Analog Quantum Teleportation

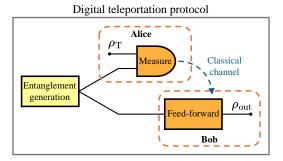
Uesli Alushi, 1, * Simone Felicetti, 2, 3 and Roberto Di Candia 1, 4, †

¹Department of Information and Communications Engineering, Aalto University, Espoo 02150, Finland
²Institute for Complex Systems, National Research Council (ISC-CNR), Via dei Taurini 19, 00185 Rome, Italy
³Physics Department, Sapienza University, P.le A. Moro 2, 00185 Rome, Italy
⁴Dipartimento di Fisica, Università degli Studi di Pavia, Via Agostino Bassi 6, I-27100, Pavia, Italy

Digital teleportation protocols make use of entanglement, local measurements and a classical communication channel to transfer quantum states between remote parties. We consider analog teleportation protocols, where classical communication is replaced by transmission through a noisy quantum channel. We show that analog teleportation protocols outperform digital protocols if and only if Alice and Bob are linked by a channel that does not reduce the entanglement of its input state. We first derive general analytical results in the broader context of Gaussian-channel simulation. Then, we apply it to the quantum teleportation of a uniformly distributed codebook of coherent states, showing that an analog protocol is optimal for a wide range of communication channel transmissivities. Our result is relevant in the intermediate case when the communication channel is far from being ideal but is not too lossy, as is the case of cryogenic links in microwave superconducting circuits.

Introduction—Quantum teleportation allows two parties situated in distant locations, Alice and Bob, to transfer an unknown quantum state using entanglement, local operations and a classical communication channel as resources [1–3]. The protocol consists of four steps: (i) Alice and Bob share an entangled state; (ii) Alice performs a joint (Bell) measurement of the unknown state and her part of the entangled state; (iii) Alice sends the measurement results to Bob via a classical communication channel; (iv) Bob performs a local operation on his part of the entangled state, depending on Alice classical message. Since the early theoretical proposals, there have been experimental implementations of quantum teleportation on several quantum platforms [4, 5]. The performance dependence on steps (i)-(ii) and (iv) has already been well studied in the literature: The protocol is limited by the amount of pre-shared entanglement, the quantum memory of Bob, the fidelity of the Bell measurement at Alice and the noise of the decoding operation at Bob [4, 5]. Usually, step (iii) is treated as noise-free as one can perform digital error correction, which means that the classical information can be transmitted virtually without errors. Nevertheless, a fundamental question has never been addressed: Can the quantum teleportation performance be improved if the parties share a noisy quantum channel?

In this paper, we answer this question in a continuous-variable setting. We investigate an analog protocol that leverages the quantum nature of the communication channel by replacing the digital error correction step in standard quantum teleportation with an analog error correction scheme. In this approach, the Bell measurement is replaced by an encoding operation implemented



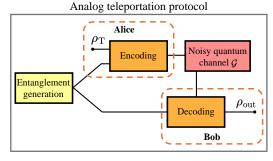


FIG. 1. Sketch of digital and analog protocols. Both protocols leverage an entangled resource state to transfer obliviously a state from Alice to Bob. In the analog protocol, we exploit the quantum description of the communication channel $\mathcal G$ to implement an optimal encoding/decoding scheme, which replaces the measure/feedforward operations of the digital protocol.

via a quantum-limited two-mode squeezer. The encoded output is then transmitted directly to Bob, who applies a decoding operation to retrieve the teleported state, see Fig. 1. In the limit of infinite squeezing, the performance of the analog protocol becomes equivalent to that of the digital protocol. When there is no pre-shared entanglement, the analog protocol represents direct state transfer enhanced by local encoding/decoding operations.

