Quality of Service in quantum networks

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Tomorrow's quantum networks will allow one to connect multiple users scattered around the world. The large distances separating the users as well as limited physical resources will require using different paths to exchange information. The resources can be then distributed over channels of different length and transmitted to the end's users. Optimizing the distribution of such resources over those channels is then a fundamental issue that must be considered to achieve efficient long-distance quantum communication. One can then think of a routing mechanism that assigns the available channels to the users across which they can share unencoded and encoded information. Here we explore the assignment issue of a router in a quantum aggregation scenario, in which the users exchange unencoded information. Then we consider that the information is encoded using an error correction code. We give a simple example for both cases with whom we estimate the fidelities of several different configurations.

I. INTRODUCTION

Large scale quantum networks will connect multiple users distributed over distant locations by establishing entangled states between them [1–3]. Entanglement swapping operations performed at the middle nodes will allow then to connect further nodes aiming at the creation of a global quantum internet [4]. This will open up a completely new scenario offering users the possibility of utilizing several quantum technologies that will exceed the performance of the current classical technologies. Connecting distributed quantum processors over the nodes in the network will enhance the computational power of the resulting quantum computer [5–12]. Then, connecting users across remote nodes will enable them to exchange information in a much more secure way [5, 13-17]. Further, linking several nodes will increase the accuracy and precision of quantum measurements used in quantum clocks and quantum remote sensing [18–20].

Although profoundly different, quantum networks will share many similarities with the existing classical networks (especially as the quantum networks must be supported by conventional telecommunication networks). In particular both current networks and future quantum networks have a restricted number of physical resources available at each node potentially leading to congestion. This aspect might be quite detrimental for the optimal functioning of the network reducing the rate at which users exchange information or lowering the quality of the exchanged states. To tackle this issue in classical networks, a set of methods and technologies are used to manage the traffic of information to ensure high performance of applications with limited network capacity [21–23]. They are called the Quality of Service (QoS) and measures four specific features of a network: the bandwidth (the speed at which users exchange information); the loss (the amount of data lost); the delay (the time for a packet of information to go from its source to its destination) and the jitter (the irregular speed of the transmitted information resulting in a random arrival of the packets across a network). Given that a resource constrained scenario will also be a fundamental issue in quantum communication technologies, future quantum networks must consider a quantum version of QoS to guarantee an optimal performance of those technologies. Transposing the QoS into the quantum world must include all the differences compared to the classical world. For instance, one big difference is that the delay of a quantum packet will require the storage of such a state into quantum memories. Here, memory relaxation will affect the stored state fidelity, leading to information loss (unlike what happens with classical packets).

Here, we focus on selecting a few features of a quantum network that might pave the way for a future in-depth analysis. To this goal we model a quantum network in which two distant nodes containing multiple users and with a limited number of communication channels want to exchange quantum information with a certain fidelity. Selecting the fidelity as main figure of merit might be seen as a limitation in the use of the network, however, the transmitted quantum state must retain a certain quality to achieve higher performance when compared to classical technologies. We also assume that these two remote nodes are connected by several paths of different length each containing a certain number of lossy channels over which the quantum state travels. To protect the physical state from losses one can think of using a quantum error correction code encoding the state in an higher dimensional system adding more qubits or qudits [24–31]. In this scenario, quantum aggregation [32] allows one to distribute the encoded state over different paths to optimize the fidelity of the transmitted state. In quantum aggregation the quantum packets used to encode the initial state are distributed and sent over different channels. At the receiver's node the encoded packets are then recombined and decoded to retrieve the initial information.

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Assigning the channels in which the information travels to reach the far end node is therefore a crucial step for satisfying the requests of the users. One can consider a routing mechanism that distributes the encoded state over multiple channels upon specific requests made by the users. It is then fundamental to design a router protocol that optimize the assignment of the channels to the users in the sender node. The QoS might be an useful tool to consider for such an optimization and therefore it is natural to define the features of the quantum networks QoS.

