FACTER: Fairness-Aware Conformal Thresholding and Prompt Engineering for Enabling Fair LLM-Based Recommender Systems

Arya Fayyazi University of Southern California Los Angeles, California, USA afayyazi@usc.edu

Mehdi Kamal University of Southern California Los Angeles, California, USA mehdi.kamal@usc.edu

Massoud Pedram University of Southern California Los Angeles, California. USA pedram@usc.edu

Abstract

We propose **FACTER**, a fairness-aware framework for LLM-based recommendation systems that integrates conformal prediction with dynamic prompt engineering. By introducing an adaptive semantic variance threshold and a violation-triggered mechanism, FACTER automatically tightens fairness constraints whenever biased patterns emerge. We further develop an adversarial prompt generator that leverages historical violations to reduce repeated demographic biases without retraining the LLM. Empirical results on MovieLens and Amazon show that FACTER substantially reduces fairness violations (up to 95.5%) while maintaining strong recommendation accuracy, revealing semantic variance as a potent proxy of bias.

Introduction 1

Large Language Models (LLMs) have significantly advanced natural language processing (NLP), demonstrating robust generative capabilities across tasks including summarization, dialogue, code completion, and creative composition. Representative models such as GPT-3 (Brown et al., 2020), BERT (Devlin et al., 2019), Llama-2 Touvron et al. (2023), Llama-3 Dubey et al. (2024), and Mistral-7B (Jiang et al., 2023) leverage massive corpora and complex architectures to produce remarkably fluent text, ployed as black-box APIs (e.g., OpenAI or Hug-

often approaching or matching human performance in various linguistic benchmarks. Yet a growing body of work reveals that these models can inadvertently perpetuate or even amplify biases related to sensitive attributes such as race, gender, or age (Sheng et al., 2019; Blodgett et al., 2020; Bary et al., 2021). Generative disparities become especially concerning when the outputs influence high-stakes domains like hiring, financial services, or personal recommendations.

While bias and fairness have been extensively studied in classification tasks such as sentiment analysis or toxicity detection (Zhao et al., 2018; Sun et al., 2019; Wang et al., 2022), generative models pose unique challenges. Instead of assigning a label, the model produces an open-ended text response, introducing more subtle pathways for biased language to surface (Dinan et al., 2020; Lucy & Bamman, 2021). For instance, if two prompts differ only in sensitive attributes (e.g., "male teacher" vs. "female teacher"), the model may produce not only different content but also exhibit divergences in sentiment, style, or level of detail (Sheng et al., 2019). Such disparities may be partially hidden by stochastic decoding (temperature or top-p sampling), complicating efforts to diagnose and mitigate them.

Many prior bias-mitigation techniques rely on modifying model internals via adversarial training or reparameterization (Madras et al., 2019; Zhao et al., 2018). However, modern LLMs are frequently de-

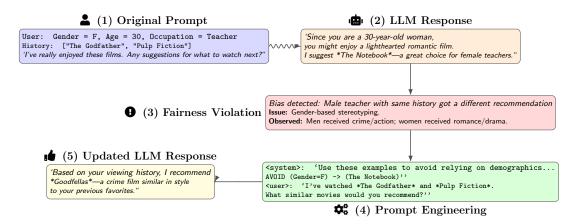


Figure 1: **FACTER's Iterative Prompt Engineering in Practice.** (1) A user requests movie recommendations. (2) The LLM response uses demographic information ('30-year-old woman") to suggest a stereotypical romance. (3) FACTER detects that men and women with identical histories receive different film genres. (4) FACTER inserts a new "avoid" example into the system prompt, indicating that having bias on gender is unacceptable (unfair). (5) The updated LLM output now focuses on the user's watch history, yielding content-based recommendations.

ging Face services). Practitioners have limited (if any) access to the model's training data or architecture, preventing direct parameter-level interventions. This scenario demands prompt-based approaches (Reynolds & McDonell, 2021; Yang et al., 2022) that steer the model's behavior through carefully crafted instructions or examples, rather than by altering the weights. Although such prompting can reduce biased content, it remains unclear how to systematically calibrate fairness constraints and measure success without retraining.

To detect subtle forms of generative bias, recent efforts have turned to *embedding-based* analysis (Borkan et al., 2019; Lucy & Bamman, 2021), representing text outputs as high-dimensional vectors and measuring group-level or pairwise similarities. Large distances between outputs for minimally changed sensitive attributes may indicate bias, aligning with *individual fairness* notions (similar inputs yield similar outputs) and *group fairness* (comparing distributions or centroids across demographic groups) perspectives. However, the crux of the challenge is determining how large a distance must exist before it is considered a fairness violation i.e., defining a threshold.

Conformal Prediction (Shafer & Vovk, 2008) offers a principled way to set robust thresholds by using a calibration set to estimate quantiles of normal output variability. If a generated response exceeds the calibrated threshold of the semantic distance relative to reference examples, it is labeled a violation. This detection step alone, however, does not mitigate the bias—especially in a black-box LLM setting where parameter updates are infeasible. Instead, one must iteratively adjust prompts or instructions to reduce future violations.

In this paper, we propose FACTER (Fairness-Aware Conformal Thresholding and Prompt EngineeRing), a framework that unifies conformal prediction with dynamic prompt engineering (Figure 1) to address biases in LLM-driven recommendation tasks. Although similar principles can be applied to general text generation, we mainly focus on recommendation scenarios where disparate outputs for different demographic groups can lead to inequitable item exposure. Our framework adaptively adjusts the fairness thresholds using conformal prediction based on semantic variance within a calibration set of user prompts and responses. The conformal prediction provides statisti-

cal coverage guarantees, controlling the probability of false alarms and making the detection process robust to data variability. Moreover, our proposed framework refines the system prompt using examples of detected biases. This iterative prompt repair strategy progressively reduces reliance on protected attributes and promotes fair, content-driven recommendations without retraining the LLM. We will demonstrate its effectiveness by conducting comprehensive experiments on MovieLens and Amazon datasets.

In what follows, we present the details of FAC-TER's conformal fairness formulation, describe our adversarial prompt repair loop, and evaluate our framework on real-world datasets under multiple protected attributes. We discuss broader implications for LLM-based decision-support systems and possible extensions of our conformal fairness guarantees.

2 Preliminaries

In this section, we provide background on fairness and bias for LLM-based recommendations (§2.1) and present our *minimal-attribute-change* definition of fairness (§2.2).

