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Gravitational lensing: towards combining the multi-messengers

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The next generation of gravitational wave detectors and electromagnetic telescopes are beckoning the onset of the multi-messenger era and the exciting science that lies ahead. Multi-messenger strong gravitational lensing will help probe some of the most important questions of the Universe in an unprecedented manner. In particular, understanding the nature of gravitational wave sources, the underlying physical processes and mechanisms that produce emissions well before or right until the time of the merger, their associations to the seemingly distinct populations of gamma ray bursts, fast radio bursts and kilonovae. Not to mention, multi-messenger lensing will offer unique probes of test of gravity models and constraints on cosmological parameters complementary to other probes. Enabling multimessenger science calls for concerted follow-up efforts and development of new and shared resources required in the community.

1. Introduction:

We are currently in the era where two of the important predictions of Einstein's theory of General relativity, Gravitational lensing and Gravitational waves, have not only been observationally confirmed but are being routinely discovered, studied and hold great promise for the next few decades.

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Gravitational lensing is the phenomenon, due to the gravitational potential of a massive object, which causes deflection of the light rays coming from distant galaxies. In strong lensing, multiple, distorted, (de-)magnified images of the distant background source can be seen under suitable conditions. Since the light rays, corresponding to the multiple images, traverse different geometric path lengths and experience different gravitational lens potential, the observer sees a delay in the arrival times of these light rays. The multiple images thus appear with some time delays between them. For general theory and review, see e.g. [1–3].

Gravitational waves (GW) have enabled a novel way to study another dimension of the Universe. Over the last decade, the GW detectors have discovered numerous GW events arising from compact binary coalescence systems, namely, binary black holes (BBH), binary neutron stars (BNS) or neutron star - black holes (NSBH) e.g. [4–7]. Lensing of Gravitational waves was hypothesised and the corresponding theory presented long before even the confirmation of Gravitational waves e.g. [8–10]. The last few years has seen dedicated development and implementation of search algorithms to find lensed GW signals in data taken from the LIGO–Virgo–Kagra collaboration e.g. [11–18].

The detection of GW170817 is a remarkable moment in astrophysics, demonstrating the impact of multi-messenger astronomy in its truest sense [19,20]. It is the first observation of gravitational wave from a merging BNS. Besides, the signal was rapidly followed up by associated electromagnetic (EM) signatures across a wide spectrum, including gamma-ray bursts (GRBs), X-rays, ultraviolet, optical, infrared and radio waves e.g. [21–25]. Such multifaceted astrophysical information provided insights into the physics of neutron stars e.g. [26], the speed of gravitational waves e.g. [27], and presented a new technique for measuring the expansion rate of the universe e.g. [28].

This article attempts to summarise and highlight the importance of lensing in the multi-messenger studies that are either triggered by GW discoveries or help in better understanding sources that will produce GW.

2. Multi-messenger science with lensing

There are numerous applications of combining the multi-messenger observations which would be impossible otherwise. In this section, we briefly describe a few such applications that have been explored and stated in the literature.

(a) Cosmology

The standard cosmological model fits plethora of observations very well although there are still a few outstanding problems to which there are no clear explanations which leaves scope for alternate models to be viable. For instance, the nature of dark matter. Lensing due to low-mass sub-galactic population of objects referred to as microlensing (lensing deflections $\sim \mathcal{O}(10^{-6})$ arcsec) and millilensing (lensing deflections $\sim \mathcal{O}(10^{-3})$ arcsec) arising from stars, stellar remnants, primordial black holes and dark matter clumps (or sub-halos) are ideal as the different cosmological models tend to make more distinct predictions for the properties of dark matter at smaller masses and smaller spatial scales e.g. [29–33].

Another interesting problem is the tension in the measurement of the Hubble constant (H_0) arising from differences in the constraints from the early Universe probes, for instance, cosmic microwave background measured by the Planck satellite $(H_0=67.4\pm0.5~{\rm km/s/Mpc}, [34])$ versus the late Universe probes such as the distance ladder calibrated using Cepheids $(H_0=73.0\pm1.0~{\rm km/s/Mpc}, [35])$. [36] proposed the idea that the measurement of time delays between the multiply lensed images of a distant supernova could help in constraining the Hubble Constant. Historically, time-delay measurements of lensed quasars have emerged as a powerful cosmological probe. Studies, for example, from the TDCOSMO collaboration [37–40] report an H_0 determination with precision at the level of $\sim 2\%$, for example, $H_0=65^{+23}_{-14}~{\rm km/s/Mpc}$.

These measurements are consistent with early-Universe probes but amplify the tension with late-Universe values. Lensed supernovae (SNe) have proved intractable to be discovered and be used for cosmology, owing to their low rate of occurrence coupled with lack of suitable telescope surveys and observations, until very recently e.g., [41,42] although this is expected to change with upcoming surveys like Rubin-LSST e.g., [43,44].