Our manuscript is arranged as follows: in Section II we define the main features of the QoS when applied to a quantum network. Then in Section III we devise a quantum router protocol for the optimal assignment of the channels connecting to remote nodes. In Section IV we show an example of assignment of the channels when quantum aggregation is in use and we describe the effects of temporal delay on the assignment of the selected channels. We conclude in Section V.

II. QUALITY OF SERVICE OF A QUANTUM NETWORK

Let us now transpose the features of QoS defined in the Introduction to a quantum network beginning with some basic definitions. In general, a network can be represented by a graph, which is a set of vertices connected by edges, as shown in Fig. 1. We will refer to the ver-



Figure 1. A generic quantum network can be represented by a set of vertices (nodes) connected by edges (links). Each link can contain multiple channels. The set of links that connects two nodes is a path.

tices of a quantum network as nodes and to its edges as links. We call the link the physical or logical connection between two nodes. Then, the channel identifies the physical medium being used. More specifically a channel refers either to a physical transmission medium or to a logical one over a multiplexed medium. For instance, a channel can be a generic optical fiber or free space connection. Since multiple channels can be embedded into a link, we define the capacity of a link as the sum of the capacities of the channels of that link. Then a path

refers to the sequence of links and devices that a packet of data must traverse to reach its destination. Hence, the length of a path, L, is the sum of the lengths of all its links. In aggregated quantum networks, the information is distributed over multiple paths that differ in length and all connect a sender and a receiver, making, thus, an aggregated path. In this scenario some features of the QoS can have a slight different definition.

Now let us define the features of QoS of quantum networks and outline their main features.

- First the bandwidth of a path as the smallest capacity of its links. Similarly, the bandwidth for an aggregated path is the sum of the lowest capacities of all those paths. The bandwidth, hence, informs the users on the largest amount of information that can be sent across that path per unit of time when unencoded states are transmitted. Alternatively, when the information is encoded, the bandwidth can be associated with the highest quality of a transmitted quantum data packet.
- Second, the loss of information transmitted across noisy channels can be measured in several ways. Here, we adopt the infidelity of the transmitted state, 1-F, where F is the fidelity of the state received and processed at the receiver's node. The infidelity might depend on several errors, such as the channel loss, parametrized by the transmission coefficient $p = \eta e^{-L/L_{att}}$, where L_{att} is the attenuation length that depends on the type of channel in use, and η is a loss term due to interactions with other components of the network, such as quantum memories (QMs) and frequency converters. In a quantum aggregated network the infidelity depends also on the decoherence time of the QMs used to store the quantum states at the receiver's node [33].
- Third the delay, τ_P , of a path is defined as the time interval between the time the quantum data packets are sent by the sender node and the time they arrive at the receiver node. It is given by $\tau_P = L/c + t_c$, where L is the length of the path, c is the speed of light traveling in the medium (optical fiber or free space) while t_c is the congestion time. The latter is the extra time due to the waiting time some packets might require at the intermediate nodes and the processing time. In fact, one can think that when the physical resources travel across multiple nodes before reaching the far end, some of those intermediate nodes might be not be immediately accessible or could take some time to process the quantum data packet. Therefore, some delay line must be added before the resources can pass through that node. This time can be then considered as the sum of all those extra delays. This definition of delay can be generalized by including more temporal terms due to the queuing times. [34]

• Fourth the *jitter* is defined as the time difference of the arrival times of the information packets. Whereas in classical networks such a time difference will cause a delay in the information flow, in quantum networks that information must be stored into QMs. Therefore, the stored states will undergo a dephasing process that will deteriorate the fidelity of the final state, resulting in a loss of information. Here we have assumed the jitter is negligible. However, in a real implementation the jitter can be different from zero and, in this case, must be considered.

