QOSF Task 1: Switching Rotation Gates

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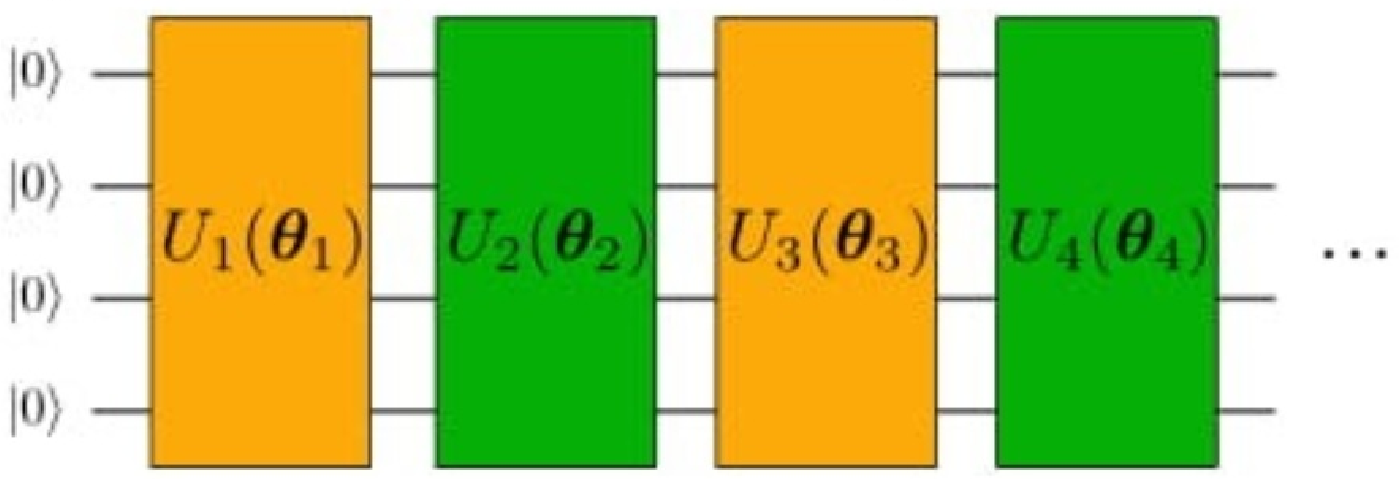
We will be implementing the following 4 qubit state $|\psi(\theta)\rangle$



where the number of layers, denoted with L, is a parameter. We call "Layer" the combination of 1 yellow + 1 green block, so, for example, U1 + U2 is a layer.

In [1]:

```
# installations
!pip install --upgrade qiskit
!pip install pylatexenc

from IPython.display import clear_output
clear_output()

# import
import qiskit
import numpy as np
import random
import scipy
import matplotlib.pyplot as plt

from qiskit import QuantumCircuit
from qiskit import Aer, execute
from qiskit.circuit.random import random_circuit
from numpy import linalg as LA
from timeit import default_timer as timer

pi = np.pi
```

We will be switching the Rotation Z gates to Rotation X gates for the green block implementation. (previous implementation found in QOSF_Task1)

In [2]:

```
# Implement Green Block - Even Block
def green_block(qcirc, rlist, index):

    '''
    Creates Green Block (Even) and appends to input circuit

    Inputs:
      qcirc - input circuit
      rzlist - list of angles for the rotation z gates
      index - index of block to place green block
    '''
    # Throw error for an odd index
    if (index % 2 != 0):
          print("Error: Must be an even index to apply a green block")
          return

    # initialize quantum circuit
    qc = QuantumCircuit(qcirc.num_qubits)

    # add rotation blocks - rz
    for i in range(qcirc.num_qubits):
        # qc.rz(rlist[i], i)  # add rz gate with specified angle
        qc.rx(rlist[i], i)  # add rz gate with specified angle

    # add CZ gates
    for i in range(qcirc.num_qubits):
        for j in range(qcirc.num_qubits):
            if (j > i):
                qc.cx(i,j)   # add CZ gate

    # turn above gates to green block and append to qcirc
    qc = qc.to_gate()
    qc.name = "$R_z(theta_%d)$" % (index)
    qcirc.append(qc, range(qcirc.num_qubits))
```