^{*} uesli.alushi@aalto.fi

[†] rob.dicandia@gmail.com

[‡] simone.felicetti@cnr.it

We first frame the problem within the broader context of Gaussian channel simulation, where quantum teleportation corresponds to the case of noisy simulation of a phase-insensitive channel [6, 7]. Here, the goal is to reproduce the effect of a given channel with a minimal amount of added noise. We show that the analog protocol performs better than the digital one if and only if the communication channel between Alice and Bob does not reduce the entanglement of the resource state. We showcase the performance of the analog teleportation protocol applied to a codebook of Gaussian distributed coherent states. We find that the analog protocol offers a significant advantage in the intermediate transmission regime, where the communication channel is neither ideal nor excessively lossy.

So far, the analog feedforward has only been considered as a technical solution to speed up the protocol implementation. Indeed, the absence of a measurement step means that no analog-to-digital conversion and viceversa is performed at any point of the protocol, reducing the waiting time at Bob's side. Based on [8], recent experiments in the microwave regime have already proven the working mechanism of the analog feedforward in the strong-squeezing limit [9, 10], effectively replicating the performance of the digital teleportation scheme. In this paper, we argue that when the communication channel losses are limited, the optimal encoding squeezing is *finite*, and we provide its value. This result is of practical relevance for solid-state quantum technologies, e.g., in superconducting quantum circuits connected via criolinks [11, 12]. Accordingly, our analysis has direct applications in modular quantum computing [13, 14] and quantum communication [15].

Gaussian formalism— An m-mode bosonic system [16] is described by the self-adjoint canonical operators \hat{x}_j and \hat{p}_j , with j=1,...,m, satisfying the canonical commutation relations. If we define $\hat{r}=(\hat{x}_1,\hat{p}_1,...,\hat{x}_m,\hat{p}_m)$, the canonical commutation relations read $[\hat{r}_j,\hat{r}_k]=i\Omega_{jk}$, where $\Omega=\bigoplus_{j=1}^m i\sigma_y$ is the symplectic form. Here and in the following, $\sigma_x,\sigma_y,\sigma_z$ are the Pauli matrices. Gaussian states ρ_G are fully characterized by their first-moments vector $v=\mathrm{Tr}[\rho_G\hat{r}]$ and their covariance matrix Γ with components $\Gamma_{jk}=\mathrm{Tr}[\rho_G\{\hat{r}_j-v_j,\hat{r}_k-v_k\}]$. In this representation, the Robertson-Schrödinger uncertainty relation $\Gamma+i\Omega>0$ must be satisfied.

In the following, we consider the entanglement resource to be a two-mode Gaussian state with null first moments and covariance matrix

$$\Gamma_{AB} = \begin{pmatrix} a \mathbb{I}_2 & -c\sigma_z \\ -c\sigma_z & b \mathbb{I}_2 \end{pmatrix}, \qquad (1)$$

with $a,b \ge 1$ and $0 \le c \le \sqrt{ab-1-|a-b|}$. This state can be generated using two-mode squeezing operations applied to uncorrelated thermal states [9, 10, 17–19]. The entanglement can be quantified using the logarithmic negativity $E_{\mathcal{N}}(\rho_{\mathrm{AB}}) = \ln \|\rho_{\mathrm{AB}}^{T_{\mathrm{A}}}\|_1$ [20], where ρ_{AB} is the density matrix of the resource state and T_{A} denotes the partial transpose. For two-mode Gaussian states, this

corresponds to $E_{\mathcal{N}} = \max\{0, -\ln(\nu_{-})\}$, where ν_{-} is the lowest symplectic eigenvalue of the partial transposed covariance matrix $(\mathbb{I}_2 \oplus \sigma_z) \Gamma_{AB} (\mathbb{I}_2 \oplus \sigma_z)$ [21]. Other measures of entanglement can be considered to quantify the teleportation resource [7]. However, here we consider the logarithmic negativity as it will give a simple optimality condition for the analog protocol.