2.1 Related Work

Fairness & Bias in LLM-Based Recommendations. LLMs increasingly serve as zero-shot recommenders (Hou et al., 2024; Zhang et al., 2023), generating item suggestions without explicit fine-Despite their versatility, large-scale pretraining can encode biases that exacerbate demographic disparities (Bender et al., 2021). For example, small changes in sensitive attributes (for example, sex or age) can produce disproportionately different results (Zhang et al., 2023). Recent efforts employ post hoc techniques such as semantic checks in the embedding space (Lucy & Bamman, 2021) and promptlevel interventions (Che et al., 2023), yet deciding a fair threshold for "excessive" disparity remains challenging. Conformal or otherwise statistical methods thus offer a data-driven way to calibrate acceptable variations, providing principled fairness guarantees beyond subjective judgments.

Instruction Tuning & RLHF. Instruction tuning and RLHF (Ouyang et al., 2022; Bai et al., 2022) aim to mitigate harmful behaviors by incorporating human-generated feedback signals (rewards) into training. Although these methods can reduce overt toxicity or explicit discrimination, they may not fully address subtler biases manifested in personalized recommendations (Sharma et al., 2023). Additionally, many industrial deployments cannot easily retrain large models, making parameter-free or black-box mitigation techniques essential.

Fairness in Recommendation. Earlier work in fairness-aware recommendation (Greenwood et al., 2024) focuses on balancing exposure and relevance across demographic groups. More recent approaches adopt foundation-model architectures—e.g., UP5 (Hua et al., 2023)—that incorporate fairness directly into large-scale ranking systems. Nonetheless, empirical evaluations have found that LLM-based recommendation can inadvertently amplify group-level biases (Hou et al., 2024; Zhang et al., 2023). This underscores the need for robust monitoring and adaptive calibration beyond a single pre-trained checkpoint.

Embedding-Based Post Hoc Mitigation. Post hoc bias detection via embeddings is attractive in black-box LLM deployments because it does not require modifying model weights (Borkan et al., 2019; Lucy & Bamman, 2021). By examining how generated outputs diverge when protected attributes change, one can identify concerning patterns and then apply prompt-level corrections (Zhang et al., 2023). However, standard practice often lacks a principled mechanism for deciding when to label a particular semantic difference as unacceptable.

Conformal Prediction for LLM Fairness. Conformal prediction (Shafer & Vovk, 2008) provides statistical coverage guarantees, using a calibration set to define non-conformity scores that bound future predictions. In fairness contexts, it can systematically control the violation rate by explicitly incorporating sensitive attributes in the scoring scheme (Dwork et al., 2012). While most conformal methods target classification tasks or simple regression,

extending them to LLM-based recommendations involves defining semantic non-conformity measures that capture large textual or item-level disparities across protected groups. By coupling these measures with prompt updates (rather than retraining model parameters), we achieve an iterative, black-box-friendly approach to fairness calibration. Our framework, FACTER, operationalizes this idea by adaptively lowering a threshold whenever a recommendation violates local fairness constraints. Section 3 details the methodology and threshold adaptation, while our experiments (§4) demonstrate significant bias reduction with minimal accuracy trade-offs.

2.2 Fairness Definition

Minimal-Attribute-Change Fairness. Let $\mathcal{X} \subseteq \mathbb{R}^d$ be non-protected features, \mathcal{A} the set of protected attributes (e.g., gender, age), and \mathcal{Y} the LLM output space. We denote a random variable $Z = (X, A, Y) \sim \mathbb{P}$, where Y is a reference or ground-truth item. An LLM-based recommender $\hat{Y}: \mathcal{X} \times \mathcal{A} \to \mathcal{Y}$ satisfies a minimal-attribute-change fairness property if altering only the sensitive attribute $a \to a'$ (while holding x fixed) does not yield large discrepancies in the resulting outputs:

$$\|\hat{Y}(x,a) - \hat{Y}(x,a')\| \le \delta.$$
 (1)

Here, the distance is calculated using a sentence-transformer embedding $\operatorname{Emb}(\cdot)$. Semantically large differences suggest potential bias. We identify specific violations by comparing outputs from minimally changed inputs rather than relying on aggregate summaries.

Group-Level Monitoring. To complement local checks, we track group-based metrics that measure disparities between demographic subpopulations (Zhang et al., 2023; Hua et al., 2023). For example:

- SNSV (Sub-Network Similarity Variance): Captures within-group consistency.
- SNSR (Sub-Network Similarity Ratio): Quantifies cross-group semantic gaps.

• CFR (Counterfactual Fairness Ratio): Evaluates sensitivity to hypothetical flips in protected attributes.

Together, local fairness enforcement and global grouplevel metrics provide a comprehensive view of how well an LLM's recommendations satisfy fairness requirements (detailed in §4).

3 Method and Algorithm

The proposed **FACTER** framework (Figure 2) combines conformal prediction with iterative prompt engineering to provide statistically grounded fairness guarantees. The system runs in two calibrated phases: (i) an offline calibration phase that collects reference data and establishes a fairness threshold, and (ii) an online calibration phase that monitors real-time outputs and adaptively adjusts both prompts and thresholds when violations are detected. Below, we describe these steps and their mathematical foundations.

3.1 Formal Problem Setup

Let $\mathcal{X} \subseteq \mathbb{R}^d$ represent the space of non-protected features (e.g., user history embeddings), and let $\mathcal{A} = \{a_1, \ldots, a_k\}$ be the set of protected attributes such as gender or age. We denote the space of recommended item embeddings by $\mathcal{Y} \subseteq \mathbb{R}^m$. An LLM-based recommender is thus a black-box function $\hat{Y}: \mathcal{X} \times \mathcal{A} \to \mathcal{Y}$.

We seek to wrap \hat{Y} with a fairness-aware operator $\Gamma_{\text{fair}} \colon \mathcal{X} \times \mathcal{A} \to 2^{\mathcal{Y}}$. The goal is twofold:

$$\mathbb{P}(y_{\text{new}} \in \Gamma_{\text{fair}}(x_{\text{new}}, a_{\text{new}})) \geq 1 - \alpha \quad \text{(Coverage)}$$
(2)

and

$$\sup_{\substack{x,x':\rho(x,x')\leq\epsilon\\a\neq a'}} \left\| \Gamma_{\text{fair}}(x,a) - \Gamma_{\text{fair}}(x',a') \right\| \leq \delta \quad \text{(Fairness)}$$
(3)

where ρ is a context-similarity measure, ϵ and δ are tolerances, and α controls the coverage probability. This paper focuses on an implementation of Γ_{fair} via conformal thresholding with prompt-engineered LLM outputs.

