1

$$T = \begin{array}{c|ccc} & s_1 & s_2 & s_3 \\ \hline s_1 & 1/4 & 1/4 & 1/2 \\ s_2 & 1/2 & 1/4 & 1/4 \\ s_3 & 1/4 & 1/2 & 1/4 \end{array} \qquad O = \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline 1/4 & 0 & 1/4 & 1/4 & 1/4 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 \end{array}$$

given

$E_1 = e_4, \; E_2 = e_3, \; E_3 = e_1$

$$\begin{bmatrix} P(S_1 = s_1 \mid E_1 = e_4) \\ P(S_1 = s_2 \mid E_1 = e_4) \\ P(S_1 = s_3 \mid E_1 = e_4) \end{bmatrix} = \langle D^0 S^0 \rangle = \left\langle \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \right\rangle$$

$$= \left\langle \begin{bmatrix} 1/12 \\ 1/6 \\ 0 \end{bmatrix} \right\rangle = \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}$$

$$S^1 = \begin{bmatrix} P(S_2 = s_1 \mid E_1 = e_4) \\ P(S_2 = s_2 \mid E_1 = e_4) \\ P(S_2 = s_3 \mid E_1 = e_4) \end{bmatrix} = T \langle D^0 S^0 \rangle$$

$$= \left\langle \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} \frac{1}{12} + \frac{2}{12} \\ 1/6 + \frac{2}{12} \\ 1/12 + 2/6 \end{bmatrix} \right\rangle$$

$$= \left\langle \begin{bmatrix} 3/12 \\ 4/12 \\ 5/12 \end{bmatrix} \right\rangle = \begin{bmatrix} 3/12 \\ 4/12 \\ 5/12 \end{bmatrix} \text{ (or) } \begin{bmatrix} 1/4 \\ 1/3 \\ 5/12 \end{bmatrix}$$

②

$$\begin{bmatrix} P(S_2=s_1 \mid E_1, E_2 = e_4, e_3) \\ P(S_2=s_2 \mid E_1, E_2 = e_4, e_3) \\ P(S_2=s_3 \mid E_1, E_2 = e_4, e_3) \end{bmatrix} = \langle D^1 S^1 \rangle = \left\langle \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/3 \\ 5/12 \end{bmatrix} \right\rangle$$

$$= \left\langle \begin{bmatrix} 1/16 \\ 0 \\ 5/48 \end{bmatrix} \right\rangle = \begin{bmatrix} 3/8 \\ 0 \\ 5/8 \end{bmatrix}$$

$$S^2 = \begin{bmatrix} P(S_3=s_1 \mid E_1, E_2 = e_4, e_3) \\ P(S_3=s_2 \mid E_1, E_2 = e_4, e_3) \\ P(S_3=s_3 \mid E_1, E_2 = e_4, e_3) \end{bmatrix} = T \langle D^1 S^1 \rangle = \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 3/8 \\ 0 \\ 5/8 \end{bmatrix}$$

$$S^2 = \left\langle \begin{bmatrix} 3/32 + 5/16 \\ \frac{3}{16} + 5/32 \\ 3/32 + 5/32 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 13/32 \\ 11/32 \\ 8/32 \end{bmatrix} \right\rangle = \begin{bmatrix} 13/32 \\ 11/32 \\ 8/32 \end{bmatrix}$$

$$\begin{bmatrix} P(S_3=s_1 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \\ P(S_3=s_2 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \\ P(S_3=s_3 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \end{bmatrix} = \langle D^2 S^2 \rangle = \left\langle \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 13/32 \\ 11/32 \\ 8/32 \end{bmatrix} \right\rangle$$

$$= \left\langle \begin{bmatrix} \frac{13}{128} \\ 0 \\ 8/128 \end{bmatrix} \right\rangle = \begin{bmatrix} 13/21 \\ 0 \\ 8/21 \end{bmatrix}$$