Gaussian-preserving channels are defined by two real matrices X and Y acting on the first-moments vector and covariance matrix as

$$v \to Xv$$
 , $\Gamma \to X\Gamma X^T + Y$. (2)

The complete positivity condition is $Y + i\Omega \ge iX\Omega X^T$ [16]. For single-mode Gaussian channels, this condition reduces to $\sqrt{\det(Y)} \ge |1 - \det(X)|$ and $Y \ge 0$.

Throughout this work, we consider the case in which Alice and Bob share a Gaussian communication channel \mathcal{G} , which can be "diagonalized" as

$$\mathcal{G}[\rho] = \hat{W} \left(\mathcal{E} \left[\hat{U} \rho \hat{U}^{\dagger} \right] \right) \hat{W}^{\dagger}. \tag{3}$$

Here, \hat{U} , \hat{W} are two unitary operations and \mathcal{E} is the Gaussian channel in the canonical form [22]. Since the operations \hat{U} and \hat{W} can be reversed in the encoding and decoding stages, we can restrict the analysis to channels in their canonical form. Indeed, in the following we focus only on non-degenerate channels, i.e., those channels that do not erase the information carried on one quadrature. Specifically, we consider phase-insensitive channels defined by

$$X = \sqrt{x} \mathbb{I}_2 \quad , \quad Y = y \mathbb{I}_2 \,, \tag{4}$$

where x > 0 is the transmissivity and y is the added noise [23]. The complete positivity condition for this channel is $y \ge |1-x|$. If $y \ge e^{-2r}(1+x)$, then the channel reduces the entanglement of any input state with logarithmic negativity larger than 2r. A particular case is r = 0, for which $y \ge 1 + x$ means the channel is entanglement-breaking.

Analog protocol— Alice wants to transmit to Bob an unknown state $\rho_{\rm T}$ with first and second statistical moments $v_{\rm T}$ and $\Gamma_{\rm T}$, respectively. She shares with Bob an entangled state $\rho_{\rm AB}$ with covariance matrix (1). The first-moments vector and the covariance matrix of the initial three-mode state are then $v = v_{\rm T} \oplus (0,0)_{\rm AB}^T$ and $\Gamma = \Gamma_{\rm T} \oplus \Gamma_{\rm AB}$. The analog protocol consists of three steps, see Fig. 2: (i) Alice applies an encoding operation to $\rho_{\rm T}$ and her part of the entangled state; (ii) Alice transmits the encoded signal to Bob through the Gaussian communication channel, defined in (4); (iii) Bob applies a decoding operation to retrieve a "noisy" version of $\rho_{\rm T}$.

The difference between the analog and the digital protocol stands in the application of the encoding operation instead of a Bell measurement. This operation is needed to correct, at least partially, the noise added by the communication channel \mathcal{G} . This encoding operation (labeled

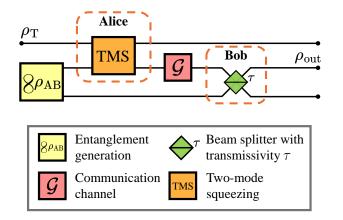


FIG. 2. Analog teleportation scheme. Alice wants to transfer an unknown state $\rho_{\rm T}$ to Bob using as a resource the pre-shared entangled state $\rho_{\rm AB}$. Alice applies a two-mode squeezing operation to her part of the entangled state and $\rho_{\rm T}$. She transmits one of the resulting modes to Bob through the communication channel \mathcal{G} . Finally, Bob performs a decoding operation using his part of the entangled state, obtaining $\rho_{\rm out}$, which is a "noisy" version of the state $\rho_{\rm T}$.

TMS in Fig. 2) is chosen as

$$X_{\rm enc} = \begin{pmatrix} \sqrt{d} \mathbb{I}_2 & \sqrt{d-1}\sigma_z \\ \sqrt{d-1}\sigma_z & \sqrt{d} \mathbb{I}_2 \end{pmatrix} \oplus \mathbb{I}_2 , \quad Y_{\rm enc} = 0, \quad (5)$$

where $d \geq 1$. It corresponds to a two-mode squeezing operation acting only on Alice's modes. Notice that measuring the quadratures x and p on one of the outputs of the encoding operation is equivalent to a Bell measurement if d is infinite. Alice neglects one mode and transmits the other to Bob via the channel (4).

