COSC1285/2123: Algorithms & Analysis

Greedy Techniques

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Lecture 9

Overview

Levitin – The design and analysis of algorithms

This week we will be covering the material from Chapter 9.

Learning outcomes:

- Understand and be able to apply the greedy approach to solving problems.
- Examples:
  - spanning tree – Prim's algorithm
  - spanning tree – Kruskal's algorithm
  - single source shortest-path – Dijkstra's algorithm
  - data compression

Outline

1. Overview
2. Prim's Algorithm
3. Kruskal's Algorithm
4. Dijkstra's Algorithm
5. Data Compression
6. Summary

Overview

Greedy Algorithms build up a solution piece by piece, always choosing the next piece that offers the most immediate and obvious benefit.

Sometimes such an approach can lead to an inferior solution, but in other cases it can lead to a simple and optimal solution.

Overview

Minimum Spanning Tree

Spanning Tree Problem

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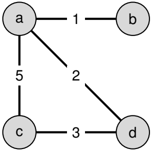
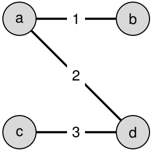
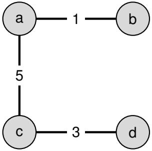


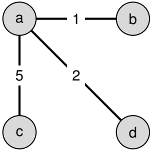
Graph



$w(T_1) = 6$



$w(T_2) = 9$



$w(T_3) = 8$

Applications of Minimum Spanning Tree

- Designing networks (phones, computers etc): Want to connect up a series of offices with telephone or wired lines, but want to minimise cost.
- Approximate solutions to hard problems: travelling salesman
- Generation of perfect mazes

Prim's Algorithm – Sketch

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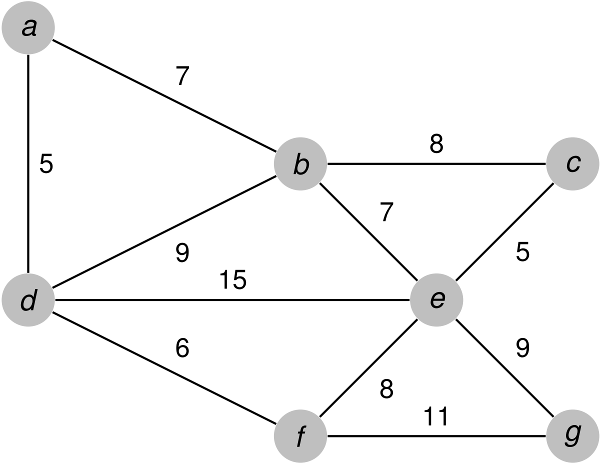
- Use a min priority queue to quickly find this neighbouring vertex with minimum edge weight (in literature, the neighbour set is sometimes called the frontier set).
- When adding, we may need to update the smallest edge weight to a vertex in neighbour set as there may be a smallest edge weight from updated tree to new neighbour set.

Prim's Algorithm – Example



$V_T = \{\}$, $PQ = \{\}$

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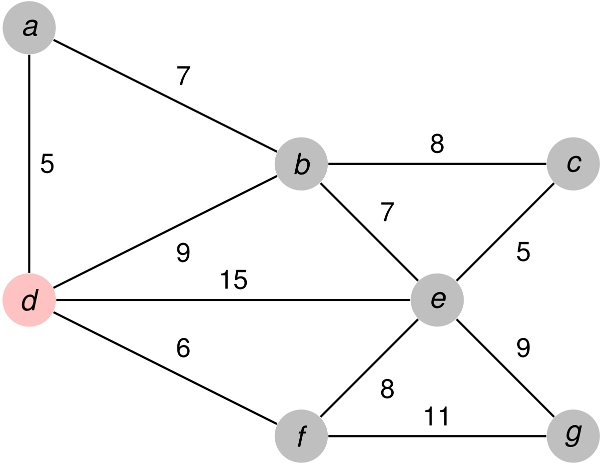