Since lensed GW events will have extremely high precision in the time delay measurements compared to optically lensed quasars and lensed SNe, [45] noted that cosmological constraints from lensed GW combined with EM data will be far superior. Similarly, lensed FRBs are also expected to be excellent probes for high precision H0 measurements [46]. The time delays between images in lensed GW and FRB systems can be measured with more precision and accuracy, often reaching the millisecond range, compared to the days (or hours, at best) for lensed quasars and supernovae. Furthermore, certain systematics such as the uncertainty in the determination of the relative Fermat potential 1 , $\delta\Delta\psi=(\psi_{i_{\rm true}}-\psi_{i_{\rm mod}})-(\psi_{j_{\rm true}}-\psi_{j_{\rm mod}})$, at the location of a pair of lensed images (i,j) are proposed to be smaller which could lead to sub-percent precision on H_0 e.g., [45].

Systematic investigations of actual uncertainties in the relative Fermat potential for optical images, similar to the resolution of the Hubble Space Telescope, are carried out (Ali & More 2024, in prep.). Preliminary analyses indicate that the lens systems containing two images (i.e. doubles) have smaller uncertainties compared those containing four images (i.e. quads) as shown in Fig. 1. Also, among quads, certain configurations give much better accuracy in $\delta \Delta \psi$ across different combinations of image pairs. These results are tested and valid in idealised scenarios and are consistent with the uncertainties stated in [45] but are expected to increase substantially in realistic noise conditions.

Strongly lensed point sources are almost always going to be affected by microlensing by stars and stellar remnant population embedded in the strong lens galaxy [47]. Microlensing due to stellar and stellar remnant population embedded in the strong lensing galaxy will introduce modulations in the GW signals [48]. Both parameter estimation of GW signals (e.g. the luminosity distance, chirp masses and effective spins) and detection of strongly lensed GW signals may get affected due to strong distortions produced by microlensing [49]. Also, the fraction of strong lenses in which severe microlensing (strong lensing magnification > 10) are expected to be present are about 50% for the current GW detector sensitivities which will decrease for next generation detectors [50]. Thus, if the lens models fail to account for the microlensing component, these may ultimately lead to certain systematics in the inference of the cosmological parameters.

One interesting possibility to consider is that the microlensing effects from the same stellar (remnant) population will influence the GW source as well as its EM counterpart, say the kilonova. However, their imprints or observable effects will be different because, at GW frequencies, the wave effects (interference and diffraction) will cause frequency-dependent microlensing distortions whereas, at optical frequencies, geometric-optics limit will apply and produce a constant magnification due to microlensing for each of the strongly lensed image. Therefore, joint analysis, combining the data from two messengers, will give unique constraints on the microlens population. This can further be combined with imaging of the lensed host galaxy (unaffected by microlensing) to obtain accurate lens mass models which is essential for doing precision cosmology.

(b) Test of General Relativity

Properties such as graviton mass, speed, polarisation and propagation of GW are some of the tests of general relativity. Constraints on these properties have been placed with the help of GW signals from compact binary coalescences. However, in presence of electromagnetic counterparts

¹The Fermat potential allows us to determine the arrival times of light rays or signals from a distant source which are affected due to the presence of an intervening gravitational potential. It depends on the extra geometrical path traversed by the light rays or signals and the lensing potential experienced by them.

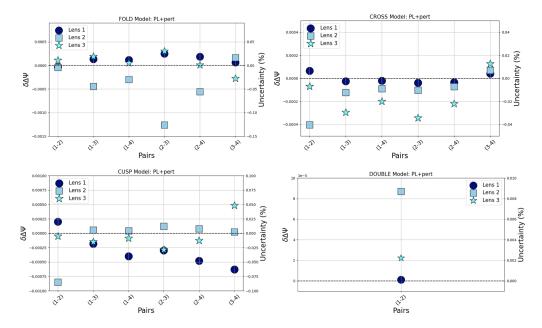


Figure 1. The uncertainties in $\Delta\psi$ shown for different combinations of pairs of lensed images for four lensed image configurations (3-Quads: Fold, Cross and Cusp and 1-Double). In a fold and a cusp configuration, two and three of the four images, respectively, are nearly merging with each other whereas in a cross configuration, all of the four images are almost equidistant, similar to the ends of a cross symbol. Three lenses (Lens 1-3) are randomly generated to capture variations across different lenses per image configuration. The simulated lenses are assumed to follow a power-law mass density profile and a contribution from external shear or perturbation (PL+pert). The same model is assumed when finding the best-fit model although all of the parameters are kept free. In the subsequent iteration, this assumption will be relaxed which will lead to a further increase in the uncertainties (see Ali & More 2025, in prep. for details).

to GW sources and with lensing, further interesting studies with improved constraints have been proposed, as discussed below.

