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CS 20 Laboratory 2: Application of Kirchhoff's Laws

1. KCL and KVL Exercise

(a) Enumerate all effective nodes of the circuit; include all components connected to each node.

Let us denote the effective nodes in the circuit as follows:

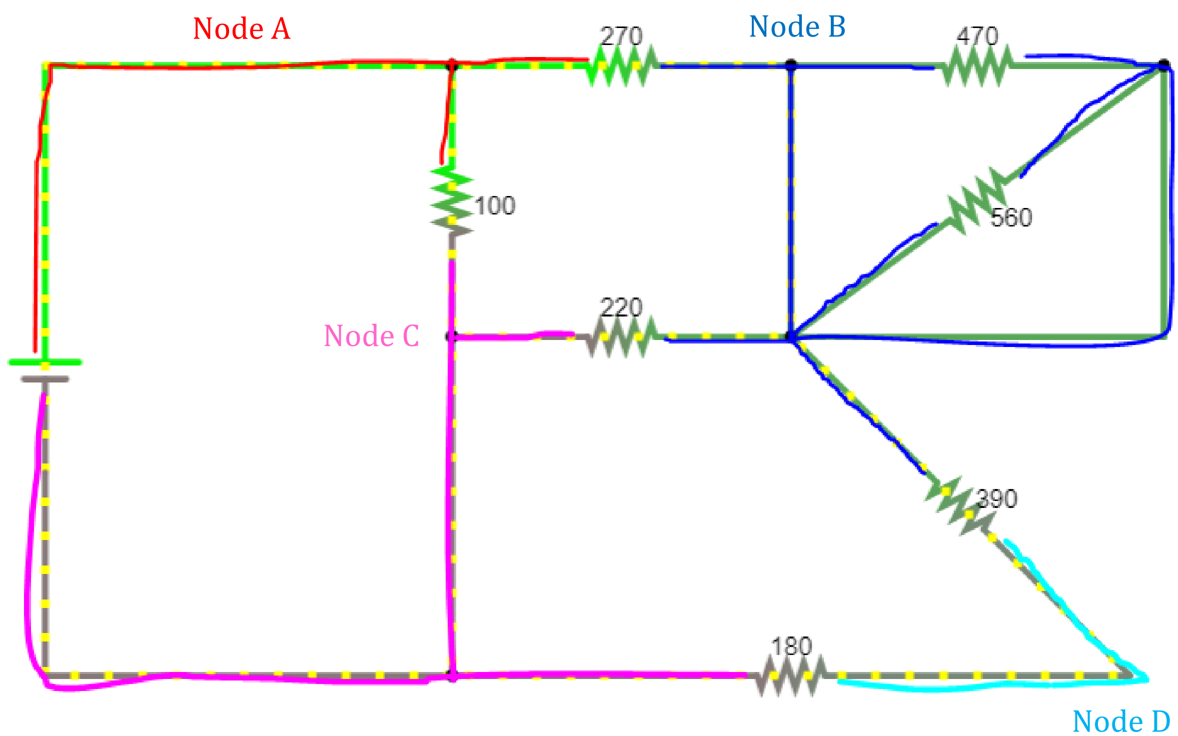


Figure 1: Visualization of effective nodes

The components of each node are:

- Node A – 5V voltage source, 100 Ω resistor, and 270 Ω resistor
- Node B – 270 Ω resistor, 470 Ω resistor, 220 Ω resistor, 390 Ω resistor, and 560 Ω resistor
- Node C – 5V voltage source, 100 Ω resistor, 220 Ω resistor, and 180 Ω resistor
- Node D - 180 Ω resistor and 390 Ω resistor