$V_T = \{d\}$, $PQ = \{(a, 5), (f, 6), (b, 9), (e, 15)\}$

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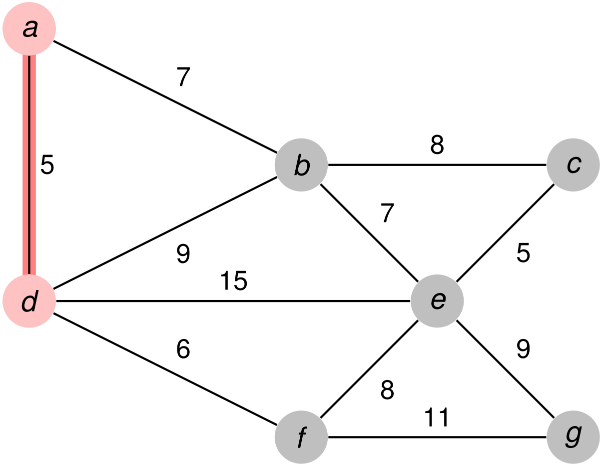

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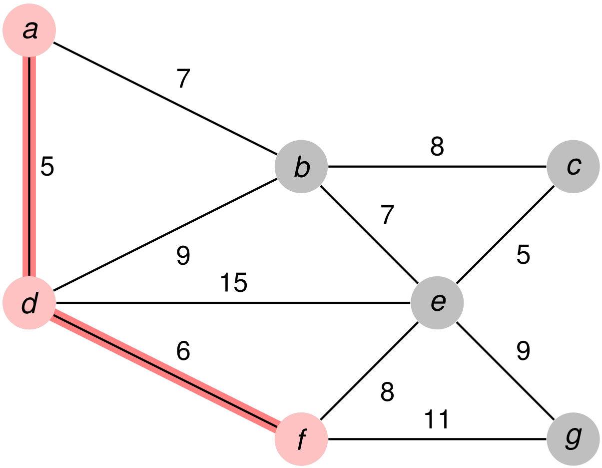

$V_T = \{d, a, f\}$, $PQ = \{(b, 7), (e, 8), (g, 11), (e, 15)\}$

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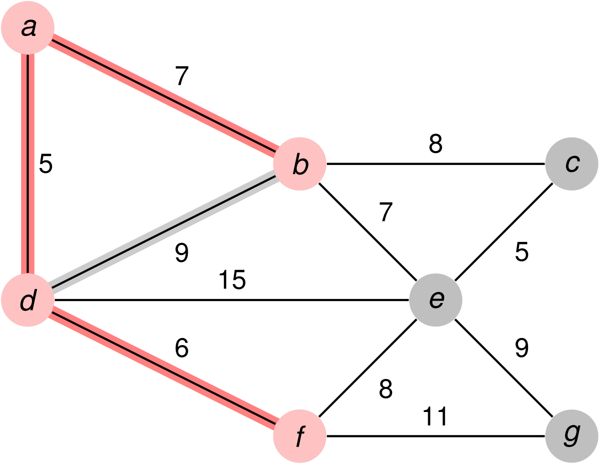

$V_T = \{d, a, f, b\}$, $PQ = \{(e, 7), (e, 8), (c, 8), (g, 11)\}$

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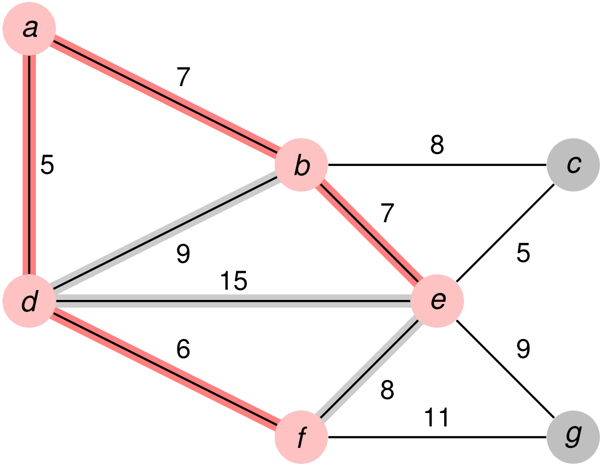