Finally, as decoding, Bob performs a beamsplitter operation on his part of the entangled state and the received state. This decoding operation is defined by the matrices

$$X_{\text{dec}} = \mathbb{I}_2 \oplus \begin{pmatrix} \sqrt{\tau} \mathbb{I}_2 & \sqrt{1-\tau} \mathbb{I}_2 \\ \sqrt{1-\tau} \mathbb{I}_2 & -\sqrt{\tau} \mathbb{I}_2 \end{pmatrix} , \quad Y_{\text{dec}} = 0 , \quad (6)$$

with $0 < \tau \le 1$. This operation can be implemented, e.g., with a directional coupler in a microwave experiment [9, 10, 24]. One of the two outputs will be

$$v_{\text{out}} = gv_{\text{T}}$$
 , $\Gamma_{\text{out}} = g\Gamma_{\text{T}} + G\mathbb{I}_2$, (7)

where $g = dx\tau$ is the total gain of the simulated channel and

$$G = \left(g - \frac{g}{d}\right)a + \left(1 - \frac{g}{dx}\right)b$$
$$-2\sqrt{\left(1 - \frac{g}{dx}\right)\left(g - \frac{g}{d}\right)}c + \frac{gy}{dx} \tag{8}$$

is the total added noise. Notice that, since $0 < \tau \le 1$, we must have $0 < g/dx \le 1$ and, consequently, a squeezing parameter $d \ge \max\{g/x, 1\}$.

In practice, the analog protocol simulates the action of a phase-insensitive Gaussian channel with gain g and noise G onto the state $\rho_{\rm T}$. In the limit of large d, we get $G_{\rm dig}=ga+b-2\sqrt{g}c$, which matches the added noise via the digital protocol [6]. We point out that Eq. (7) can be interpreted as follows: for a given channel gain g, that can be modulated via d and τ , the analog protocol will provide a noise G given in (8).

The optimization of G with respect to the triplet (a,b,c) and the squeezing parameter d is challenging. There are trivial examples, such as the g=x case, for which $G=G_{\rm dig}+(y-G_{\rm dig})/d$. This expression is linear in 1/d and is therefore minimized by d=1 or $d\to\infty$ depending on whether $G_{\rm dig}$ is greater or smaller than y. In this example, the problem reduces to optimize $G_{\rm dig}$. In the following, we strive to obtain a necessary and sufficient condition for a *finite* d to be optimal in a general setting.

Performance analyses— We want to determine the regimes in which an analog protocol with finite squeezing performs better than the digital one in simulating a phase-insensitive channel. Such performance is quantified by the amount of added noise G, i.e., the less noise the better the simulation. In the following, we find a necessary and sufficient condition for G to be minimized at finite d, given a constraint on the logarithmic negativity of the resource state.

An optimization of G_{dig} considering as entanglement resource a state with $E_{\mathcal{N}}(\rho_{AB}) = 2r$ gives the triplet

$$a = \frac{b + e^{-2r}(g - 1)}{g} , \quad c = \frac{b - e^{-2r}}{\sqrt{g}},$$

$$b \ge \frac{e^{2r}g + e^{-2r} - |g - 1|}{g + 1 - e^{2r}|g - 1|} \equiv b^*,$$
 (9)

where the solution holds under the constraint $\tanh(r) \leq g \leq \coth(r)$ [6]. This triplet has been found by first setting $c = \sqrt{(a - e^{-2r})(b - e^{-2r})}$, such that $E_{\mathcal{N}}(\rho_{\mathrm{AB}}) = 2r$; then, by fixing a to have $G_{\mathrm{dig}} = e^{-2r}(1+g) \equiv G_{\mathrm{dig}}^*$, which is the boundary of the channel accessible with the digital protocol, as shown in [6]. Finally, $b \geq b^*$ is needed to satisfy the constraint that ρ_{AB} is physical. Notice that $b = b^*$ corresponds to the optimal ρ_{AB} with minimal energy.