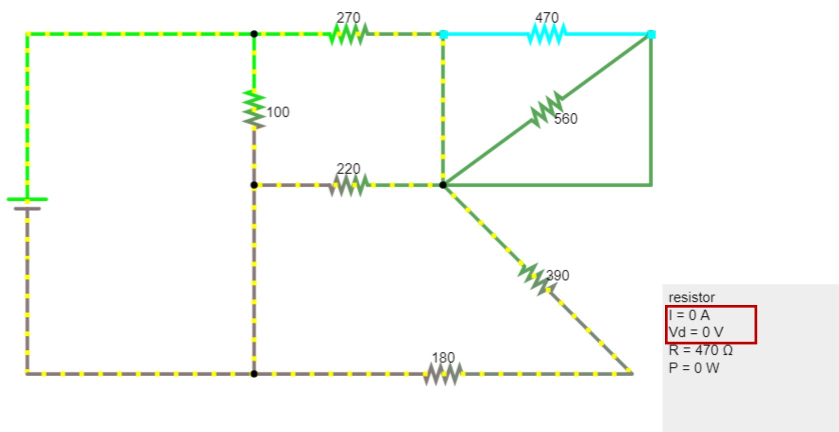
(b) Fill up the table below. Show relevant equations as well as screenshots from Falstad showing that your calculations are correct:

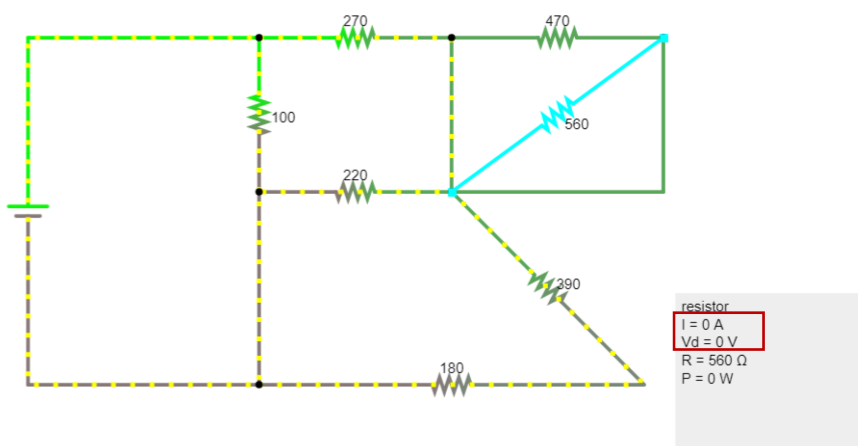
Element	Voltage across	Current through
$R_{100\Omega}$	5V	$0.05\ A\ or\ 50mA$
$R_{180\Omega}$	$0.58459\ V\ or\ 584.59\ mV$	$0.0032\ A\ or\ 3.25\ mA$

$R_{220\Omega}$	$1.85\ V$	$0.0084\ A\ or\ 8.41\ mA$
$R_{270\Omega}$	$3.15\ V$	$0.01166\ A\ or\ 11.66\ mA$
$R_{390\Omega}$	$1.27\ V$	$0.0032\ A\ or\ 3.25\ mA$
$R_{470\Omega}$	0 V	0 A
$R_{560\Omega}$	0 V	0 A
Source	5V	$0.6166\ A\ or\ 61.66\ mA$

Solutions:

In order to obtain the desired quantities, we must simplify the circuit. We can neglect $R_{470\Omega}$ and $R_{560\Omega}$ already because they do not affect the circuit and have no current.





As shown in figure 1, $R_{180\Omega}$ and $R_{390\Omega}$ are in series so we can simplify their resistance. Let R_{s1} be their simplification given by

$$R_{s1} = R_{180\Omega} + R_{390\Omega} = 570\ \Omega$$

Furthermore, $R_{220\Omega}$ and R_{s1} are in parallel so we can further simplify them. Let R_{s2} be their simplification given by

$$R_{s2} = \left(\frac{1}{R_{s2}}\right)^{-1} = \left(\frac{1}{R_{220\Omega}} + \frac{1}{R_{s1}}\right)^{-1} = \left(\frac{79}{12540}\right)^{-1} = 158.7341772\ \Omega \approx 158.73\ \Omega$$

