

CS 2051: Honors Discrete Mathematics

Spring 2023 `generate_cfg_english` Solution

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This document aims to cover the solution to the `generate_cfg_english` function. As a refresher, the function asked us to create a context-free grammar (CFG) capable of generating a few english sentences. To do this, we will go through each of the sentences one by one, and continually add / update our production rules to fit our requirements.

To simplify our CFG, we use the following abbreviations:

| | |
|------|-------------------|
| ART | Article |
| CONJ | Conjunction |
| NP | Noun Phrase |
| ADV | Adverb |
| ADJ | Adjective |
| NN | Noun |
| Vi | Intransitive Verb |
| Vt | Transitive Verb. |

Here, to simplify notation, we have combined four categories into one:

$$\text{Noun} = \text{Singular Noun} \cup \text{Plural Noun} \cup \text{Proper Noun} \cup \text{Pronoun}.$$

Now that we have defined our parts of speech, we can begin to develop our CFG:

1. **The man swims.** The sentence consists of a subject, an article, and an intransitive verb. We use the traditional notation of splitting sentences up into Noun Phrases (NP) and Verb Phrases (VP), leading to the following structure:

$$\begin{aligned} S &\rightarrow \text{NP} \quad \text{VP} \\ \text{NP} &\rightarrow (\text{ART}) \quad \text{NN} \\ \text{VP} &\rightarrow \text{Vi} \end{aligned}$$

2. **The trainer carried the dumbbells.** We now introduce transitive verbs. In this sentence, “the dumbbells” is an example of a direct object, the recipient of a transitive verb. We modify our verb phrase as follows:

$$\text{VP} \rightarrow \text{Vt} \quad \text{NP}$$

3. **Ron ate his cold, delicious protein shake and The very extremely tall, intelligent woman deadlifted.** I mentioned this one in the instructions, but we can add a potentially infinite number of adjectives to modify our direct object without any fault:

$$\begin{aligned} \text{NP} &\rightarrow (\text{ART}) \quad (\text{ADJs}) \quad \text{NN} \\ \text{ADJs} &\rightarrow \text{ADJ} \quad (\text{ADJs}) \end{aligned}$$

4. **Reese gave her bar to Paul.** Again we must modify our verb phrase. We do so using prepositional phrases. We modify our rules as follows:

$$\begin{aligned} \text{VP} &\rightarrow \text{Vi} \quad (\text{PP}) \\ \text{VP} &\rightarrow \text{Vt} \quad \text{NP} \quad (\text{PP}) \\ \text{PP} &\rightarrow \text{PREP} \quad \text{NP} \end{aligned}$$

Again the parentheses around PP means it is optional.

In addition, note that “her” is a possessive noun, not an article. As a hack, we can add the rule

$$\text{ART} \rightarrow \text{NN}$$

However, this is not too elegant, and could be improved by differentiating possessive nouns in the `parts_of_speech` dictionary (see ‘Final Remarks’ section below).

5. **She ran on the treadmill quickly and enthusiastically.** Adverbs are the verbal equivalent to adjectives. However, in this case we must also include a conjunction between each adverb.

$$\begin{aligned} \text{VP} &\rightarrow \text{Vi} && (\text{PP}) && (\text{ADV}_{s1}) \\ \text{VP} &\rightarrow \text{Vt} && \text{NP} && (\text{PP}) && (\text{ADV}_{s1}) \\ \text{ADV}_{s1} &\rightarrow \text{ADV} && (\text{ADV}_{s2}) \\ \text{ADV}_{s2} &\rightarrow \text{CONJ} && \text{ADV}_{s1} \end{aligned}$$

Note that adverbs can also be placed before the prepositional phrase, i.e. the phrase “She quickly and enthusiastically ran on the treadmill” would be perfectly valid.

6. **They exercised with the ab-roller and the jump-rope.** Here two noun phrases are being combined with a conjunction. We accommodate this change by allowing noun phrases to split into multiple noun phrases.

$$\begin{aligned} \text{NP} &\rightarrow (\text{ART}) && (\text{ADJs}) && \text{NN} && (\text{NPs}) \\ \text{NPs} &\rightarrow \text{CONJ} && \text{NP} \end{aligned}$$

7. **The motivated fellow lifted the weights, but the unmotivated fellow dropped them.** We can combine sentences using conjunctions like so:

$$S \rightarrow S \text{ CONJ } S$$

The final CFG is shown below:

$$\begin{aligned} S &\rightarrow S && \text{CONJ} && S \\ S &\rightarrow \text{NP} && \text{VP} \\ \text{NP} &\rightarrow (\text{ART}) && (\text{ADJs}) && \text{NN} && (\text{NPs}) \\ \text{ART} &\rightarrow \text{NN} \\ \text{ADJs} &\rightarrow \text{ADJ} && (\text{ADJs}) \\ \text{NPs} &\rightarrow \text{CONJ} && \text{NP} \\ \text{VP} &\rightarrow \text{Vi} && (\text{PP}) && (\text{ADV}_{s1}) \\ \text{VP} &\rightarrow \text{Vt} && \text{NP} && (\text{PP}) && (\text{ADV}_{s1}) \\ \text{ADV}_{s1} &\rightarrow \text{ADV} && (\text{ADV}_{s2}) \\ \text{ADV}_{s2} &\rightarrow \text{CONJ} && \text{ADV}_{s1} \\ \text{PP} &\rightarrow \text{PREP} && \text{NP} \end{aligned}$$

Testing it with the appropriate `parts_of_speech` and the sentences yields favorable results.

Final Remarks

If I were to do it again, I would do a few things differently.

1. I would split nouns into ‘possessive nouns’ and “common nouns”, just like verbs were split up into ‘transitive verbs’ and ‘intransitive verbs’. This would allow for better differentiation for parts of speech such as pronouns (“her” vs “she”).

2. I would modify some of the test cases. The original goal was for each test case to test a different structure, but some either tested the same structure or were exceedingly difficult to implement (the latter ones were removed).
3. Finally, I would make the exposition a bit more detailed. It's always a bit of a time crunch to make these with school and everything.