Scanned by CamScanner

$$S^3 = \begin{bmatrix} P(S_4 = s_1 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \\ (S_4 = s_2 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \\ (S_4 = s_3 \mid E_1, E_2, E_3 = e_4, e_3, e_1) \end{bmatrix}, = T \langle D^2 S^2 \rangle$$

$$= \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 13/21 \\ 0 \\ 8/21 \end{bmatrix} = \begin{bmatrix} 13/84 + 8/42 \\ 13/42 + 8/84 \\ 13/84 + 8/84 \end{bmatrix}$$

$$S^3 = \begin{bmatrix} 29/84 \\ 34/84 \\ 21/84 \end{bmatrix}$$

(2) 4

a_1	
3/4	1/4
1/4	3/4

a_2	
1/4	3/4
3/4	1/4

a_3	
1/8	7/8
1/2	1/2

$u^{(0)}(s_1) = 1, \; u^{(0)}(s_2) = 0, \; \gamma = 4/5$

$u_1 = B(u_0) = r + \gamma \max_a P_a u_0$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max_a \left\{ \frac{1}{4} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \frac{1}{4} \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \frac{1}{8} \begin{bmatrix} 1 & 7 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max \left\{ \frac{1}{4} \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \frac{1}{4} \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} \frac{1}{8} \\ \frac{1}{2} \end{bmatrix} \right\}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \left\{ \begin{bmatrix} 3/4 \\ 3/4 \end{bmatrix} \right\}$

$u_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{3}{5} \\ 3/5 \end{bmatrix} = \begin{bmatrix} -2/5 \\ 8/5 \end{bmatrix}$

$\frac{\gamma}{1-\gamma} \| u_1 - u_0 \| = \frac{4/5}{1-4/5} \left\| \begin{bmatrix} -2/5 - 1 \\ 8/5 - 0 \end{bmatrix} \right\|$

$= 4 \left\| \begin{matrix} -7/5 \\ 8/5 \end{matrix} \right\| = \max \{ 28/5, 32/5 \}$

$= 32/5$

$\| u_1 - u^* \| < 32/5$

$u_2 = B(u_1) = r + \gamma \max_a P_a u_1$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max_a \left\{ \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2/5 \\ 8/5 \end{bmatrix}, \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -2/5 \\ 8/5 \end{bmatrix}, \begin{bmatrix} 1/8 & 7/8 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} -2/5 \\ 8/5 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max_a \left\{ \frac{1}{20}\begin{bmatrix} -6+8 \\ -2+24 \end{bmatrix}, \frac{1}{20}\begin{bmatrix} -2+24 \\ -6+8 \end{bmatrix}, \begin{bmatrix} -\frac{2}{40} + \frac{56}{40} \\ -\frac{2}{10} + \frac{8}{10} \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max \left\{ \begin{bmatrix} \frac{1}{10} \\ \frac{11}{10} \end{bmatrix} ✓, \begin{bmatrix} \frac{11}{10} \\ \frac{1}{10} \end{bmatrix}, \begin{bmatrix} \frac{27}{20} \\ 6/10 \end{bmatrix} ✓ \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5}\begin{bmatrix} 27/20 \\ 11/10 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{27}{25} \\ 44/50 \end{bmatrix}$$

$$\underline{u_2} = \begin{bmatrix} \frac{2}{25} \\ 94/50 \end{bmatrix} = \begin{bmatrix} 2/25 \\ 47/25 \end{bmatrix}$$

$$\frac{\gamma}{1-\gamma}\|u_2 - u_1\| = \frac{4/5}{1/5}\left\|\begin{bmatrix} 2/25 - \left(\frac{-2}{5}\right) \\ 94/50 - \frac{8}{5} \end{bmatrix}\right\| = 4\left\|\begin{bmatrix} 12/25 \\ 14/50 \end{bmatrix}\right\|$$

$$= \max\{48/25, 56/50\} = 48/25$$

$\therefore \|\underline{u_2 - u^*}\| < 48/25$

(6)

$$u_3 = Bu_2 = r + \gamma \max_a P_a u_2$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max \left\{ \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2/25 \\ 47/25 \end{bmatrix}, \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2/25 \\ 47/25 \end{bmatrix}, \begin{bmatrix} \frac{1}{8} & 7/8 \\ 1/2 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2/25 \\ 47/25 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max \left\{ \frac{1}{100}\begin{bmatrix} 6+47 \\ 2+141 \end{bmatrix}, \frac{1}{100}\begin{bmatrix} 2+141 \\ 6+47 \end{bmatrix}, \begin{bmatrix} \frac{2}{200} + \frac{329}{200} \\ 47/50 + 47/50 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5} \max \left\{ \begin{bmatrix} 53/100 \\ 143/100 \end{bmatrix}, \begin{bmatrix} 143/100 \\ 53/100 \end{bmatrix}, \begin{bmatrix} \frac{331}{200} \\ 94/50 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{4}{5}\begin{bmatrix} 331/200 \\ 94/50 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 331/250 \\ 376/250 \end{bmatrix}$$