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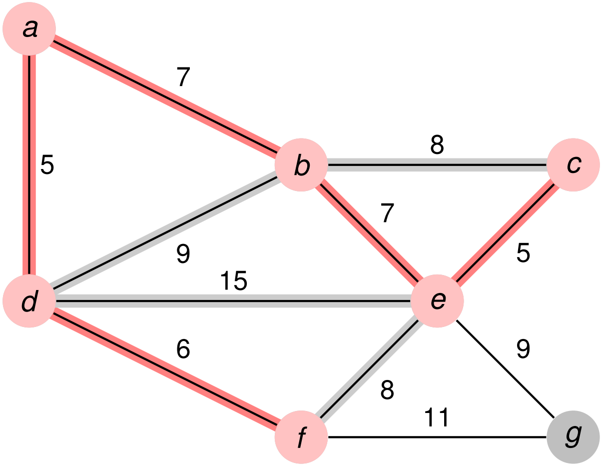

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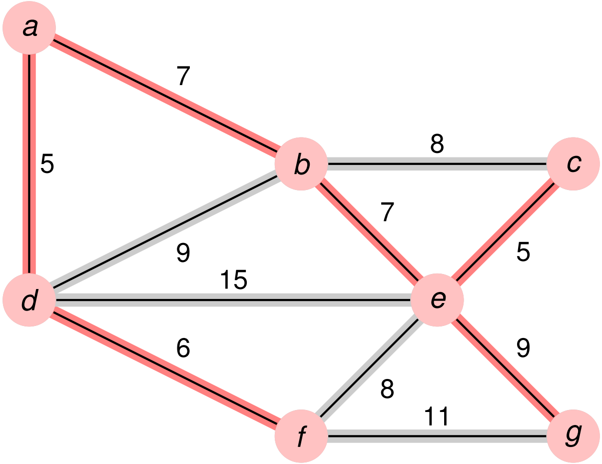

$V_T = \{d, a, f, b, e, c, \textcolor{red}{g}\}$, $PQ = \{\}$

Prim's Algorithm – Summary

- The algorithm always gives the optimal solution, regardless of where you start. See Levitin Chp 9.1 for the proof.

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- The algorithm always gives the optimal solution, regardless of where you start. See Levitin Chp 9.1 for the proof.
- The efficiency of the algorithm using a min-heap and an adjacency list is $O(|E| \log |V|)$.
- The efficiency of the algorithm using a Fibonacci heap and an adjacency list is $O(|E| + |V| \log |V|)$.

Overview

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Krushal's algorithm is an alternative method to finding minimum spanning trees. (We will explore why have two algorithms at end of this section)

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Note:

- To be fast in edge selection, we initially sort all edges from smallest to largest by weight.
- Note that the nodes are not always connected in the intermediate stages of the algorithm.

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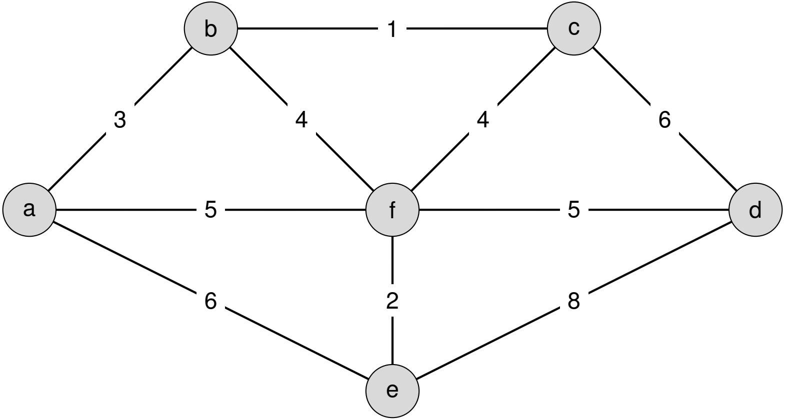

All Edges :	bc	ef	ab	bf	cf	af	df	ae	cd	de
	1	2	3	4	4	5	5	6	6	8
Tree Edges :	$\emptyset$									
	$\emptyset$									