Based on the above quantum network definitions of QoS, we can now qualitatively estimate the requirements of a few quantum applications, as shown in Table I. In a

	Bandwidth	Loss	Delay	Jitter
QKD	high	medium	low	low
MA-QKD	medium	low	low	low
Purification	medium	medium	medium	medium
Error correction	high	medium	high	high
Quantum	low	medium	medium	medium
teleportation				
Quantum	medium	high	medium	medium
remote sensing				
Distributed quantum	high	medium	high	high
computation				

Table I. QOS feature requirements for different applications.

conventional quantum key distribution (QKD) protocol the rate at which two users can share a secret key is the main figure of merit. Therefore, the bandwidth plays a fundamental role and it must be high. Similarly, for distributing quantum computing high bandwidth is necessary to connect multiple nodes faster. For memory assisted (MA) QKD and conventional purification protocols the bandwidth can be slightly lower, as quantum memories store the quantum states increasing the probability of successfully sharing information. Also, for quantum remote sensing, we need to generate complex states like GHZ states, which need higher quality (fidelity). A lower bandwidth is required for quantum teleportation since a single entangled state is sufficient for its realization. Then, a small delay and the jitter for QKD protocols do not affect the efficiency of these systems since a delayed secret key and an irregular transmission of the arriving packets do not compromise the security of the shared key. A medium requirement on both the delay and jitter is, on the other hand, needed for purification, teleportation and remote sensing as storing entangled states might affect the quality of the final states due to decoherence processes in the QMs. Distributing quantum computing is highly affected by delay and jitter as it is fundamental that the packets arrive regularly and at precise times at each node to be processed correctly. Finally, the loss of information has a low impact on a MA-QKD protocol because the states are stored into QMs before extracting a secret key.

It has a medium effect on conventional QKD systems as it might reduce the secret key rate. Analogously for purification the loss of information has a medium impact on the efficiency of this protocol because several entangled states need to be created to increase the fidelity of the final state. Then, for distributing quantum computation the quantum error correction codes in use can restore the loss of information with some probability, having, thus, a medium impact on these systems. On the contrary, losses have a more detrimental effect on the sensitivity of a sensing protocol and can move us away from the Heisenberg limit [35]. This is especially true as the kind of resource states we are generating can be completely destroying if a single qubit is lost.

Having defined a quantum network QoS, a natural question now arises: what is the best way of assigning channels between a sender and a receiver node with limited resources? In the next Section we give an answer to this question by introducing the routing protocol.

III. THE ROUTING PROTOCOL

Let us now describe our routing protocol for a simple wide area network (WAN) as illustrated in Fig. 2. We

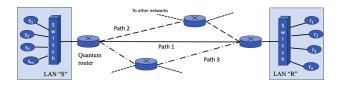


Figure 2. Schematic illustration of a wide area network (WAN) with quantum routers. Here two local area networks (LANs) are connected by three paths represented by a solid line (path 1), dashed lines (path 2) and dash-dotted lines (path 3) that different in length each containing an arbitrary number of channels, over which the information can be transmitted from LAN "S" to LAN "R". A quantum router assigns the channels over which distribute the quantum packets according to a specific regime of operation based on the users' requests or denies the requests if the resources are not sufficient to meet their expectations. Each path might contain multiple routers that connect other networks.

assume here that two local area networks (LANs), LAN "S" (the sender) and LAN "R" (the receiver) with m users in S and n users in R, respectively, are both connected to a quantum router. These routers are, in turn, connected by paths of different delay containing an arbitrary number of channels over which information is transmitted. Although in a real network scenario multiple paths can be established between two remote LANs, in the scheme of Fig. 2 three paths are shown, represented by the solid line (path 1), dashed lines (path 2) and dash-dotted lines (path 3). The goal is allocating the available resources

contained in the LAN "S" among the various channels connecting the two nodes. To this goal one can think of a routing mechanism that performs the following protocol. Initially the users in the LAN "S" make requests to exchange information with the users in the LAN "R" giving specific requirements. For instance, they might require to obtain a desired threshold fidelity of the information received in the LAN "R" or to exchange information at a certain rate. All these requests are input into the quantum router (request step), which will determine whether the available resources in both LANs are sufficient to satisfy the expected requests (processing step). If such requests cannot be met by the current capabilities of the nodes then the router will hold the requests adding them to a queue list for a certain number of time steps. Otherwise, if the capabilities of the quantum network are in general insufficient to satisfy the user's request then the quantum router will send a denial signal to inform the users that the communication cannot be established and the quantum router waits for the next input. In the other case, the quantum router will move to the assignment step, in which it will distribute the resources over the available channels. Let us now examine these steps in more details.

