GATE 2021 BM

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Q:27A unit step input is applied to a system with impulse response H(s) = $\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$ at t=0. The output of the system $y(t)$ at t=0^+ is:

a) 1
b) $-\frac{\omega_z}{\omega_p}$
c) $-\frac{\omega_p}{\omega_z}$
d) 0

Solution:
Given, input signal

$$x(t) = u(t)$$

$$Y(s) = H(s).X(s) \tag{1}$$
$$= \frac{1}{s}.\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}} \tag{2}$$

By initial value theorem,

$$y(t=0^+) = \lim_{s\to\infty} sY(s) \tag{3}$$
$$= \lim_{s\to\infty} \frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}} \tag{4}$$
$$= \lim_{s\to\infty} \frac{\frac{1}{s}-\frac{1}{\omega_z}}{\frac{1}{s}+\frac{1}{\omega_p}} \tag{5}$$
$$= -\frac{\omega_p}{\omega_z} \tag{6}$$

Hence, option (c) is correct