On the one hand, if we fix the entanglement to 2r, Eq. (8) can be written as

$$G = \left[\left(g - \frac{g}{d} \right)^{\frac{1}{2}} (a - e^{-2r})^{\frac{1}{2}} - \left(1 - \frac{g}{dx} \right)^{\frac{1}{2}} (b - e^{-2r})^{\frac{1}{2}} \right]^{2} + \frac{g}{dx} \left[y - e^{-2r} (1 + x) \right] + G_{\text{dig}}^{*}.$$
(10)

Therefore, if $y \ge e^{-2r}(1+x)$ we have $G \ge G_{\text{dig}}^*$ for any choice of a, b, and d, and the digital protocol is optimal.

On the other hand, we can expand G to the first order

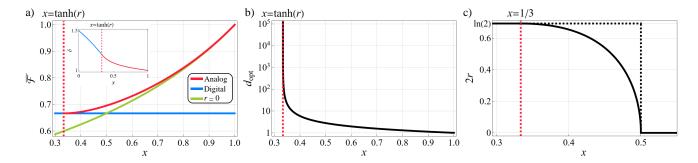


FIG. 3. Teleportation fidelity, optimal encoding operation and no-cloning threshold. a) Plot of the average fidelity $\overline{\mathcal{F}}$ of the analog (red), digital (blue) and entanglement-free (green) protocols optimized for a codebook of uniformly distributed coherent states. Here, we consider a quantum-limited attenuator with transmissivity x as a communication channel, and the triplet $(a,b,c)=(\cosh(2r),\cosh(2r),\sinh(2r))$ optimizing the digital case. In the inset, we plot the ratio between the infidelities $\delta=(1-\overline{\mathcal{F}}_{\rm ef})/(1-\overline{\mathcal{F}}_{\rm an})$, showing the non-trivial advantage of the analog protocol over the entanglement-free case, even in the high-transmissivity regime. For $x\leq \tanh(r)$ (blue), $\overline{\mathcal{F}}_{\rm an}$ is optimized for $d\to\infty$ and matches the digital case. For $x>\tanh(r)$ (red), $\overline{\mathcal{F}}_{\rm an}$ is optimized at $d=[x-\tanh^2(r)]/[x^2-\tanh^2(r)]$, which is finite (see b) plot). In a) and b) we consider $2r=\ln(2)\simeq 0.693$, such that $\overline{\mathcal{F}}_{\rm dig}=2/3$ matches the no-cloning threshold. Changing r results in qualitatively similar plots. c) Plot of the logarithmic negativity 2r needed to reach the no-cloning threshold, as a function of the transmissivity x, for the analog (continuous) and digital (black dashed) protocols.

in 1/d for $d \to \infty$, obtaing

$$G = G_{\text{dig}} + \frac{G'}{d} + o\left(\frac{1}{d}\right), \tag{11}$$

$$G' = -g\left(a + \frac{b}{x}\right) + \frac{c\sqrt{g}(g+x)}{x} + \frac{gy}{x}.$$
 (12)

The condition G' < 0 computed using any triplet in Eq. (9) is sufficient for $G < G_{\text{dig}}^*$ to hold for a finite d. Such condition is equivalent to $y < e^{-2r}(1+x)$.

Main Result. Consider a Gaussian communication channel \mathcal{G} given in (4), with transmissivity x > 0 and added noise y. An analog teleportation protocol with a finite value of encoding squeezing, optimized over the set of entangled resource states with logarithmic negativity 2r, outperforms the optimal digital protocol in the task of simulating Gaussian phase-insensitive channels with gain $g \in [\tanh(r), \coth(r)]$, if and only if $y < e^{-2r}(1+x)$.

