ME 36

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QUESTION: The value of Integral

$$\oint \left(\frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5} \right) dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z = \jmath$, where $\jmath$ is the imaginary unit,is

1) $(-1 + \jmath)\pi$
2) $(1 + \jmath)\pi$
3) $2(1 - \jmath)\pi$
4) $(2 + \jmath)\pi$

(GATE 2022 ME)

Solution:

Given $z = \jmath$ is only enclosing pole

$$\oint \left(\frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5} \right) dz = \oint \left(\frac{\frac{6z}{2z^3+(2\jmath-3)z^2+(5-3\jmath)z+5\jmath}}{z - \jmath} \right) dz \tag{1}$$

$$= 2\pi\jmath \left(\frac{6z}{2z^3 + (2\jmath - 3)z^2 + (5 - 3\jmath)z + 5\jmath} \right) \text{ At } z = \jmath \text{ (Cauchy's integral formula)} \tag{2}$$

$$= 2\pi\jmath \left(\frac{6\jmath}{2\jmath^3 + (2\jmath - 3)\jmath^2 + (5 - 3\jmath)\jmath + 5\jmath} \right) \tag{3}$$

$$= 2\pi\jmath \left(\frac{\jmath}{\jmath + 1} \right) \tag{4}$$

$$= -2\pi \frac{\jmath - 1}{\jmath^2 - 1} \tag{5}$$

$$= (-1 + \jmath)\pi \tag{6}$$