In the request step the main figure of merit is assessed. Upon an input request, firstly the quantum router determines whether some sort of error protection must be provided to the state to be transmitted to guarantee the desired fidelity. In the affirmative case, the quantum router then determines in which regime of operation it needs to operate. There might be several regimes of operations based on the requirements the users demand. For instance, the quantum router can prioritize a single user or a small number of users, limiting, thus, the access to the paths to the majority of the users. Alternatively the quantum router can ensure that most of the users can use the available paths. Multiple scenarios can develop but in the next Section we will focus on three main scenarios leaving a more exhaustive analysis for a future work.

After the request step has finished the router moves to the processing step. Here the router analyze the available resources in the LAN "S", the coherence times of the QMs in the LAN "R" and the paths connecting both LANs to assign the channels to the users in the LAN "S" under a specific regime of operation. This step is crucial for optimizing the performance of the protocol and we give a simple example on how this can be achieved in the next Section. Finally, after the processing step, the quantum router will inform the switch device to connect the users to the network to send their data. Then the quantum router is initialized to start a new cycle.

IV. ROUTING ASSIGNMENT IN THE UNENCODED SCENARIO

Selecting a specific channel assignment might optimize the performance of the network. The features of QoS

contribute to such a selection. In the following we focus on two features of QoS: the bandwidth and the loss. We assume that the channels have limited capacity and some information is lost during the transmission or deteriorated when stored into QMs. In this section we give an example of how a router can assign multiple channels between two distant nodes, S and R, both containing a variable number of users that want to exchange some information. We assume that S and R are connected by by N_c channels with $N_c/2$ channels contained into a path with delay τ_1 and the remaining $N_c/2$ channels in a path with delay τ_2 , with $\tau_2 > \tau_1$. Let us also further assume that in node S a router assigns to the users some channels, across which they send their state to node R. A natural question is: what is the optimal assignment? To answer this question one can think of three possible scenarios. In scenario a, called the greedy regime, a single user aims at maximizing the quality (fidelity defined as the scalar product of the final state with the initial state) of their transmitted state. In scenario b, we aim to satisfy the requirements of as many users as possible (balanced regime) and, in scenario c, a restricted number of users aim at maximizing their fidelities under the condition that the difference between them is small (restricted regime). Depending on the selected regime the router will assign the available channels accordingly. The question is now: what is the best assignment strategy in those three regimes? In this Section we show with an example what is the best assignment when users send a certain number of unencoded packets. Let us consider that $N_c = 10$ and the capacity of each channel can transmit a qudit of dimension 7 per time unit. Let us also assume that user send five qudits and, for simplicity, that $\eta_1 = \eta_2 = 1$ and $t_{c_1} = t_{c_2} = 0$. Now, Table II contains all the possible assignments of the channels of path 1 and path 2 among N_u users, with N_u the highest number of users that can be served given the above conditions. For

Assign	ment 1	Assign		
path 1	path 2	path 1		N_u
5	0	0	5	2
4	1	1	4	2
3	2	2	3	2
10	10	0	0	4

Table II. Assignments of the number of qudits traveling in two paths in the unencoded network scenario. Each row corresponds to a possible assignment of channels among N_u users. In red the configurations affected by decoherence occurring in the QMs located at the R node. The QMs are used to store the packets arriving earlier. These stored quantum states must wait for the quantum packets traveling in the longer delay path to be processed. In the first three rows two users are sending qudits of dimension 7 whereas in the last row up to four users can send qudits of dimension 3.

