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$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$\det(A - \lambda I) = 0$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \to \lambda I = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$$

let $M =$

$$A - \lambda I = \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

$\det(M) = (4-\lambda)(\det M_{1,1}) - 8(\det M_{1,2}) + (-1)(\det M_{1,3})$
$- (-2)(\det M_{1,4})$

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$$M_{11} = \begin{bmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

$$\Big[(-9-\lambda)\big[(5-\lambda)(-13-\lambda)-(-10)(-14)\big]\Big] - \Big[(-2)\big[(10)(13-\lambda) - (10)(-13)\big]\Big] + \Big[(-4)\big[(10)(-14)-(5-\lambda)(-13)\big]\Big]$$

$$\Big[(-9-\lambda)(-65-5\lambda+13\lambda+8\lambda^2-140)\Big] - \Big[(-2)(10\lambda-260)\Big] + \Big[(-4)(-75-13\lambda)\Big]$$

$$\Big[(-9-\lambda)(\lambda^2+8\lambda-205)\Big] - \Big[20\lambda+520\Big] + \Big[300+52\lambda\Big]$$

$$\Big[-\lambda^3-17\lambda^2+133\lambda+1845-20\lambda-520+300+52\lambda\Big]$$

$$= \Big[-\lambda^3-17\lambda^2+165\lambda+1625\Big]$$

$$M_{12} = \begin{bmatrix} -2 & -2 & -4 \\ 0 & 5-x & -10 \\ -1 & -14 & -13-x \end{bmatrix}$$

$$\Big[(-2)\big[(5-x)(-13-x)-(-10)(-14)\big] - \big((-2)\big[(0)(-13-x)-((-1)(-10))\big]\big]$$

$$+\big[(-4)\big[(0)(-14)-(5-x)(-1)\big]\big]$$

$$\big[(-2)\big[x^2+8x-205\big]\big]-\big((-2)(0)-(10)\big)+\big((-4)((0)-(-5+x))\big)$$

$$\big[(-2x^2-16x+410)\big]-(20)+\big(-4(\cancel{+5}\,5-x)\big)$$

$$\big[-2x^2-16x+410-20-20+4x\big)$$

$$\big[-2x^2-12x+370\big]$$

$$M_{13} = \begin{bmatrix} -2 & -9-x & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-x \end{bmatrix}$$

$$\big[(-2)\big[(10)(-13-x)-(-10)(-13)\big]\big]-\big[(-9-x)\big[(0)(-13-x)-((-1)(-10))\big]\big]$$

$$+\big[(-4)\big[(0)(-13)-((10)(-1))\big]\big]$$

$$\big[(-2)(-10x-260)\big]-\big[(-9-x)(-10)\big]+\big[(-4)(10)\big]$$

$$\big[(20x+520)-(90+10x)+(-40)\big]$$

$$\big[20x+520-90-10x-40\big]$$

$$\big(10x+390\big]$$

$$M_{14} = \begin{bmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{bmatrix} \qquad \begin{matrix} -5+\lambda \\ 5-\lambda \end{matrix}$$

$$\Big[(-2)\big[(10)(-14)-(5-\lambda)(-13)\big]\Big] - \Big[(-9-\lambda)\big[(0)(-14)-(5-\lambda)(-1)\big]\Big]$$

$$+ \Big[(-2)\big[(0)(-13)-(10)(-1)\big]\Big]$$

$$\Big[(-2)\big[-75-13\lambda\big]\Big] - \Big[(-9-\lambda)\big[(-5+\lambda)\big]\Big] + \Big[(-2)\big[-(-10)\big]\Big]$$

$$(-9-\lambda)(6-\lambda)$$

$$\Big[(150+26\lambda) - \big[-9(-5\pm\lambda) \mp \lambda(+5\pm\lambda)\big]\Big] + \big[-20\big]$$

$$\big[150+26\lambda\big] - (45 \mp 9\lambda \mp 5\lambda + \lambda^2) + (-20$$

$$\big[150+26\lambda+45 \mp 4\lambda \mp \lambda^2 - 20\big]$$

$$\big[-\lambda^2+22\lambda+175\big]$$

$$\det(A-\lambda I) = (4-\lambda)(-\lambda^3-17\lambda^2+165\lambda+1625) - 8(-2\lambda^2-12\lambda+370)$$

