Charged quantum Oppenheimer-Snyder model

S. Habib Mazharimousavi*

Department of Physics, Faculty of Arts and Sciences, Eastern Mediterranean University, Famagusta, North Cyprus via Mersin 10, Türkiye (Dated: February 18, 2025)

In the framework of loop quantum cosmology, particularly within the quantum Oppenheimer-Snyder model, the semiclassical Ashtekar-Pawlowski-Singh (APS) metric is associated with a static, spherically symmetric black hole that incorporates quantum effects derived from the APS metric. This quantum-corrected black hole can be interpreted as a modified Schwarzschild black hole, where the Schwarzschild metric function is adjusted by an additional term proportional to $\frac{M^2}{r^4}$, with r denoting the radial coordinate and M, the black hole mass. In this study, we show that such a quantum-mechanically modified black hole can arise in the context of nonlinear electrodynamics with either electric or magnetic charge. This charged, quantum-corrected solution is then matched to a dust ball of constant mass M_{APS} , governed by the APS metric, at a timelike thin-shell possessing nonzero mass m and electric charge Q or magnetic charge P. Analytically, it is demonstrated that the thin-shell oscillates around an equilibrium radius $r = R_{eq}$, which is expressed in terms of M_{APS} , m, and Q or P.

I. INTRODUCTION

Recently, in [1], a nonsingular black hole was proposed within the framework of loop quantum gravity that is called the quantum Oppenheimer-Snyder (qOS) model. This model of nonsingular black hole is composed of two distinct regions - the interior and the exterior - which are joined together at a timelike spherical interface hypersurface. The interior region is a dust ball described by the semiclassical Ashtekar-Pawlowski-Singh (APS) metric [2], with the line element

$$ds_{APS}^2 = -d\tau^2 + a\left(\tau\right)^2 \left\{ d\tilde{r}^2 + \tilde{r}^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\},\tag{1}$$

which applies to the region inside the hypersurface $\tilde{r} = \tilde{r}_0$. Here, the expansion parameter $a(\tau)$ satisfies the modified Friedmann equation given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),\tag{2}$$

where ρ is the energy density and ρ_c is the critical energy density. Additionally, the conservation of energy, $\nabla_{\mu}T^{\mu\nu} = 0$ with

$$T^{\mu\nu} = diag[\rho, 0, 0, 0],$$
 (3)

implies

$$\frac{d\rho}{d\tau} + \frac{3\dot{a}}{a}\rho = 0,\tag{4}$$

leading to

$$\rho = \frac{M}{\frac{4}{3}\pi\tilde{r}_0^3 a^3},\tag{5}$$

where M represents the total constant mass of the interior spacetime, which extends from r = 0 to $r = \tilde{r}_0 a$. In (2), ρ_c is the critical energy density such that in the classical regime (i.e., $\rho \leq \rho_c$), the modified Friedmann equation (2) reduces to the standard Friedmann equation. Conversely, in the quantum regime where ρ becomes comparable to ρ_c ,

^{*}Electronic address: habib.mazhari@emu.edu.tr

the term $\frac{\rho}{\rho_c}$ in (2) prevents the energy density from diverging. This can be verified by substituting Eq. (5) into Eq. (2) and solving for a. Although Eq. (2) is solvable, its structure indicates that ρ_c represents the maximum attainable energy density for ρ .

The exterior spacetime is described by a quantum-mechanically corrected black hole, with the line element

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{\alpha M^{2}}{r^{4}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{\alpha M^{2}}{r^{4}}} + r^{2}\left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right),\tag{6}$$

where

$$\alpha = \frac{3}{2\pi G \rho_c}. (7)$$

is a constant, and M is the ADM mass of the black hole. It can be shown that the two metrics, (1) and (6), smoothly match at the hypersurface $r = \tilde{r}_0 a$, where both the first and second fundamental forms are continuous across the hypersurface, provided that Eq. (7) holds. Let us add that an observer in the exterior region measures an energy-momentum tensor given by

$$T^{\nu}_{\mu} = \frac{3\alpha M^2}{r^6} [-1, -1, 2, 2],$$
 (8)

which satisfies all energy conditions for $\alpha>0$, except the dominant energy condition (DEC). To demonstrate this, we recall the definitions of the energy conditions: i) the null energy condition (NEC) requires $\rho+p_i\geq 0$, ii) the weak energy condition (WEC) requires $\rho\geq 0$, and $\rho+p_i\geq 0$, iii) the strong energy condition (SEC) requires $\rho+p_i\geq 0$, and $\rho+\sum p_i\geq 0$, and iv) the dominant energy condition (DEC) requires $\rho\geq |p_i|$. By defining the energy density and pressure components as $\rho=-T_t^t=\frac{3\alpha M^2}{r^6}$, $p_r=T_r^r=-\rho$, and $p_\theta=p_\phi=T_\theta^\theta=T_\phi^\phi=2\rho$, it is straightforward to verify that NEC, WEC, and SEC are satisfied, while DEC is not.