(i) Speed of Gravitational waves

General relativity predicts that light and gravitational waves travel at the same speed, take same path and experience the same Shapiro-time delay.

If a GW source with an EM counterpart is strongly lensed by a foreground galaxy, then the time delays between multiple GW events can readily be compared with the time delays between their respective multiple EM images e.g. [51,52]. By measuring the times of appearance of a pair of images from, say, EM observations and knowing the time of the first event in GW, the time of the second GW event can be predicted accurately. If the prediction does not match the observations then it may indicate a violation of GR. Such a method will bypass the need of any knowledge of the intrinsic time delays between the GW source and its EM counterpart.

It is noted though that the arrival times of GW compared to their EM counterparts may differ for lenses at $M < 10^5 M_{\odot}$ without violating general relativity [53]. This happens because of the longer wavelengths of GW signals compared to the deflections produced by the low mass lenses wherein wave optics effects such as diffraction and interference occur and the signal does not experience a Shapiro time delay. For the EM counterpart, however, one can use geometric optics approximation wherein the multiple images are formed at the stationary points following Fermat's principle and the images experience a Shapiro time delay. In such circumstances, there will be a difference in the $\Delta t_{\rm GW}$ compared to $\Delta t_{\rm EM}$ although when dealing with typical strong lensing effects by foreground galaxies (masses much larger than $10^5 M_{\odot}$), both the GW and EM

signals can be treated under geometric optics approximation and any differences seen in the Δt between GW and EM can be attributed to departure from General relativity.

(ii) Modified Gravitational wave propagation

During the propagation of the GW signals over large cosmological distances, their amplitudes may get attenuated by a different manner unlike the expectation in GR for the amplitudes to be inversely proportional to the scale factor. As a result, the luminosity distance measured via the GW observations ($D_{\rm L,GW}$) of the source may not remain consistent with its luminosity distance measured via electromagnetic observations ($D_{\rm L,EM}$). Thus, for alternate theories of gravity, $D_{\rm L,GW}$ may show differences from $D_{\rm L,EM}$. For instance, $D_{\rm L,GW} = D_{\rm L,EM} F(\theta)$ where the $F(\theta)$ may depend both on the parameters associated with deviation from GR models and the cosmological parameters e.g. [54,55].

Suppose Δ denotes the relative difference in the luminosity distances as $|D_{\rm L,GW}-D_{\rm L,EM}|/D_{\rm L,EM}$. The dependence of Δ on the parameters corresponding to three non-GR gravity models and on the redshift are shown in Fig. 2 taken from [55]. These models are i) large extra spatial dimensions (D) ii) a model that captures different propagation effects corresponding to various theories of gravity alternative to general relativity (Ξ) and iii) a running Planck mass (c_M) . Further details of the parametrization can be understood from [55]. For reference, GW170817 is shown with a green vertical line. At such low redshifts, the parameters need to deviate substantially to be able to discriminate various models.

Whereas if the GW signals are strongly lensed then the lensing magnification will enable detection of sources from much higher redshifts. For instance, the lensed BBH signals will typically originate from redshifts much higher than z=1-2. Such lensed signals might be better probes of the modified GW propagation as they can probe cosmologically large distances. Since the various models, in Fig. 2, show larger differences in the predictions at higher redshifts, the lensed BBH may have more discriminatory power in terms of constraining different gravity models. Given that BBHs are less likely to have direct EM counterparts, it might prove observationally challenging to use BBH for this proposed methodology which requires existence of detectable multi-messenger signals.

Contrarily, the detectable lensed BNSs, even though rarer than the lensed BBHs, will likely have extremely high magnifications e.g. [56,57], making them also high-redshift detections compared to the unlensed BNS population. Furthermore, the lensed BNSs are more likely to have detections in the EM domain provided suitably deep, high angular resolution and prompt follow-up observations are carried out e.g. [56]. Given these arguments lensed BNSs seem much more promising to pursue for the studies of modified GW propagation.

(c) Understanding the nature and physical mechanisms of the Sources

The EM sources such as the short Gamma Ray Bursts (sGRBs) [58], Kilonovae (KNe) [59] and Fast Radio Bursts (FRBs) (still speculative [60]) are thought to be associated with mergers of binary compact objects comprising one or both components as neutron stars. If such events, or strongly lensed images of such events, are detected, then one can learn about the i) nature of the progenitors responsible for the emission in the multi-messenger domain ii) time delay in the emission between the different messengers (e.g. GW vs GRBs/FRBs/Kilonovae) can give us insights into the physical processes and their causal connections iii) once the associated host galaxy is identified, detailed investigations of the transient-host relation, their angular separations and global properties of the host such as the star formation rate, metallicity and stellar masses can be conducted.