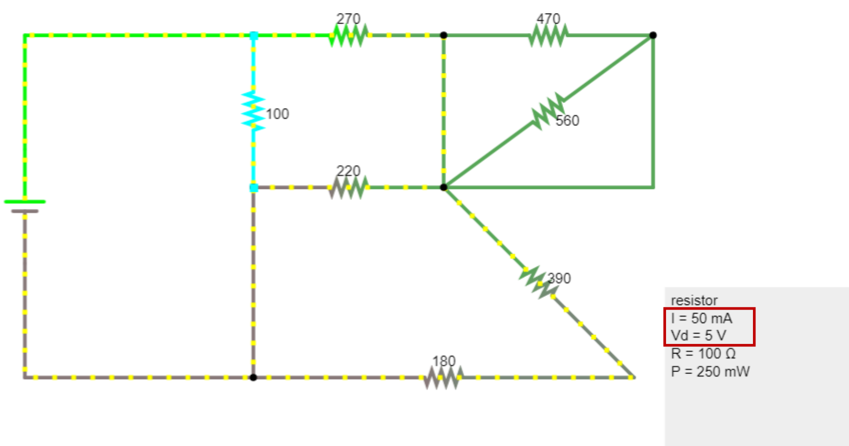
Now $R_{270\Omega}$ and R_{s2} are in series so we can further simplify them. Let R_{s3} be their simplification given by

$$R_{s3} = R_{270\Omega} + R_{s2} = 428.7341772\ \Omega \approx 428.73\ \Omega$$

Finally, $R_{100\Omega}$ and R_{s3} will be left in parallel with the source. This is the simplified version of the circuit.

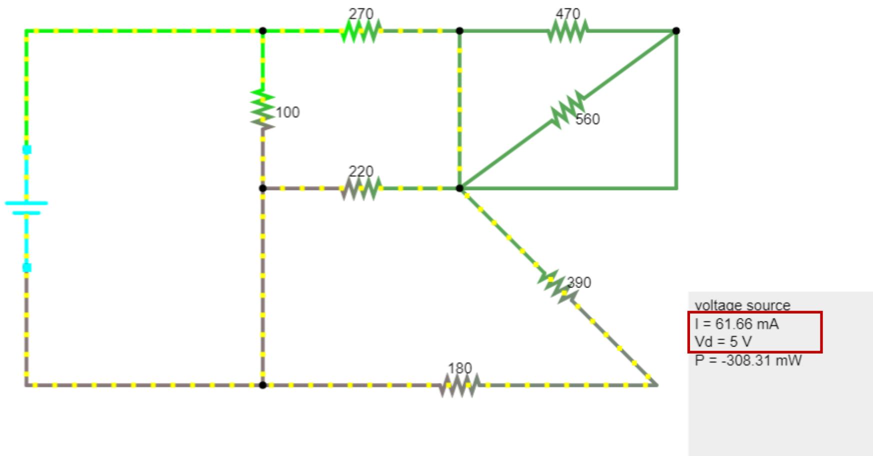
- For $R_{100\Omega}$ and source

  Since it is in parallel with the voltage source, by KVL, the voltage is also 5V. Using Ohm's Law, $I_{100\Omega} = \frac{V_{100\Omega}}{R_{100\Omega}} = \frac{5\ V}{100\ \Omega} = 0.05\ A\ or\ 50mA$



Furthermore, by KCL, we can add the current of $R_{100\Omega}$ and R_{s3}

$$I = \frac{V_{100\Omega}}{R_{100\Omega}} + \frac{V_{s3}}{R_{s3}} = \frac{5\ V}{100\ \Omega} + \frac{5\ V}{428.73\ \Omega} = 0.06166223797\ A$$
$$\approx 0.6166\ A\ or\ 61.66\ mA$$