instance, the first line gives the number of qudits traveling in each path for two assignments given to two users. In this case, with the assignment 1 a single user can send a packet of five qudits of dimension 7 along the channels of path 1 while another user sends their five qudits of dimension 7 across the channels of path 2. This configuration corresponds to the greedy regime, in which the router assigns the channels with the lowest loss to a single user leaving the channels in path 2 to the other user. Therefore, the fidelity of the "greedy" user is the highest reachable with the available resources and with $p_1 = 0.95$ and no decoherence, as shown by the solid line in Fig. 3. The fidelity of the packets sent by the other user, on the other hand, is the lowest, as show by the asterisk markers in Fig. 3. Next, in the restricted regime the router as-

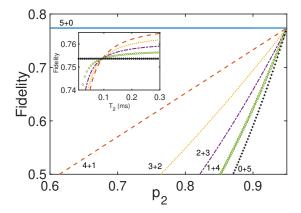


Figure 3. Fidelities of the packets at the receiver node versus the transmission probability of path 2, p_2 , corresponding to the assignments of qudits of Table II at $p_1=0.9$ and $T_2\to\infty$. The label of each curve refers to the number of qudits sent through path 1 and path 2, corresponding to the assignments of Table II. The inset shows the fidelities of the packets at the receiver node versus the coherence time, T_2 , at $p_1=0.95$ and $p_2=0.945$.

signs the channels to the users aiming at minimizing the difference of their fidelities. This condition is met by the dotted line and the dash-dotted line. Then, the balanced regime can be one of the three configurations listed in Table II as the highest number of users that can use the network is two in this case.

It is now interesting to see the effects of a finite coherence time in such a scenario. The configurations, highlighted in red in Table II, require quantum memories at the receiver's node due to the fact that the paths have different delay (length in this case) and, therefore, the receiver node must store the quantum states arriving earlier before the total packet can be processed. However, at low coherence times, the decoherence might affect a configuration with the majority of the qudits traveling in the shorter path much more than a configuration with a larger number of qudits traveling in the longer path, as shown in the inset of Fig. 3. One can see that at $T_2 < 0.1 \text{ ms}$ and at $p_2 = 0.945$ the fidelity of the transmitted packet corresponding to the assignment 4+1 (3+2) is higher than the fidelity corresponding to the assignment 1+4 (2+3). For these configurations it is then fundamental that the router is informed with the coherence time of the QMs at the receiver node to optimally assign the channels to the users. Remarkably, the inset shows that the fidelities of those configurations all cross with the 0+5 configuration at $T_2^* = 0.1$ ms. This means that for $T_2 < T_2^*$ the aggregation scenario should not be used for a possible router's assignment. Finally, the regimes defined above are not affected by finite coherence times.

V. ROUTING ASSIGNMENT IN THE AGGREGATION SCENARIO

In this Section we show with an example how a router assigns the channels between two nodes by selecting between a conventional network in which the information is encoded and transmitted across one path or an aggregated network in which the encoded information is distributed over multiple paths that differ in delay.

Let us consider two distant nodes, S and R, exchanging information using the quantum Reed-Solomon (QRS) code [36] in a quantum aggregation scenario. Similarly to the unencoded case, we assume here that the number of users in S is a variable and node S and R are connected by path 1 and path 2. For a fixed transmission probability p_1 of the channels in path 1, the best assignment depends on the transmission probability of the channels in path 2 and on the coherence time of the QMs at node R. Consider that $N_c = 10$ and the capacity of each channel can transmit a qudit of dimension 7 per time unit. For simplicity, we also assume that $\eta_1 = \eta_2 = 1$ and $t_{c_1} = t_{c_2} = 0$. Now, Table III contains all the possible assignments of