(i) Gamma Ray Bursts

GRBs are some of the most energetic and luminous objects observed in the universe. In the prompt phase, gamma-rays are produced from the collimated relativistic outflow that pushes through the

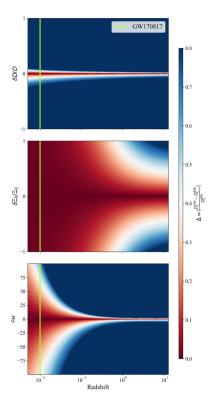


Figure 2. The trends in Δ as a function of parameters of three different non-GR gravity models (top, middle and bottom) and redshift. The blue shaded regions indicate larger differences in Δ as compared to red shaded regions and is dominantly seen at higher redshifts for the parameters of most of the alternate models. Some model parameters versus redshift show stronger contrast in the range of Δ at all redshifts (top panel). Figure taken from [55].

interstellar medium and is powered by a central engine. It is followed by an afterglow phase, detectable right from the X-ray to radio wavelengths, arising from both the expanding outflow as well as the continuous energy injection by the central engine (see reviews e.g. [61,62]). Short GRBs (sGRBs), traditionally defined as bursts lasting ≤ 2 seconds, are associated with compact binary mergers (BNS or NSBH). This classification has evolved though, as events like GRB 211211A and GRB 230307A, lasting over 10 seconds, have been robustly linked to neutron star mergers ([63–68]). However, for simplicity, this analysis focuses on sGRBs with durations < 2 seconds.

Lensing of GRBs helps unveil the internal structure and mechanics of jets, as well as the composition and dynamics of ejected materials [9]. Gravitational lensing amplifies and replicates GRB light curves, uncovering subtle characteristics such as internal shocks, energy distributions, and jet dynamics that would otherwise remain hidden. It also offers a unique opportunity to test high-energy astrophysics theories [69]. Furthermore, since the lensed GRBs are detected from much higher redshifts, it gives us insights into the early universe's evolution, star formation rates, and rate of occurrence of GRBs e.g. [70].

(ii) Kilonovae

KNe were long hypothesized to be the thermal transients associated with the mergers of binaries visible in the optical and infrared wavelengths e.g., [71–73]. In the "r-process", the heating of the non-relativistic outflows by the decays of heavy nuclei formed through rapid neutron captures is

detected as KNe e.g., [74,75]. The KNe light curve is made up of emission from a bunch of ejecta masses arising from dynamical ejecta (during the merger) or neutrino/magnetically driven winds ejecta (remnant discs in the post-merger phase). The properties of the ejecta such as their masses, velocities and opacities can be determined using some of the binary parameters such as the chirp mass, mass ratio, tidal deformability parameter and the equation-of-state (EOS) e.g. [76,77].

Lensing will allow detection of KNe from higher redshifts z > 1 whose light curves will undergo cosmological time dilation making them detectable for a longer period of time. If early warning of the upcoming lensed KNe images (and/or GW events) can be made, then one can constrain the ejecta properties in the early evolutionary phases of KNe which are impossible to without lensing until the era of third generation GW detectors [57]. As a result, combined analyses of lensed GWs and lensed KNe can help in better constraining the EOS models of the component neutron stars and the degree of tidal deformation found in the BNS/NSBH signals.

(iii) Fast Radio Bursts

FRBs are the milliseconds-duration radio signals, arising from extra-galactic sources, dispersed by passage through an ionized plasma in the line of sight. Even after the discovery of $\mathcal{O}(100)$ FRBs e.g. [78–80], their origins are still a mystery, see reviews e.g. [81,82]. A small fraction of the FRBs are found to be repeating with no particular periodicity e.g. [83,84] although it is not confirmed yet that the remaining FRBs are genuinely not repeaters or it is an observational bias. Some of the promising models for the nature of the progenitors of the FRBs are i) a magnetized rotating young neutron star e.g. [85,86]) ii) BNS mergers - which may produce coherent radio emission by magnetic braking, similar to radio pulsars [87]. Nevertheless, we note that the majority of FRBs cannot come from BNS mergers, since most FRBs seem to be repeating over much longer timescales than the lifetime of the merger remnant consistent with observations and reasonable theoretical models so far.

Quantifying the delay between the emission from FRB and GW sources (BNS or NSBH) can help constrain the mechanisms responsible for the production of FRBs which is much more feasible to achieve if the sources are lensed [88]. Similarly, different progenitor models lead to varied (lensed) FRB rates. Reversing the problem, the abundances of lensed FRBs could help identify which progenitor models are likely viable [89]. If there are multiply lensed images of a repeating FRB source with sufficiently high angular resolution (e.g. very large baseline interferometry in the radio), then any non-uniform motion inferred, based on accurate timing study, can reveal the nature of the FRB as well as the properties of its environments such as jet-medium interactions [90].