We will be switching the Rotation X gates to Rotation Y gates for the yellow block implementation. (previous implementation found in QOSF_Task1)

In [3]:

```
# Implement Yellow Block - Odd
def yellow_block(qcirc, rlist, index):

    '''
    Creates Green Block (Even) and appends to input circuit

    Inputs:
      qcirc - input circuit
      rzlist - list of angles for the rotation z gates
      index - index of block to place green block
    '''

    # Throw error for an even index
    if (index % 2 != 1):
          print("Error: Must be an odd index to apply a yellow block")
          return

    # initialize quantum circuit
    qc = QuantumCircuit(qcirc.num_qubits)

    # add rotation blocks - rx
    for i in range(qcirc.num_qubits):
        # qc.rx(rlist[i], i)  # add rz gate with specified angle
        qc.rz(rlist[i], i)  # add rz gate with specified angle

    # turn above gates to yellow block and append to qcirc
    qc = qc.to_gate()
    qc.name = "$R_x (theta_%d)$" % (index)
    qcirc.append(qc, range(qcirc.num_qubits))
```

The angles $\theta_{i,n}$ are variational parameters, lying in the interval $(0, 2\pi)$, which is initialized at random. The function *random_angle_parameters* outputs an array of random θs from 0 to 2π that will be used to initialize the circuit of layers of yellow and green blocks.

In [4]:

```
# Output a random angle from 0 to 2pi
def random_angle_parameters(num_qubits, num_layers):
    '''
    Creates Initial Angle Parameters Randomly

    Inputs:
      num_qubits - number of qubits
      num_layers - number of layers

    Returns:
      Output an array of random angles from 0 to 2pi based on inputs
    '''

    theta = []
    for i in range(num_qubits * num_layers * 2):
        theta.append(2 * pi * random.uniform(0,1))

    return theta
```

We will need to obtain the statevector of the circuit. By using one of Qiskit's simulations to view the state of the system, we can get the final statevector using *get_statevector* function.

In [5]:

```
# get statevector
def output_statevector(qc):

    '''
    Get statevector from the input circuit

    Inputs:
      qcirc - input circuit

    Returns:
      Statevector for the given input circuit
    '''

    # Run the quantum circuit on a statevector simulator backend
    backend = Aer.get_backend('statevector_simulator')

    # Create a Quantum Program for execution
    job = execute(qc, backend)
    result = job.result()
    outputstate = result.get_statevector(qc)

    return outputstate
```

The function *initialize_circuit* takes in an array of angles, a quantum circuit, the number of layers, and the number of qubits specified and outputs the statevector of the system, along with the modifed circuit with the layers appended.

In [6]:

```
def initialize_circuit(angles, qc, num_layers, num_qubits):

    '''
    Initializes circuit with random angles initialized

    Parameters:
      qc - input Quantum Circuit
      num_layers - number of layers
      num_qubits - number of qubits

    Returns:
      Modifies input qc with the number layers specified
      Outputs the statevector of the circuit
    '''

    # reshape array
    theta = np.reshape(angles, (num_layers * 2, num_qubits))
    for i in range(num_layers):
        # Initialize random thetas
        yellow_theta = theta[(2*i)]
        green_theta = theta[(2*i)+1]
        # append yellow and green blocks
        yellow_block(qc, yellow_theta, (2*i)+1)   # odd index
        green_block(qc, green_theta, (2*i)+2)    # even index

    # compute return the statevector
    qc_vector = output_statevector(qc)

    return qc_vector
```

Starting with One Layer

We will first start with one layer. We will create a random array of angles from 0 to 2π then initialize the circuit. We can see the amplitudes of each of the states possible for 4 qubits, shown when *qc_vector* is printed. The first element of the *qc_vector* array represents the amplitude of the $|0000\rangle$ state, the second element represents the amplitude of the $|0001\rangle$ state, etc., and the 16th element represents the amplitude of the $|1111\rangle$ state. The state of the layered circuit is denoted as $\psi(\theta)$.