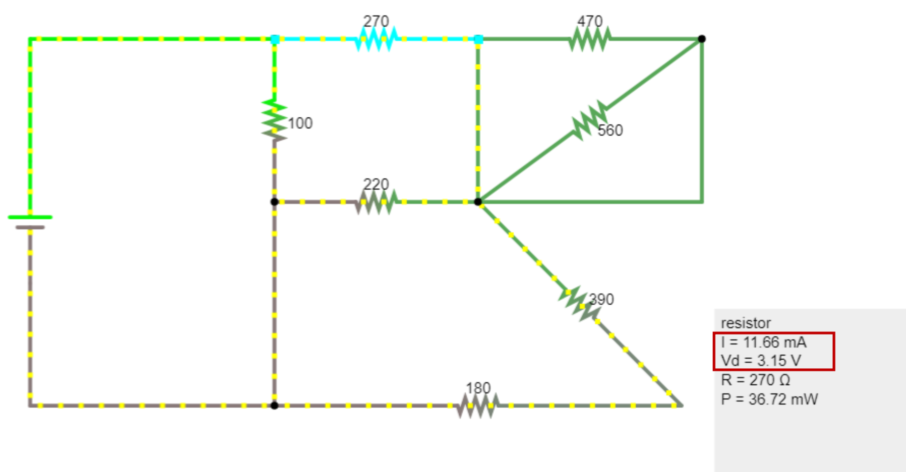
- For $R_{270\Omega}$

  Similarly, R_{s3} is in parallel with the voltage source so by KVL, its voltage is also 5V. Since it is the simplification of R_{s2} and $R_{270\Omega}$ in series, we can use voltage division such that

$$V_{270\Omega} = \left(\frac{R_{270\Omega}}{R_{s3}}\right)(V_{s3}) = \left(\frac{270\ \Omega}{428.73\ \Omega}\right)(5V) = 3.148804252\ V\ \ \approx 3.15\ V$$

Using Ohm's Law, $I_{270\Omega}\ = \frac{V_{270\Omega}}{R_{270\Omega}} = \frac{3.15\ V}{270\ \Omega} = 0.01166223797\ A$

$$\approx 0.01166\ A\ or\ 11.66\ mA$$

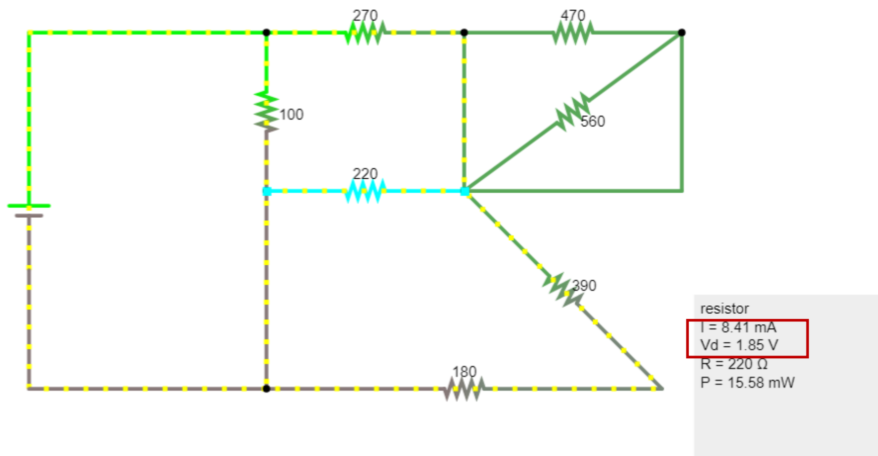


- For $R_{220\Omega}$

  R_{s2} is in series with $R_{270\Omega}$ so by KCL, they have the same current. Furthermore, it is also the simplification of $R_{220\Omega}$ and R_{s1} in parallel, so by KVL, both have the same voltage as R_{s2}, which can be obtained using Ohm's Law

$$V_{220\Omega} = V_{s2} = I_{s2}R_{s2} = (11.66\ mA)(158.73\ \Omega) = 1.851195748\ V \approx 1.85\ V$$

$$I_{220\Omega} = \frac{V_{220\Omega}}{R_{220\Omega}} = \frac{1.85\ V}{220\ \Omega} = 0.008414526129\ A \approx 0.0084\ A\ or\ 8.41\ mA$$