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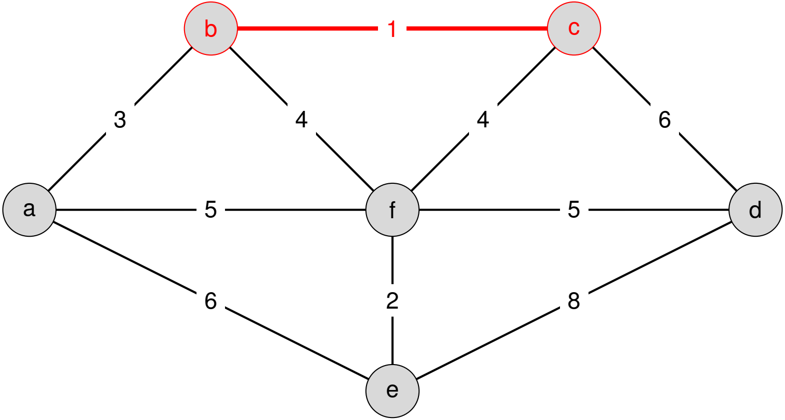

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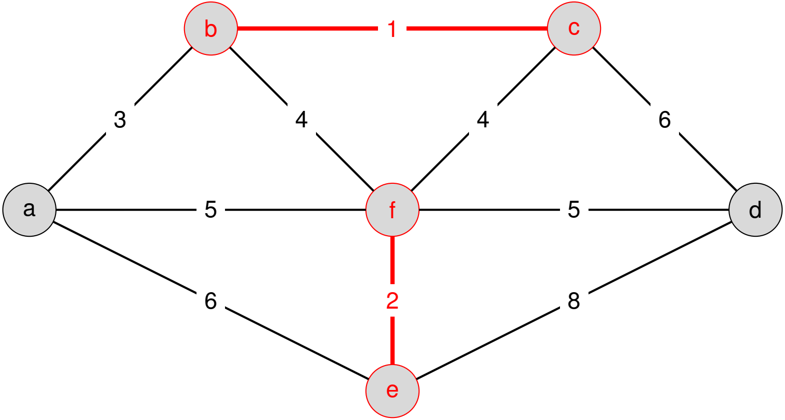

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	1									

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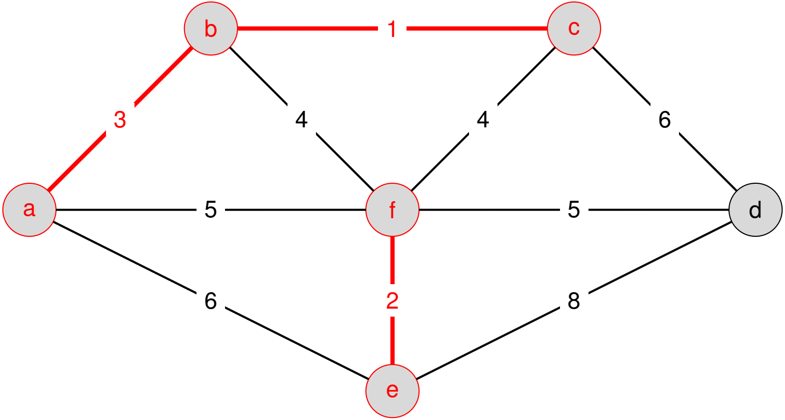

All Edges :	bc	ef	ab	bf	cf	af	df	ae	cd	de
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Tree Edges :	bc	ef								
	1	2								

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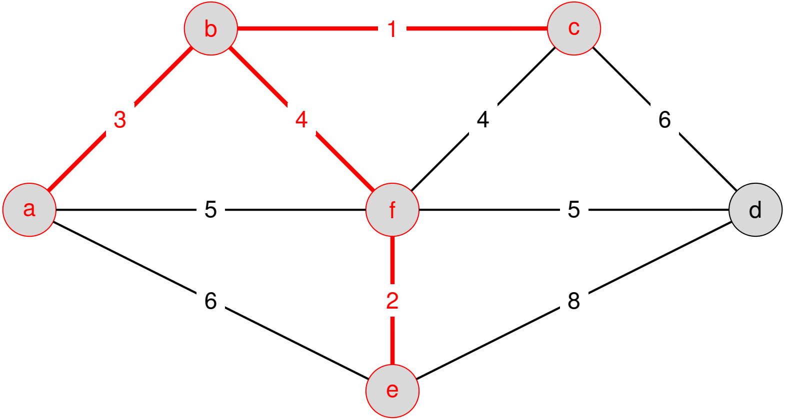