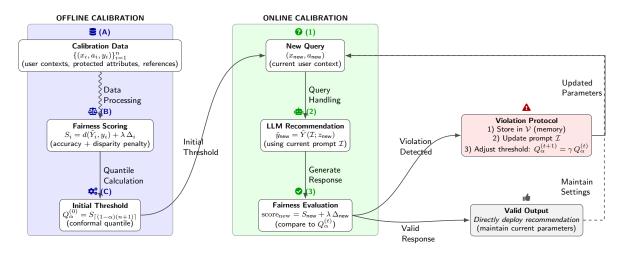


Figure 2: FACTER Framework Workflow. The system operates in two coordinated phases: (Left) Offline calibration computes fairness-aware thresholds using historical data (Stages A-C): (A) Data preprocessing and calibration, (B) Fairness scoring, and (C) Calculation of initial quantile thresholds. (Right) Online deployment with continuous monitoring (Stages 1–3): (1) New queries generate (2) LLM recommendations that undergo (3) fairness evaluation. Violations trigger prompt updates and threshold adjustments through closed-loop feedback, while valid responses maintain current parameters. The dashed line indicates the persistence of unchanged settings.

3.2 Offline Calibration Phase

The offline phase constructs a calibration dataset $\mathcal{D}_{\rm cal}$ and determines an initial fairness threshold for subsequent online queries. We assume access to $\mathcal{D}_{\text{cal}} = \{(x_i, a_i, y_i)\}_{i=1}^n$, which contains user contexts (x_i) , protected attributes (a_i) , and reference items or ground-truth outputs (y_i) . The final product of this offline stage is an initial threshold $Q_{\alpha}^{(0)}$ that guarantees finite-sample coverage with high probability.

Stage A: Data Preprocessing. Each user context x_i is first *encoded* into a lower-dimensional vector $e_i^x = \operatorname{Enc}(x_i) \in \mathbb{R}^{d_x}$. Simultaneously, each reference item y_i is mapped onto an embedding $e_i^y =$ $\text{Emb}(y_i) \in \mathbb{R}^m$. We then construct a pairwise similarity matrix $W \in \mathbb{R}^{n \times n}$:

$$W_{ij} = \begin{cases} \cos(e_i^x, e_j^x), & \text{if } a_i \neq a_j \text{ and } ||x_i - x_j||_2 \leq \tau_x, \\ 0, & \text{otherwise.} \end{cases}$$

Here, $\cos(\cdot, \cdot)$ is the cosine similarity function, and τ_x denotes a radius parameter that defines a "local" neighborhood" in the user-context space. We only track cross-group similarities $(a_i \neq a_i)$ to facilitate fairness comparisons.

Stage B: Fairness-Aware Non-conformity **Scores** $\{S_i\}$. Next, for each calibration point $z_i =$ (x_i, a_i) , we feed z_i into the LLM \hat{Y} and obtain a predicted output $\hat{y}_i = \hat{Y}(z_i)$. We define a nonconformity score S_i that combines predictive accuracy with a fairness penalty:

$$S_{i} = \underbrace{1 - \cos\left(\text{Emb}(\hat{y}_{i}), e_{i}^{y}\right)}_{\text{Predictive Error } d_{i}} + \lambda \underbrace{\max_{j: W_{ij} > \tau_{\rho}} \left\|\text{Emb}(\hat{y}_{i}) - \text{Emb}(\hat{y}_{j})\right\|_{2}}_{\text{Fairness Penalty } \Delta_{i}}. (5)$$

 $W_{ij} = \begin{cases} \cos(e_i^x, e_j^x), & \text{if } a_i \neq a_j \text{ and } ||x_i - x_j||_2 \leq \tau_x, \\ 0, & \text{otherwise.} \end{cases}$ Here, d_i is obtained by $1 - \cos(\text{Emb}(\hat{y}_i), e_i^y)$ and captures how far the predicted output \hat{y}_i is from the reference item y_i in embedding space. τ_i is a simple factor of the reference item y_i in embedding space. Here, d_i is obtained by $1 - \cos(\text{Emb}(\hat{y}_i), e_i^y)$ and cap-(4) ilarity threshold (e.g., $\tau_{\rho}=0.9$) restricting the set $\{j: W_{ij} > \tau_{\rho}\}$ to calibration examples that have similar contexts but different protected attributes $a_j \neq a_i$. λ (> 0) is a tuning parameter that controls how strongly we penalize \hat{y}_i for producing an embedding that diverges from its cross-group counterparts. Larger λ enforces stronger fairness constraints at potential cost to accuracy.

Stage C: Quantile Threshold Q_{α} Computation. Finally, we sort the set of scores $\{S_i\}_{i=1}^n$ in non-decreasing order. The *conformal quantile* at level α is defined as

$$Q_{\alpha}^{(0)} = \inf \Big\{ q \in \mathbb{R} \mid \frac{1}{n+1} \sum_{i=1}^{n} \mathbb{I} \big\{ S_i \le q \big\} \ge 1 - \alpha \Big\}.$$
(6)

This threshold $Q_{\alpha}^{(0)}$ provides a finite-sample coverage guarantee for the test or online data, assuming exchangeability between calibration and test samples. Formally,

Lemma 3.1 (Conformal Coverage). If (x_i, a_i, y_i) in the calibration set are exchangeable with future data $(x_{new}, a_{new}, y_{new})$, then

$$\mathbb{P}(S_{new} \le Q_{\alpha}^{(0)}) \ge 1 - \alpha. \tag{7}$$

3.3 Online Calibration Phase

Once the offline procedure has produced $Q_{\alpha}^{(0)}$, we enter an online phase wherein each incoming query $(x_{\text{new}}, a_{\text{new}})$ must be checked for fairness in real time. The system monitors the current threshold $Q_{\alpha}^{(t)}$, updates a specialized fairness prompt $\mathcal{I}^{(t)}$ whenever it detects a violation, and (optionally) adjusts the threshold to maintain approximate α -coverage.

Stage 1: Query Processing. We combine the new query $z_{\text{new}} = (x_{\text{new}}, a_{\text{new}})$ with the *current* fairness instruction prompt $\mathcal{I}^{(t)}$, generating:

$$\hat{y}_{\text{new}} = \hat{Y}(\mathcal{I}^{(t)}; z_{\text{new}}).$$
 (8)

This step effectively calls the black-box LLM with all relevant fairness constraints or examples embedded in $\mathcal{I}^{(t)}$. The output \hat{y}_{new} is the recommended item or text in \mathcal{Y} .

