COSC 2123/1285 Algorithms and Analysis
Tutorial
Maths and Algorithm Analysis Workshop

Questions

1 Simplify the following summation:

$$\sum_{j=2}^{n+1} 10j$$

Answer:

$$\begin{aligned}
\sum_{j=2}^{n+1} 10j &= 10\sum_{j=2}^{n+1} j \\
&= 10[\sum_{j=2}^{n+1} j + 1 - 1] \\
&= 10[\sum_{j=0}^{n+1} j - 1] \\
&= 10[\sum_{j=0}^{n} j + (n+1) - 1] \\
&= 10[\sum_{j=0}^{n} j + n] \\
&= 10[\frac{n(n+1)}{2} + n] \\
&= 10[\frac{n^2 + 3n)}{2}] \\
&= 5n^2 + 15n
\end{aligned}$$

2 Consider the following recurrence relation. Simplify it to an expression of n variable only, i.e., no recurrence terms in your final expression.

$$C(n) = C(n-1) + 2,\ C(3) = 1$$

Answer:

$$\begin{aligned}
C(n) &= C(n-1) + 2 \\
&= C(n-2) + 2 + 2 \\
&= C(n-3) + 2 + 2 + 2 \\
&= C(n-3) + 3 * 2 \\
&= \dots \\
&= C(n-i) + i \cdot 2 \\
&= \dots \\
&= C(3) + (n-3) \cdot 2 \\
&= 1 + 2n - 6 \\
&= 2n - 5
\end{aligned}$$

3 Consider the following mystery function:

Algorithm 1 RecurMystery($A[0 \dots n-1]$)

Input: an array A of size n
Output: T

if $n == 1$ **then**
    return 1
end if
$T_1 = \text{RecurMystery}(A[0 \dots n-2])$
$T = T_1 + T_1$
return T

a) What is this function computing?

Answer: Setup a recurrence for this. Let the mystery function be denoted by $F(n)$. Each interation the output of $F(n)$ is twice the output of $F(n-1)$. Therefore the recurrence is:

$$F(n) = 2F(n-1),\ F(1) = 1$$

Solving this (this is not examinable, but for your interest):

$$\begin{aligned} F(n) &= 2F(n-1) \\ &= 2^2F(n-2) \\ &= 2^3F(n-3) \\ &= \dots \\ &= 2^iF(n-i) \\ &= \dots \\ &= 2^{n-1}F(1) \\ &= 2^{n-1} \end{aligned}$$

Hence this function calculates 2^{n-1}.

b) What is the basic operation in this recursive algorithm?

Answer: Addition or comparison. Assignment is possible, but addition is generally more expensive than assignment, and also the 2 assignments can be collapsed to one line and become one assignment.

c) Write the recurrence relation for the number of basic operations executed by this algorithm, including the base/termination case.

Answer: Using addition:

$$C(n) = C(n-1) + 1,\ C(1) = 0$$

d) Simplify the recurrence relation.

Answer: Solving this:

$$\begin{aligned}
C(n) &= C(n-1) + 1 \\
&= [C(n-2) + 1] + 1 \\
&= C(n-2) + 2 \\
&= [C(n-3) + 1] + 2 \\
&= C(n-3) + 3 \\
&= \dots \\
&= C(n-i) + i \\
&= \dots \\
&= C(1) + (n-1) \\
&= 0 + n - 1 \\
&= n - 1
\end{aligned}$$