Assign	ment 1	Assignment 2		Assign		
path 1	path 2	path 1	path 2	path 1	path 2	N_u
5	2	0	3	0	0	3
4	3	1	2	0	0	3
3	4	2	1	0	0	3
2	5	3	0	0	0	3
5	0	0	5	2	1	3
4	1	1	4	2	1	3
3	2	2	3	2	1	3
3	0	2	1	0	3	6
3	0	1	2	1	2	6
2	1	2	1	1	2	6

Table III. Assignments of the number of qudits traveling in two paths paths in the aggregation scenario. Each row corresponds to a possible assignment of channels among N_u users. In red the configurations affected by decoherence occurring in the QMs located at the R node. The QMs are needed when the number of qudits arrived in R in the shorter path is not enough to decode the state.

the channels of path 1 and path 2 among N_u users, with N_u the highest number of users that can be served given the above conditions. The first line gives the number of

qudits traveling in each path for two assignments. In assignment 1 a single user encodes their information with a 7 dimensional QRS code, with 5(2) qudits of dimension 7 traveling in path 1(2), respectively. Assignment 2 allows two users to encode their information with a 3 dimensional QRS code by sending 3 qutrits across path 2. In this case, the highest capacity have been achieved by the transmitted qudits therefore no more assignments are possible. We define the degeneracy of an assignment as the highest number of users that can encode their state with the quantum packets of that specific assignment. In the example above, the degeneracy of assignment 1 is 1 whereas the degeneracy of assignment 2 is 2 as two users can encode their state with a 3 dimensional QRS code. For simplicity, for the assignment i $(i \in \mathbb{N}^*)$ with degeneracy higher than 1 we refer to user i as a single user that represents that assignment.

Now we analyze into more details the various assignments and to which regimes they can be associated. Firstly, due to the different length of the two paths some assignments (red lines in the table) might be affected by a temporal delay as shown in [33]. This can change quite drastically the desired fidelity depending on the coherence time, T_2 , of the quantum memories in use. Another factor to be considered is the length of path 2. Let us start our analysis with the asymptotic case $T_2 \to \infty$ (ideal memories). Figure 4(a) shows the fidelities of the transmitted states of user 1 and user 2, respectively, for the first four configurations of Table III. Here the curves of the same colors correspond to the fidelities of each single row and the solid (dashed) curves are the fidelities of the transmitted states of user 1 (2), respectively, for a transmission coefficient of path 1 $p_1 = 0.9$. One can see that, when more qudits are traveling in the shorter channel (red and blue curves), user 1 can always obtain better fidelities than user 2 whereas when more qudits are traveling in path 2 (purple and yellow curves) the advantage of user 1 is only for certain values of p_2 (crossing points between curves of the same color). This can be explained by the fact that in the aggregation scenario the fidelities of the configurations in which most of the packets are traveling in the longer channels strongly depend on the length of this path. The crossing points of Fig. 4(a) can be used to determine the optimal distribution of quantum packets in the restricted regime in which we tend to minimizing the difference of fidelity between the users.

Next Fig. 4(b) shows the fidelities of the transmitted state for the corresponding configurations of Table III. In this case the curves do not cross because the number of qudits is equal for both users with user 1 utilizing more channels in the shorter path than user 2. The results in Fig. 4(b) can be used for the greedy regime by selecting the communication path with the highest fidelity and for the restricted regime by analyzing the plot of the difference of the fidelities. For simplicity, to determine the optimal configuration for the restricted regime we plot in Figure 5 only the difference of the fidelities of the transmitted states of user 1 and user 2 of the 3+2 and

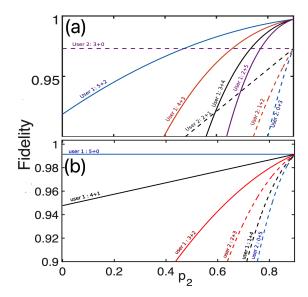


Figure 4. Fidelities of the decoded states at the receiver node versus the transmission probability of path 2, p_2 corresponding to the assignments of qudits of Table III at $p_1 = 0.9$ and $T_2 \to \infty$. The curves of the same colour correspond to each row of the Table III. The solid (dashed) curves refer to user 1 (2), respectively. One can use the fidelities in (a) to determine the greedy regime because some configurations use the highest dimension of the qudits, the restricted regime because the fidelities of some configurations cross (minimizing, thus, their difference) and the balanced regime because the there are configurations with the lowest number of qudits. Next (b) shows the fidelities that can be used to determine the restricted and the the greedy regime. In fact, the restricted regime can be given by those configurations whose fidelities are very similar, and the greedy regime can be given by those configurations that do not depend on the coherence time.