$$u_3 = \begin{bmatrix} 81/250 \\ 626/250 \end{bmatrix}$$

$$\frac{\gamma}{1-\gamma}\|u_3 - u_2\| = \frac{4/5}{1/5}\left\| \begin{bmatrix} 81/250 - 2/25 \\ 626/250 - 47/25 \end{bmatrix} \right\|$$

$$= 4 \left\| \begin{bmatrix} 61/250 \\ 156/250 \end{bmatrix} \right\| = \max \left\{ 122/125, \; 312/125 \right\}$$

$$= 312/125$$

$$\therefore \|u_3 - u^*\| < 312/125$$

3 $\pi^{(0)}(s_1) = a_2,\ \pi^{(0)}(s_2) = a_3,\ \gamma = 2/5$

$$\pi_0 = \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$P_{\pi_0} = \begin{bmatrix} 1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix}$$

$$u_0 = r + \gamma P_{\pi_0} u_0$$

$$u_0 = (I - \gamma P_{\pi_0})^{-1} r$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{2}{5} \begin{bmatrix} 1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{2}{20} & \frac{6}{20} \\ 2/10 & 2/10 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ 2/10 & 2/10 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -2/10 & 8/10 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= 10 * \frac{1}{72 - 6} \begin{bmatrix} 8 & 3 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u_0 = \frac{10}{66} \begin{bmatrix} -5 \\ 8 \end{bmatrix} = \frac{2}{33} \begin{bmatrix} -5 \\ 8 \end{bmatrix}$$

$\pi_1 = \arg\max_a P_a u_0 =$

$= \arg\max \left\{ \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \frac{2}{33}\begin{bmatrix} -5 \\ 8 \end{bmatrix}, \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \frac{2}{33}\begin{bmatrix} -5 \\ 8 \end{bmatrix}, \right.$

$\left. \begin{bmatrix} 1/8 & 7/8 \\ 1/2 & 1/2 \end{bmatrix} \frac{2}{33}\begin{bmatrix} -5 \\ 8 \end{bmatrix} \right\}$

$= \arg\max \left\{ \frac{2}{132}\begin{bmatrix} -15+8 \\ -5+24 \end{bmatrix}, \frac{2}{132}\begin{bmatrix} -5+24 \\ -15+8 \end{bmatrix}, \right.$

$\left. \frac{2}{33}\begin{bmatrix} -\frac{5}{8} + \frac{56}{8} \\ -5/2 + 8/2 \end{bmatrix} \right\}$

$= \arg\max \left\{ \frac{2}{132}\begin{bmatrix} -7 \\ 19 \end{bmatrix}, \frac{2}{132}\begin{bmatrix} 19 \\ -7 \end{bmatrix}, \frac{2}{33}\begin{bmatrix} \frac{51}{8} \\ 3/2 \end{bmatrix} \right\}$

$\pi_1 = \begin{bmatrix} \frac{102}{264} \\ 38/132 \end{bmatrix} = \begin{bmatrix} a_3 \\ a_1 \end{bmatrix}$

$P_{\pi_1} = \begin{bmatrix} 1/8 & 7/8 \\ 1/4 & 3/4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1/2 & 7/2 \\ 1 & 3 \end{bmatrix}$

$u_1 = (I - \gamma P\pi_1)^{-1} r$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{2}{5} \times \frac{1}{4}\begin{bmatrix} 1/2 & 7/2 \\ 1 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{2}{20}\begin{bmatrix} \frac{1}{2} & 7/2 \\ 1 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{20} & \frac{7}{20} \\ 2/20 & 6/20 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Scanned by CamScanner

$$= \left(\begin{bmatrix} \frac{19}{20} & -7/20 \\ -2/20 & 14/20 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= 20 * \frac{1}{(19*14)-(-2*-7)} \begin{bmatrix} 14 & 7 \\ 2 & 19 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u_1 = \frac{20}{252} \begin{bmatrix} -14+7 \\ -2+19 \end{bmatrix} = \frac{5}{63} \begin{bmatrix} -7 \\ 17 \end{bmatrix}$$

$$\pi_2 = \underset{a}{\text{argmax}}\ P_a u_1$$

$$= \text{argmax} \left\{ \frac{1}{4} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \frac{5}{63} \begin{bmatrix} -7 \\ 17 \end{bmatrix}, \frac{1}{4} \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \frac{5}{63} \begin{bmatrix} -7 \\ 17 \end{bmatrix}, \begin{bmatrix} 1/8 & 7/8 \\ 1/2 & 1/2 \end{bmatrix} \frac{5}{63} \begin{bmatrix} -7 \\ 17 \end{bmatrix} \right\}$$

