The report of coursework2

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1 Introduction

In coursework2, we have to build the agent-based model of the early 2021 GameStop short squeeze. I use some references to help me build the model. In this report, I will explain the implementation of the simplistic model and give some analysis. In section 2, I will explain how to build the hedge fund class. In section 3, I will give some types of Reddit agents and the implementation of building the Reddit agent class. In section 4, I will create a financial environment where all the agents can interact with each other. In section 5, I will use some real data to validate the ABM model I build. In section 6, I will discuss the implementation of the agent-based model and analyze some very important parameters in the model. In section 7, I will explain the performance and the limitation of the model.

2 Hedge Fund

2.1 Nature of agents

Hedge Fund class is a type of fundamentalists whose target fundamental price is 0. They bet on the price of the GME to go to $0. At the same time, this type of agents can short sell. When the price is high, the hedge fund class can borrow shares from the market and sell them. They bet on the price to go down. When the price is down, they can buy the shares and return the shares to the market. In this case, they can gain a profit from the short selling.

2.2 List of variables describing their state

In order to build the Hedge Fund class, I use some parameters to describe this type of agent. In the class Hedge_Fund, I define the number of the agents, the number of the trades they transfer everyday, a parameter to describe short selling (open short sell, 0:close the position), a parameter to describe the price when the agent decide to short sell, a parameter to save the time when agents close the position, the probability of keeping short selling and some other parameters which are used to calculate the utility value.

2.3 List of actions the agents can perform

The actions of this type of agents are short selling and closing the position. They can open a short sell at anytime. I write a function called **open_short_sell()** to describe the action. Apart from that, they can change their behaviors. I use a function called **change_behavior()** to describe the action. I apply the prospect theory to calculate the values they gain at time t and time $t+1$. Then, according to the values, agents can decide which action they will take at time t: short selling or closing the position. The equations are below:

$$v(x) = -\lambda x^{\alpha} \tag{1}$$

$$v(x_{loss}, x_{gain}, p_{loss}, p_{gain}) = p_{gain} x_{gain}^{\alpha} - p_{loss} \lambda x_{loss}^{\beta} \tag{2}$$

λ is equal to 2.55 and α and β are equal to 0.88 from the empirical researches. When agents open a short sell position, there is a parameter to save the position and also a parameter to save the price at this time. We can use the predicted fundamental price at time t and time t+1 to calculate the utility values. After that, agents can decide whether to change behavior or not.

2.4 Structure of their interaction with other agents

The interaction between hedge fund agents and other agents is that hedge fund agents can buy(close the position) and short sell stocks in the financial markets. When the hedge fund agents sell the stocks, other agents can buy the stocks in the financial market and vice versa.

3 Reddit Agent

3.1 Nature of agents

In Lucchini et al. [5], it said that The collective action originated from Reddit and eventually succeeded in GameStop stock short squeeze driven by a few determined individuals. [5]. In my mind, this small number of committed individuals should be fundamentalists which are different from the hedge fund agents. The fundamentalist agents tend to stabilize the market and drive the price towards the fundamental price. This kind of traders is the long-term traders. They know the knowledge related to financial market and know the strategic plan of hedge funds institutions through the market price and speculation. In this case, they can start collective actions on Reddit in order to raise the price of the GME so that the hedge funds agents will face huge losses.

Apart from that, retail investors in equity markets are rapidly growing due to the rise of the GME price. [5]. From my point of view, this type of agent should be chartists. Chartists are the short-term traders who tend to detect a trend in price fluctuations. This kind of agent tries to make profits from detecting a trend and so they are responsible for the formation of market bubbles and crashes, which undermines the stability of the market [2]. They came into the GME stock market because the price of the stock went so high. They do not know much information about the financial market. So they were willing to take some advice from the professional people on Reddit, who are the fundamentalists on this platform. Instead, there are also two types of chartists: optimists and pessimists. Optimists always tend to buy and pessimists always tend to sell [2].

I build the Reddit Agent class with two main categories of agents: fundamentalists and chartists. And the category chartists have two types of agents: optimists and fundamentalists.

3.2 List of variables describing their state

In the model. there are three categories: fundamentalists, optimists and pessimists. There are some variables to describe their state: the number of fundamentalists, the number of optimists, the number of pessimists, the number of chartists, the number of trading days, the probability of one type of agent switching to another type of agent, a list to describe whether the agents buy or sell and some parameters which are used to calculate.

