \tag{6}$$

parameter	laplace transform
$\frac{dy}{dx}$	$sY(s) - y(0)$
y	$Y(s)$
$cos(x)$	$\frac{s}{s^2+1}$
$sin(x)$	$\frac{1}{s^2+1}$
e^x	$\frac{1}{s-1}$

TABLE 0
TRANSFORMATION

$$\implies sY(s) - y(0) - Y(s) = \frac{s}{s^2+1} \tag{7}$$

$$Y(s)(s-1) = y(0) + \frac{s}{s^2+1} \tag{8}$$

$$Y(s) = \frac{s + y(0)\left(s^2+1\right)}{(s-1)(s^2+1)} \tag{9}$$

$$Y(s) = \frac{A}{s-1} + \frac{Bs+C}{s^2+1} \tag{10}$$

$$A = y(0), B = \frac{-1}{2}, c = \frac{1}{2} \tag{11}$$

$$\implies Y(s) = \frac{y(0)}{s-1} + \frac{-s+1}{2(s^2+1)} \tag{12}$$

apply inverse laplace transform

$$\mathcal{L}^{-}(Y(s)) = \mathcal{L}^{-}\left(\frac{y(0)}{s-1}\right) + \mathcal{L}^{-}\left(\frac{-s+1}{2(s^2+1)}\right) \tag{13}$$

$$y(x) = y(0)e^{x} + \frac{sin(x) - cos(x)}{2} \tag{14}$$

according to options

$$y(0) = c \tag{15}$$

$$\implies y = \frac{sin(x) - cos(x)}{2} + ce^{x} \tag{16}$$

Option A is correct