

GATE NM-50 2022

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Q: Let $y(x)$ be the solution of the differential equation

$$y'' - 4y' - 12y = 3e^{5x} \quad (1)$$

satisfying $y(0) = \frac{18}{7}$ and $y'(0) = -\frac{1}{7}$.

Then $y(1)$ is _____ (rounded off to nearest integer).

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Solution:

Parameter	Description	Value
$y'' - 4y' - 12y = 3e^{5x}$	Differential equation	none
$y(x)$	Solution of differential equation	$y(0) = \frac{18}{7}$
$y'(x)$	First order derivative of solution of differential equation	$y'(0) = -\frac{1}{7}$

TABLE 0

INPUT PARAMETERS

$$y''(t) \xleftrightarrow{\mathcal{L}} s^2 Y(s) - sy(0) - y'(0) \quad (2)$$

$$y'(t) \xleftrightarrow{\mathcal{L}} sY(s) - y(0) \quad (3)$$

$$y(t) \xleftrightarrow{\mathcal{L}} Y(s) \quad (4)$$

$$e^{at} \xleftrightarrow{\mathcal{L}} \frac{1}{s-a} \quad (5)$$

Applying Laplace transform on both sides of (1),

$$\mathcal{L}(y''(t) - 4y'(t) - 12y(t)) = \mathcal{L}(3e^{5x}) \quad (6)$$

From (2), (3), (4), (5)

$$Y(s)(s^2 - 4s - 12) - y(0)(s - 4)$$

$$-y'(0) = \frac{3}{s-5} \quad (7)$$

$$Y(s)(s^2 - 4s - 12) - \frac{(18s - 73)}{7} = \frac{3}{(s-5)} \quad (8)$$

$$Y(s) = \frac{3}{(s-5)(s^2 - 4s - 12)} + \frac{(18s - 73)}{7(s^2 - 4s - 12)} \quad (9)$$

$$\Rightarrow Y(s) = \frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{1}{(s+2)} \quad (10)$$

$$\frac{1}{s-a} \xleftrightarrow{\mathcal{L}^{-1}} e^{at} \quad (11)$$

Now finding Inverse Laplace Transform on both sides of (10),

From (11)

$$\Rightarrow y(t) = \left(e^{6t} - \frac{3}{7}e^{5t} + 2e^{-2t} \right) u(t) \quad (12)$$

$$\Rightarrow y(1) = e^6 - \frac{3}{7}e^5 + 2e^{-2} \quad (13)$$

$$\therefore y(1) = 340 \quad (14)$$

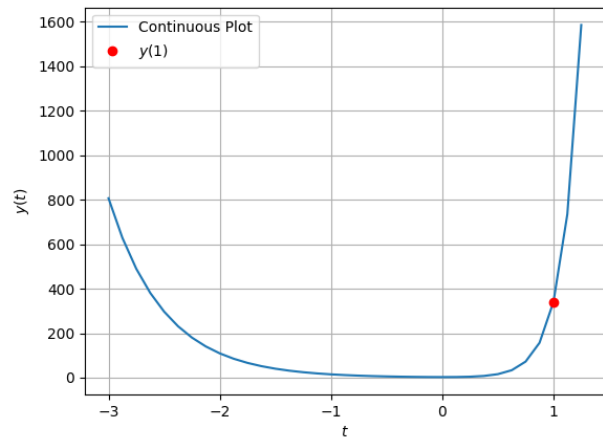


Fig. 0.