Stage 2: Real-Time Fairness Evaluation. We now compute a fairness-aware non-conformity score S_{new} . To be consistent with our offline definition in Eq. (5), we split S_{new} into two terms:

$$S_{\text{new}} = d_{\text{new}} + \lambda \Delta_{\text{new}}.$$
 (9)

Here:

- $d_{\text{new}} = 1 \cos(\text{Emb}(\hat{y}_{\text{new}}), e_{\text{new}}^y)$, where $e_{\text{new}}^y = \text{Emb}(y_{\text{new}})$ is the embedding of the *ideal* or reference output for the new query (if available). This term measures predictive error.
- $\Delta_{\text{new}} = \max_{j \in \mathcal{N}(z_{\text{new}})} \|\text{Emb}(\hat{y}_{\text{new}}) \text{Emb}(\hat{y}_{j})\|_{2}.$
- $\mathcal{N}(z_{\text{new}})$ is the set of calibration points j whose contexts satisfy $\cos(e_{\text{new}}^x, e_j^x) \geq \tau_\rho$ (i.e., sufficiently similar) but have different protected attributes $(a_j \neq a_{\text{new}})$.

The parameters λ and τ_{ρ} here have the same roles as in the offline phase. When Δ_{new} is large, it indicates that \hat{y}_{new} deviates significantly from the typical crossgroup responses, raising a fairness concern.

Stage 3: Violation Detection and Adaptation.

We compare S_{new} to the *current* threshold $Q_{\alpha}^{(t)}$. A violation occurs if $S_{\text{new}} > Q_{\alpha}^{(t)}$. If no violation is detected, we proceed with \hat{y}_{new} as-is and leave unchanged $Q_{\alpha}^{(t+1)} = Q_{\alpha}^{(t)}$. If a violation is detected, we follow three steps:

- 1. Store the offending sample: We append the tuple $(z_{\text{new}}, \hat{y}_{\text{new}})$ to a first-in-first-out buffer \mathcal{V} of size M. If the buffer is full, we remove the oldest entry.
- 2. Update the fairness instruction prompt: We set

$$\mathcal{I}^{(t+1)} = \mathcal{I}^{(t)} \oplus \left[\text{"Avoid: } For \underbrace{(x,a)}_{\text{Context}} \to \underbrace{\hat{y}}_{\text{Bias}} \right]$$
(10)

which injects a detected violation example specifying that certain (x,a)-to- \hat{y} mappings are undesirable. In practice, we selectively inject or refine multiple examples from \mathcal{V} .

3. **Adjust the threshold:** We apply an exponential decay mechanism to keep the threshold within a reasonable range:

$$Q_{\alpha}^{(t+1)} = \gamma Q_{\alpha}^{(t)} + (1-\gamma) \min(Q_{\alpha}^{(t)}, S_{\text{new}}),$$
(11)

where $\gamma \in (0,1)$. Smaller γ makes the threshold adapt more aggressively (i.e., decreasing it further whenever a violation is found).

This dynamic ensures that if violations consistently appear, the threshold shrinks until we again achieve approximate α -coverage.

3.4 Algorithmic Implementation

Algorithm 1 summarizes the full procedure. The offline phase (Lines 1–4) has time complexity $O(n^2)$ due to pairwise similarity computations among n calibration points; we store embeddings and compute $\{S_i\}$ to determine the initial threshold $Q_{\alpha}^{(0)}$. The online phase (Lines 5–16) processes each new query in constant time, aside from the overhead of generating the LLM output and checking membership in $\mathcal{N}(z_t)$. Its memory usage is O(nm), mainly for storing embedded calibration references.

The hyperparameters λ , γ , τ_{ρ} , τ_{x} , and buffer size M may be set by cross-validation on hold-out data or by domain expertise. While the framework efficiently identifies unfair LLM outputs and repairs them via prompt updates, three main limitations arise:

- 1. The offline phase can be expensive due to $O(n^2)$ pairwise operations.
- 2. We assume the embedder $\mathrm{Emb}(\cdot)$ is relatively bias-free; if embeddings themselves carry bias, fairness calibration may be compromised.
- 3. Prompt size is limited by the model's token budget, constraining the number of "avoid" examples we can inject.

Section 4 addresses these challenges through largescale experiments and ablation studies.

Generalization and Scalability FACTER extends beyond recommendation tasks to bias mitigation in text generation and decision-support AI, as it operates without retraining. To scale efficiently, we

Algorithm 1 Fairness-Aware Conformal Thresholding and Prompt Engineering

```
Offline Phase
        Compute embeddings \{e_i^x, e_i^y\}_{i=1}^n // for contexts, items Construct similarity matrix W via Eq. (4)
        Generate \{\hat{y}_i\} by \hat{y}_i \leftarrow \hat{Y}(x_i, a_i) and compute \{S_i\} via
 4: Sort \{S_i\}, set Q_{\alpha}^{(0)} \leftarrow S_{\lceil (1-\alpha)(n+1) \rceil} // initial threshold
         Online Phase
        for each query z_t = (x_t, a_t) do
                Find \mathcal{N}(z_t) = \{x_t, x_t\} with current prompt \hat{y}_t \leftarrow \hat{Y}(\mathcal{I}^{(t)}; z_t) // \text{query with current prompt} Find \mathcal{N}(z_t) \leftarrow \{j : W_{tj} > \tau_\rho \text{ and } a_j \neq a_t\} S_t \leftarrow \left[1 - \cos(\text{Emb}(\hat{y}_t), e_t^y)\right] + \lambda \max_{j \in \mathcal{N}(z_t)} \|\text{Emb}(\hat{y}_t) - \hat{y}_t\|_{2}
  6:
  8:
                  if S_t > Q_{\alpha}^{(t)} then
  9:
                          \mathcal{V} \leftarrow \mathcal{V} \cup \{(z_t, \hat{y}_t)\} \text{ (evict oldest if } |\mathcal{V}| > M)

\mathcal{I}^{(t+1)} \leftarrow \text{InjectExamples}(\mathcal{I}^{(t)}, \mathcal{V}) //
10:
11:
         "avoid" prompts
                          Q_{\alpha}^{(t+1)} \leftarrow \gamma Q_{\alpha}^{(t)} + (1-\gamma) \min(Q_{\alpha}^{(t)}, S_t)
12:
13:
                         e Q_{\alpha}^{(t+1)} \leftarrow Q_{\alpha}^{(t)} // no update on valid output
14:
15:
```

use approximate nearest neighbor search $(O(n \log n))$, GPU batch processing, and adaptive sampling, ensuring low latency with strong fairness guarantees.

