GATE: ES - 36.2022

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Question

Given, $y = f(x)$; $\frac{d^2y}{dx2} + 4y = 0; y(0) = 0; \frac{dy}{dx}(0) = 1$.
The problem is a/an

(a) initial value problem having soluition $y = x$
(b) boundary value problem having soluition $y = x$
(c) initial value problem having soluition $y = \frac{1}{2}\sin 2x$
(d) boundary value problem having soluition $y = \frac{1}{2}\sin 2x$

(GATE 2022 ES)

Solution:
The above equation can be written as,

$$y''(t) + 4y(t) = 0 \tag{1}$$

Using the Laplace transformation pairs,

$$y''(t) \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) \tag{2}$$

$$y(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s) \tag{3}$$

$$\sin at \overset{\mathcal{L}}{\longleftrightarrow} \frac{a}{a^2 + s^2} \tag{4}$$

Applying Laplace transform for the equation we get,

$$s^2Y(s) - 1 + 4Y(s) = 0 \tag{5}$$

$$\implies Y(s) = \frac{1}{4 + s^2} \tag{6}$$

Now, applying inverse laplace transform we get,

$$y(t) = \frac{1}{2}\sin 2t \quad \text{(from (4))} \tag{7}$$

Since, the conditions at the same point(0) are mentioned, it is an initial valued problem having solution $y = \frac{1}{2}\sin 2x$.

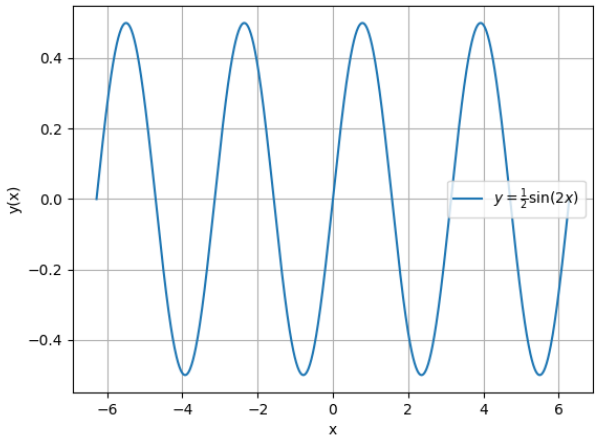

Fig. 1. $y(x)$ *vs* x graph