$$= \text{argmax} \left\{ \frac{5}{252} \begin{bmatrix} -21+17 \\ -7+51 \end{bmatrix}, \frac{5}{252} \begin{bmatrix} -7+51 \\ -21+17 \end{bmatrix}, \frac{5}{63} \begin{bmatrix} -7/8 + \frac{119}{8} \\ -7/2 + 17/2 \end{bmatrix} \right\}$$

$$= \text{argmax} \left\{ \frac{5}{252} \begin{bmatrix} -4 \\ 44 \end{bmatrix}, \frac{5}{252} \begin{bmatrix} 44 \\ -4 \end{bmatrix}, \frac{5}{63} \begin{bmatrix} \frac{112}{8} \\ 10/2 \end{bmatrix} \right\}$$

$$\pi_2 = \begin{bmatrix} \frac{112}{8} * 5/63 \\ \frac{5}{252} * 44 \end{bmatrix} = \begin{bmatrix} a_3 \\ a_1 \end{bmatrix}$$

$\pi_2 == \pi_1$, $\therefore$ We stop here

4) given

$$[(1,1),(2,2),(1,1),(1,2),(1,2),(2,2),(2,2),(2,1)]$$

$$\theta_1^{MLE} = \frac{N_1}{N_1+N_2} = \frac{4}{8} = \frac{1}{2}$$

$$\theta_2^{MLE} = \frac{N_2}{N_1+N_2} = \frac{4}{8} = \frac{1}{2}$$

$$\theta_{11}^{MLE} = \frac{N_{11}}{N_{11}+N_{12}} = \frac{2}{2+2} = \frac{1}{2}$$

$$\theta_{12}^{MLE} = \frac{N_{12}}{N_{11}+N_{12}} = \frac{2}{2+2} = \frac{1}{2}$$

$$\theta_{21}^{MLE} = \frac{N_{21}}{N_{21}+N_{22}} = \frac{1}{1+3} = \frac{1}{4}$$

$$\theta_{22}^{MLE} = \frac{N_{22}}{N_{21}+N_{22}} = \frac{3}{1+3} = \frac{3}{4}$$

5

$$\gamma_{bf} = P(B=b \mid F=f) \propto P(F=f \mid B=b)\,P(B=b)$$
$$\approx \theta^{0}_{bf}\,\theta^{0}_{b}$$

$$\gamma_{b1} = \left\langle \begin{bmatrix} \theta^{0}_{11} & \theta^{0}_{1} \\ \theta^{0}_{21} & \theta^{0}_{2} \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 1/2 * 1/2 \\ 1/2 * 1/2 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 1/4 \\ 1/4 \end{bmatrix} \right\rangle$$

$$= \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \gamma_{11} \\ \gamma_{21} \end{bmatrix}$$

$$\gamma_{b2} = \left\langle \begin{bmatrix} \theta^{0}_{12} & \theta^{0}_{1} \\ \theta^{0}_{22} & \theta^{0}_{2} \end{bmatrix} \right\rangle = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \gamma_{12} \\ \gamma_{22} \end{bmatrix}$$

$$\theta^{1}_{11} = \frac{N_1 \gamma_{11}}{N_1\gamma_{11} + N_2\gamma_{12}} = \frac{4 * 1/2}{4*1/2 + 4*1/2} = \frac{1}{2}$$

$$\theta^{1}_{21} = \frac{N_1 \gamma_{21}}{N_1\gamma_{21} + N_2\gamma_{22}} = \frac{4 * 1/2}{4*1/2 + 4*1/2} = \frac{1}{2}$$

$$\theta^{1}_{12} = 1 - \theta^{1}_{11} = \cancel{2/3}\ 1 - 1/2 = 1/2$$

$$\theta^{1}_{22} = 1 - \theta^{1}_{21} = 1 - 1/2 = 1/2$$

$$\theta^{1}_{1} = \frac{N_1\gamma_{11} + N_2\gamma_{12}}{N_1\gamma_{11} + N_2\gamma_{12} + N_1\gamma_{21} + N_2\gamma_{22}} = \frac{4*\frac{1}{2} + 4*\frac{1}{2}}{4*\frac{1}{2} + 4*\frac{1}{2} + 4*1/2 + 4*\frac{1}{2}}$$

$$\theta^{1}_{1} = \frac{1}{2}$$

$$\theta^{1}_{2} = 1 - \theta^{1}_{1} = 1/2$$