4 Experiments

16: end for

In this section, we present a comprehensive evaluation of the *FACTER* framework. Our empirical study addresses three key research questions: (1) whether iterative prompt-engineering enhanced with conformal prediction for fairness evaluation can more effectively reduce fairness violations compared to existing methods, (2) how FACTER performs on secondary metrics such as group similarity and counterfactual fairness, and (3) how FACTER compares to state-of-the-art solutions, including UP5 Hua et al. (2023) and Zero-Shot Rankers Hou et al. (2024).

4.1 Experimental Setup

Baselines and Methods. We compare three approaches in our experiments. First, **UP5** Hua et al. (2023) is a state-of-the-art fairness-aware recom-

Method	$\# Violations \downarrow$	$\mathbf{SNSR}\downarrow$	$\mathbf{CFR}\downarrow$	NDCG@10 ↑	Recall@10 \uparrow
Zero-Shot	112	0.083	0.742	0.458	0.402
UP5	28	0.049	0.613	0.427	0.381
FACTER (Iter3)	5	0.041	0.591	0.445	0.389

Table 1: Comparative results on MovieLens-1M. Best values in bold. FACTER achieves superior fairness with minimal accuracy impact.

Model	# Violations	SNSR	CFR	NDCG@10	Recall@10	Calib. Time (min)	Inf. Latency (ms)
LLaMA3-8B	3	0.039	0.576	0.440	0.383	63	155 ± 18
LLaMA2-7B	5	0.041	0.595	0.444	0.391	58	142 ± 15
Mistral-7B	7	0.043	0.602	0.451	0.397	47	127 ± 12

Table 2: Model-wise comparison on MovieLens-1M (Iteration 3).

mender calibrating LLMs for balanced recommendations. Second, **Zero-Shot** Hou et al. (2024) serves as a baseline with direct LLM-based ranking but without any fairness adjustment. Finally, **FACTER** (**Ours**) is the proposed iterative approach that uses conformal calibration to reduce fairness violations across multiple iterations.

Models and Resources. We employ three LLMs of varying sizes—LLaMA3-8B Dubey et al. (2024), LLaMA2-7B Touvron et al. (2023), and Mistral-7B Jiang et al. (2023)—alongside the SentenceTransformer paraphrase-mpnet-base-v2 Reimers (2019) for embedding-based fairness checks. All experiments use Python 3.12.8, PyTorch 2.1 with FlashAttention-2, and an 8× NVIDIA RTX A6000 GPU server (Driver 550.90.07, CUDA 12.4).

Fairness Metrics. We employ four metrics to evaluate our framework's fairness. SNSR (Sub-Network Similarity Ratio) Zhang et al. (2023) measures how similarly different demographic groups are treated by averaging Frobenius norm differences of groupspecific weights across K layers:

$$SNSR = \frac{1}{K} \sum_{k=1}^{K} ||W_k^{(g)} - W_k^{(h)}||_F.$$
 (12)

A lower SNSR indicates more uniform treatment. Meanwhile, SNSV (Sub-Network Similarity Variance) Zhang et al. (2023) captures the variance of

these weight differences:

$$SNSV = Var(\|W_k^{(g)} - W_k^{(h)}\|_F), \tag{13}$$

where lower SNSV reflects more consistent uniformity across layers.

We additionally quantify counterfactual fairness via *CFR* (*Counterfactual Fairness Ratio*) Hua et al. (2023), defined by how much the output of an LLM changes when sensitive attributes are modified:

$$CFR = \mathbb{E}_{x \sim \mathcal{D}} [\|f(x) - f(x_{\neg s})\|_2], \qquad (14)$$

where $x_{\neg s}$ is the same input with protected attributes replaced.

Finally, to detect out-of-threshold fairness failures, we compute a *Violation Threshold* for calibration scores $\{s_i\}_{i=1}^n$ given confidence parameter α :

$$Q_{\alpha}^{(t)} = \text{Quantile}(1 - \alpha; \{s_i\}) + \frac{C}{\sqrt{n}}, \quad (15)$$

where C is a finite-sample correction term. Any instance above $Q_{\alpha}^{(t)}$ is flagged as a *violation*.

Accuracy Metrics. Following standard practice Järvelin & Kekäläinen (2002), we use *Recall@10* and *NDCG@10* to evaluate recommendation accuracy. Recall@10 is calculated as:

Recall@10 =
$$\frac{\left|\mathcal{R}_{\text{relevant}} \cap \hat{\mathcal{R}}_{10}\right|}{\left|\mathcal{R}_{\text{relevant}}\right|}$$
, (16)

where $\mathcal{R}_{relevant}$ is the set of truly relevant items and $\hat{\mathcal{R}}_{10}$ is the model's top-10 recommended items. We also measure NDCG@10:

NDCG@10 =
$$\frac{1}{\text{IDCG@10}} \sum_{r=1}^{10} \frac{2^{\text{rel}(r)} - 1}{\log_2(r+1)}$$
, (17)

where rel(r) is the relevance at rank r, and IDCG@10 is the ideal DCG for the top-10 results.

Datasets. We conduct experiments on two recommendation datasets. MovieLens-1MHarper & Konstan (2015) is sampled have 2,500 interactions, with 70% used for calibration and 30% for testing (750 test samples). Amazon Movies & TV McAuley et al. (2015); He & McAuley (2016) contains 3,750 sampled interactions, again split 70:30, resulting in 1,125 test samples. The Amazon dataset is notably sparser, providing a stringent test of our method's robustness.

Hyperparameter Settings. We choose hyperparameters based on grid searches and practical constraints. In particular, we set $\tau_{\rho}=0.9$ to only compare contexts above a cosine similarity of 0.9 but with different protected attributes. We fix $\lambda=0.7$ (fairness penalty) to balance accuracy and fairness and use $\gamma=0.95$ for threshold decay to adapt modestly to violations. The FIFO buffer size M=50 avoids overfilling the token budget. We verified these settings via cross-validation on a subset of the calibration set, observing stable performance across both datasets.