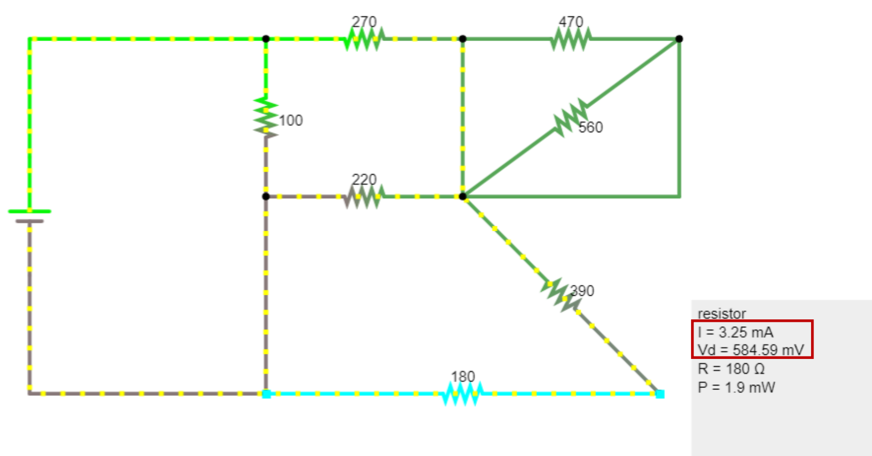
- For $R_{180\Omega}$ and $R_{390\Omega}$

  R_{s1} is in parallel with $R_{220\Omega}$ so by KVL, they have the same voltage. Furthermore, it is also the simplification of $R_{180\Omega}$ and $R_{390\Omega}$ in series, so by KCL, both have the same current as R_{s1}, which can be obtained using Ohm's Law

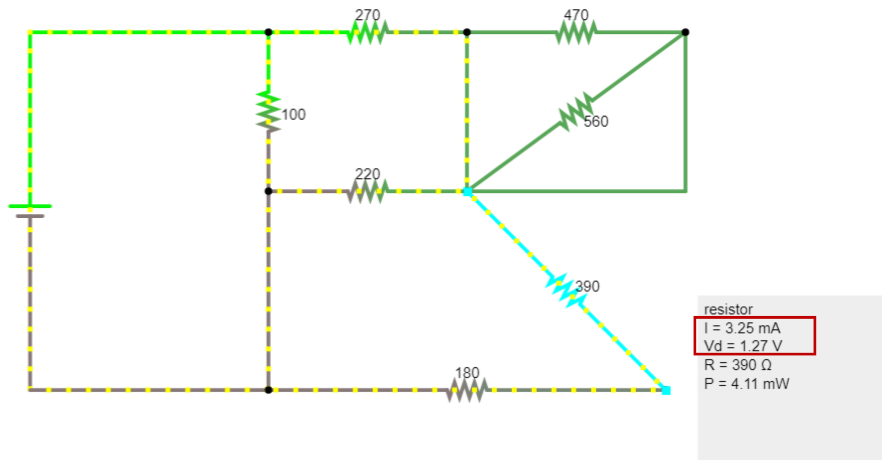
$$I_{180\Omega} = I_{390\Omega} = I_{s1} = \frac{V_{s1}}{R_{s1}} = \frac{1.85\ V}{570\ \Omega} = 0.003247711839\ A \approx 0.0032\ A\ or\ 3.25\ mA$$

Since they are in series, they will have different voltages

$$V_{180\Omega} = I_{180\Omega}\,R_{180\Omega} = (3.25\ mA)(180\ \Omega) = 0.5845881311\ V \approx 0.58459\ V\ or\ 584.59\ mV$$



$$V_{390\Omega} = I_{390\Omega}\, R_{390\Omega} = (3.25\ mA)(390\ \Omega) = 1.266607617\ V \approx 1.27\ V$$



(c) Are there any resistors in which no current passes through? Identify which resistor(s), if any. Why is there no current passing through the said resistor(s)?

