CS 2051: Honors Discrete Mathematics
Spring 2023 Homework 6 Supplement

Nithya Jayakumar

1. A countable set is a set that has a one-to-one mapping with the set of natural numbers. Prove that the set of positive rational numbers is countable by setting up a function that assigns to a rational number p/q with $\gcd(p, q) = 1$ the base 11 number formed by the decimal representation of p followed by the base 11 digit A, which corresponds to the decimal number 10, followed by the decimal representation of q.

2. Define a Carmichael number as a composite number n which satisfies the following relation: $b^n \equiv b \pmod{n}$, for all integers b. Show that if $n = p_1 p_2 \cdots p_k$, where $p_1, p_2, \ldots, p_k$ are distinct primes that satisfy $p_j - 1 | n - 1$ for $j = 1, 2, \ldots, k$, then n is a Carmichael number.