Notice that this result can be generalized to communication channels described by Eq. (3) that have a canonical form given in (4). Notice also that our result depends uniquely on the structure of the communication channel. It states that a necessary and sufficient condition for the digital protocol to be optimal is that the communication channel reduces the entanglement of the resource state.

Entaglement-free case— Let us consider the specific analog protocol when there is no pre-shared entanglement between Alice and Bob, that is r=0. We choose the triplet corresponding to a pure state, i.e., (a,b,c)=(1,1,0). This case represents direct state transfer enhanced by local encoding/decoding operations. The encoding operation corresponds to a quantum-limited phase-insensitive amplification of the state $\rho_{\rm T}$. The protocol is implemented by sending the amplified state di-

rectly through the communication channel, obtaining

$$G_{\text{ef}} = 1 + g + \left[\frac{y - (1+x)}{x} \right] \frac{g}{d}.$$
 (13)

This can be optimized over d with the constraint $d \ge \max\{g/x, 1\}$. Since (13) is linear in 1/d, $G_{\rm ef}$ is minimized for $d \to \infty$ if $y \ge 1 + x$ (i.e., the channel is entanglement-breaking), or by $d = \max\{g/x, 1\}$ if y < 1 + x.

Notice that $G_{\rm ef}$ can be lower than $G_{\rm dig}$ if the transmissivity of the communication channel is large enough and the added noise is sufficiently small. In fact, while $G_{\rm dig}$ is limited by the entanglement, $G_{\rm ef}$ is limited by the communication channel parameters. The analog protocol establishes a connection between digital teleportation and the entanglement-free approaches.

Quantum teleportation— Eq. (7) can be directly used for computing the average fidelities of a quantum teleportation protocol. We consider quantum teleportation of a Gaussian distributed codebook of coherent states $|\alpha\rangle$. The fidelity averaged on the codebook can be computed as $\overline{\mathcal{F}} = \int_{\mathbb{C}} \frac{\lambda}{\pi} e^{-\lambda |\alpha|^2} \langle \alpha | \rho_{\text{out}} | \alpha \rangle d^2 \alpha$ with $\lambda > 0$. One easily obtains $\overline{\mathcal{F}} = 2\lambda/[2(1-\sqrt{g})^2 + \lambda(1+g+G)]$ [6]. This expression shall be optimized with respect to g and G. However, it is clear that for a fixed g, a lower G is beneficial and our main result can be directly applied.

For the sake of the discussion, let us consider the $\lambda \to 0$ limit, i.e., the limit of a uniformly distributed codebook. In this case, $\overline{\mathcal{F}}$ is zero for $g \neq 1$, while it is 2/(2+G) for g=1 [6]. Let us focus on the relevant case of the quantum-limited attenuator as a communication channel, defined by Eq. (4) with $0 < x \le 1$ and y=1-x. This channel plays a crucial role in, e.g., superconducting microwave technology, where the communication can be implemented with a cryogenic link [11, 12]. We set

g=1 and consider the triplet in Eq. (9) optimizing $G_{\rm dig}$ with $b=b^*$, i.e., $(a,b,c)=(\cosh(2r),\cosh(2r),\sinh(2r))$. With this choice, the average teleportation fidelity is $\overline{\mathcal{F}}_{\rm dig}=1/(1+e^{-2r})$ for the digital protocol. For the analog protocol optimized over d, we obtain $\overline{\mathcal{F}}_{\rm an}=\overline{\mathcal{F}}_{\rm dig}$ for $x\leq \tanh(r)$ and $\overline{\mathcal{F}}_{\rm an}=2x/[1+x(2-x)-(1-x)^2\cosh(2r)]$ for $x>\tanh(r)$. This shall be compared with the average fidelity of the entanglement-free case, i.e., $\overline{\mathcal{F}}_{\rm ef}=1/(2-x)$. Notice that $\overline{\mathcal{F}}_{\rm ef}\leq\overline{\mathcal{F}}_{\rm an}$ always, and the equality is achieved at x=1. However, $\overline{\mathcal{F}}_{\rm ef}$ is larger than $\overline{\mathcal{F}}_{\rm dig}$ as soon as $x>1-e^{-2r}$.

