GENERATION AND TELEPORTATION OF THREE AND FOUR PARTICLE W STATE

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In this paper, we introduced circuits for three- and four-particle quantum systems to generate W states with any arbitrary coefficients and phases. Subsequently, each qubit was transmitted separately through a four-qubit entangled channel. Before transmission, the sender performed pre-processing on their qubits to minimize the resources required for transmission. Additionally, the receiver applied post-processing using the ancilla qubit(s) to recover the final states. To further improve efficiency, it is preferable to implement the protocol in a bidirectional manner, as this allows the unknown qubits initially held by the users to be utilized ancilla qubit(s). Finally, we compared our protocol with similar works and validated the correctness of the protocol by simulating it using Qiskit, a tool provided by IBM.

 $\textit{Keywords} \hbox{: } \text{bidirectional quantum teleportation;} W \hbox{ state entangled generation }.$

1. Introduction

With advancements in quantum technologies, there is a growing need to evolve quantum communication protocols. Recently,in the field of quantum broadcasting 1,2, quantum secret sharing³ And one of the most important of them is quantum teleportation.

Quantum teleportation is a protocol for transferring the quantum state of one or multiple separate or entangled qubits without physically transmitting them. It was first proposed theoretically in 1993 by Bennett and his colleagues 4 . The first experimental implementation was carried out in 1997 by Anton Zeilinger using optical devices 5 . Following this practical realization, extensive research has been conducted to advance quantum protocols further.

A bidirectional protocol in which two qubits are transmitted simultaneously between two users,⁶ a bidirectional protocol that allows the transmission of an arbitrary number of qubits between two users⁷ and also, the channel is noisy,⁸Performance Analysis of Hardware-Efficient Algorithms in Noisy

Intermediate-Scale Quantum Devices⁹ ,an arbitrary number of qubits in the GHZ state,¹⁰ a controlled asymmetric protocol where two qubits are sent from one side and three qubits are sent from the other side¹¹ ,bidirectional quantum teleportation of an arbitrary number of qubits over a noisy quantum channel,¹² an asymmetric three-party protocol¹³ ,Quantum teleportation with distributed gates,¹⁴ a controlled protocol with the ability to change the receiver¹⁵ and also, a bidirectional controlled protocol for an arbitrary number of qubits transmitted through a multi-hop network.¹⁶

Quantum teleportation is performed through an entangled channel. W states have been used as a channel in numerous studies. The standard W quantum state for an arbitrary number of n qubits is represented as $|W_n\rangle = \frac{1}{\sqrt{n}}\left(|100\ldots0\rangle + |010\ldots0\rangle + \cdots + |000\ldots1\rangle\right)$ Articles 17–19 have proposed schematics for teleporting an unknown qubit through a W state, examines the effect of noise on teleportation when using GHZ or W states²⁰, multi-hop teleportation protocols based on W states and EPR pairs. 21, 22 a protocol for teleporting an arbitrary and unknown two-qubit state from a sender to one of two receivers using W state and GHZ state.²³, transmits two qubits through channels based on pseudo-GHZ and W states.²⁴

Numerous articles have explored the generation of W states on various platforms, including superconducting platforms 25–27, optical platforms 28–30, and trapped ions 31–33. Also, the theoretical construction of the GHZ state³⁴ And the detector of entangled state using machine learning³⁵ has been introduced. In the second section of this paper, we generated the W state for three-particle systems, and in the third section, the W state for four-particle systems using quantum circuits, In the fourth section, we addressed the teleportation of the W state for three-particle systems, and in the fifth section, we focused on the teleportation of the W state for four-particle systems, to increase efficiency in these protocols, we employed data processing techniques. This processing is particularly useful for multi-hop protocols, as it minimizes the quantum and classical resources required at the repeater stage, in section six, we demonstrated that the efficiency further increases if the protocol is bidirectional. This is because qubits that were initially in the W state and later transformed to the $|0\rangle$ state after processing can be used as ancilla qubits In the final section, we compare the efficiency of the protocols introduced in this paper with similar works.