Yes, $R_{470\Omega}$ and $R_{560\Omega}$ does not have any current passing. As shown in figure 1, $R_{470\Omega}$ is connected to a single node such that both its terminals are connected to only one node. This is also the case for $R_{560\Omega}$. Since these resistors just loop at a single node, and is neither in parallel nor in series, we can neglect them as they do not affect the circuit.

2. Equivalent Resistors exercise

(a) Show a screenshot of your implementation of the circuit in Falstad.

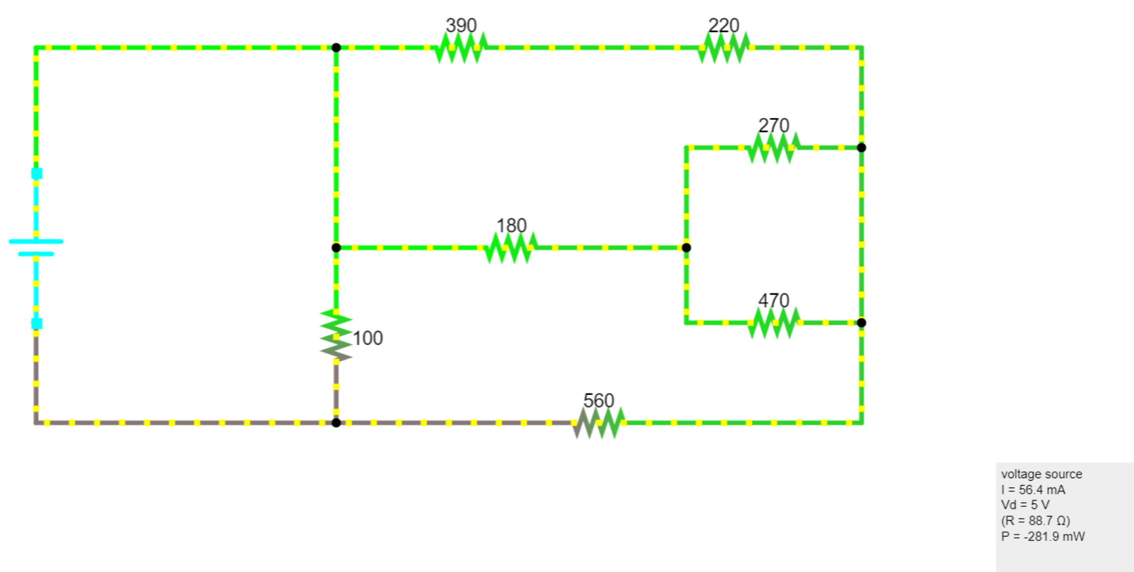


Figure 2: New Circuit Implemented in Falstad

(b) Prove using resistor collapsing that your implementation has an equivalent resistance of about 88.7Ω. Include step-by-step equations and screenshots.