In [7]:

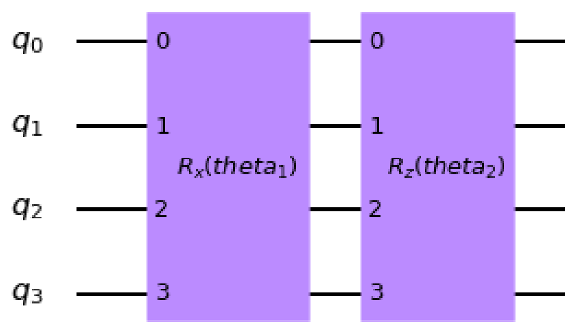
```
# Create new circuit
num_qubits = 4
num_layers = 1

# Create array of random angles
theta_list = random_angle_parameters(num_qubits, num_layers)

# Initialize circuit with layers using random angles
qc = QuantumCircuit(num_qubits)
qc_vector = initialize_circuit(theta_list, qc, num_layers, num_qubits)
qc_vector = output_statevector(qc)
print(qc_vector)
qc.draw('mpl')
```

```
[-1.74624475e-01+0.00000000e+00j  7.13217978e-02+1.74688023e-17j
  1.52169098e-17-8.28369989e-02j  9.20627396e-18-1.50349864e-01j
  2.66109992e-01+3.25890750e-17j -4.68021941e-02-5.73161572e-18j
 -3.32850447e-18+5.43586031e-02j -4.20882838e-17+2.29117946e-01j
  4.40601126e-18-7.19556245e-02j  3.17954341e-17-1.73086282e-01j
 -2.01031782e-01-2.46192928e-17j  6.19530472e-02+7.58706010e-18j
  3.95441645e-17-6.45805215e-01j  3.54264764e-18-1.92852755e-02j
 -2.23989635e-02-2.74308189e-18j  5.56031599e-01+6.80942318e-17j]
```

Out[7]:



Now we want to create a random state ϕ, which will be fixed throughout the run. We can create a random circuit, using Qiskit's *random_circuit* and then obtain its statevector. The circuit of the random state is drawn below.

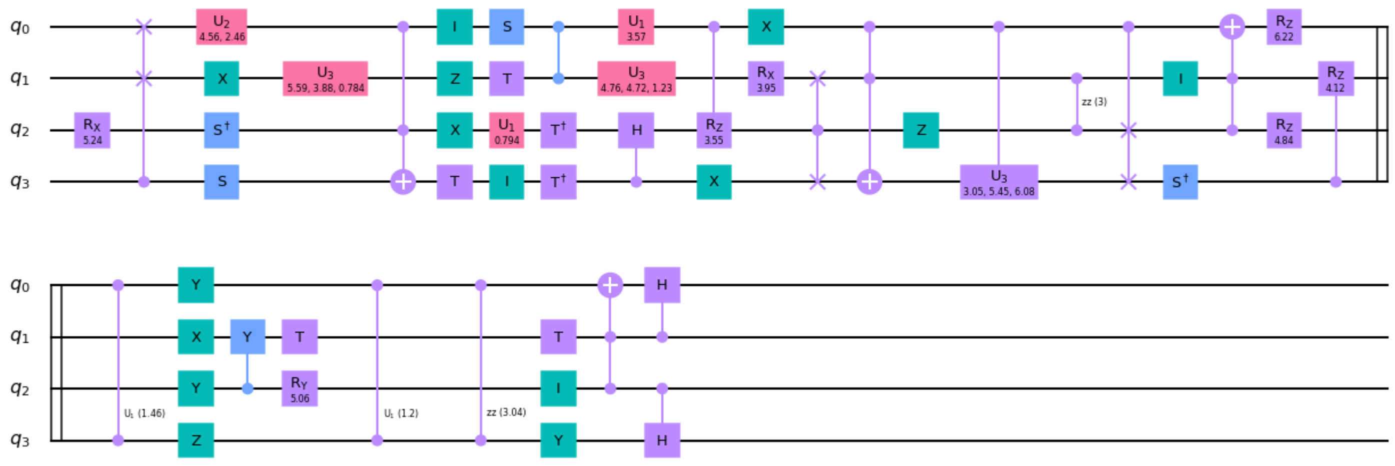
In [8]:

```
# Create random state
rand_circ = random_circuit(4, 20, measure=False)
rand_vector = output_statevector(rand_circ)
```

```
print(rand_vector)
rand_circ.draw('mpl')
```

```
[ 0.3155487 -0.05337014j  0.00296201+0.08588435j  0.31121512+0.12243474j
  0.13135437+0.08915757j -0.10602331+0.1811614j  -0.01664159+0.05816758j
  0.24433421+0.06968392j -0.19879905-0.13467321j  0.4966355 -0.21339754j
 -0.06766645+0.11554671j -0.05486919-0.12303528j  0.06500499+0.13739861j
 -0.20634713-0.12832881j  0.01370941-0.14318689j  0.11371369+0.12759027j
 -0.33729801-0.09554297j]
```