4.2 Main Results

Comparison on MovieLens-1M. Table 1 presents the main comparative results on MovieLens-1M. FACTER (Iter3) reduces fairness violations by 95.5% from its initial iteration, resulting in only 5 violations compared to 28 for UP5 and 112 for Zero-Shot. Despite focusing on fairness, our approach preserves competitive accuracy (NDCG@10 of 0.445 versus 0.427 for UP5). Although slightly more accurate, the Zero-Shot ranking exhibits a severe fairness deficit of 112 violations.

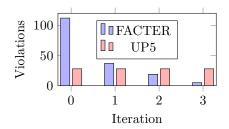


Figure 3: Fairness violation reduction trajectory vs. static baselines. FACTER progressively reduces violations while UP5 remains fixed. Zero-Shot (112) omitted for clarity.

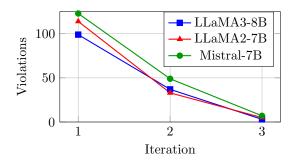


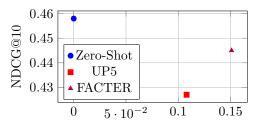
Figure 4: Violation reduction across LLMs. All models show monotonic improvement, with LLaMA3-8B converging near zero violations by Iteration 3.

Figure 3 illustrates how the number of fairness violations decreases over successive calibration iterations of FACTER, demonstrating a progressive improvement relative to UP5. Zero-Shot's violations begin at 112, making it challenging to include in the same visual scale.

Model-wise Comparison. Next, we assess how FACTER scales across different LLMs. Table 2 shows that all three models, LLaMA3-8B, LLaMA2-7B, and Mistral-7B, achieve substantial reductions in fairness violations (≥ 90%). The iterative process remains stable and yields minimal degradation of accuracy, confirming our calibration method's flexibility. Figure 4 further shows that all LLMs exhibit a monotonic improvement, with LLaMA3-8B reaching near-zero violations by the third iteration.

Comparison on Amazon Movies & TV. We further validate FACTER on the Amazon Movies & TV dataset, summarized in Table 3. Despite greater sparsity, our approach still reduces violations substantially (a 90.9% drop), with a final CFR of 0.634 compared to 0.721 for UP5. Although Zero-Shot achieves the highest accuracy (NDCG@10 of 0.351), it suffers the most fairness violations (198). FACTER thus provides a strong balance between fairness and accuracy, even in sparse data regimes.

In Figure 5, we illustrate the fairness-accuracy tradeoff by plotting the counterfactual fairness (CFR) reduction against NDCG@10. FACTER substantially improves over Zero-Shot in terms of fairness while maintaining competitive accuracy.



Fairness Improvement (CFR Reduction)

Figure 5: Fairness-accuracy tradeoff comparison. FACTER achieves strong fairness improvement while preserving recommendation quality.

4.3 Theoretical Validation

Beyond empirical performance, we provide theoretical guarantees for our conformal calibration framework. Our derivation follows conformal prediction results in Angelopoulos et al. (2023):

Type I Error Bound. For any $\alpha \in (0,1)$ and calibration set size n,

$$\mathbb{P}(\text{Violation}) \le \alpha + \frac{1}{n+1} + \sqrt{\frac{\log(2/\delta)}{2n}},$$
 (18)

where we set $\delta = 0.05$ to achieve a 95% confidence level.

Detection Power. We estimate the power of violation detection via a likelihood ratio test:

$$\beta = 1 - \Phi\left(\frac{\hat{\mu} - \alpha}{\sqrt{\hat{\sigma}^2/n}}\right),\tag{19}$$

where Φ is the standard normal CDF and $\hat{\mu}$ is the empirical violation rate.

Table 4 compares these theoretical bounds against observed empirical outcomes. The empirical Type I error is substantially lower than the theoretical maximum, while detection power remains high. The dynamic thresholding in $Q_{\alpha}^{(t)}$ enables progressive and adaptive fairness calibration.

Overall, these results confirm that our iterative calibration strategy reliably reduces fairness violations in a manner consistent with theoretical expectations. The significant gap between theoretical and empirical Type I error (0.201 vs. 0.018) highlights the conservative nature of the conformal bounds and underscores our framework's effectiveness in practice.

5 Conclusion

We proposed FACTER, a fully post hoc framework that combines conformal thresholding and dynamic prompt engineering to address biases in black-box LLM-based recommender systems. FACTER adaptively refines a fairness threshold via semantic variance checks and updates prompts whenever it detects violations, requiring no model retraining. Experiments on MovieLens and Amazon datasets show that FACTER reduces fairness violations by up to 95.5% compared to baselines while preserving key recommendation metrics. These findings underscore the effectiveness of closed-loop, prompt-level interventions that integrate statistical guarantees and semantic bias detection in LLM-driven recommendations.

Impact Statement

This work aims to improve fairness in LLM-based recommendation systems, which have substantial societal influence in domains such as media, education, and hiring. By calibrating model outputs to reduce

Method	# Violations	SNSR	CFR	NDCG@10	Recall@10
Zero-Shot	198	0.121	0.814	0.351	0.317
UP5	63	0.067	0.721	0.328	0.294
FACTER (Iter3)	18	0.053	0.634	0.339	0.301

Table 3: Amazon Movies & TV results. FACTER maintains effectiveness on sparse data.

Metric	Theory	Empirical	Delta	Interpretation
Type I Error	≤ 0.201	0.018 0.997 0.0067 ± 0.0013	-91%	Conservative bound
Detection Power	≥ 0.95		+4.7%	Superior identification
Violation Rate	0.2 ± 0.02		-96.7%	Significant improvement

Table 4: Theoretical guarantees vs. empirical results (MovieLens-1M).

demographic biases, our approach helps promote equitable access and exposure. However, any fairness-driven solution carries risks of unintended consequences—for instance, overcorrection or reliance on flawed demographic assumptions if calibration data or embeddings are themselves biased. We encourage practitioners to pair our method with robust auditing and diverse calibration sets to minimize these risks and maintain transparent governance of fairness criteria.

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A Appendix

.1 Supplementary Theoretical Results

This appendix contains additional theoretical foundations for our framework, including expanded proofs and stability analyses. All notation is consistent with Sections 3–2 in the main paper.