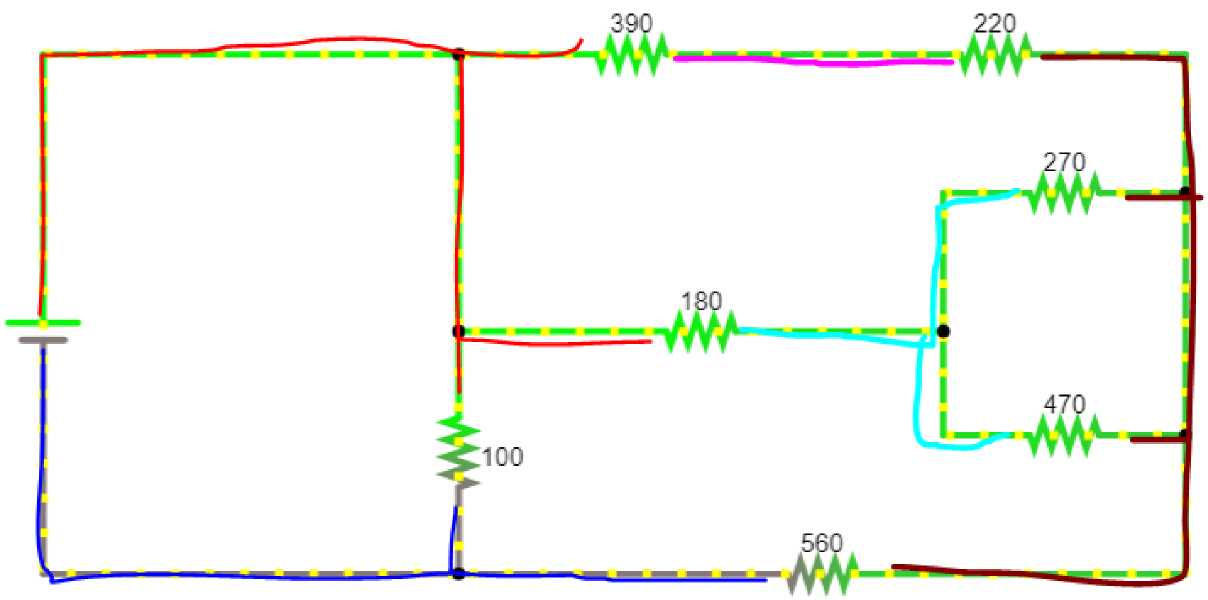
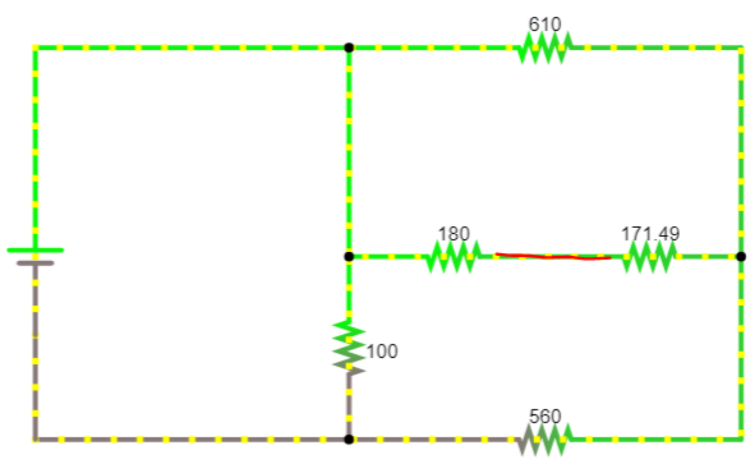


Figure 3: Visualization of nodes of new circuit

As shown in the figure above (fig 3), $R_{390\Omega}$ and $R_{220\Omega}$ are in series while $R_{270\Omega}$ and $R_{470\Omega}$ are in parallel. We can get the equivalent of these resistors as R_{e1} and R_{e2}, respectively.

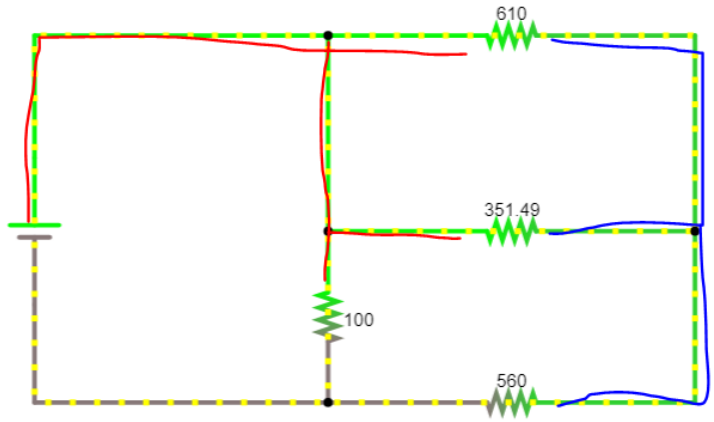
$$R_{e1} = R_{390\Omega} + R_{220\Omega} = 610\ \Omega$$

$$R_{e2} = \left(\frac{1}{R_{e2}}\right)^{-1} = \left(\frac{1}{R_{270\Omega}} + \frac{1}{R_{470\Omega}}\right)^{-1} = \left(\frac{37}{6345}\Omega\right)^{-1} = 171.4864865\ \Omega \approx 171.49\ \Omega$$