Out[8]:



We want to measure the difference between the two states. This can be done by using numpy's linear algebra function *norm*. We first subtract the amplitudes of the initial state prepared with random angles and the random state prepared using a random circuit. The distance is given as

$$\epsilon = \min_\theta$$
$$|| |\psi(\theta)\rangle$$
$$- |\phi\rangle ||$$

In [9]:

```
initial_epsilon = LA.norm(np.subtract(qc_vector, rand_vector))
```

Optimization of Angles

The correct set of the variational parameters $\theta_{i,n}$ need to found such that ϵ defined above is the minimum. We will be using *scipy.optimize.minimize*, a Scipy function that minimizes scalar functions of one or more variables. We first need to create an objective function to be minimized, in this case *optimize_angles*. The default method solver will be used, BFGS (quasi-Newton method of Broyden, Fletcher, Goldfarb, and Shanno), which uses first derivatives.

In [10]:

```
# Optimize Variational Parameters
def optimize_angles(angles):

    '''
    Function to optimize angles

    Parameters:
      angles - array of angles

    Returns:
      Statevector for the given input circuit
    '''

    global rand_vector
    global num_qubits
    global num_layers
    qc = QuantumCircuit(num_qubits)
    qc_vector = initialize_circuit(angles, qc, num_layers, num_qubits)

    return LA.norm(np.subtract(qc_vector, rand_vector))
```

We will input the objective function *optimize_angles* and the initial angles into *scipy.optimize.minimize*. This will output an *OptimizeResult* object, which has several attributes, such as *x*, which is the the solution array, and *nfev*, which is the number of evaluations of the objective function.

In [11]:

```
# Optimize variational parameters
final_optimized_angles = scipy.optimize.minimize(optimize_angles, theta_list)
print(final_optimized_angles)
```

```
      fun: 1.1041356152154627
 hess_inv: array([[  1.        ,   0.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ],
       [  0.        ,   1.        ,   0.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ],
       [  0.        ,   0.        ,   1.        ,   0.        ,
          0.        ,   0.        ,   0.        ,   0.        ],
       [  0.        ,   0.        ,   0.        ,   1.        ,
          0.        ,   0.        ,   0.        ,   0.        ],
       [  0.        ,   0.        ,   0.        ,   0.        ,
         23.1807517 ,  -0.52455125,  13.50881326, -11.08567799],
       [  0.        ,   0.        ,   0.        ,   0.        ,
         -0.52455125,  10.32702197,  -1.28604655,   2.32738306],
       [  0.        ,   0.        ,   0.        ,   0.        ,
         13.50881326,  -1.28604655,  19.07885429,  -5.7001272 ],
       [  0.        ,   0.        ,   0.        ,   0.        ,
        -11.08567799,   2.32738306,  -5.7001272 ,  14.8644615 ]])
      jac: array([ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
        1.49011612e-06, -3.14414501e-06, -1.63912773e-06, -4.47034836e-07])
  message: 'Optimization terminated successfully.'
     nfev: 250
      nit: 18
     njev: 25
   status: 0
  success: True
        x: array([4.27094175, 5.59708867, 5.43994707, 5.86510939, 8.81247741,
       0.60757949, 3.0482194 , 5.85815974])
```

Now that we have the solution angles from the optimizer, we can set up the circuit with the optimized angles, and compare the ϵ between the initial random angles and the optimized angles given by the optimizer.

