CIS 467/667: Introduction to Articial Intelligence Homework 1

1)

1. a) $$\sum_{n=0}^{D} x^n = \frac{1-x^D}{1-x}$$

b) $$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

c) $$\sum_{n=0}^{D} n x^{n-1} = \frac{d}{dx} \sum_{n=0}^{D} x^n = \frac{d}{dx}\left(\frac{1-x^D}{1-x}\right) = \frac{1-x^{D+1}}{(1-x)^2} - \frac{(D+1)\,x^D}{1-x}$$

d) $$\sum_{n=0}^{\infty} n x^{n-1} = \frac{d}{dx} \sum_{n=0}^{\infty} x^n = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$$

e) $$\sum_{n=0}^{\infty} n x^n = \underline{(d) \times x} = \frac{x}{(1-x)^2}$$

f) $$\sum_{n=0}^{\infty} n(n-1) x^{n-2} = \frac{d}{dx} \sum_{n=0}^{\infty} n x(n-1) = \frac{d}{dx}\left(\frac{1}{(1-x)^2}\right)$$

$$= \frac{2}{(1-x)^3}$$

g) $$\sum_{n=0}^{\infty} (n+2)(n+1) x^n = \frac{2}{(1-x)^3}$$

h) $$\sum_{n=0}^{\infty} (n^2+2n+2) x^n = (g) - (e) = \sum_{n=0}^{\infty} (n+2)(n+1) x^n - \sum_{n=0}^{\infty} n x^n$$

(i) $$\sum_{n=0}^{\infty} (n+1)^2 x^n = (h) - (c) = \sum_{n=0}^{\infty} (n^2+2n+2) x^n - \sum_{n=0}^{\infty} x^n$$

$$= \frac{1+x}{(1-x)^3}$$

2) Suppose Algorithm 1 below is used to generate a search tree. Suppose each tree node has at most b children, and that when the algorithm terminates, the deepest leaves of the tree are at depth D, where the initial node is at depth d = 0, its children are at depth d = 1, etc. Use the numeric values of b and D assigned to your NetID for question 2 in individualized.pdf. [9 points]

Given, b= 8, D = 7

a. If a FIFO (first-in, first-out) queue is used, then in the worst case, what is the largest number of nodes simultaneously stored in the queue at any given time?

At any point tree will have b^D nodes stored in queue

= 8^7 = 2097152

b. Repeat the previous question if a LIFO (last-in, first-out) queue is used instead of FIFO.

At any point tree will have (b-1)*D+1 nodes stored in queue

= (7)*7 + 1 = 50

c. Whichever type of queue is used, in the worst case, what is the largest number of nodes popped off the frontier before the algorithm terminates?

We need to check every element in the tree for a goal state, so we will be popping size(tree) nodes from the frontier
Size of the tree is 1 + b^1 + b^2 + b^3 + ……+b^D
Which is in geometric series, so sum =
= (1-b^(D+1))/(1-b)
=(1-8^8)/(1-8)
= 117,440,505

3) Suppose you run a uniform cost search on a graph where each step costs at least s, each node has at most b children, and the cost of the optimal path is C∗. Look up the values of s, b, and C∗ for your NetID in individualized.pdf. What is the worst case asymptotic running time in your case? [2 points]