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	1	2	3							

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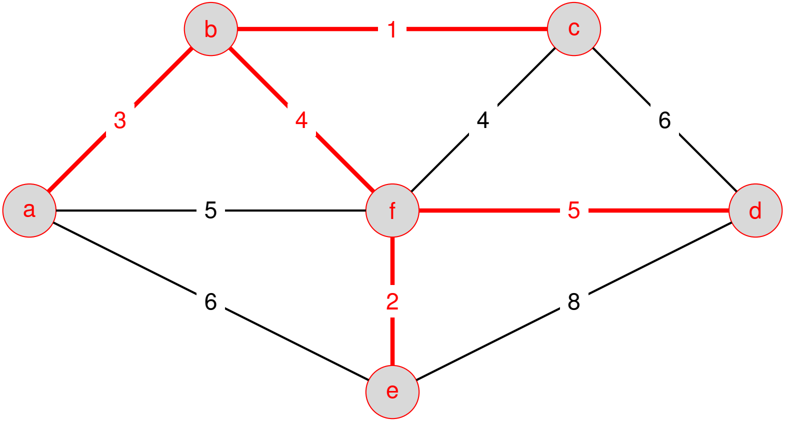

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Tree Edges :	bc	ef	ab	bf						
	1	2	3	4						

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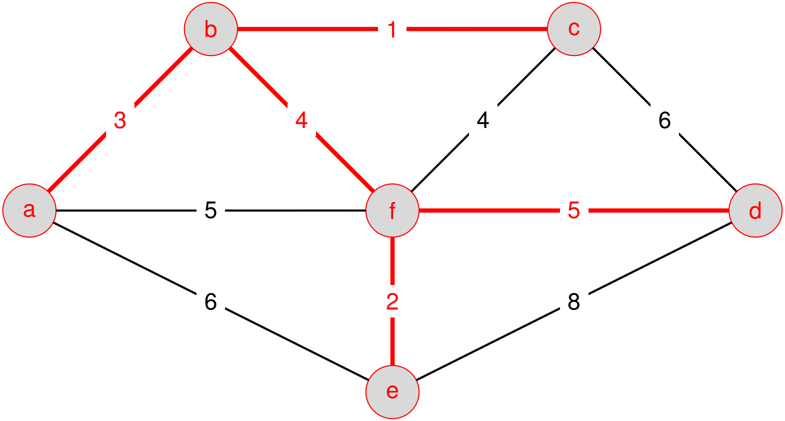

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	1	2	3	4	5					

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- If do not have to sort edges, runs in $O(|E| \log |V|)$.
- Note if graph is sparse, $|E|$ same order of magnitude as $|V|$, than Krushal's algorithm can be faster than Prim's (recall $O(|E| + |V| \log |V|)$). Otherwise, Prim's algorithm can be faster.

Minecraft Maze Generation

https://www.youtube.com/watch?v=5mnOClCO_9o

Overview

Shortest Paths in Graphs

Problem

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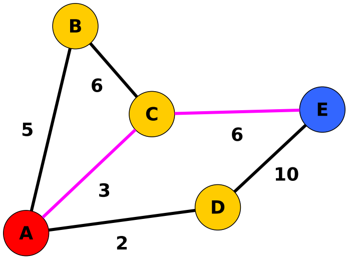

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Idea:

- At all times, we maintain our best estimate of the shortest-path distances from source vertex to all other vertices.

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- Initially we do not know, so all these distance estimates are ∞.
- But as the algorithm explores the graph, we update our estimates, which converges to the true shortest path distance.

Dijkstra's Algorithm – Sketch

Maintain a set S of vertices whose final shortest-path weights from the source s have already been determined.