2. Generation of a Three-Particle W State

Initially, the first qubit is initialized using the $\text{Ry}(\theta_0)$ and $\text{Rz}(\varphi_0)$ gates to bring it to the $|\psi\rangle = \left(\cos\left(\frac{\theta_0}{2}\right)|0\rangle - e^{i\varphi_0}\sin\left(\frac{\theta_0}{2}\right)|1\rangle\right)|00\rangle$ state, Next, a controlled-Ry(θ_1) gate is applied between the first(as the controller) and second(as the target) qubits to create a linear superposition of three states, a Rz(φ_1) gate is then applied to introduce a phase on the second qubit, allowing for arbitrary phases to be applied to all states, $|\psi\rangle = \cos\left(\frac{\theta_0}{2}\right)|00\rangle - e^{i\varphi_0}\sin\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\theta_1}{2}\right)|10\rangle +$

 $e^{i(\varphi_0+\varphi_1)}\sin\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\theta_1}{2}\right)|11\rangle$ then, the CNOT (second qubit, third qubit), CNOT (first qubit, second qubit), and X (first qubit) gates are applied as shown in Fig.1 to obtain the final state:

$$|\psi\rangle = \alpha_0|100\rangle + \alpha_1|010\rangle + \alpha_2|001\rangle \tag{1}$$

where the Eq. (1); $\alpha_0 = \cos\left(\frac{\theta_0}{2}\right), \alpha_1 = -e^{i\varphi_0}\sin\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\theta_1}{2}\right), \alpha_2 = e^{i(\varphi_0 + \varphi_1)}\sin\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\theta_1}{2}\right)$ Additionally, the coefficients are normalized $|\alpha_0|^2 + |\alpha_1|^2 + e^{i(\varphi_0 + \varphi_1)}\sin\left(\frac{\theta_1}{2}\right)$ $|\alpha_2|^2 = 1$

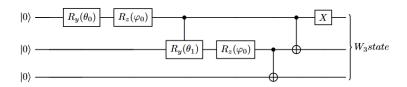


Fig. 1. Circuit for generating the three-particle W state

3. Generation of a Four-Particle W State

The state of the system after applying the $Ry(\theta_0)$, $Rz(\varphi_0)$, $Ry(\theta_1)$ and $Rz(\varphi_1)$ gates to initialize the first and second qubits will be as follows: $|\psi\rangle$ $\left(\cos\left(\frac{\theta_0}{2}\right)|0\rangle - e^{i\varphi_0}\sin\left(\frac{\theta_0}{2}\right)|1\rangle\right)\left(\cos\left(\frac{\theta_1}{2}\right)|0\rangle - e^{i\varphi_1}\sin\left(\frac{\theta_1}{2}\right)|1\rangle\right)|00\rangle$ Subsequently, by applying the gates shown in Fig.2 , we arrive at the Eq. 2

$$|\psi\rangle = \beta_0|0010\rangle + \beta_1|0100\rangle + \beta_2|1000\rangle + \beta_3|0001\rangle$$
 (2)

where in the relation, $\beta_0 = \cos\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\theta_1}{2}\right), \ \beta_1 = -e^{i\varphi_1}\cos\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\theta_1}{2}\right), \ \beta_2 = -e^{i\varphi_0}\sin\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\theta_1}{2}\right), \ \beta_3 = e^{i(\varphi_0+\varphi_1)}\sin\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\theta_1}{2}\right)$ And also, the coefficients are normalized $|\beta_0|^2 + |\beta_1|^2 + |\beta_2|^2 + |\beta_3|^2 = 1$

4. Teleportation of a Three-Particle W State

Suppose Alice has the quantum state $|\phi\rangle_{A_0A_1A_3}=\alpha_0|100\rangle+\alpha_1|010\rangle+\alpha_2|001\rangle$ and wants to transmit this quantum state to Bob, To achieve this, Alice first performs a pre-processing with the X(first qubit), CNOT (first qubit, second qubit), CNOT (second qubit, third qubit) gates on her qubits to assign the unknown coefficients to her first and second qubits, sending only those two. The third qubit will be set to $|0\rangle$ and will be disregarded. Thus, Alice's quantum state becomes as follows: $|\phi'\rangle_{A_0A_1} = (\alpha_0|00\rangle + \alpha_1|10\rangle + \alpha_2|11\rangle$