3.3 List of actions the agents can perform

Actions that the agents have been to buy and sell the stocks in the financial market when there are some stocks available in the financial market. So I build two functions to describe the actions which are mentioned above.

3.4 Structure of their interaction with other agents

There are three types of agents in the class: fundamentalists, optimists and pessimists. They can switch to other categories according to the price in the financial market. I use some equations which are proposed in Alfarano Lux [1] to build the model. The model is initialized randomly by designating each chartist either an optimist or a pessimist. For each time step, each optimist has a probability P_{op} to switch to a pessimist and each pessimist has a probability P_{po} to switch to an optimist. The equations are below:

$$P_{op} = v_1 \Delta t \frac{N_p}{N} \qquad P_{po} = v_1{}_\Delta t \frac{N_o}{N} \tag{3}$$

Δt is the time span. v_1 is the parameter. N_p is the number of pessimists.N_o is the number of optimists.N is the number of chartists. As pessimists, optimists and chartists will switch to each other due to the price fluctuations, for each time step, the number of optimists, pessimists and chartists is flexible.

At the same time, each fundamentalist has a probability P_{fc} to switch to a chartist and each chartist has a probability P_{cf} to switch to a fundamentalist. The equations are below:

$$P_{fc} = v_2 \Delta t e^{-\alpha\rho} \qquad P_{cf} = v_2 \Delta t(1 - e^{-\alpha\rho}) \tag{4}$$

α is a free parameter, and

$$\rho = \frac{|p_f - p|}{p_f} \tag{5}$$

is the absolute percent deviation of the share price from the fundamental price. I will explain how to calculate the share price in the section Market environment.
On average, we want the switching probability of the fundamentalists and chartists to be roughly equal. So we can use this to select a value for α [3].

$$e^{-\alpha\rho} \approx 1 - e^{-\alpha\rho} \tag{6}$$

For instance, we allow for the 10% deviation from the fundamental price [1]. In this case, we can calculate the parameter α [3]:

$$\alpha \approx \frac{0.1}{1} ln \frac{1}{2} \approx 6.9 \tag{7}$$

In the ABM model, I build the Reddit agent class which contains three types of agents: fundamentalists, optimists and pessimists. Optimists and pessimists belong to chartists. In this class, I define the number of fundamentalists and chartists who can trade in the financial market. I also define some parameters to calculate the switching probability of each type of agent. The action of the agents is that they can switch to different categories according to the price in the market. In this case, I create a function to describe this action. I calculate the P_{op},P_{po},P_{cf},P_{fc} and then switch each type of agents to another type of agents according to the probabilities above.

4 Market environment

In this part, I build a market environment in which agents can buy or sell stocks. I define some variables in order to describe some actions: the number of trades by fundamentalists, the number of trades by chartists, the price and the fundamental price and some parameters which are used to calculate.

4.1 Price

In fact, traders submit buying and selling instructions to brokers, and brokers execute transactions by matching all traders' orders and determining the equilibrium (market) price. This agent behavior is modeled by an ordinary differential equation (ODE) [3]. This equation takes the number of each type of trader at a given time. The dynamics of the price can be calculated by the equation below [1]:

$$p_{t+1} = p_t + \beta[N_f T_f (p_{f_t} - p_t) + N_c T_c x]p_t \tag{8}$$

where

$$x = \frac{N_o - N_p}{N_c} \tag{9}$$

p_{t+1} means the stock price at time $t+1$. p_t is the stock price at time $t.\beta$ is the parameter which controls the speed of the price adjustment per unit time. T_f means the number of stocks traded by each fundamentalist and T_c means the number of stocks traded by each chartist. N_f is the number of fundamentalists and N_c is the number of chartists. p_{f_t} is the fundamental price at time t. We can use the following equation to calculate the fundamental price at a given time [1]:

$$p_{f_{t+1}} = p_f + \mu p_f \tag{10}$$

4.2 Interaction

In the market environment, the number of fundamentalists and chartists is dynamic at each given time because the fundamentalists and the chartists can switch to each other. At the same time, optimists and pessimists can switch to each other. This is due to the change in the price at a given time. Because of the price change, the hedge fund agents and other agents can decide to buy or sell the shares, which will influence the stock price the next time.