2+3 configuration (red curve), 4+3 and 1+2 configuration (blue curve) and 3+4 and 2+1 configuration (black curve) versus p_2 . As one can see the graph can be divided into three parts for certain range of p_2 . Specifically for $0.65 < p_2 < 0.71$ the configurations that minimize the difference between the fidelities of user 1 and user 2 correspond to distributing the resource according to 3+2 the configuration for user 1 and 2+3 for user 2 (red curve). Next, for $0.71 < p_2 < 0.8$ the black curve minimizes the difference of the fidelities corresponding to the 3+4 configuration for user 1 and 2+1 for user 2. Finally at $0.8 < p_2$ the smallest difference of fidelities is given by the 4+3 (1+2) configuration for user 1 (2) respectively.

Further, the fidelities of the balanced regime corresponding correspond to the communication paths in the last three rows of table. For the amount of resources considered 6 users can be served as each assignment has degeneracy 2. Even for this regime sub cases can exist in which users might want to minimize the difference of their fidelities (fair balanced) or assign most of the resources traveling in the shorter channel to one user (unfair balanced regime).

Next we consider the case in which the router selects

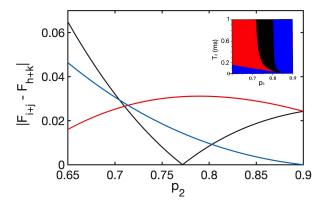


Figure 5. Difference of the fidelities of the transmitted states in which i, j, k and k correspond to the following configurations: 3+4 and 2+1 (red curve), 3+2 and 2+3 (blue curve), and 2+5 and 3+0 (black curve). In the restricted regime the router will then assign the 3+4 and 2+1 configuration (red curve) for $0.65 < p_2 < 0.71$, the 2+5 and 3+0 configuration (black curve) for $0.71 < p_2 < 0.81$, and the 4+3 and 1+2 configuration (blue curve) for $p_2 > 0.81$. Inset: optimal configurations with coherence time, T_2 .

the best configuration in the greedy regime. For instance, if user 1 requires the highest fidelity, the router will select the router will assign to user 1 with most of the resources traveling along path 1. To this aim we then compare the fidelity of the configuration 5+2 with the fidelity of the configuration 5+0 as shown in Fig. 6. Interestingly, the

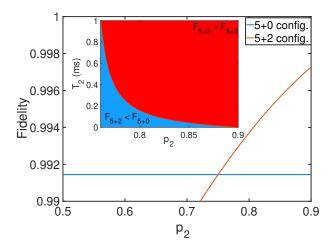


Figure 6. Fidelity of the transmitted state corresponding to the 5+2 configuration (red curve) and 5+0 configuration (blue line). In the inset we show the optimal configurations with coherence time, T_2 .

advantage of using higher dimensional code is achieved for $p_2 > 0.75$. For lower values of p_2 it is more advantageous to encode the state in a lower dimensional code. This might seem a bit counterintuitive because the 7 dimensional QRS code can tolerate the loss of three qudits while the 5 dimensional code can tolerate the loss of only 2 qudits. Therefore, one might think that at $p_2 = 0$,

i.e., when the two qudits traveling in path 2 are lost, the fidelity of the encoded state using the 7 dimensional code is equal to the fidelity of the 5 dimensional code. This not the case because when the two qudits in path 2 are lost the 7 dimensional code can tolerate the loss of one qudit traveling in path 1 (or, obviously, zero qudit). Therefore the contribution to the fidelity in this case are proportional to p_1^5 (no loss in path 1) and $p_1^4(1-p_1)$ (1 qudit loss in path 1). On the other hand, when the state has been encoded with a 5 dimensional QRS code with all qudits traveling in path 1, the fidelity will also have a contribution proportional to $p_1^3(1-p_1)^2$ added to the above contributions. These results are obtained for ideal QMs at the receiver's node, in which the coherence time is infinite.