(d) Improved efficiency of searches

Searches for the multi-messenger sources conducted with data from any one messenger alone may be less efficient than searching for them jointly. It is suggested that if these sources are also lensed, there will be additional constraints, leading to further improvement in the efficiency of their searches. We recognise that lensing will be seen only for a small subset of multi-messenger sources but find it important to describe their advantages as given below.

(i) GWs-FRBs: Efficient discoveries with lensing

As noted in Sec (c), the GW sources may potentially be the progenitors of (subset of) the FRBs. Active searches to find time-coincident events in these two domains are therefore needed e.g. [91,92]. Searching for the association of FRBs with GWs is currently inefficient, primarily due to the following reasons. Firstly, the delay time between the emission of an FRB from the time of GW event is uncertain, especially, since not even a single confirmed association exists. Thus, the time window within which counterparts should be searched are unknown. Another factor is the lack of accurate distance measurements from both GW e.g. [93] and FRB e.g. [94] sources which introduces uncertainties in associating them spatially in the direction of line-of-sight. Lastly, the

sky-localization uncertainties of the GW sources, with current network of GW detectors, can vary from $\mathcal{O}(10)-\mathcal{O}(100)$ sq. deg. making it difficult to find the true FRB counterpart even though FRBs have far better angular resolution.

It is possible to overcome the above challenges if we are dealing with lensed FRBs and GWs as suggested in [88] owing to the presence of repeated arrival of multiply lensed GW signals which will have the same time delays in the associated lensed FRB images, provided they are detectable. In some cases, even if one of the lensed counterparts is weaker or fainter in one of the messengers (referred to as subthreshold in GW), it may be possible to still discover the weaker lensed counterpart based on the association in the data from the other messenger (also, see Sec ii below).

The top row of Fig. 3 shows the time delay distribution for an intrinsic population of BNS mergers (blue curves) in comparison to that for the detectable fraction from GW (orange curves) and the same from FRB telescopes - CHIME (green, [95]) and BURSTT (red, [96]) telescopes. The left panel shows the detectable population from current generation of detectors e.g. the projected fifth observing run (O5) of LIGO–Virgo–Kagra and the right panel from the future third generation (3G) GW detectors and equivalent updated sensitivities to the FRB telescopes. Not only the detected lensing fractions increase with increasing sensitivity, the peak of the time delay distribution also shifts to longer values due to higher redshift sources entering the horizon [88].

The false alarm probability (FAP) of randomly associating a detected lensed FRB with the corresponding lensed GW pairs is found to scale linearly with the number of discovered lensed FRBs [88]. The FAP distributions arising from measurement uncertainties in time delays are shown for increasing number of discovered lensed FRBs (see bottom left panel of Fig. 3). The FAP to obtain a correct association for 1 lensed FRB is about 10^{-8} .

Even in the instances where the GW is being lensed by low mass lenses such that the wave-optics effects will modulate the GW waveforms rather than producing time-resolved multiple events, [88] proposed that the correct association between the GW and the FRB sources can be made. An example of this is shown in the bottom right panel of Fig. 3 for the time delay posterior of a lensed NSBH merger. The time measurement for FRBs is extremely accurate resulting in a narrow posterior. Matching the independently measured time delays between the lensed GWs and lensed FRBs can establish that they are counterparts.

(ii) GRBs-GWs: Detecting sub-threshold GW counterparts

Similar to the FRBs, the time-delay based association between the lensed GRBs and lensed GWs can be extremely useful in detecting the counterparts ². There have been several efforts to identify lensed GRBs. Some studies involved analyses of decade-long data from the Fermi Gamma-ray Burst Monitor (GBM), examining around several hundreds of GRBs to identify potential lensed events [97,98]. Additionally, specific event(s) have been explored to find evidence of lensing in individual GRBs e.g. [99,100].

Two scenarios are presented for searches based on GRB-GW association 1) if lensed GRBs are detected then finding the associated counterpart lensed GW events and 2) if lensed GW events are detected then finding the associated lensed GRBs. It is assumed that all detectable sGRBs originate from BNS mergers and that the luminosity produced is similar to that of GRB 170817A to simplify the simulation. Additionally, the inclination angle of the GW is expected to be aligned with the GRB jet axis [20,101]. This alignment is crucial as it influences the detectability and characteristics of the observed GRB, given the highly beamed nature of these emissions.

For viewing angle $\theta < \theta_c$, i.e. within the core angle of 5 deg, all (unlensed) GRBs out to z=5 are expected to be detected by Fermi and Swift [102]. For off-axis case, $\theta > \theta_c$, the probability of detection becomes a function of θ and z and is given by

²In this section, GRBs and short GRBs are used interchangeably to mean the same. Similarly, for GWs and BNSs may be used interchangeably.