We plot these fidelities in Fig. 3 for $2r = \ln(2)$, which makes the digital protocol reach the no-cloning threshold $\overline{\mathcal{F}}_{\rm dig} = 2/3$. Fig. 3(a) illustrates how the analog protocol outperforms the digital protocol for a wide range of the attenuation x, while Fig. 3(b) highlights how the optimal value of d is finite in that range. In Fig. 3(c), we show that the analog protocol requires less entanglement than the digital protocol to reach the no-cloning threshold. Here, we identify the 1/3 < x < 1/2 region, where the needed entanglement is $2r = 2 \arcsin \left[\sqrt{x(1-2x)/2(1-x)^2} \right]$, as transitional to the region where entanglement is no longer necessary. In an analog protocol, this transition occurs smoothly, whereas in the digital protocol it is abrupt.

Conclusions— We have found a clear necessary and

sufficient condition for analog teleportation to be optimal in the task of oblivious quantum state transfer through a Gaussian channel. Our result is relevant as it concerns the case of limited entanglement resources and/or limited losses in the communication channel. In this context, we have shown how an analog protocol improves the average teleportation fidelities compared to the digital protocol, and outperforms always the entanglement-free case. From a fundamental perspective, our result defines a classicality criterion for the communication channel in quantum teleportation. It shows that, in this context, such a criterion is not absolute but resource-dependent and is weaker than the entanglement-breaking condition. From the experimental point of view, our results have direct application in, e.g., microwave quantum teleportation [9], where Alice and Bob can be connected by a cryogenic link [11, 12], resulting in a quantum-limited attenuator as the communication channel.

Acknowledgments— U.A. and R.D. acknowledge financial support from the Academy of Finland, grants no. 353832 and 349199. S.F. acknowledges financial support from PNRR MUR project PE0000023-NQSTI financed by the European Union – Next Generation EU, and from CQSense project financed by Fondazione Compagnia di San Paolo.

- [1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an unknown quantum state via dual classical and einstein-podolskyrosen channels, Phys. Rev. Lett. 70, 1895 (1993).
- [2] L. Vaidman, Teleportation of quantum states, Phys. Rev. A 49, 1473 (1994).
- [3] S. L. Braunstein and H. J. Kimble, Teleportation of continuous quantum variables, Phys. Rev. Lett. 80, 869 (1998).
- [4] S. Pirandola, J. Eisert, C. Weedbrook, A. Furusawa, and S. L. Braunstein, Advances in quantum teleportation, Nat. Photonics 9, 641 (2015).
- [5] X.-M. Hu, Y. Guo, B.-H. Liu, C.-F. Li, and G.-C. Guo, Progress in quantum teleportation, Nat. Rev. Phys. 5, 339 (2023).
- [6] P. Liuzzo-Scorpo, A. Mari, V. Giovannetti, and G. Adesso, Optimal continuous variable quantum teleportation with limited resources, Phys. Rev. Lett. 119, 120503 (2017), Erratum available at DOI: 10.1103/Phys-RevLett.120.029904.
- [7] S. Tserkis, J. Dias, and T. C. Ralph, Simulation of gaussian channels via teleportation and error correction of gaussian states, Phys. Rev. A 98, 052335 (2018).
- [8] R. Di Candia, K. Fedorov, L. Zhong, S. Felicetti, E. Menzel, M. Sanz, F. Deppe, A. Marx, R. Gross, and E. Solano, Quantum teleportation of propagating quantum microwaves, EPJ Quantum Technol. 2, 1 (2015).
- [9] K. G. Fedorov, M. Renger, S. Pogorzalek, R. Di Candia, Q. Chen, Y. Nojiri, K. Inomata, Y. Nakamura, M. Partanen, A. Marx, et al., Experimental quantum telepor-