A. The effect of finite coherence of QMs

Let us now consider the case in which the QMs at the receiver's node are not perfect and have a finite coherence time T_2 . This might drastically change the optimal assignment made the router as illustrated in the following example. Let us assume we are in the greedy regime. in which a single user demands that its transmitted state reach the highest fidelity with the available resources. Looking at Fig. 6 one notice that at $p_2 < 0.72$ the configuration 5+0 is the most advantageous. We want to determine then for which values of T_2 the configuration 5+2 has a better performance than the configuration 5+0for $p_2 > 0.75$. The inset of fig. 6 shows the red(blue) area delimited by the coherence time T_2 and by p_2 to obtain a fidelity of the transmitted state higher(lower) in the 5+2 configuration compared to the 5+0 configuration. At $T_2 > 1$ ms the asymptotic regime is achieved and the coherence time only slightly affects the fidelity in the 5+2configuration. However, at $T_2 < 1$ ms one can see that higher values of p_2 are needed to guarantee that the 5+2configuration gives an advantage compared to the 5+0 configuration.

Similarly the inset of Fig. 5 shows that the coherence time plays a relevant role in assigning the channels in the restricted regime as well. Firstly one can notice that at $T_2=1$ ms the asymptotic regime is achieved as well as in the greedy regime. Next, at lower values of $T_2>0.2$ ms, the 3+4 and 2+1 configuration (red area) have more similar fidelities than the 2+5 and 3+0 configuration (black area). It is remarkable to notice that at $T_2<0.2$ ms the 3+2 and 2+3 have much more similar fidelities than all the other configurations under consideration.

These results highlight the fact that the coherence time might play a big role in determining the optimal assignment of channels, therefore it is crucial that the router is informed of the coherence time of QMs used at the remote end to meet the expectations of the users.

VI. CONCLUSION

In this work we introduce the features of the classical QoS for a quantum network and an aggregated quantum network. Based on this features we qualitatively describe their requirements on a few quantum applications. We then introduce a routing protocol for the optimal assignment of channels connecting two remote nodes with multiple users. Next we give an example of an optimal assignment based on three specific regimes of operation in two cases. In the first case, the users exchange information by sending multiple unencoded quantum states. In this case, the coherence time of the QMs does not effect the optimal assignment in the three regimes. However, for a specific example, we find a threshold value of the coherence time below which using quantum aggregation is not convenient. In the second case, we show that, when users exchange information using the quantum Reed-Solomon code, the optimal assignments strongly depend on the transmission probability of the channels as well as on the coherence time of the quantum memories at the receiver's node. From this analysis we estimate the values of the coherence time in which decoherence does not affect the quality of the final quantum state for the greedy and the restricted regime. In these regimes, we obtain that for a coherence time higher than 1 ms the fidelity of the final state is only affected by the transmission probability of the second path given that the sender and the receiver node are connected by a shorter path of length 2.32 km ($p_1 = 0.9$), assuming that the qudits travel in optical fibers with 0.2 dB losses. Our results then give a quantitative analysis on which quantum memories can be used for such a network. For instance, current quantum memories based on ion-qubit [37], superconductive cavity [38] and ensemble-based quantum memories [39] can be used.

In this work we show that the bandwidth and the loss are two important features of the QoS and allows us to determine the optimal assignment in a small aggregated network. A more detailed analysis including the delay and the jitter might enhance the performance of such a network with a different optimal channel assignment. Therefore, we believe that the QoS for quantum networks introduce in this work is an important figure of merit that can be used to optimize the performance of future quantum networks.

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