The black hole referenced in (6) is widely recognized in the literature as the quantum-mechanically corrected black hole or quantum Oppenheimer-Snyder (qOS) black hole. In [3] Cao et al. examined the stability of the inner horizon of the qOS black hole. Using both a test scalar field analysis and the generalized Dray-'t Hooft-Redmond relation, the authors demonstrate that the inner (Cauchy) horizon of this black hole is unstable, with flux and energy density diverging as free-falling observers approach it. This instability leads to mass inflation and the emergence of a null singularity, supporting the strong cosmic censorship hypothesis. In [4], Yang et al. investigate the effects of quantum corrections on black hole shadows and stability within the framework of loop quantum gravity. The study reveals that quantum corrections reduce the radius of black hole shadows and analyzes the stability of these quantum-corrected black holes by calculating quasinormal modes (QNMs). The results indicate that the quantum-corrected black holes are stable against scalar and vector perturbations, with the QNMs showing increased oscillation frequencies and decreased damping rates compared to classical Schwarzschild black holes. Recently, Dong et al. in [5] conducted a detailed investigation of this black hole, which introduces quantum corrections to overcome the limitations of the classical model, especially the issue of the Big Bang singularity. Their research delves into multiple facets of the modified black hole metric, such as its thermodynamic properties, Hawking radiation, quasi-normal modes, and the topological characteristics of photon spheres and thermodynamic potentials. In another recent study Gong et al. in [6] investigated the QNMs spectra of the qOS black hole using scalar perturbations. They found that the fundamental QNM modes of the qOS BH exhibit two key properties: (1) a nonmonotonic behavior concerning the quantum correction parameter for zero multipole number and (2) slower decay modes due to quantum gravity effects. Moreover, in [7] a novel cosmological model called the quantum Oppenheimer-Snyder-Swiss Cheese (qOSSC) model, which combines the qOS and quantum Swiss Cheese (qSC) models within the framework of loop quantum cosmology, was presented. According to [1], the qOS model describes a collapsing matter ball inside a deformed Schwarzschild black hole, while the qSC model represents a deformed Schwarzschild black hole surrounded by a quantum-modified Friedmann-Robertson-Walker (FRW) universe. Both models use the APS metric, a semiclassical solution in loop

In this article, we first focus on the qOS black hole in (6). Since this solution emerges from Einstein's equations with an accompanying energy-momentum tensor, we suggest that nonlinear electrodynamics (NED) could be the origin of the correction term. As we will demonstrate, the NED model that aligns with our objectives is the power Maxwell law (PML), introduced by Hassaine and Martínez in [8, 9]. This model has garnered attention across various fields of physics. For instance, black holes in Lovelock theory coupled with MPL were explored in [10], and wormholes in Einstein-Gauss-Bonnet gravity with PML were examined in [11]. Additionally, holographic superconductors within the framework of PML were investigated in [12, 13], and numerous studies have explored different aspects of the model [14–23] (and references therein).

After deriving the charged quantum-mechanically corrected black hole in the first part of the paper, we proceed to reconstruct the qOS model to include electric or magnetic charge. This new framework, referred to as the *charged* $qOS \ model \ (cqOS)$, involves matching the exterior and interior spacetimes at a massive and charged timelike spherical thin-shell.

II. QUANTUM-MECHANICALLY CORRECTED BLACK HOLE IN NED

We now consider the action in Einstein's theory of gravity minimally coupled to the PML NED, as proposed by Hassaine and Martínez in [5], which is expressed as

$$I = \int d^4 \sqrt{-g} \left[\frac{R}{2\kappa} + \beta \mathcal{F}^s \right], \tag{9}$$

where $\kappa = 8\pi G$, $\mathcal{F} = F_{\alpha\beta}F^{\alpha\beta}$ is the Maxwell invariant, β is a coupling constant, and $s \in \mathbb{R}$. The corresponding Einstein field equations are given by

$$G^{\nu}_{\mu} = \kappa T^{\nu}_{\mu},\tag{10}$$

where the energy-momentum tensor T^{ν}_{μ} takes the form

$$T^{\nu}_{\mu} = 4\beta \left(\frac{1}{4} \mathcal{F}^s \delta^{\nu}_{\mu} - s \mathcal{F}^{s-1} F_{\mu\lambda} F^{\nu\lambda} \right). \tag{11}$$