$$+(-1)(10\lambda+390) - (-2)(-\lambda^2+22\lambda+175)$$

$$= \Big[4(-\lambda^3-17\lambda^2+165\lambda+1625) - \lambda(-\lambda^3-17\lambda^2+165\lambda+1625)\Big]$$

$$+ (16\lambda^2+96\lambda-2960) + (-10\lambda-390) + (-2\lambda^2+44\lambda+350)$$

$$= \Big[(-4\lambda^3-68\lambda^2+660\lambda+6500) + (\lambda^4+17\lambda^3-165\lambda^2-1625\lambda)$$

$$+(16\lambda^2+96\lambda-2960) + (-10\lambda-390) + (-2\lambda^2+44\lambda+350)$$

$$= \Big[\lambda^4+13\lambda^3-233\lambda^2-965\lambda+6500\Big] + (16\lambda^2+96\lambda-2960) + (-10\lambda-390)$$

$$+(-2\lambda^2+44\lambda+350$$

$x^4 + 13x^3 - 219x^2 - 835x + 3500$

if $a_1 = 13$, $a_2 = -219$, $a_3 = -835$ $a_4 = 3500$

it becomes $x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$

Cubic Resolvent equation $\rightarrow y^3 - a_2y^2 + (a_1a_3 - 4a_4)y + (4a_2a_4 - a_3^2 - a_1^2a_4) = 0$

$y^3 - (-219)y^2 + [(13)(-835) - 4(3500)]y + [4(-219)(3500) - (-835)^2 - (13)^2(3500)] =$

$y^3 + 219y^2 + (-10{,}855 - 14{,}000)y + (-3{,}066{,}000 - 697{,}225 - 591500) = 0$

$y^3 + 219y^2 - 24{,}855y - 4{,}354{,}725 = 0$

To find the cubic equation root, we did some testing to find y, which is the root.

$y_1 = 147.9479 \simeq \cancel{148}\ 147.9479$

to find the quartic root, we use

$z^2 + \frac{1}{2}\left(a_1 \pm \sqrt{(a_1)^2 - 4a_2 + 4y_1}\right)z + \frac{1}{2}\left(y_1 \pm \sqrt{(y_1)^2 - 4a_4}\right) = 0$

$z^2 + \frac{1}{2}\left(13 \pm \sqrt{(13)^2 - (4(-219)) + 4(147.9479)}\right)z + \frac{1}{2}\left(147.9479 \pm \sqrt{(147.9479)^2 - 4(350}\right.$ $= 0$

$z^2 + \frac{1}{2}(13 \pm 40.45)z + \frac{1}{2}(147.9479 \pm 88.86) = 0$

we have 2 equations

$z^2 + \frac{1}{2}(13 + 40.45)z + \frac{1}{2}(147.9749 + 88.86) = 0$

$z^2 + \frac{1}{2}(13 - 40.45)z + \frac{1}{2}(147.9749 - 88.86) = 0$

$z^2 + 26.73z + 118.40$

$z^2 - 13.73z + 29.54$

using $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

for equation 1, $z = \dfrac{-26.73 \pm \sqrt{(26.73)^2 - 4(1)(118.40)}}{2 \times 1}$

$$z = \frac{-26.73 \pm \sqrt{240.83}}{2}$$

$$= \frac{-26.73 \pm 15.51}{2}$$

$$= \cancel{-26.73 + 7.755 \text{ or } -26.73 - 7.765}$$

$$= \frac{-26.73 + 15.51}{2} \text{ or } \frac{-26.73 - 15.51}{2}$$

$$= -5.61 \text{ or } -21.12$$

For equation 2: $z - 13.73z + 29.54$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-13.73) \pm \sqrt{(-13.73)^2 - 4(1)(29.54)}}{2}$$

$$z = \frac{13.73 \pm \sqrt{70.43}}{2}$$

$$z = \frac{13.73 + 8.39}{2} \text{ or } \frac{13.73 - 8.39}{2}$$

$$z = 11.054 \text{ or } 2.67$$

So $r_1 = -21.12$

$r_2 = -5.61$

$r_3 = 2.67$

$r_4 = 11.054$

$\lambda_3 = 2.67$

$(A - \lambda I)v = 0$

$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - 2.67 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - \begin{pmatrix} 2.67 & 0 & 0 & 0 \\ 0 & 2.67 & 0 & 0 \\ 0 & 0 & 2.67 & 0 \\ 0 & 0 & 0 & 2.67 \end{pmatrix}$$