5 Validation

In this section, I will validate the ABM model by using different methods. First, I will show the logarithmic returns, which is used to analyze the price dynamics in financial markets [6]. Second, I characterize the statistical characteristics of price changes, including the heavy tails and the long memory of the returns. Last, I examine the autocorrelation of the returns.

5.1 Data

In order to validate the ABM model, I use the true daily prices of GME from the yahoo website between 01-December-2020 and 04-February-2021. To perform the analysis, I use the changes of the close price of GME. I calculate the returns from the ABM simulation market to examine the characteristics of the distributions of returns. Then, I compute other statistical properties and analyse them.

5.2 Analysis of returns

In this part, I calculate the logarithmic returns in the real GME financial market and the ABM simulation financial market. Then I compare the differences between them and give some analysis.

I use the daily log returns, r_t to evaluate my model, where:

$$r_t = log(\frac{p_t}{p_{t-1}}) \tag{11}$$

p_t is the price at time t, and p_{t-1} is the price at time $t-1$. Log returns is a good method to analyze financial time series, because it has a lot of advantages over using raw or percentage returns [6]. The rate of return is a concept of compound interest, and the logarithmic rate of return can be used to intuitively feel the rate of return for a period of time, compared to the percentage returns.

The time series and histogram of the real daily log returns calculated using the real closing price from 01-December-2020 and 04-February-2021 are shown in red in Figure 1. The time series and histogram of the log returns calculated using the simulated price are shown in blue in Figure 2.

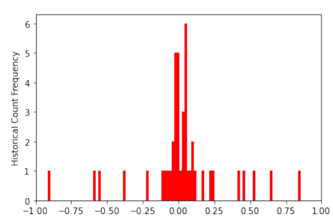

Figure 1: real log return

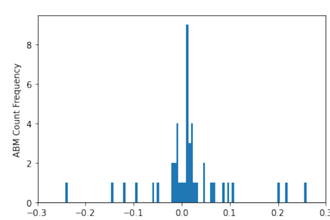

Figure 2: ABM log returns

From Figure 1 and Figure 2, we can see that there are some similarities between the two figures: there is a lot of data distributed around 0. But there are some differences between the two figures. In Figure 1, the distribution of the returns is so wide, for -1 $<r_t<$1. But in Figure 2, the distribution of the returns is too narrow as -0.3 $<r_t<$0.3. This may be because We use a formula to define the price in the agent-based model, which is different from the real market price of stocks. This may cause errors and inaccuracies.

5.3 Statistical characteristics

In this part, I will analyse distribution of returns by using different methods, including heavy tails and the long memory of returns.

5.3.1 Statistical analysis

I calculate the mean, variation coefficient, skewness and kurtosis of returns in agent-based model. I list them in Table 1.

	Mean	$c_v(\frac{\sigma}{\mu})$	Skewness	Kurtosis
ABM market	0.015	5.328	0.236	3.059
real market	0.027	10.417	-0.244	2.917

Table 1: Summary statistics for the distribution of log returns in ABM

Variation coefficient is a normalized measure of the degree of dispersion of the probability distribution. The advantage of the variation coefficient is that it does not need to refer to the average value of the data. Therefore, when comparing two sets of data with different dimensions or different means, the coefficient of variation should be used instead of the standard deviation for comparison. From Table 1, we can see that the variation coefficient of the ABM market is a little bit different from the variation coefficient of the real financial market.

For the skewness, I find the value of the skewness of the ABM market is opposite to that of the real financial market, which means that price changes in the ABM market are more likely to be positive, compared to the real financial market, but the downward movements of the market tend to be stronger [6]. The skewness of the ABM market is greater than 0, the distribution is skewed to the right, that is, there is a long tail on the right.

The values of mean and kurtosis of the two markets are similar. Kurtosis is a statistic that studies the steepness or smoothness of data distribution. By measuring the kurtosis coefficient, we can find whether the data is steeper or gentler than the normal distribution. The value of the kurtosis is larger than 0, which indicates that the peak state of distribution is steep. This suggests that the distribution of log returns might have heavy tailed [6].

5.3.2 Analysis of the Hurst exponent

Hurst exponent H is introduced by Hurst [4]. I use the Hurst exponent in order to examine the presence of long-range memory in ABM markets. Hurst exponent reflects the autocorrelation of time series, especially the hidden long-term trend in the series, which is statistically called long-term memory.

