CS 2051: Honors Discrete Mathematics
Spring 2023 Homework 8 Supplement

Gerandy Brito, Nithya Jayakumar, Sarthak Mohanty

Note: A template is provided in the `.tex` file. Make sure to state which type of induction you are using!

Question 1

(Warm-up) The concept of induction is often compared to the domino analogy. It starts with the assumption that the first domino falls, and from there, we can infer that if the nth domino falls, then the $(n + 1)$th domino will also fall. This leads to the eventual falling of all the dominoes. But what if the dominoes fell in the opposite direction?

Suppose we have proven the following facts with respect to some predicate $P(n)$:

$$P(1) \tag{1}$$
$$\forall n \in \mathbb{N}^+, P(n) \implies P(n-1) \tag{2}$$
$$\forall n \in \mathbb{N}, P(n) \implies P(kn) \tag{3}$$

Let k be an integer such that $|k| > 1$. Prove that $(\forall n \in \mathbb{N})(P(n))$.

Question 2

(Brito) Let $S = \{s_1, s_2, \ldots, s_{2n-1}\}$ natural numbers, where $n = 2^k > 1$. Show that one can choose a subset $S' \subset S$ of cardinality $|S'| = n$ such that the sum of the elements in S' is a multiple of n.