$$\begin{pmatrix} 1.33 & 8 & -1 & -2 \\ -2 & -11.67 & -2 & -4 \\ 0 & 10 & 2.33 & -10 \\ -1 & -13 & -14 & -15.67 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1.33 & 8 & -1 & -2 \\ -2 & -11.67 & -2 & -4 \\ 0 & 10 & 2.33 & -10 \\ -1 & -13 & -14 & -15.67 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}$$

$$0.754\left(\begin{array}{cccc|c} 1.33 & 8 & -1 & -2 & 0 \\ -2 & -11.67 & -2 & -4 & 0 \\ 0 & 10 & 2.33 & -10 & 0 \\ -1 & -13 & -14 & -16.67 & 0 \end{array}\right)\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$2\times\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.67 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right)$$

$$1\times\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right)$$

$$2.518\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.984 & -14.754 & -17.184 & 0 \end{array}\right)$$

$$
\begin{array}{c}
\\
\times -10 \\
\\
\\
\end{array}
\left(\begin{array}{cccc|c}
1 & 6.036 & -0.754 & -1.509 & 0 \\
0 & 1 & -8.834 & -17.669 & 0 \\
0 & 10 & 2.325 & -10 & 0 \\
0 & -6.964 & -14.754 & -17.184 & 0
\end{array}\right)
$$

$$
\begin{array}{c}
\\
\times 6.964 \\
\\
\\
\end{array}
\left(\begin{array}{cccc|c}
1 & 6.036 & -0.754 & -1.509 & 0 \\
0 & 1 & -8.834 & -17.669 & 0 \\
0 & 0 & 90.669 & 166.688 & 0 \\
0 & -6.964 & -14.754 & -17.184 & 0
\end{array}\right)
$$

$$
\begin{array}{c}
\times 0.011 \\
\\
\\
\\
\end{array}
\left(\begin{array}{cccc|c}
1 & 6.036 & -0.754 & -1.509 & 0 \\
0 & 1 & -8.834 & -17.669 & 0 \\
0 & 0 & 90.669 & 166.688 & 0 \\
0 & 0 & -76.278 & -140.231 & 0
\end{array}\right)
$$

$$
\begin{array}{c}
\\
\times 76.278 \\
\\
\\
\end{array}
\left(\begin{array}{cccc|c}
1 & 6.036 & -0.754 & -1.509 & 0 \\
0 & 1 & -8.834 & -17.669 & 0 \\
0 & 0 & 1 & 1.838 & 0 \\
0 & 0 & -76.278 & -140.231 & 0
\end{array}\right)
$$

$$
\begin{array}{cc}
 & \left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \\
C \times 8.834 &
\end{array}
$$

$$
\begin{array}{cc}
 & \left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \\
2 \times 0.754 &
\end{array}
$$

$$
\begin{array}{cc}
 & \left(\begin{array}{cccc|c} 1 & 6.036 & 0 & -0.122 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \\
1 \times (-6.036) &
\end{array}
$$

$$
\left(\begin{array}{cccc|c} 1 & 0 & 0 & 8.494 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)
$$

$$
\left.\begin{array}{l} x_1 + 8.494 x_4 = 0 \\ x_2 - 1.428 x_4 = 0 \\ x_3 - 1.838 x_4 = 0 \end{array}\right) \Leftrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}
$$

$0 x_4 = 0$

$x_4 = 0$

$x_1 = -8.404\, x_4$

$x_2 = 1.428\, x_4$

$x_3 = 1.838\, x_4$

$\cancel{x_4 = 1}$ $\quad x_4 = x_4$

2

Eigen vectors
$$\begin{pmatrix} -8.494\, x_4 \\ 1.428\, x_4 \\ 1.838\, x_4 \\ x_4 \end{pmatrix}$$

$$x_4 \begin{pmatrix} -8.484 \\ 1.428 \\ 1.838 \\ 1 \end{pmatrix}$$

To find the eigen vector corresponding to $\lambda_1 = -5.604$ we solve the equation $(A - \lambda I) = 0$

First we substract λ from the diagonal of A:

$$\begin{pmatrix} 4+5.604 & 8 & -1 & -2 \\ -2 & -9+5.604 & -2 & -4 \\ 0 & 10 & 5+5.604 & -10 \\ -1 & -13 & -14 & -13+5.604 \end{pmatrix} = \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & 7.396 \end{pmatrix}$$

then we perform Gaussian elimination to find the null space of $A - \lambda I$

final matrix

$$\left(\begin{array}{cccc|c} 9.604 & 8 & -1 & -2 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right)$$