Additionally, with the electromagnetic two-form defined by

$$\mathbf{F} = \frac{1}{2} F_{\mu\nu} dx^{\mu} \wedge dx^{\nu} \tag{12}$$

the nonlinear Maxwell equation becomes

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}\frac{\partial}{\partial\mathcal{F}}\left(\beta\mathcal{F}^{s}\right)F^{\mu\nu}\right)=0. \tag{13}$$

Our objective is to determine the value of s in equation (11) such that the energy-momentum tensor aligns with equation (8). To achieve this, we begin by considering a pure electric field described by

$$\mathbf{F} = E(r) dt \wedge dr, \tag{14}$$

in conjunction with the line element

$$ds^{2} = -f(r) dt^{2} + \frac{dr^{2}}{f(r)} + r^{2} \left(d\theta^{2} + \sin^{2}\theta d\phi^{2} \right).$$
 (15)

From this, we derive

$$\mathcal{F} = -2E^2. \tag{16}$$

To ensure the action is physically meaningful, we redefine $\beta = (-1)^s \xi$, so that $\beta \mathcal{F}^s$ becomes $\xi (-\mathcal{F})^s$, where $\xi \in \mathbb{R}$. Furthermore, Maxwell's equation (13) leads to

$$E = \frac{C}{r^{\frac{2}{2s-1}}},\tag{17}$$

where C is an integration constant. Substituting Eq. (17) into Eq. (11), we obtain

$$T^{\nu}_{\mu} = (2)^{s} \xi \frac{C^{2s}}{r^{\frac{4s}{2s-1}}} diag \left(1 - 2s, 1 - 2s, 1, 1\right). \tag{18}$$

By setting $\frac{4s}{2s-1} = 6$, we find $s = \frac{3}{4}$, and thus

$$T^{\nu}_{\mu} = \xi \frac{C^{3/2}}{\sqrt[4]{2}r^6} diag(-1, -1, 2, 2).$$
 (19)

By comparing equations (19) and (8), we deduce that

$$3\alpha M^2 \equiv \xi \frac{C^{3/2}}{\sqrt[4]{2}}.\tag{20}$$

Consequently, the metric function for the electrically charged black hole solution is derived as

$$f(r) = 1 - \frac{2M}{r} + \frac{\xi C^{3/2}}{3\sqrt[4]{2}r^4}.$$
 (21)

In (21), M represents the ADM mass of the black hole as measured by an observer at $r \to \infty$. Additionally, the integration constant C is related to the electric charge of the black hole, which can be determined using Gauss's law

$$q = \frac{1}{4\pi} \int_{S^2} \frac{\partial}{\partial \mathcal{F}} \left(\xi \left(-\mathcal{F} \right)^s \right) F^{tr} r^2 \sin\theta d\theta d\phi = s\xi 2^{s-1} C^{2s-1}, \tag{22}$$

where $s=\frac{3}{4}.$ Hence, we find $C=\frac{16\sqrt{2}}{9}\frac{Q^2}{\xi^2},$ leading to the metric function

$$f(r) = 1 - \frac{2M}{r} + \frac{3\alpha Q^2}{r^4},\tag{23}$$

where $\alpha = \frac{2^{13/2}}{3^5} \xi^2$. In this reformulated metric function, the electric charge Q directly assumes the role of the mass M in (9), while the coupling constant ξ mimics the quantum correction parameter α .

After determining s in equation (9) for a purely electric NED, we now seek s for a purely magnetic electromagnetic field. To this end, we assume

$$\mathbf{F} = P\sin\theta d\theta \wedge d\phi \tag{24}$$

which ensures that both the nonlinear Maxwell equation and the Bianchi identity are trivially satisfied. Here, the constant parameter P represents the magnetic charge, and the radial magnetic field is given by $B = \frac{P}{r^2}$. Similar to the purely electric case, we redefine $\beta = -\xi$, with $\xi \in \mathbb{R}$. The Maxwell invariant is then calculated as

$$\mathcal{F} = \frac{2P^2}{r^4} \tag{25}$$

and the corresponding energy-momentum tensor becomes

$$T^{\nu}_{\mu} = \xi \frac{2^{s} P^{2s}}{r^{4s}} diag\left(-1, -1, -1 + 2s, -1 + 2s\right). \tag{26}$$