4 Generation and Teleportation of three and four particle W state

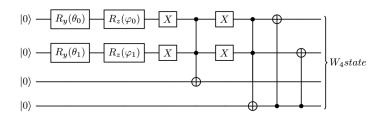


Fig. 2. Circuit for generating the four-particle W state

For the quantum channel, each user shares two qubits (a_0, a_1) by Alice and b_0, b_1 by Bob), preparing the qubits in the quantum $|\omega\rangle_{\mathbf{a}_0\mathbf{b}_0\mathbf{a}_1\mathbf{b}_1} = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)$

Thus, the entire system $|\phi'\rangle \otimes |\omega\rangle$ can be expressed as Eq. (3).

$$|\psi\rangle_{A_0A_1a_0a_1b_0b_1} = \frac{1}{2} (\alpha_0|000000\rangle + \alpha_0|000101\rangle + \alpha_0|001010\rangle + \alpha_0|001111\rangle + \alpha_1|100000\rangle + \alpha_1|100101\rangle + \alpha_1|101010\rangle + \alpha_1|1011111\rangle + \alpha_2|110000\rangle + \alpha_2|110101\rangle + \alpha_2|111010\rangle + \alpha_2|1111111\rangle)$$
(3)

In the next step, Alice applies the $CNOT(A_0, a_0)$ and $CNOT(A_1, a_1)$ gates, and the system state will become as follows Eq. (4).

$$|\psi'\rangle_{A_0A_1a_0a_1b_0b_1} = \frac{1}{2}(\alpha_0|000000\rangle + \alpha_0|000101\rangle + \alpha_0|001010\rangle + \alpha_0|001111\rangle + \alpha_1|101000\rangle + \alpha_1|101101\rangle + \alpha_1|100010\rangle + \alpha_1|100111\rangle + \alpha_2|111100\rangle + \alpha_2|111001\rangle + \alpha_2|110110\rangle + \alpha_2|110011\rangle)$$
(4)

After applying the operators in Table 1, Bob's quantum state becomes $|\psi''\rangle_{b_0b_1}=\alpha_0|00\rangle+\alpha_1|10\rangle+\alpha_2|11\rangle$. Then, using an ancilla qubit, Bob, through post-processing on his qubits, which includes the X(first qubit), CNOT(first qubit, second qubit), and CNOT (second qubit, ancilla qubit) gates, reconstructs Alice's initial quantum state on his own qubits. $|\psi'''\rangle_{b_0b_1b_{ancilla}}=\alpha_0|100\rangle+\alpha_1|010\rangle+\alpha_2|001\rangle$ The schematic of this protocol is shown in Fig.3.

5. Teleportation of a Four-Particle W State

Consider the unkhown quantum Four-Particle W State $|\phi\rangle_{B_0B_1B_2B_3} = \beta_0|0010\rangle + \beta_1|0100\rangle + \beta_2|1000\rangle + \beta_3|0001\rangle$ on Bob's qubits.Initially, a pre-processing step is performed by Bob on the unknown qubits, including the CNOT(fourth qubit , second qubit),CNOT(fourth qubit , firest qubit).TOFFOLI(firest qubit , second qubit , fourth qubit),X(firest qubit),X(second qubit),TOFFOLI(firest qubit , second qubit , third qubit),X(firest qubit),X(second qubit) gates. After this pre-processing, two of the unknown qubits are reset to $|0\rangle$. Now, Bob only needs to send the