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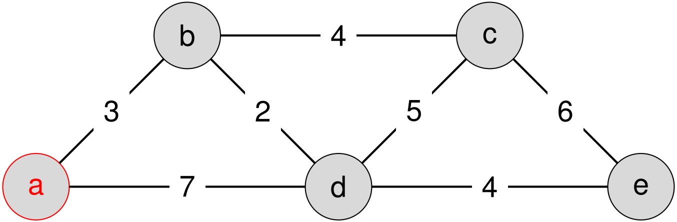
5. Repeat from step 2, until all vertices have been added to S.

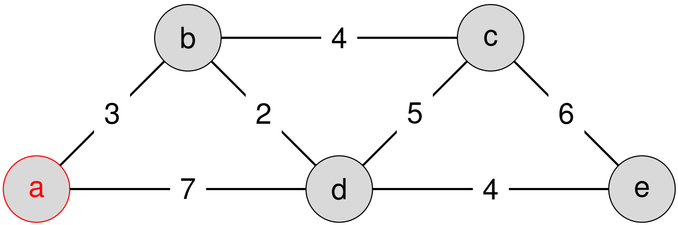
Dijkstra’s Algorithm – Example



a(a,0) b(-,∞) c(-,∞) d(-,∞) e(-,∞)

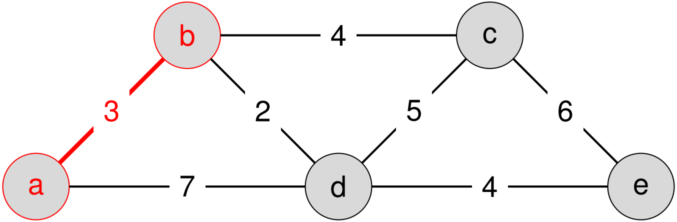
S = { }

Dijkstra's Algorithm – Example



b(a,3) c(-,∞) d(a,7) e(-,∞)

S = {a(a,0)}



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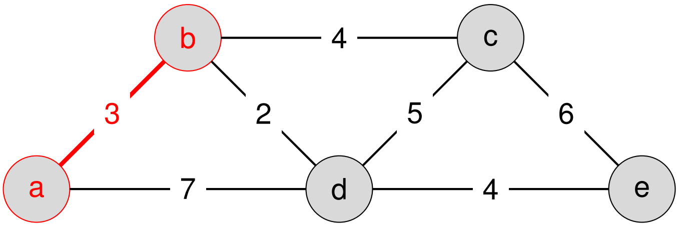
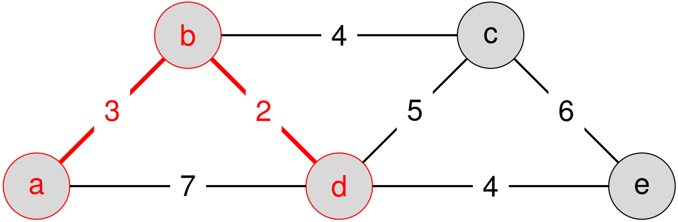


c(b,3 + 4) d(b,3 + 2) e(-,∞)

S = {a(a,0), b(a,3)}

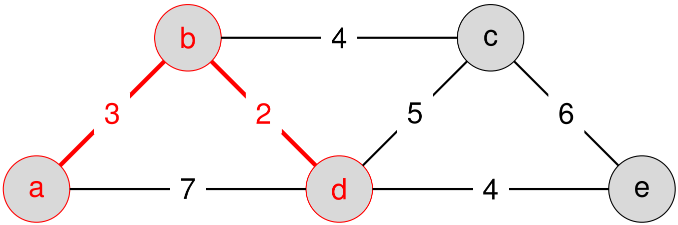
Dijkstra's Algorithm – Example



c(b,7) d(b,5) e(-,∞)

S = {a(a,0), b(a,3)}

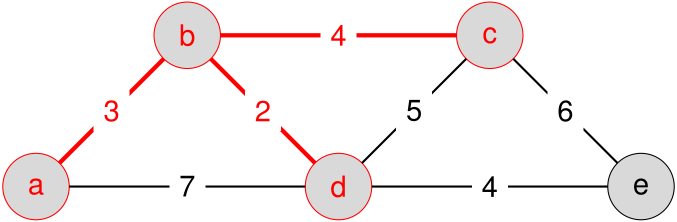
Dijkstra's Algorithm – Example



c(b,7) e(d,5+4)