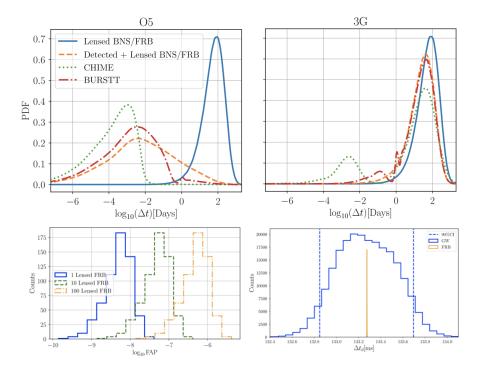


Figure 3. Top: Time delay distributions for lensed BNS mergers for GW observing scenarios – O5 (left) and 3G (right) including detectability from CHIME and BURSTT radio telescopes for lensed FRBs. Bottom Left: Distribution of FAPs of lensed FRBs that can be associated with GW signals arising from lensing GW time-delay uncertainties. Bottom Right: Time delay posterior inferred for a microlensed NSBH (blue) and corresponding lensed FRB (orange). Figure taken from [88].

$$P_{\rm det}(\theta,D_L) = \begin{cases} 1 & \text{if } \theta \le 5 \text{ deg and } D_L \le 46652 \, {\rm Mpc} \ (z \sim 5) \\ 1 & \text{if } \frac{1.61 \times 10^8}{4\pi D_L^2} \exp(-\frac{\theta^2}{2 \times 21.2^2}) \ge 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2.1}$$

The unlensed GRBs will only be detectable if $P_{\rm det} = 1$ whereas for the lensed GRBs, at least two of the images must satisfy the detectability criteria.

Similarly, the (lensed or unlensed) GW population is considered detectable for a network comprising the three LIGO–Virgo detectors, operating at their ultimate design sensitivity, with a combined network signal-to-noise ratio (SNR) greater than 6 (scenario 1) or greater than 8 (scenario 2). GW events with a network SNR greater than 8 are considered super-threshold, while those with lower SNR are termed sub-threshold events from the standard search pipelines. For lensed populations, at least two events need to meet the detectability condition.

In Fig. 4, scenarios 1 (left) and 2 (right) show the unlensed and lensed populations of GRBs and GWs along with the associated counterparts in the redshift vs viewing angle parameter space to give insights into their distributions. In scenario 1, the unlensed GRB population is generated with Population I/II star merger-rate density following [103,104] and with the luminosity function taken from [105]. The combined detection with SWIFT and FERMI detectors is assumed to have a 50% duty cycle, meaning half of the sky is always visible. Through extensive sampling and simulation, along with the application of the detectability criteria, it is found that the number of GRB lens systems that meet the detectability conditions is approximately 10 yr⁻¹. This assumes a viewing angle range of $\theta_v \in [0, 90]$ degree and redshifts $z \in [0, 10]$. Out of the detectable lens

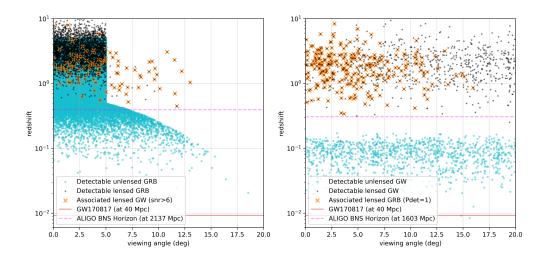


Figure 4. Scenario 1 (left): Detectable lensed GRBs and the associated detectable lensed GW events, lying in the sub/super-threshold regime (SNR>6) at times, as a function of their redshifts and viewing angle. For reference, the unlensed GRBs are also shown along with the BNS horizon (assuming SNR>6) at Advanced LIGO design sensitivity (purple dashed horizontal line) and the distance at which GW170817 was located (red solid horizontal line). Scenario 2 (right): Similar to the left panel although starting with detectable lensed GW events (super-threshold with SNR>8) and their associated detectable lensed GRBs. For reference, the unlensed GWs are also shown along with the BNS horizon (assuming SNR>8) at Advanced LIGO design sensitivity (purple dashed horizontal line) and distance of GW170817 as before (Phurailatpam et al.,in prep). See Sec. 2 d ii for more details.

systems, 1 GW lens out of 1030 GRB lens systems in the sub/super-threshold domain (with SNR>6) could potentially be recovered with this approach.