Alice's results	Bob's states	Bob's operators
$\mathbf{A}_0\mathbf{A}_1\mathbf{a}_0\mathbf{a}_1$	$\mathbf{b}_0\mathbf{b}_1$	
+ + 00	$\alpha_0 00\rangle + \alpha_1 10\rangle + \alpha_2 11\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_I\otimes\sigma_I)$
+ + 01	$\alpha_0 01\rangle + \alpha_1 11\rangle + \alpha_2 10\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_I\otimes\sigma_x)$
+ + 10	$\alpha_0 10\rangle + \alpha_1 00\rangle + \alpha_2 01\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_x\otimes\sigma_I)$
+ + 11	$\alpha_0 11\rangle + \alpha_1 01\rangle + \alpha_2 00\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_x\otimes\sigma_x)$
+ - 00	$\alpha_0 00\rangle + \alpha_1 10\rangle - \alpha_2 11\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_I\otimes\sigma_I)$
+ - 01	$\alpha_0 01\rangle + \alpha_1 11\rangle - \alpha_2 10\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_I\otimes\sigma_x)$
+ - 10	$\alpha_0 10\rangle + \alpha_1 00\rangle - \alpha_2 01\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_x\otimes\sigma_I)$
+ - 11	$\alpha_0 11\rangle + \alpha_1 01\rangle - \alpha_2 00\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_x\otimes\sigma_x)$
- + 00	$\alpha_0 00\rangle - \alpha_1 10\rangle - \alpha_2 11\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_I\otimes\sigma_I)$
- + 01	$\alpha_0 01\rangle - \alpha_1 11\rangle - \alpha_2 10\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_I\otimes\sigma_x)$
- + 10	$\alpha_0 10\rangle - \alpha_1 00\rangle - \alpha_2 01\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_x\otimes\sigma_I)$
- + 11	$\alpha_0 11\rangle - \alpha_1 01\rangle - \alpha_2 00\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_x\otimes\sigma_x)$
00	$\alpha_0 00\rangle - \alpha_1 10\rangle + \alpha_2 11\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_I\otimes\sigma_I)$
01	$\alpha_0 01\rangle - \alpha_1 11\rangle + \alpha_2 10\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_I\otimes\sigma_x)$
10	$\alpha_0 10\rangle - \alpha_1 00\rangle + \alpha_2 01\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_x\otimes\sigma_I)$
11	$\alpha_0 11\rangle - \alpha_1 01\rangle + \alpha_2 00\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_x\otimes\sigma_x)$

Table 1. Bob's states and operators for the system are determined based on Alice's measurements

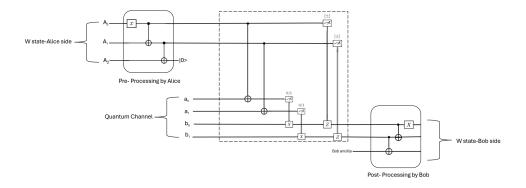


Fig. 3. Teleportaion protocol for 3 particle W state

remaining two qubits through the $|\omega\rangle_{b_0b_1a_0a_1}$ channel , this channel is no different from the channel in the previous section. The general state of the system can be expressed according to Eq. (5).

$$|\psi\rangle_{B_0B_1b_0b_1a_0a_1} = \frac{1}{2} (\beta_0|000000\rangle + \beta_0|000101\rangle + \beta_0|001010\rangle + \beta_0|001111\rangle + \beta_1|010000\rangle + \beta_1|010101\rangle + \beta_1|011010\rangle + \beta_1|011111\rangle + \beta_2|100000\rangle + \beta_2|100101\rangle + \beta_2|101010\rangle + \beta_2|101111\rangle + \beta_3|110000\rangle + \beta_3|110101\rangle + \beta_3|111010\rangle + \beta_3|111111\rangle)$$
(5)

In the next step, Bob applies the $CNOT(B_0,b_0)$ and $CNOT(B_1,b_1)$ gates, and the system state will become as follows Eq. (6).

$$|\psi'\rangle_{\mathbf{B}_{0}\mathbf{B}_{1}\mathbf{b}_{0}\mathbf{b}_{1}\mathbf{a}_{0}\mathbf{a}_{1}} = \frac{1}{2} \left(\beta_{0}|000000\rangle + \beta_{0}|000101\rangle + \beta_{0}|001010\rangle + \beta_{0}|001011\rangle + \beta_{1}|010111\rangle + \beta_{1}|011010\rangle + \beta_{1}|011010\rangle + \beta_{1}|011011\rangle + \beta_{2}|101001\rangle + \beta_{2}|101101\rangle + \beta_{2}|100010\rangle + \beta_{2}|100111\rangle + \beta_{3}|111100\rangle + \beta_{3}|111100\rangle + \beta_{3}|110110\rangle + \beta_{3}|110011\rangle \right)$$
(6)

then, Bob performs X basis measurements (Hadamard basis) on her qubits B_0 and B_1 , and Z basis measurements (standard basis) on the qubits of her channel, Table 2 shows the system's collapsed states and the unitary operators that Alice must apply to reconstruct her qubits.

