Honors Discrete Mathematics: Exam 2 Extra Credit

Due on Sunday, April 24, 2022 at 11:59pm

Professor Gerandy Brito Spring 2022.

IMPORTANT: For this assignment, you may NOT collaborate with other students, nor use resources outside the ones from class. Think of it as a take-home exam that is open book, open notes.

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Exercise 1 [1 pt]

Every time you visited The Farm, you went to the barn to get some fresh milk. To milk the cows, the farmer brings in 23 distinct cows from the pasture into the barnyard and lines them up.

You notice that two adjacent cows never both give milk on the same day, but no more that two adjacent cows ever give no milk on the same day. On a given day, how many different ways can the farmer get milk? (An example would be "Cow 1 gives milk, Cow 2 does not, Cow 3 does not, etc.")

Exercise 2 [1 pt]

Consider the grid shown below in Figure 1.

```
            S
          S A S
        S A R A S
      S A R T R A S
    S A R T H T R A S
  S A R T H A H T R A S
S A R T H A K A H T R A S
  S A R T H A H T R A S
    S A R T H T R A S
      S A R T R A S
        S A R A S
          S A S
            S
```

Figure 1: A diamond-shaped, narcissistic grid of letters.

Suppose you start at any S, and can move only left, right, down, or up to adjacent letters. In how many ways can the palindrome SARTHAKAHTRAS be read if

(a) the same letter can be used more than once in each sequence?

(b) ~~the same letter cannot be used more than one in each sequence?~~

Exercise 3 [1 pt, Challenge]

Let $\mathcal{P}$ be the set of all prime numbers. Let M be a subset of $\mathcal{P}$ with at least 3 elements. Choose any proper subset A of M. Consider the number

$$n_A = -1 + \prod_{p \in A} p.$$

Suppose that any prime divisor of n_A lies in M for all $A \subseteq M$. Show that $M = \mathcal{P}$.

*Hint: **Dirchlet's Theorem** states that "For any two coprime positive integers a and d, there exist infinitely many primes of the form $a + nd$, where $n \in \mathbb{N}$." More information can be found here.*

Exercise 4 [0 pts, Fall '21 Challenge]

Let N be a natural number, written in base 10. Show that there is a power of two such that its first digits are exactly those in N, in the same order. For example,

for $N = 3$ we have $2^5 = 32$;
for $N = 12$ we have $2^7 = 128$;
for $N = 102$ we have $2^{10} = 1024$.