In scenario 2, the detectable unlensed GW events are expected to be 21 for the Advanced LIGO design sensitivity over an observing run of 1 year. Applying the lensing rates [106,107], about 1 GW lens out of 400 unlensed GW events are detectable with the 3-detector network SNR> 8 (see right panel of Fig. 4). Using the simulated source and lens parameters for GW events, the corresponding $P_{\rm det}$ value for the lensed GRBs is determined. Searching for such lensed GRB counterparts within a time window of 2 sec around the time of the GW events, gives 1 GRB lens out of 13 GW lenses (see [108] for detailed analysis).

3. Towards enabling multi-messenger science

As some of the EM counterparts to the GW events occur within a short time window or are sufficiently bright only for a limited timescales (e.g. GRBs and KNe), concerted efforts among the multi-messenger community and development of new tools, software and databases will be imperative for enabling timely multi-messenger investigations and analyses. Some ideas and thoughts in these directions are presented below which are not exhaustive but useful to consider.

(a) Importance of additional GW detectors

Multi-messenger studies depend critically on the angular resolution of the GW observations. One of the biggest advantages of having additional GW detectors in the network is the improvement in the sky localisation [109–112] which is a decisive factor in the efficacy of the successful EM follow-up observations. The sky localisation uncertainties of the LIGO–Virgo network are (O)10-(O)1000 sq. deg. The fraction of sources with sky localisation of 20 sq.deg. increases by

a factor of 1.6 (or 3) when including a fourth GW detector in India (or a total of 5 GW detectors with both LIGO-India and Kagra) [110].

Furthermore, not only the sky coverage of the detected sample improves i.e. the events can be detected from a larger fraction of the sky, at any given time, but also the duty cycle of observations becomes better, in other words, the fraction of time detectors are online in the network within an observing run. More number of detectors also impacts the sensitivity because one essentially obtains more observations of the same event which can be combined (accounting for the antenna pattern) to get better SNR, allowing signals from farther out to be detected [110,111]. For example, comparison of two networks i) two LIGO detectors (Hanford-H and L-Livingston) and ii) three LIGO detectors (H, L and A-LIGO India) shows that the sky localisation may improve by over 90% for a GW150914-like event, duty cycle by 20% under multi-detector coincidence SNR criteria and better sky coverage which results in a higher detection rates of BBH signals by 40%–50% [111].

Improved constraints on certain other GW parameters, for instance, the luminosity distance and inclination angle becomes feasible [111–113] which are hampered otherwise owing to inherent degeneracies. It is noted that many of the advantages of having additional GW detectors are also achievable by a GW source that is lensed since we get more observations of the same event at multiple epochs. Regardless, in an expanded network of detectors, the lensed GW events are not only benefitted by many of the aforementioned improvements but will also see reduction in the false positives owing to the improvements in sky localisation. This can also have a positive impact on the search efficiency of the EM counterparts e.g., [114,115]. Moreover, the improved constraints on the parameter estimation of the lensed GW events will constrain the lens mass models better.

(b) Methodologies and tools for model predictions and analyses in low latency

Methodologies or tools that could help speed up and improve efficiencies of follow-up studies in multi-messenger domains will either need to be developed or are being developed. Some ideas are presented below.

While detection of GW170817 and subsequent detailed studies have established BNS to be the progenitors of (some) sGRBs and KNe where the latter are proved to be responsible for the production of the heavy nuclei through the "r-process", the exact degree of contribution of NSBH mergers to this universal r-process production is not yet well understood which may vary from none to being the most dominant compared to the BNSs. Only through timely multimessenger studies, that provide constraints on GW merger rates along with (non-)detection of the counterpart EM sources, we will be able to ultimately place constraints on the r-process yields. With NSBH as the progenitors, the KNe light curves may not reach their peak at optical wavelengths for about a week after the merger event, for some viewing angles. Therefore, early observations of the rising light curves of KNe, particularly, when the corresponding GW detection may not be informative, are of supreme importance [116]. Furthermore, early observations of the emergent KNe, within about 2 days, are also critical in discriminating between BNS and NSBH mergers in such cases. (Semi-)Analytical frameworks have been developed for generation of KNe light curves based on physical parameters of the BNS or NSBH signals in low latency [76,116]. As a result, rapid follow-up with suitable observational set-up can be proposed for efficient detection of the EM counterpart at optical and infrared wavelengths.