Bob's results	Aice's states	Alice's operators
$\mathbf{B}_0\mathbf{B}_1\mathbf{b}_0\mathbf{b}_1$	$\mathbf{a}_0\mathbf{a}_1$	
+ + 00	$\beta_0 00\rangle + \beta_1 01\rangle + \beta_2 10\rangle + \beta_3 11\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_I\otimes\sigma_I)$
+ + 01	$\beta_0 01\rangle + \beta_1 00\rangle + \beta_2 11\rangle + \beta_3 10\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_I\otimes\sigma_x)$
+ + 10	$\beta_0 10\rangle + \beta_1 11\rangle + \beta_2 00\rangle + \beta_3 01\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_x\otimes\sigma_I)$
+ + 11	$\beta_0 11\rangle + \beta_1 10\rangle + \beta_2 01\rangle + \beta_3 00\rangle$	$(\sigma_I\otimes\sigma_I)(\sigma_x\otimes\sigma_x)$
+ - 00	$\beta_0 00\rangle - \beta_1 01\rangle + \beta_2 10\rangle - \beta_3 11\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_I\otimes\sigma_I)$
+ - 01	$\beta_0 01\rangle - \beta_1 00\rangle + \beta_2 11\rangle - \beta_3 10\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_I\otimes\sigma_x)$
+ - 10	$\beta_0 10\rangle - \beta_1 11\rangle + \beta_2 00\rangle - \beta_3 01\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_x\otimes\sigma_I)$
+ - 11	$\beta_0 11\rangle - \beta_1 10\rangle + \beta_2 01\rangle - \beta_3 00\rangle$	$(\sigma_I\otimes\sigma_z)(\sigma_x\otimes\sigma_x)$
- + 00	$\beta_0 00\rangle + \beta_1 01\rangle - \beta_2 10\rangle - \beta_3 11\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_I\otimes\sigma_I)$
- + 01	$\beta_0 01\rangle + \beta_1 00\rangle - \beta_2 11\rangle - \beta_3 10\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_I\otimes\sigma_x)$
- + 10	$\beta_0 10\rangle + \beta_1 11\rangle - \beta_2 00\rangle - \beta_3 01\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_x\otimes\sigma_I)$
- + 11	$\beta_0 11\rangle + \beta_1 10\rangle - \beta_2 01\rangle - \beta_3 00\rangle$	$(\sigma_z\otimes\sigma_I)(\sigma_x\otimes\sigma_x)$
00	$\beta_0 00\rangle - \beta_1 01\rangle - \beta_2 10\rangle + \beta_3 11\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_I\otimes\sigma_I)$
01	$\beta_0 01\rangle - \beta_1 00\rangle - \beta_2 11\rangle + \beta_3 10\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_I\otimes\sigma_x)$
10	$\beta_0 10\rangle - \beta_1 11\rangle - \beta_2 00\rangle + \beta_3 01\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_x\otimes\sigma_I)$
11	$\beta_0 11\rangle - \beta_1 10\rangle - \beta_2 01\rangle + \beta_3 00\rangle$	$(\sigma_z\otimes\sigma_z)(\sigma_x\otimes\sigma_x)$

Table 2. Bob's states and operators for the system are determined based on Alice's measurements

After reconstructing the qubits, Alice performs post-processing on his qubits using two ancilla qubits to construct the final state, as shown in Fig.4.