In [12]:

```
# Extract angle from optimization
theta = np.array(final_optimized_angles.x)

# Set up circuit with optimized angles
final_qc = QuantumCircuit(num_qubits)
final_statevector = initialize_circuit(theta, final_qc, num_layers, num_qubits)
final_epsilon = LA.norm(np.subtract(final_statevector, rand_vector))

print("Initial Epsilon: " + str(initial_epsilon) + ", Final Epsilon: " + str(final_epsil
on))
```

```
Initial Epsilon: 1.5665162962590253, Final Epsilon: 1.1041356152154627
```

Increasing the Number of Layers

Now, we will iteratively increase the number of layers to see the effects on the minimum distance between $\psi(\theta)$ and ϕ.

In [13]:

```
max_layers = 8
num_qubits = 4

optimized_epsilon = []
initial_epsilon = []
times = []
num_fev = []

for i in range(1,max_layers):
    # Create new circuit
    num_layers = i
    qc = QuantumCircuit(num_qubits)
    theta_list = random_angle_parameters(num_qubits, num_layers)
    qc_vector = initialize_circuit(theta_list, qc, num_layers, num_qubits)

    # time the process of the optimization
    start = timer()
    # optimize the thetas (variational parameters)
    final_optimized_angles = scipy.optimize.minimize(optimize_angles, theta_list)
    end = timer()
    time_taken = end - start
    print("Number of Layers: " + str(i) + ", Time Taken: " + str(time_taken))

    # print(final_optimized_angles)

    # Extract the solution of theta from optimization
    theta = np.array(final_optimized_angles.x)

    # Set up circuit with optimized angles
    final_qc = QuantumCircuit(num_qubits)
    final_statevector = initialize_circuit(theta, final_qc, num_layers, num_qubits)

    # Calculate epsilon after optimization
    final_epsilon = LA.norm(np.subtract(final_statevector, rand_vector))

    # Save additional variables to to printed in plots
    initial_epsilon.append(LA.norm(np.subtract(qc_vector, rand_vector)))
    optimized_epsilon.append(final_epsilon)
    num_fev.append(final_optimized_angles.nfev)
    times.append(time_taken)
```

```
Number of Layers: 1, Time Taken: 5.106738250999996
Number of Layers: 2, Time Taken: 16.815667085
Number of Layers: 3, Time Taken: 39.764421698999996
Number of Layers: 4, Time Taken: 168.43986264200004
Number of Layers: 5, Time Taken: 399.78162879300004
Number of Layers: 6, Time Taken: 440.07049219099997
Number of Layers: 7, Time Taken: 494.796713684
```

Results

In [14]:

```
# Print out arrays
print("Initial epsilon: " + str(initial_epsilon))
print("Optimized epsilon: " + str(optimized_epsilon))
print("Number of function Evaluations: " + str(num_fev))
```

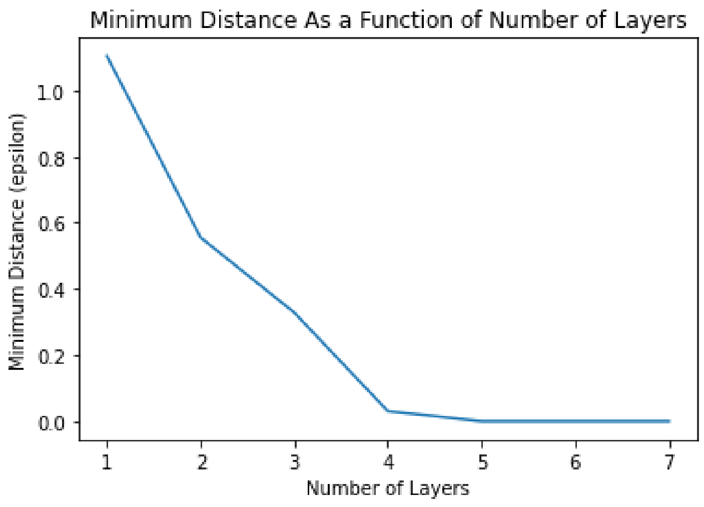
```
Initial epsilon: [1.4258002124473674, 1.2648101708546076, 1.3661888259311927, 1.534822348
5194699, 1.5391798408388422, 1.3831171987874287, 1.2013942704054956]
Optimized epsilon: [1.1041356151449497, 0.5556345840867547, 0.3286072023362954, 0.0303159
74808402774, 1.536789107031715e-06, 1.9624987364435 82e-06, 2.6717844880400473e-06]
Number of function Evaluations: [280, 684, 1274, 4454, 8790, 8612, 8711]
```

We can now see the effects of the additional layers on the minimum distance between $\psi(\theta)$ and ϕ.

