

GATE-AE.1

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Question: Consider the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$ and the boundary conditions $y(0) = 1$ and $\frac{dy}{dx}(0) = 0$. The solution to equation is: (GATE.AE-1.2021)

Solution:

Symbol	Values	Description
$Y(s)$	-	y in s domain
$y(x)$	-	y in x domain
$y(0)$	1	y at $x = 0$
$y'(0)$	0	$y'(x)$ at $x = 0$
$u(x)$	$= \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{o.w} \end{cases}$	unit step function

TABLE I
PARAMETERS

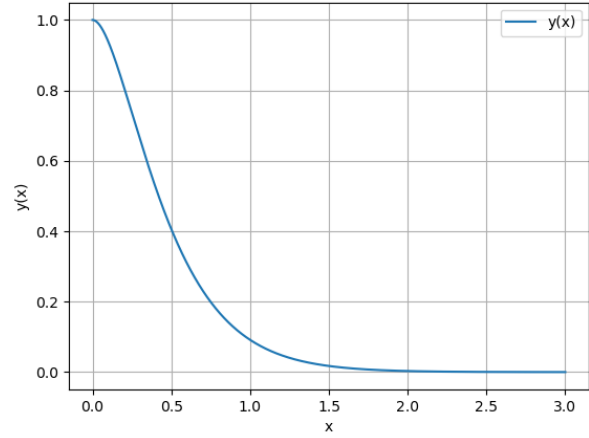


Fig. 1. Plot of $y(x)$

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) + 8sY(s) - 8y(0) + 16Y(s) \quad (1)$$

$$Y(s)(s^2 + 8s + 16) = s + 8 \quad (2)$$

$$\Rightarrow Y(s) = \frac{s + 8}{s^2 + 8s + 16} \quad (3)$$

$$= \frac{1}{s + 4} + \frac{4}{(s + 4)^2} \quad (4)$$

For inversion of $Y(s)$ in partial fractions-

$$\frac{b}{(s + a)^n} \xleftrightarrow{\mathcal{L}^{-1}} \frac{b}{(n - 1)!} \cdot x^{n-1} e^{-ax} \cdot u(x) \quad (5)$$

Applying Laplace inverse-

From (4),(5)

$$y(x) = \frac{1}{0!} e^{-4x} \cdot u(x) + \frac{4}{1!} x \cdot e^{-4x} \cdot u(x) \quad (6)$$

$$= (1 + 4x) e^{-4x} u(x) \quad (7)$$