- tation of propagating microwaves, Sci. Adv. 7, eabk0891 (2021).
- [10] B. Abdo, W. Shanks, O. Jinka, J. Rozen, and J. Orcutt, Teleportation and entanglement swapping of continuous quantum variables of microwave radiation, arXiv preprint arXiv:2501.05537 (2025).
- [11] P. Magnard, S. Storz, P. Kurpiers, J. Schär, F. Marxer, J. Lütolf, T. Walter, J.-C. Besse, M. Gabureac, K. Reuer, A. Akin, B. Royer, A. Blais, and A. Wallraff, Microwave quantum link between superconducting circuits housed in spatially separated cryogenic systems, Phys. Rev. Lett. 125, 260502 (2020).
- [12] W. Yam, M. Renger, S. Gandorfer, F. Fesquet, M. Handschuh, K. Honasoge, F. Kronowetter, Y. Nojiri, M. Partanen, M. Pfeiffer, et al., Cryogenic microwave link for quantum local area networks, arXiv preprint arXiv:2308.12398 (2023).
- [13] J. Eisert, K. Jacobs, P. Papadopoulos, and M. B. Plenio, Optimal local implementation of nonlocal quantum gates, Phys. Rev. A 62, 052317 (2000).
- [14] S. Bravyi, O. Dial, J. M. Gambetta, D. Gil, and Z. Nazario, The future of quantum computing with superconducting qubits, J. App. Phys. 132 (2022).
- [15] V. C. Usenko, A. Acín, R. Alléaume, U. L. Andersen, E. Diamanti, T. Gehring, A. A. Hajomer, F. Kanitschar, C. Pacher, S. Pirandola, et al., Continuous-variable quantum communication, arXiv preprint arXiv:2501.12801 (2025).
- [16] A. Serafini, Quantum Continuous Variables: A Primer of Theoretical Methods (CRC Press, 2017).

- [17] E. Menzel, R. Di Candia, F. Deppe, P. Eder, L. Zhong, M. Ihmig, M. Haeberlein, A. Baust, E. Hoffmann, D. Ballester, et al., Path entanglement of continuousvariable quantum microwaves, Phys. Rev. Lett. 109, 250502 (2012).
- [18] E. Flurin, N. Roch, F. Mallet, M. H. Devoret, and B. Huard, Generating entangled microwave radiation over two transmission lines, Phys. Rev. Lett. 109, 183901 (2012).
- [19] S. Pogorzalek, K. Fedorov, M. Xu, A. Parra-Rodriguez, M. Sanz, M. Fischer, E. Xie, K. Inomata, Y. Nakamura, E. Solano, et al., Secure quantum remote state preparation of squeezed microwave states, Nat. Commun. 10, 2604 (2019).
- [20] G. Vidal and R. F. Werner, Computable measure of entanglement, Phys. Rev. A 65, 032314 (2002).
- [21] A. Serafini, F. Illuminati, and S. D. Siena, Symplectic

- invariants, entropic measures and correlations of gaussian states, J. Phys. B: At. Mol. Opt. Phys. **37**, L21 (2003).
- [22] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. 84, 621 (2012).
- [23] Referring to Table I of Ref. [22], we do not consider quantum channels in the classes A_1 and A_2 as they are degenerate. The class B_1 can be treated as a phase-insensitive channel by adding noise in the x quadrature. Classes B_2 and \mathcal{C} are of the form given in Eq. (4). Class D acts as a noisy complex conjugation, which can be corrected at the encoding stage in a noisy way.
- [24] K. G. Fedorov, L. Zhong, S. Pogorzalek, P. Eder, M. Fischer, J. Goetz, E. Xie, F. Wulschner, K. Inomata, T. Yamamoto, Y. Nakamura, R. Di Candia, U. Las Heras, M. Sanz, E. Solano, E. P. Menzel, F. Deppe, A. Marx, and R. Gross, Displacement of propagating squeezed microwave states, Phys. Rev. Lett. 117, 020502 (2016).