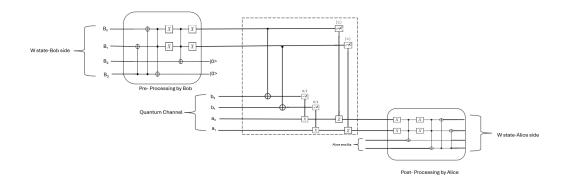


Fig. 4. Teleportaion protocol for 4 particle W state

6. Bidirectional teleportation of three and four particle W state

Alice wants to send the $|\psi\rangle_{A_0A_1A_3} = \alpha_0|100\rangle + \alpha_1|010\rangle + \alpha_2|001\rangle$ state to bob . After performing the pre-processing introduced in Section four, her state transforms into $|\psi\rangle_{A_0A_1A_3} = (\alpha_0|00\rangle + \alpha_1|10\rangle + \alpha_2|11\rangle)|0\rangle$. On the other side, Bob has the $|\phi\rangle_{B_0B_1B_2B_3} = \beta_0|0010\rangle + \beta_1|0100\rangle + \beta_2|1000\rangle + \beta_3|0001\rangle$ state and simultaneously applies the preprocessing introduced in Section five to his qubits to transform into the $|\phi\rangle_{B_0B_1B_2B_3} = (\beta_0|00\rangle + \beta_1|01\rangle + \beta_2|10\rangle + \beta_3|11\rangle)|00\rangle$

For the quantum channel, Alice and Bob each share four qubits, the $a_0a_1a_2a_3$ belong to Alice ,while the $b_0b_1b_2b_3$ qubits belong to Bob, And sequentially, the CNOT gates is applied between Alice's qubits as the control and Bob's qubits as the target, so that the channel state can be expressed as $|\omega\rangle_{a_0a_1a_2a_3b_0b_1b_2b_3}=\frac{1}{4}\sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\sum_{l=0}^{1}|ijkl\rangle|ijkl\rangle$

In the next step, Alice applies the $CNOT(A_0, a_0)$, $CNOT(A_1, a_1)$ And Bob applies the $CNOT(B_0, b_2)$, $CNOT(B_1, b_3)$ And they also perform measurements in the X basis on $A_0A_1B_0B_1$ And measure the $a_0a_1b_2b_3$ qubits in the Z basis. Then, based on Bob's results and regardless of her own measurement outcomes, Alice can refer to Table 2 And apply the unitary operators on the a_2a_3 qubits, Bob also simultaneously refers to Table 1 and, based on Alice's results, applies the unitary operators to the b_0b_1 . After applying the unitary gates, Alice uses the A_2 qubit and an ancilla qubit, while Bob uses the B_2 qubit to reconstruct the qubits through post-processing. The schematic of the above protocol is shown in Fig.5.

7. Conclusion

Based on what is stated in article³⁶, equation $\eta = \frac{q_s}{q_u + b_t + q_a}$ is used to compare the efficiency between several protocols. In this equation, q_s represents the number of qubits transmitted, q_u denotes quantum resources, b_t represents classical resources, and q_a corresponds to ancilla qubits. In Table 3, the efficiency of articles 6, 37–39 has been calculated and compared with the efficiency of the protocols presented in 8 Generation and Teleportation of three and four particle W state

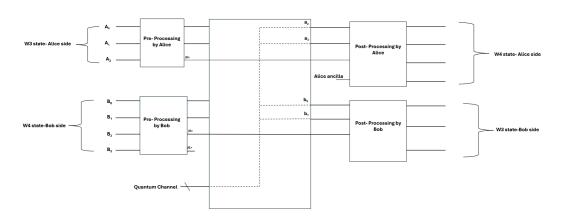


Fig. 5. schematic of bidirectional teleportation

this paper, demonstrating that our proposed protocols achieve higher efficiency.

Protocol	Transmitted Qubits	Quantum Channel	Classical Bits	Efficiency
37	three-qubit state	two four-qubit cluster states	8	$\frac{3}{16} \simeq 18.7\%$
38	three-particle W state	seven-qubit cluster state	7	$\frac{3}{14} \simeq 21.4\%$
39	one-two GHZ state (bidirectional)	five-qubit Cluster State	5	$\frac{3}{10} = 30\%$
6	two-two qubit state (bidirectional)	eight-Qubit Entangled State	8	$\frac{4}{16} = 25\%$
This paper	three-particle W state	four qubit cluster state + 1 ancilla qubit	4	$\frac{3}{9} \simeq 33.3\%$
This paper	four- particle W state	four qubit cluster state+ 2 ancilla qubits	4	$\frac{4}{10} = 40\%$
This paper	three-four particle W state(bidirectional)	eight qubit cluster state+ 1 ancilla qubit	4	$\frac{7}{17} \simeq 41.1\%$

Table 3. Comparison of different protocols with our protocols in efficiency

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