In [15]:

```
fig, ax1 = plt.subplots()
```

```
ax1.plot(range(1,max_layers), optimized_epsilon)
ax1.set_xlabel('Number of Layers')
ax1.set_ylabel('Minimum Distance (epsilon)')
ax1.set_title('Minimum Distance As a Function of Number of Layers')
fig.show()
```

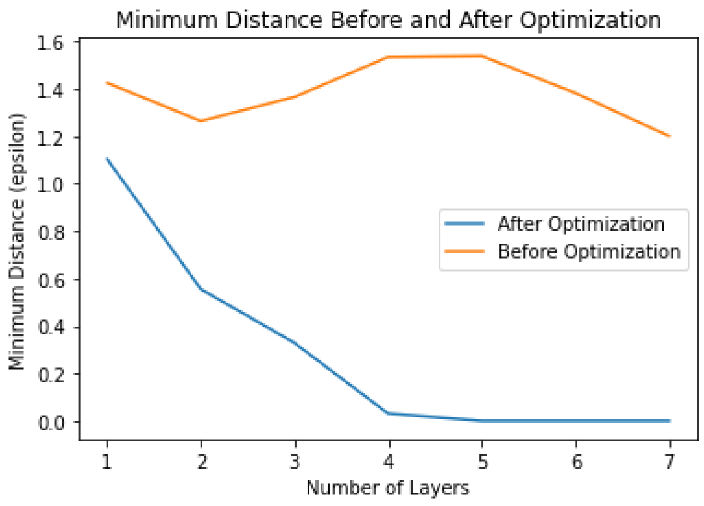


We can see the more layers added, the mininum distance between $\psi(\theta)$ and the random state ϕ generated decreases as the number of layers increases and the distance is close to zero after 4 layers. This is because there are so many parameters to tune that it's likely for $\psi(\theta)$ and ϕ to be similar. The plot below shows the ϵ before and after optimization.

In [16]:

```
fig, ax1 = plt.subplots()

ax1.plot(range(1,max_layers), optimized_epsilon, label = "After Optimization")
ax1.plot(range(1,max_layers), initial_epsilon, label = "Before Optimization")
ax1.set_xlabel('Number of Layers')
ax1.set_ylabel('Minimum Distance (epsilon)')
ax1.set_title('Minimum Distance Before and After Optimization')
ax1.legend()
fig.show()
```



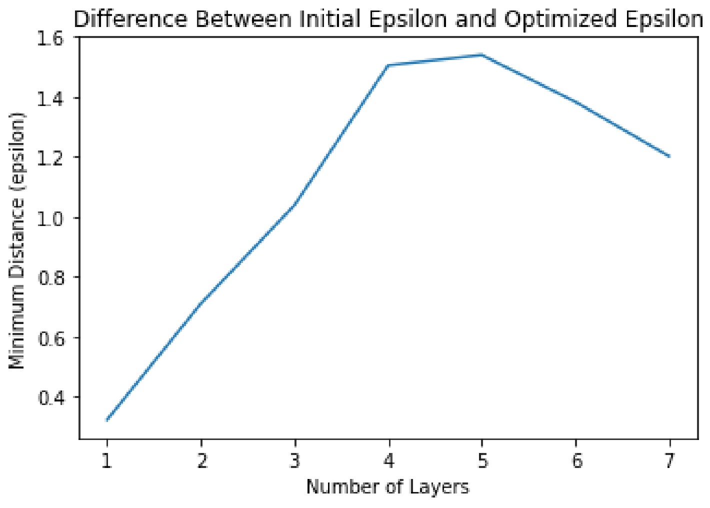
We can then plot the difference of ϵ before and after optimization of angles.