Given, s=1/19, b=2, C*=13

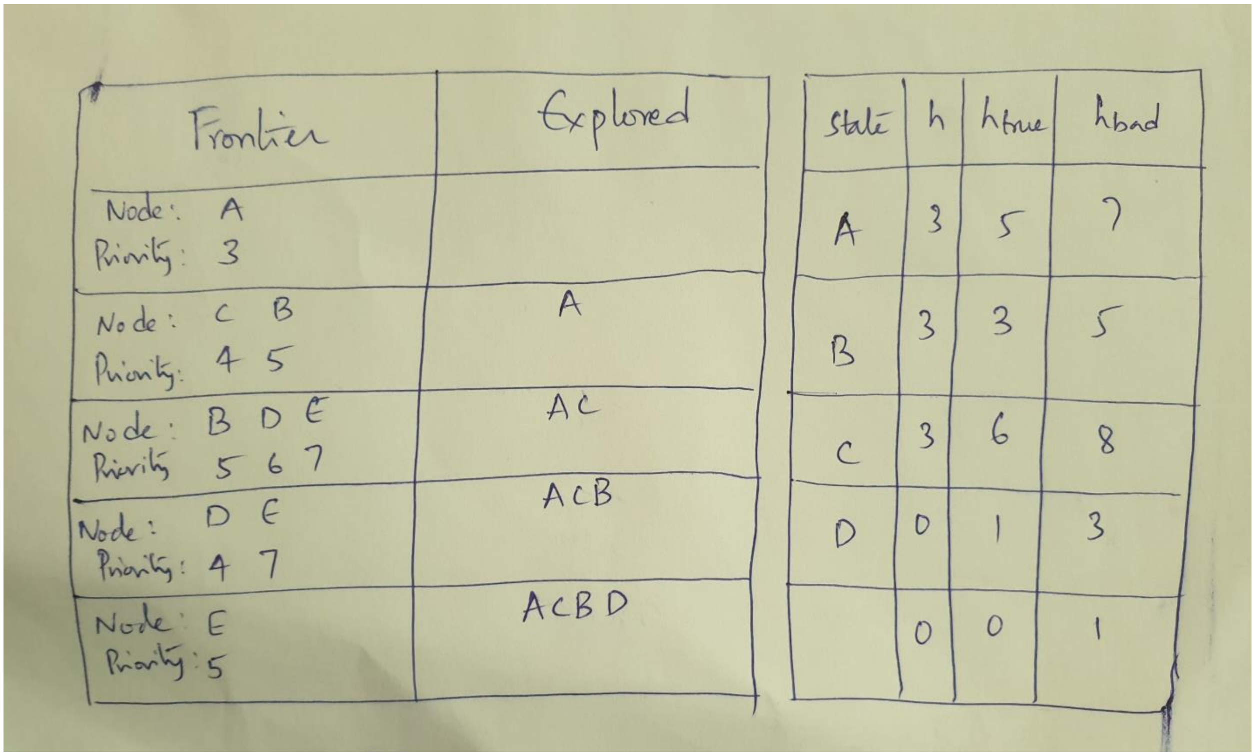
Asymptotic run time = O($b^{1+floor\left(\frac{C*}{s}\right)}$)

= O($3^{1+floor\left(\frac{13}{1/19}\right)}$)

= O($3^{1+floor(247)}$)

= O(3^248)

4)



Frontier	Explored
Node: A Priority: 3	
Node: C B Priority: 4 5	A
Node: B D E Priority 5 6 7	A C
Node: D E Priority: 4 7	ACB
Node: E Priority: 5	ACBD

State	h	htrue	hbad
A	3	5	7
B	3	3	5
C	3	6	8
D	0	1	3
	0	0	1

5)

a) Given, p1=1/21, p2=5/21, p3=3/21, p4=2/21, p5=4/21, p6=6/21

$E_1(x) = \sum x*p(x)$

= 1* 1/21 + 2*5/21 + 3*3/21 + 4*2/21 + 5*4/21 + 6*6/21

= 84/21

= 4

b)

$E_2(x) = 2 * E_1(x)$

= 2* 4

= 8

6)

p-move = 46/126

Given p-stay=1- p-move = 80/126,

p_s = p-stay

p_m = p-move

a. 2* p* p* + 2* p_s*(p_m) + 2*(p_m)*(p_m)

=2*80/126*80/126+2*80/126*46/126+2*46/126*46/126

= 1.5364

b. $p_m = 46/126$

$= 0.36507$

c. $p_s*(p_m) = 80/126*46/126$

$= 0.23179$

d. $p_s* p_s*(p_m) = 80/126*80/126*46/126$

$= 0.14717$

e.

~~(e) E(x) =~~

(e) $E(x) = P_m * \sum_{t=1}^{\infty} t*(P_s)^{t-1} = P_m * \frac{1}{P_m^2} = \frac{1}{P_m}$

$= \frac{1}{46/126} = \frac{126}{46} = 2.73913$

f. $p_m* p_m = 46/126*46/126$

$= 0.13328$

g. $p_s*(p_m)*(p_m) + (p_m)*p*(p_m) = 2*80/126*46/126*46/126 = 0.16924$

h. $3*(p_s\text{^}2)*(p_m)\text{^}2 = 3*(46/126)\text{^}2*(80/126)\text{^}2$

$= 0.48752$

i. $4*(p_s\text{^}2)*(p_m)\text{^}2 = 4*(46/126)\text{^}2*(80/126)\text{^}2$

$= 0.21491$

j.

(j) $E(x) = P_m^2 * \sum_{t=2}^{\infty} t(t-1)*(P_s)^{(t-2)}$

$= P_m^2 * \frac{2}{(P_m)^3}$

$= \frac{2}{P_m} = 2/(46/126) = \frac{2\times126}{46} = 5.47826$

8).