1. From Row 1 we divide 9.604 to make the 1st entry 1:

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 1 & -13 & -14 & -7.396 & 0 \end{array}\right)$$

2. Row 2 + 2 × Row 1, Row 4 + Row 1:

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right)$$

3. Row 2 ÷ −1.730 to make the 2nd entry 1

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & 0.208 & 0 \\ 0 & 1 & 1.296 & 2.552 & 0 \\ 0 & 10 & 10.004 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.004 & 0 \end{array}\right)$$

4. Row 3 − 10 × Row 2, Row 4 + 12.167 × Row 2:

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.552 & 0 \\ 0 & 0 & -2.156 & -35.590 & 0 \\ 0 & 0 & 1.428 & 23.448 & 0 \end{array}\right)$$

5. Row 3 ÷ −2.156 to make the 3rd entry 1, followed by Row 4 − 1.428 × Row 3:

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.552 & 0 \\ 0 & 0 & 1 & 16.472 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Finally the system reduces to:

$$\begin{cases} v_1 + 0.833v_2 - 0.104v_3 - 0.204v_4 \\ v_2 + 1.276v_3 + 2.552v_4 = 0 \\ v_3 + 16.472v_4 = 0 \end{cases}$$

and we let $v_4 = t$ then:

$$v_3 = -16.472t$$

$$v_2 = 1.276(16.472t) - 2.552t = 18.470t$$

$$v_1 = -15.386t - 1.713t + 0.208t = -16.81t$$

So the eigen vector v is: $\vec{v} = t\begin{pmatrix} -16.891 \\ 18.470 \\ -16.472 \\ 1 \end{pmatrix}$

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$\lambda_4 = 11.054$

$(A - \lambda I)\vec{v} = 0$

$$\left(\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - \left((11.054)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\right)\vec{v} = 0$$

$$\left[\begin{array}{cccc|c} -7.054 & 8 & -1 & -2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right]$$

multiply first row by -1

$$= \begin{bmatrix} 7.054 & -8 & 1 & 2 \\ 0 & -22.322 & -1.716 & \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right]$$

multiply first row by $\frac{-2}{7.054}$
subtract first row from second row

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right]$$

multiply first row by $\frac{-1}{7.054}$
subtract first row from 4th row

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.77 & 0 \end{array}\right]$$

multiply second row by $\frac{10}{-22.322}$
subtract second row by third row

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 10 \\ 0 & -22.322 & -1.716 & -3.433 & 10 \\ 0 & 0 & -6.823 & -11.538 & 10 \\ 0 & -14.134 & -13.858 & -23.72 & 10 \end{array}\right]$$

multiply second row by $\dfrac{-14.134}{-22.322}$

subtract second row from 4th row

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 10 \\ 0 & -22.322 & -1.716 & -3.433 & 10 \\ 0 & 0 & -6.823 & -11.538 & 10 \\ 0 & 0 & -12.771 & -21.596 & 10 \end{array}\right]$$

multiply Third row by $\dfrac{-12.771}{-6.823}$

subtract third row by 4th row

$$\left[\begin{array}{cccc|c} 7.054 & -8 & 1 & 2 & 10 \\ 0 & -22.322 & -1.716 & -3.433 & 10 \\ 0 & 0 & -6.823 & -11.538 & 10 \\ 0 & 0 & 0 & 0 & 10 \end{array}\right]$$

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Let $v_4 = 1$

From third row

$0v_1 + 0v_2 - 6.823v_3 - 11.538v_4 = 0$

$-6.823v_3 = 11.538v_4$

$v_3 = \frac{-6.823}{11.538} = -1.691$

From second row

$0v_1 - 22.322v_2 - 1.716v_3 - 3.433v_4 = 0$

$-22.322v_2 + 2.901 - 3.433 = 0$

$v_2 = \frac{0.532}{-22.322} = -0.0238$

From 1st row

$7.054v_1 - 8v_2 + 1v_3 + 2v_4 = 0$

$7.054v_1 - 0.1904 - 1.691 + 2 = 0$

$7.054v_1 = -0.4994$

$v_1 = \frac{-0.4994}{7.054} = -0.071$

Eigenvector $\vec{v} = \begin{bmatrix} -0.071 \\ -0.0238 \\ -1.691 \\ 1 \end{bmatrix}$