By setting 4s = 6, we obtain $s = \frac{3}{2}$, and thus equation (26) simplifies to

$$T^{\nu}_{\mu} = \frac{2^{3/2} \xi P^3}{r^6} diag(-1, -1, 2, 2). \tag{27}$$

Comparing equations (27) and (8) yields

$$2^{3/2}\xi P^3 = 3\alpha M^2,\tag{28}$$

and the metric function for the magnetic black hole solution is found to be

$$f(r) = 1 - \frac{2M}{r} + \frac{3\alpha P^2}{r^4}. (29)$$

Here, in analogy with the electric solution, we introduce $\alpha = \frac{2^{3/2}\xi P}{9}$. Furthermore, the magnetic charge corresponds to the mass through the relation $P^2 = M^2$.

To conclude this section, we note that both the electric and magnetic metric functions, given respectively in Eqs. (23) and (29), may admit no horizon, one double horizon, or two horizons, depending on the parameters' values. For the electric solution, the double horizon occurs when $M = M_c = \frac{2}{\sqrt{3}} \left(\alpha Q^2\right)^{1/4}$, with the double horizon located at $r_+ = \sqrt{3} \left(\alpha Q^2\right)^{1/4}$. For the magnetic solution, $M = M_c = \frac{2}{\sqrt{3}} \left(\alpha P^3\right)^{1/4}$, and the double horizon is located at $r_+ = \sqrt{3} \left(\alpha P^2\right)^{1/4}$. The subscript c in M_c denotes the critical mass, such that for $M < M_c$, no horizon forms, and the spacetime exhibits a naked singularity. On the other hand, for $M > M_c$, two horizons exist: the event horizon and the Cauchy horizon. In [5], the authors explored various aspects of the quantum-corrected black hole described by Eq. (6). These findings are equally applicable to the electric and magnetic black hole solutions presented in this study.

III. CQOS MODEL

In this section, we join the interior APS metric [2] to the exterior charged quantum-mechanically corrected black hole model. The interior line element is given by

$$ds_{APS}^2 = -d\tau^2 + a\left(\tau\right)^2 \left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right),\tag{30}$$

while the exterior is described by Eq. (15), where the metric function f(r) is defined by either (23) or (29). The two metrics are glued at a spherical timelike hypersurface defined by $R(\tau) = r = a(\tau) \tilde{r}_0$. The induced line elements on the inner and outer sides of the hypersurface are respectively

$$ds_{-}^{2} = -d\tau^{2} + a(\tau)^{2} \tilde{r}_{0}^{2} d\Omega_{-}^{2}, \tag{31}$$

and

$$ds_{+}^{2} = \left(-f(r)\dot{t}^{2} + \frac{\dot{R}(\tau)^{2}}{f(r)}\right)d\tau^{2} + R(\tau)^{2}d\Omega_{+}^{2}.$$
 (32)

Here, a dot denotes the derivative with respect to the proper time τ , and the subscripts \pm denote the inner (-) and outer (+) sides of the hypersurface. By imposing $-f(r)\dot{t}^2 + \frac{\dot{R}(\tau)^2}{f(r)} = -1$ and $d\Omega_-^2 = d\Omega_+^2$, we obtain

$$ds_{-}^{2} = ds_{+}^{2} = -d\tau^{2} + a(\tau)^{2} \tilde{r}_{0}^{2} d\Omega^{2}, \tag{33}$$

which satisfies the first Israel junction condition [24, 25]. The second Israel junction condition (with G=1) requires

$$\left[K_i^j\right] - \left[K\right]\delta_i^j = -8\pi S_i^j,\tag{34}$$

where $\left[K_i^j\right] = K_{i+}^j - K_{i-}^j$, with K_{i+}^j and K_{i-}^j representing the mixed extrinsic curvatures of the outer and inner sides of the hypersurface, respectively. Similarly, $\left[K\right] = \left[K_i^i\right]$ is the trace of $\left[K_i^j\right]$. Additionally, $S_i^j = diag\left[-\sigma, p, p\right]$ is the surface energy-momentum tensor. In [1], where the exterior metric is given by (6), $\sigma = p = 0$, and the black hole mass is determined by the interior energy-momentum tensor in (3), provided α satisfies Eq. (7). Our detailed calculations yield (see [26] and the references therein)

$$\sigma = \frac{1}{4\pi R} \left(1 - \sqrt{f(R) + \dot{R}(\tau)^2} \right), \tag{35}$$

and

$$p = \frac{1}{8\pi} \left(\frac{1}{R} \left(1 - \sqrt{f(R) + \dot{R}(\tau)^2} \right) - \frac{f'(R) + 2\ddot{R}}{2\sqrt{f(R) + \dot{R}(\tau)^2}} \right).$$
 (36)