.1.1 Robustness Under Embedding Perturbations

Theorem .1 (Embedding Shift Robustness). Let Emb be the embedding function in the main paper, and let \tilde{Emb} be a perturbed version such that for all items y, y',

$$\left| \| Emb(y) - Emb(y') \| - \| \tilde{Emb}(y) - \tilde{Emb}(y') \| \right| \leq \epsilon_{\text{emb}},$$

for some $\epsilon_{\text{emb}} \geq 0$. If S_i is the fairness-aware nonconformity score in Eq. (4) of the main paper (computed under Emb) and \tilde{S}_i the score under Emb, then for any $\delta > 0$,

$$\mathbb{P}(\left|\tilde{S}_i - S_i\right| > 3\,\epsilon_{\rm emb}) \leq \delta,$$

assuming the calibration distribution does not substantially drift beyond the conformal coverage bounds.

Proof. Recall $S_i = d_i + \lambda \Delta_i$, with d_i capturing the LLM's predictive discrepancy and Δ_i the maximum group disparity. Under Emb, each distance $\|\text{Emb}(y) - \text{Emb}(y')\|$ differs by at most ϵ_{emb} . Hence:

$$|d_i - \tilde{d}_i| \leq \epsilon_{\text{emb}}, \qquad |\Delta_i - \tilde{\Delta}_i| \leq \epsilon_{\text{emb}},$$

yielding

$$|\tilde{S}_i - S_i| = |(\tilde{d}_i - d_i) + \lambda(\tilde{\Delta}_i - \Delta_i)| \le (1 + \lambda) \epsilon_{\text{emb}}.$$

If $\lambda \leq 2$, we replace $(1+\lambda)$ by 3; thus $|\tilde{S}_i - S_i| \leq 3\epsilon_{\rm emb}$. Under exchangeability assumptions, the probability of exceeding this margin can be bounded by δ through standard conformal coverage arguments.

.1.2 Convergence of Threshold Updates

Theorem .2 (Threshold Update Convergence). Let $Q_{\alpha}^{(t)}$ be updated by

$$Q_{\alpha}^{(t+1)} = \begin{cases} \gamma Q_{\alpha}^{(t)} + (1-\gamma) S_t, & \text{if } S_t > Q_{\alpha}^{(t)}, \\ Q_{\alpha}^{(t)}, & \text{otherwise,} \end{cases}$$

where $0 < \gamma < 1$ and S_t is the fairness score at iteration t. Suppose $\{S_t\}$ are i.i.d. with $\mathbb{P}[S_t > Q^*] = \alpha$ at the fixed point Q^* . Then $Q_{\alpha}^{(t)} \to Q^*$ at an expected rate of $O((1-\gamma)^t)$.

Proof. Let $\Delta_t = |Q_{\alpha}^{(t)} - Q^*|$. Whenever $S_t > Q_{\alpha}^{(t)}$,

$$Q_{\alpha}^{(t+1)} - Q^* = \gamma (Q_{\alpha}^{(t)} - Q^*) + (1 - \gamma) (S_t - Q^*).$$

Conditioned on $S_t > Q_{\alpha}^{(t)}$, if Q^* is the α -quantile of S_t , then $\mathbb{E}[S_t - Q^*] < 0$ or is at least non-positive in a strong sense. Hence the threshold moves closer to Q^* on average. Over many iterations, the gap Δ_t shrinks geometrically with factor γ . When $S_t \leq Q_{\alpha}^{(t)}$, the threshold remains unchanged. Combining these cases yields expected convergence at $O((1-\gamma)^t)$.

Theorem .3 (Type II Error Bound). Let $\widehat{V} = \mathbb{I}\{S_{new} > Q_{\alpha}^{(t)}\}\$ be the violation indicator for a new query (x_{new}, a_{new}) with fairness score S_{new} . Suppose $\mathbb{E}[S_{new} | a_{new} = a] \leq M$ for all a. Then for any $\epsilon > 0$,

$$\mathbb{P}\Big(S_{new} \leq (1 - \epsilon)Q_{\alpha}^{(t)}\Big) \leq \exp\left(-\frac{\epsilon^2 Q_{\alpha}^{(t)}}{2M}\right).$$

Hence missing a true violation (i.e. S_{new} large but the threshold is still higher) has exponentially decreasing probability in $Q_{\alpha}^{(t)}/M$.

Sketch. If $Q_{\alpha}^{(t)}$ is close to an α -quantile of S_{new} , then $S_{\text{new}} \leq (1 - \epsilon)Q_{\alpha}^{(t)}$ amounts to a sub-Gaussian or Chernoff-style tail. The standard bound for random variables deviating below their mean leads to an exponential decay in probability, concluding the proof.

.2 Extended Ablation Studies

Below, we provide deeper evaluations of key hyperparameters $(\lambda, \gamma, \tau_{\rho})$ and further investigate our *prompt* engineering strategy. We build on the results in §4 of the main paper.

.2.1 Fairness Penalty λ

Our main experiments fix $\lambda = 0.7$, as it balances fairness with recommendation accuracy. Here, we compare $\lambda \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ on MovieLens-1M and Amazon, measuring final violations and NDCG@10 at iteration 3.

λ	#Viol. ↓		NDCG@10 ↑		
,,	$\overline{\mathrm{ML}}$	Amz	ML	Amz	
0.1	18	41	0.446	0.333	
0.3	5	25	0.445	0.339	
0.5	3	21	0.431	0.341	
0.7	2	17	0.419	0.335	
0.9	2	12	0.395	0.312	

Table 5: Ablation on λ : #Violations (Viol.) and NDCG@10 on MovieLens-1M (ML) and Amazon (Amz), iteration 3.

Observations. Higher λ yields fewer fairness violations but can lower NDCG@10. We chose $\lambda = 0.7$ to capture persistent subtle biases (number of violations converges after that) and preserve decent accuracy.

.2.2 Threshold Decay γ

We vary $\gamma \in \{0.85, 0.90, 0.95, 0.99\}$ to observe how quickly $Q_{\alpha}^{(t)}$ declines after repeated violations.

Key Results. Figure 6 shows that γ primarily affects early adaptation speed. By iteration 5, all variants converge to ≈ 4 violations. For practical usage, $\gamma = 0.95$ is a good default.

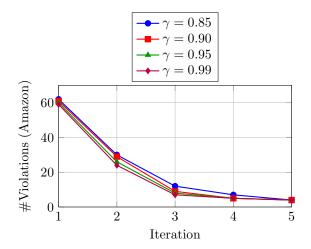


Figure 6: Ablation on γ : final violation counts on Amazon over 5 calibration rounds, $\lambda = 0.7$. All converge by iteration 5, but larger γ lowers the threshold more gradually.