In [17]:

```
fig, ax1 = plt.subplots()

diff_epsilon = np.subtract(initial_epsilon, optimized_epsilon)
ax1.plot(range(1,max_layers), diff_epsilon)
ax1.set_xlabel('Number of Layers')
ax1.set_ylabel('Minimum Distance (epsilon)')
ax1.set_title('Difference Between Initial Epsilon and Optimized Epsilon')
```

```
fig.show()
```

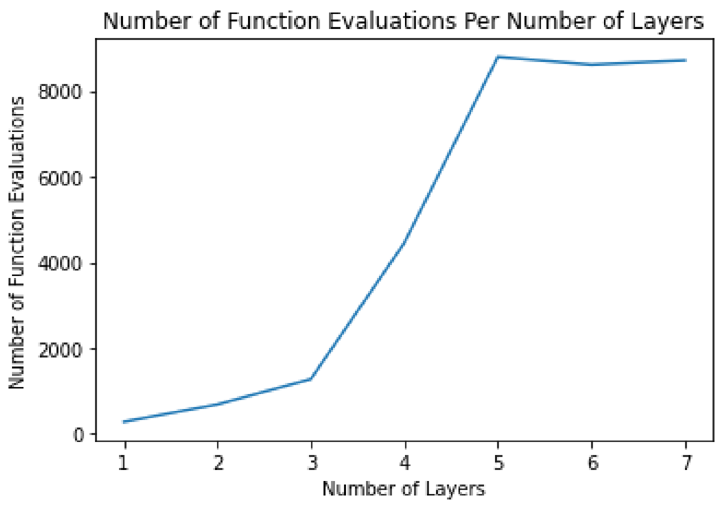


The number of times *optimize_angles* was called during optimization is plotted below. As the number of layers increase, the number of the function *optimize_angles* evaluations increases.

In [18]:

```
fig, ax1 = plt.subplots()

ax1.plot(range(1,max_layers), num_fev)
ax1.set_xlabel('Number of Layers')
ax1.set_ylabel('Number of Function Evaluations')
ax1.set_title('Number of Function Evaluations Per Number of Layers')
fig.show()
```

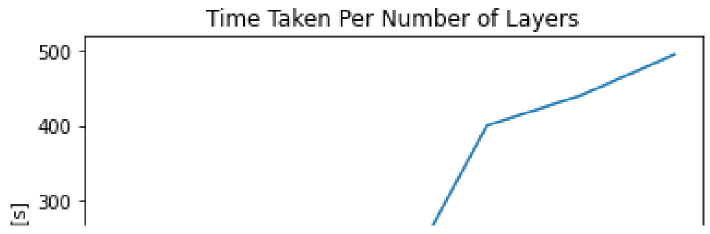


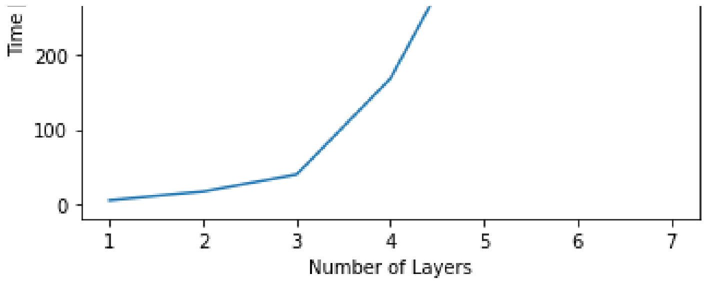
Additionally, we can analyze the runtime for the number of layers specified. As the number of layers increase, the time it takes to optimize increases.

In [19]:

```
fig, ax1 = plt.subplots()

ax1.plot(range(1,max_layers), times)
ax1.set_xlabel('Number of Layers')
ax1.set_ylabel('Time [s]')
ax1.set_title('Time Taken Per Number of Layers')
fig.show()
```





Conclusion

We implemented the 4-qubit state, which depends on layers of one yellow and one green block. These blocks depend on variational parameters, $\theta_{i,n}$, which lie in the interval $(0, 2\pi)$. These variational parameters are initialized at random, which then need to be optimized to give the minimum distance ϵ. *Scipy.optimize.minimize* is used to give the correct $\theta_{i,n}$. The number of layers are then varied to see the effects of ϵ. We concluded that the more layers added, the mininum distance between $\psi(\theta)$ and the random state ϕ generated by Qiskit's *random_circuit* function decreases as the number of layers increases and the distance is close to zero after four layers.