Hurst exponent H has three forms:
1. If H = 0.5, it means that time series can be described by random walk;

2. If 0 <H <0.5, it indicates that the time series is mean-reverting [6];
3. If 0.5 <H <1, it indicates that the time series has long-range memory;

There are several ways to calculate the Hurst exponent H. I choose to use R/S analysis. There are several steps to calculate the R/S analysis. We need to calculate the cumulative deviate series Z, the range R, the standard deviation S. The cumulative deviate series Z is defined by:

$$Z_t = \sum_{i=0}^{t} Y_i \tag{12}$$

where

$$Y_t = X_t - m \qquad m = \frac{1}{n}\sum_{i=0}^{n} X_i \tag{13}$$

The range R is calculated by :

$$R(n) = max(Z_0, Z_1, Z_2..., Z_n) - min(Z_0, Z_1, Z_2..., Z_n) \tag{14}$$

The standard deviation S is computed by :

$$S(n) = \sqrt{\frac{1}{n}\sum_{i=0}^{n}(X_i - m)^2} \tag{15}$$

Then calculate the rescaled range $R(n)/S(n)$ and average over all the partial time series of length n. A time series with a total length of n is divided into 14 shorter time series, and then the average readjustment range of each n value is calculated. The Hurst exponent is estimated by fitting the power law $[R(n)/S(n)] = Cn^H$ to the data. This can be done by plotting $log[R(n)/S(n)]$ and fitting a straight line.

	H
ABM	0.754
real market	0.848

Table 2: Hurst exponent H

From Table 2, we can see that both Hurst exponent values of log returns in the ABM market and returns in the real financial market are between 0.5 and 1. This means that the time series produced by the agent-based model has long-term memory.

6 Sensitivity Analysis

Sensitivity analysis determines how different values of an independent variable affect a particular dependent variable under a given set of assumptions. It is used to understand which variables are useful and the impact of each variable on the output.

There are many methodologies to perform sensitivity analysis, like Sensitivities based on one-factor-at-a-time (OFAT), Sensitivities based on model-free output variance decomposition, Sensitivities based on model-based output variance decomposition [7]. I choose multi-factor sensitivity analysis.

There are a lot of variables in the agent-based model. I fix some certain parameters and calibrate others. I set N = 200(the number of traders), $N_f = 100$(the number of fundamentalists), $N_c = 100$(the number of chartists) and a time step Δt = 1/45 as there are 45 trading days, with an initial probability of 0.5 for designating each chartists an optimist or pessimist. I also set $\beta = 0.1$ and $\alpha =$ 6.9. Although these parameters are a little bit arbitrary, they produce model behaviour which is very close to the behaviour in the real financial market [3]. Due to the fact that the price p fluctuates around the fundamental price p_f, therefore p_f drives the long-term returns of stocks [3]. I set μ in Equation(10) equal to the average return from the real price of the GME stock($\mu = 0.0978$) [3]. Finally, I set the initial values of p and p_f equal to 1 in order to compare the simulated price to the real one.

Despite the fixed parameters above, there are 4 parameters which needs to be calibrated: v_1, v_2, T_c and T_f.I created the Table 3 in order to calibrate:

Parameter	Range
$v_1\Delta t$	{0.1,0.2,0.3,...0.9}
$v_2\Delta t$	{0.001,0.002,0.003,...0.009}
T_c	{2,3,5,7,10}
T_f	{200,300,500,700,1000}

Table 3: Parameter ranges

Instead of choosing v_1 and v_2 values directly, I choose to set values for $v_1\Delta t$ and $v_2\Delta t$ respectively as these values should between 0 and 1 [3], which is easier to control. $v_1\Delta t$ is used to calculate the probability for a switching between optimists and pessimists. They belong to chartists. It is more likely for optimists to switch to pessimists or for pessimists to switch to optimists due to the price of the stock. $v_2\Delta t$ is used to calculate the probability for switching between chartists and fundamentalists. As they have different natures, there is less chance for chartists to switch to fundamentalists or for fundamentalists to switch to chartists. but this can also happen. In this case, I choose larger values for $v_1\Delta t$ and choose smaller values for $v_2\Delta t$. According to the natures of fundamentalists and chartists, fundamentalists usually buy and sell large amounts of shares, but chartists tend to trade a small amount of shares. So I set the different ranges of T_c and T_f.