.2.3 Neighborhood Similarity τ_{ρ}

Finally, we vary $\tau_{\rho} \in \{0.80, 0.85, 0.90, 0.95\}$ in constructing local fairness neighborhoods. Table 6 revisits MovieLens-1M at iteration 3.

$ au_{ ho}$	#Viol. \downarrow	NDCG@10 ↑	$\mathrm{CFR}\downarrow$
0.80	16	0.442	0.721
0.85	11	0.441	0.683
0.90	8	0.440	0.651
0.95	4	0.435	0.630

Table 6: Ablation on τ_{ρ} : #Violations, NDCG@10, and CFR for MovieLens-1M, iteration 3, $\lambda = 0.7$.

Observation. Higher τ_{ρ} detects narrower subgroups, reducing violations but slightly lowering NDCG@10. In the main text, $\tau_{\rho} = 0.90$ is used to balance fairness with top-N accuracy.

.3 Prompt Engineering Strategies

A distinctive aspect of our approach is updating system prompts with *concrete bias patterns* whenever a fairness violation is observed. Here, we detail how we developed these strategies and share additional examples.

.3.1 Design Variants for Prompt Updates

(1) Generic Warnings. Initially, we tried appending a short phrase such as:

<system>: "Avoid demographic-based biases."

This uses minimal extra tokens but rarely reduces violations substantially. The LLM generally fails to infer which *specific* biases to avoid.

(2) Negative Examples. A second approach enumerates specific *avoid* pairs from the FIFO buffer V. For instance:

```
<system>: "AVOID: (Gender=F) -> (Romance-Only)."
<user>: "I've watched 'The Godfather' ..."
```

This helps the LLM see explicit mistakes but may not generalize beyond those single examples.

(3) Explicit Patterns. We converge on enumerating a short list of repeated patterns that appear in V, e.g.:

```
<system>: "You must not rely on user demographics.
AVOID these biases:
   1) (Gender=F) -> (Romance-Only)
```

2) (Age=60) -> (Excluding new releases)

Focus on user history, item genre, and feedback."

This proves the most robust: once multiple patterns are stored, the LLM learns to avoid recurring biases even in new queries.

.3.2 Expanded Practical Examples

Example A: Age-Related Bias. A user scenario might be:

If the LLM recommended "light nostalgic comedy for seniors" ignoring the user's superhero interest, a violation is flagged. Our system might then append:

This directs the LLM to highlight user history, e.g. "Spider-Man: No Way Home."

Example B: Occupation Stereotyping. Another scenario:

```
<user>: "Occupation=Engineer,
  History=['The Matrix', 'Blade Runner']
  'Any new suggestions?'"
```

If the model incorrectly provides only highly technical documentaries—dismissing the user's interest in scifi—our system logs "(Occupation=Engineer) - >(Documentary Only)." The next prompt iteration might say: <system>: "Avoid: (Occupation=Engineer)->(Documentaries).
Focus on user interest in sci-fi or dystopian genres."
<user>: "Same user, which movies are similar to 'Blade Runner'?"

Thus, the LLM shifts to thematically relevant sci-fi recommendations.

Example C: Combined Patterns. If multiple biases arise simultaneously (e.g., "(Gender=F) -> (Romance-Only), (Age=60) -> (Kids Movies)"), the prompt enumerates both:

<system>: "You must not rely on these biases:

- 1) (Gender=F)->(Romance-Only)
- 2) (Age=60)->(Kid-friendly content)

Focus on user history plus item similarity."

<user>: "I've enjoyed 'Pulp Fiction' and 'Die Hard.'
Please suggest something new."

Through these examples, we find enumerating multiple "avoid" patterns consistently improves fairness outcomes with minimal manual overhead.

.3.3 Comparing Prompt Strategies

We measure final iteration violations on Amazon using $\lambda = 0.7$, $\gamma = 0.95$, $\tau_{\rho} = 0.90$. Table 7 shows that enumerating explicit patterns yields the fewest violations.

Strategy	#Viol.	CFR	NDCG@10
Generic Warnings	38	0.721	0.336
Negative Examples	24	0.661	0.334
Explicit Patterns	15	0.649	0.339

Table 7: Prompt Engineering Comparison (Iteration 3, Amazon).

Thus, explicit patterns effectively highlight repeated biases, prompting the LLM to generalize away from them in new queries.

.4 Additional Visualizations and Tables

.4.1 Iteration-Level Convergence for Prompt Variants

Figure 7 underscores that *explicit patterns* converge to the fewest violations by iteration 5, while *negative* examples remain slightly higher, and *generic warnings* do not remove repeated stereotypes as effectively.

.4.2 Calibration vs. No Calibration Revisited

As in the main text, Table 8 highlights how ignoring conformal calibration leads to more frequent biases. Ignoring calibration yields a significantly higher violation count and worsens CFR, confirming the value of data-driven threshold setting.

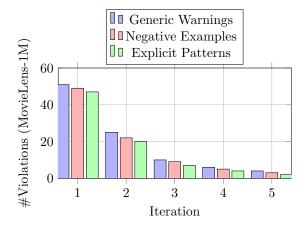


Figure 7: Convergence of fairness violations over 5 calibration rounds for different prompt strategies on MovieLens-1M, $\lambda = 0.7, \gamma = 0.95, \tau_{\rho} = 0.90$.

Method	#Viol.	CFR	NDCG@10	Recall@10
No Calib	42	0.716	0.336	0.298
Calib	15	0.649	0.339	0.305

Table 8: FACTOR (no calibration) vs. FACTER (with calibration) on Amazon, $\lambda = 0.7$ at iteration 3.

.5 Conclusion of Appendices

In summary, these detailed appendices reinforce and expand upon the main paper's conclusions:

- Theoretical insights: Theorems .1—.3 illustrate the robustness and convergence properties of our conformal fairness approach.
- Hyperparameter ablations: Varying λ , γ , and τ_{ρ} reveals predictable trade-offs between fairness (violation reduction) and recommendation accuracy (NDCG, recall). Our selected values ($\lambda = 0.7, \gamma = 0.95, \tau_{\rho} = 0.90$) offer strong overall performance.
- Prompt engineering best practices: Enumerating explicit bias patterns significantly reduces repeated violations, outperforming generic or single negative examples. Realistic scenarios (age-based or occupation-based biases) confirm that listing multiple "avoid" patterns improves generalization.

Together, these results demonstrate the flexibility and robustness of *FACTER* across varied settings, enabling black-box LLMs to adaptively mitigate demographic biases via conformal thresholding and refined prompt engineering.