

CIS 467/667: Introduction to Artificial Intelligence Homework 1

PROBLEM 1

a $\sum_{n=0}^D x^n =$

$$= \frac{1 - x^{D+1}}{1 - x}$$

b $\sum_{n=0}^{\infty} x^n =$

$$= \frac{1}{1 - x}$$

c $\sum_{n=0}^D nx^{n-1}$

$$\begin{aligned} &= \frac{d}{dx} \sum_{n=0}^D x^n = \frac{d}{dx} \left(\frac{1 - x^{D+1}}{1 - x} \right) \\ &= \frac{1 - x^{D+1}}{(1 - x)^2} - \frac{(D+1)x^D}{1 - x} \end{aligned}$$

d $\sum_{n=0}^{\infty} nx^{n-1}$

$$\begin{aligned} &= \frac{d}{dx} \sum_{n=0}^{\infty} x^n = \frac{d}{dx} \left(\frac{1}{1 - x} \right) \\ &= \frac{1}{(1 - x)^2} \end{aligned}$$

e $\sum_{n=0}^{\infty} nx^n$

Multiply option 'd' with 'x'

$$= \frac{x}{(1 - x)^2}$$

f $\sum_{n=0}^{\infty} n(n-1)x^{n-2}$

$$\begin{aligned} &= \frac{d}{dx} \sum_{n=0}^{\infty} nx^{n-1} \\ &= \frac{d}{dx} \frac{1}{(1 - x)^2} \\ &= \frac{2}{(1 - x)^3} \end{aligned}$$

g $\sum_{n=0}^{\infty} (n+2)(n+1)x^n$

multiply option (b) with x^2 and differentiate 2 times

$$= \frac{d}{dx} \frac{d}{dx} \left(\frac{x^2}{1 - x} \right)$$

$$= \frac{2}{(1 - x)^3}$$

$$\text{h} \quad \sum_{n=0}^{\infty} (n^2 + 2n + 2)x^n$$

consider option 'g' - option'e'

$$= \sum_{n=0}^{\infty} (n+2)(n+1)x^n - \sum_{n=0}^{\infty} nx^n$$

=

$$\text{i} \quad \sum_{n=0}^{\infty} (n+1)^2 x^n$$

consider option 'h' - option 'c'

$$= \sum_{n=0}^{\infty} (n^2 + 2n + 2)x^n - \sum_{n=0}^{\infty} x^n$$

$$= \frac{1+x}{(1-x)^3}$$