In order to do sensitivity analysis, it is better to use a grid search between the ordered quaternions from the parameter table to find the best set of parameter values which can achieve the lowest mean-square error(MSE) between the skewness and kurtosis of the time series in the real financial market and that produced by agent-based model. Mean-square error (MSE) is the average of the sum of squares of the differences between the data and the true value, which can be used to describe the relationship between the data series and the true value. Upon completion of the grid search, we will find the best parameter set.

From my point of view, the most important parameters are T_c and T_f. These parameters have a connection with price fluctuation. If too many traders buy stocks, the price will go up. And if a large number of traders sell stocks, the price will go down. And the switching probability is related to T_c and T_f. Therefore, I think T_c and T_f are the most important parameters.

7 Discussion

In this coursework, I build an agent-based model based on the early 2021 GameStop (GME) short squeeze. The Figures below show the results of the agent-based model.

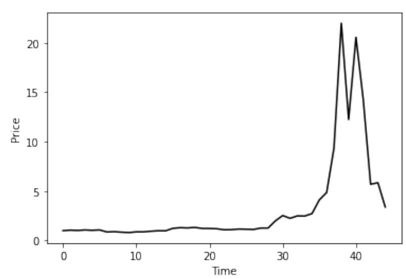

Figure 3: real price

Figure 4: ABM price & fundamental price

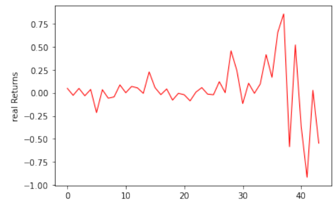

Figure 5: real returns

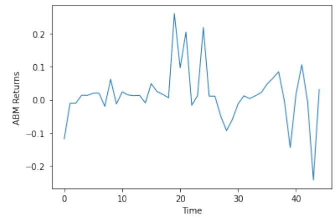

Figure 6: ABM returns

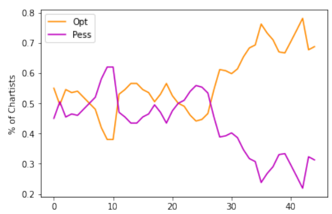

Figure 7: switching between optimists and pessimists

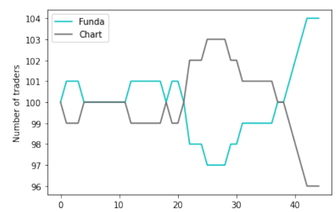

Figure 8: switching between fundamentalists and chartists

I simulated the GME price which is shown in Figure 4. From day 0 to day 30, the price of GME experienced an upward trend, despite some fluctuations. After day 30, the price rises significantly and gets to the peak on day 32. Then, the price dropped slightly and went down quickly. In the last few days, the price showed an upward trend. Compared to the real price shown in Figure 3, there are some similarities and also differences. Before 30 days, the two figures show a similar upward trend. However, the real price grew rapidly on day 32, which is 2 days later than the ABM price. This may be because a lot of new users participate in the financial market and buy GME due to the rise of the price. Apart from that, in the last few days, the real price fell but the ABM price got up. These inaccuracies may result from that the trading days are so short. We cannot get a lot of data from the financial market.

Figure 5 and Figure 6 show the returns of the real market and the market produced by the agent-based model. Before day 35, the fluctuation of returns from the real market is slow. But from day 20 to day 28, the fluctuations of returns from the ABM market are so huge, which is so different from the returns from the real market. From day 35 to the last day, the real returns and abm returns show a similar trend.

Figure 7 and Figure 8 show the changes between optimists and pessimists and the switches between fundamentalists and chartists. Before day 30, only a small number of optimists and pessimists switch to each other due to the fluctuations of GME prices. When the price rise, pessimists switch to optimists to buy the shares. But when the price falls, optimists switch to the pessimists to sell the shares. After day 30, massive pessimists switch to optimists as the price grows hugely. The changes between optimists and pessimists are in line with price fluctuations. In Figure 8, not many fundamentalists switch to chartists or chartists switch to fundamentalists due to the nature of these two agents. After day 30, some chartists change to fundamentalists. This may be because some chartists want to make long-term investments and they do not want to let the price influence their behaviour.

Although an agent-based model is a good way to simulate GameStop (GME) short squeeze and financial market. There are also some limitations. Firstly, the results are not accurate as trading days are too short and there is not a formula to define the market price. The prices and returns produced by the agent-based model can show a similar trend but there are also some differences. These are the limitations of the coursework.

References

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