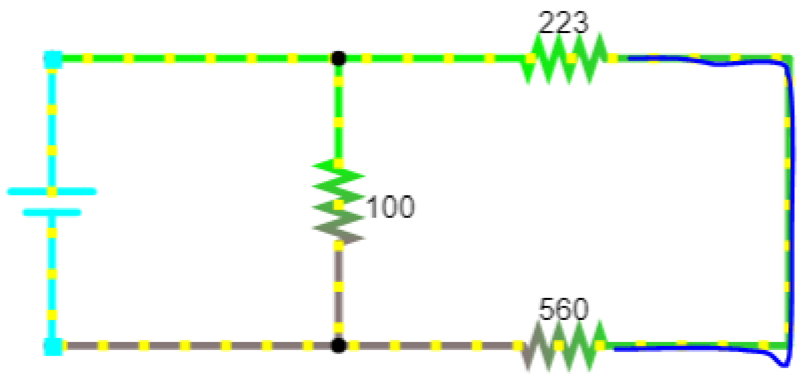
$R_{180\Omega}$ and $R_{171.49\Omega}$ are in series so we can get their equivalent R_{e3}

$$R_{e3} = R_{180\Omega} + R_{171.49\Omega} = 351.49\ \Omega$$



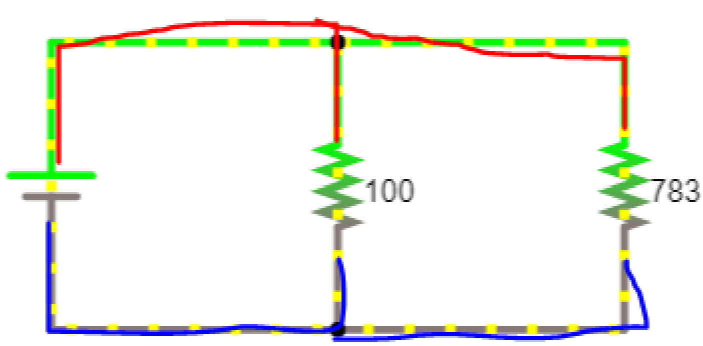
$R_{610\Omega}$ and $R_{351.49\Omega}$ are in series so we can get their equivalent R_{e4}

$$R_{e4} = \left(\frac{1}{R_{e4}}\right)^{-1} = \left(\frac{1}{R_{610\Omega}} + \frac{1}{R_{351.49\Omega}}\right)^{-1} = 222.9950808\ \Omega \approx 223\ \Omega$$



$R_{223\Omega}$ and $R_{560\Omega}$ are in series so we can get their equivalent R_{e5}

$$R_{e5} = R_{223\Omega} + R_{560\Omega} = 783\ \Omega$$



Finally, $R_{100\Omega}$ and $R_{783\Omega}$ are in series so we can get their equivalent R_{eq} which is the equivalent resistance of the whole circuit

$$R_{eq} = \left(\frac{1}{R_{eq}}\right)^{-1} = \left(\frac{1}{R_{100\Omega}} + \frac{1}{R_{783\Omega}}\right)^{-1} = 88.67497169\ \Omega \approx 88.7\ \Omega$$

Therefore, the equivalent resistance of the circuit is about $88.7\ \Omega$

Q.E.D.

(c) From the equations gathered in the previous item, show that the current supplied by the 5V source is about $\frac{5}{88.7}\ A$

To get the current supplied by the source voltage, we use Ohm's law

where R_{eq} is the equivalent resistance of the circuit computed from the previous item,

$$I = \frac{V}{R} = \frac{V}{R_{e6}} = \frac{5V}{88.7\ \Omega} \approx \frac{5}{88.7} A$$