Similarly, as soon as a potentially lensed BNS or NSBH is detected in the GW, it will be important to assess the plausibility of it being lensed and rapid predictions of the properties of both the subsequent GW lensed counterparts as well as the EM counterparts will be instrumental in the design of the follow-up observations since the lensed source is likely to arise from higher redshifts and the time delays between the subsequent lensed events are more likely to be $\mathcal O$ hoursto-days [57] (see bottom row of Fig. 5). Also, it is interesting to see that while the time delays are larger for lenses and sources situated at high redshifts, the relative time delay uncertainties do

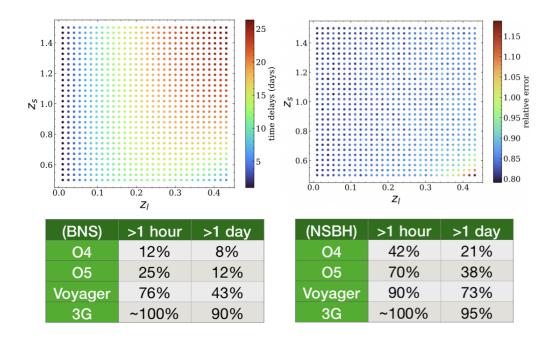


Figure 5. Time delay predictions and the relative errors as a function of the lens and source redshifts (top row). The errors in the predicted time delays arise from the astrometric uncertainties in the optical lensed images. Fractions of lensed BNSs and lensed NSBHs to have time delays above a threshold for various observing scenarios (bottom row) such as the fourth and fifth observing runs of LIGO (O4 and O5), Voyager and 3G detectors (Einstein Telescope and Cosmic Explorer). Results are taken from [57].

not change as dramatically as a function of these redshifts (top row of Fig. 5). Typical EM follow-up imaging strategies which are intended for unlensed BNS/NSBH may not go deep enough to detect the underlying host if it were actually lensed. Moreover, without rapid and accurate lensing time delay predictions, EM observations of the subsequent emergent KNe may not be scheduled in the appropriate time windows. Thus, further studies are ongoing that will help understand realistic uncertainties in the lensing time delay predictions that could be obtained from the modelling of ground-based (low angular resolution) imaging of any known (or mock) lensed host galaxies. Additionally, methodologies and resources are being developed which can actually process any newly discovered candidate BNS/NSBH combined with early EM imaging (taken from the archive or new EM data obtained with a latency of about 24 hours) to make time delay predictions within a day or two .

(c) Cross-matching investigations and Lens databases

The GW (candidate) events triggered by the GW network of observatories will have to be rapidly cross-matched with galaxy catalogs produced by electromagnetic observations from existing surveys. The lens detection pipelines will identify a pair of GW events to be promising candidates for strong lensing. The sky localisation contours of such pair of GW events are expected to cover $\mathcal{O}(100)$ sq. deg. in the sky. There are many wide-area imaging surveys conducted in the EM, primarily, in the optical, radio and infrared wavelengths with sufficient sensitivity which would find hundred thousands of galaxies within the sky localisation contours. Since many of these EM survey data also have been searched for strong lensing, there are lensed EM galaxy (or cluster) catalogs available in the literature (e.g. Master Lens Database [117]).

Cross-matching the EM lens catalogs with the sky localisation regions will produce $\mathcal{O}(100)$ matched lens systems. Early studies such as [114,118] have put forward some ideas on localisation of the counterpart EM host galaxy, however, improved and efficient methodologies are still needed. Also, there are GW-centric tools being developed such as Lenscat³ [119]. Further screening of the matched lens systems can be done by extracting additional observational constraints such as the redshifts, stellar velocity dispersions or time delays. Alternatively, the imaging data can be used for finding the lens mass models that best-fit matched lenses and their corresponding predictions of time delays and image types (e.g. Type-I or Type-II based on the topology of the Fermat's time delay surface [120]). These data constraints will help in further vetting and identification of the most likely candidates that are counterparts to the pair of GW events e.g. [18]. A systematic large-scale modelling framework that can analyse imaging data of heterogeneous angular resolution and depth for accurate and generate data products including time delays, magnifications, image types and other properties will be ideal.

4. Summary and Conclusion

Multi-messenger studies herald an exciting avenue of research that is expected to have an impact on areas of astrophysics, cosmology and fundamental physics. It will help unravel numerous mysteries such as the nature of progenitors of kilonovae and gamma ray bursts, contribution of neutron star - black holes in heavy nuclei production, potential association between gravitational wave sources and fast radio bursts, alternate gravity theories, nature of dark matter, Hubble tension, electromagnetic counterparts of supernovae and almost certainly, completely unexpected discoveries in the next few decades.

Strong lensing of gravitational waves or any of the electromagnetic counterparts, in spite of their low occurrence rates, will push the boundaries of our knowledge faster, for instance, by enabling certain investigations sooner, by improving accuracy in certain models given their repeated occurrences, by aiding in conducting optimised follow-up observations. Some of these ideas are touched upon in this article. A more comprehensive description can be found in Smith et al. (this issue). In order to make this enterprise a successful one, efficient coordination, cooperation and communication between the multi-messenger astronomy community will be essential. Moreover, building resources, such as tools, pipelines and databases that are publicly accessible, will maximise scientific output.

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³https://github.com/lenscat/lenscat

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