Next, we set $M = M_{ADS} + E$ and $Q^2 = M_{ADS}^2 + q^2$ for the electrically charged black hole exterior spacetime, where

$$M_{ADS} = \frac{4}{3}\pi \tilde{r}_0^3 a^3 \rho, \tag{37}$$

is the total constant mass of the dust ball with radius $a(\tau)\tilde{r}_0$. Here E is the asymptotic energy of the thin shell, and q is part of its electric charge (see [27, 28]). Knowing that the dust energy density ρ satisfies (4) and α is given by Eq. (7), (35) and (36) become

$$\sigma = \frac{1}{4\pi R} \left(1 - \sqrt{1 - \frac{2E}{R} + \frac{3\alpha q^2}{R^4}} \right),\tag{38}$$

and

$$p = \frac{1}{8\pi} \left(\frac{1}{R} \left(1 - \sqrt{1 - \frac{2E}{R} + \frac{3\alpha q^2}{R^4}} \right) - \frac{\frac{2E}{R^2} - \frac{12\alpha q^2}{R^5}}{2\sqrt{1 - \frac{2E}{R} + \frac{3\alpha q^2}{R^4}}} \right). \tag{39}$$

In this setup, the mass of the thin shell is given by

$$m = 4\pi R^2 \sigma = R \left(1 - \sqrt{1 - \frac{2E}{R} + \frac{3\alpha q^2}{R^4}} \right).$$
 (40)

This equation implies

$$E = m - \frac{m^2}{2R} + \frac{3\alpha q^2}{2R^3},\tag{41}$$

and after substituting $q^2 = Q^2 - M_{ADS}^2$, it becomes

$$E = m - \frac{m^2}{2R} - \frac{3\alpha M_{ADS}^2}{2R^3} + \frac{3\alpha Q^2}{2R^3}.$$

Here, the asymptotic energy consists of four terms: the actual mass, the gravitational energy of the shell, the gravitational energy of the dust ball, and the electromagnetic energy. By solving $\frac{dE}{dR} = 0$, we find the critical radius of the shell

$$R_{eq} = \frac{3\sqrt{\alpha}\sqrt{Q^2 - M_{ADS}^2}}{m}. (42)$$

At this critical radius, we find $\frac{d^2E}{dR^2} = \frac{m^5}{27\alpha^{3/2}q^3} > 0$, indicating that the asymptotic energy is minimized, and the thin shell is stable. Consequently, the shell's radius oscillates around this stable equilibrium radius without collapsing or evaporating. The stable equilibrium radius of the timelike spherical hypersurface (42) depends on the mass m and charge Q of the shell, as well as the constant mass of the dust ball M_{ADS} , and its existence relies on the condition $Q > M_{ADS}$. We note that this cqOS model reduces to its charge-neutral version, qOS, introduced in [1]. Finally, for the magnetically charged case, one can substitute Q by P, and all results remain valid.

IV. CONCLUSION

In this study, we proposed a potential origin for the quantum correction of the Schwarzschild black hole. This quantum-corrected black hole, previously utilized by Lewandowski in [1] to construct the regular qOS black hole model, is characterized by the black hole mass M and a correction parameter α . When $\alpha \to 0$, the model reverts to the standard Schwarzschild black hole. By employing the NED framework developed by Hassaine and Martínez, known as the PML, we introduced two possible sources for the correction term in the form of electric or magnetic charges. These correspond to the PML NED model with $s = \frac{3}{4}$ and $\frac{3}{2}$, respectively. Additionally, we joined this charged black hole to an interior spacetime represented by a dust ball governed by the APS metric. The two spacetimes are joined at a timelike spherical hypersurface with mass m and electric charge Q or magnetic charge P. By minimizing the asymptotic energy of the spacetime, we derived a stable equilibrium radius R_{eq} , around which the thin shell oscillates. This ensures the stability of the entire spacetime, preventing collapse. The existence of this stable equilibrium radius requires $Q > M_{ADS}$ for the electrically charged case and $P > M_{ADS}$ for the magnetically charged thin shell.

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