S = {a(a,0), b(a,3), d(b,5)}

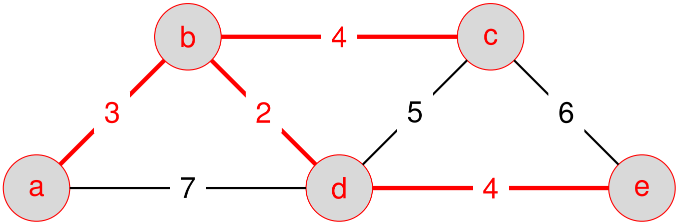
Dijkstra's Algorithm – Example



e(d,9)

S = {a(a,0), b(a,3), d(b,5), c(b,7)}

Dijkstra's Algorithm – Example



S = {a(a,0), b(a,3), d(b,5), c(b,7), e(d,9)}

Dijkstra's Algorithm – Example

So, we have the following distances from vertex *a*:

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a(a,0)   b(a,3)   d(b,5)   c(b,7)   e(d,9)

Which gives the following shortest paths:

Length	Path
3	a - b
5	a - b - d
7	a - b - c
9	a - b - d - e

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- Dijkstra's algorithm is guaranteed to always return the optimal solution. This is not necessarily true for all greedy algorithms.
- If we use an adjacency list and a min-heap, the algorithm runs in $\Theta(|E| \log |V|)$ time.

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What is data compression?

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The three principle components of a compression system: modelling, probability estimation, and coding.

Fixed Length Codes

The simplest encoding/decoding approach is to create a mapping from source alphabet to strings (codewords), where these codewords are fixed in length.

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Example (Fixed Length Coding)

Given $\mathcal{S} = \{a, c, g, t\}$, $\Sigma = \{0, 1\}$, and the encoding scheme,

$$\begin{aligned} a &\mapsto 00, \\ c &\mapsto 01, \\ g &\mapsto 10, \\ t &\mapsto 11, \end{aligned}$$

then $\phi(gattaca) = 10001111000100$.

Fixed Length Codes – ASCII

ASCII

American Standard Code for Information Interchange is a fixed length character encoding scheme over an alphabet of 128 characters.

ASCII Code Chart

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

```
char* string = "AARDVARK"
```

A	A	R	D	V	A	R	K
0x41	0x41	0x52	0x44	0x56	0x41	0x52	0x4B
1000001	1000001	1010010	1000100	1010110	1000001	1010010	1001011

Variable Length Codes

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- Why? The frequency of appearance of each member of the source alphabet may not be uniformly distributed.
- Consider the letters `e` and `z` in natural language text, and using the same length codewords to represent both.
- e.g., "zee"
- Using ascii where each letter represented by 7 bits, this is 3 * 7 = 21 bits

Character Frequency

Character	Frequency	Probability
e	24,600,752	0.0880
t	18,443,242	0.0660
a	17,379,446	0.0621
⋮	⋮	⋮
j	671,765	0.0024
q	264,712	0.0009
z	186,802	0.0007

The frequency of appearance of characters from the English alphabet extracted from a 267 MB segment of SGML-tagged newspaper text drawn from the *WSJ* component of the TREC data set.

Variable Length Codes

Solution?

- A variable length code maps each member of a source alphabet to a codeword string, but the length of codewords is no longer fixed.
- E.g., use a shorter codeword for ’e’ and a larger one for ’z’.

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- “zee” (hypothetically use 2 bit code for ’e’ and ’10’ bit code for z, this is 10 + 2*2 = 14 bits)

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- "zee" (hypothetically use 2 bit code for 'e' and '10' bit code for z, this is 10 + 2*2 = 14 bits)
- However, not all possible variable length coding schemes are decodeable.

Variable Length Codes – Decoding

Symbol	a	b	c	d	e	f	g
Frequency	25	12	9	4	3	2	1

Symbol	Codeword	ℓ_i
a	0	1
b	1	1
c	00	2
d	01	2
e	10	2
f	11	3
g	110	3

Decode: `001010001000011001011`

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g	`110`	3

Decode: `0010100010000111001011`

Variable length codes must be chosen so text is uniqly decodeable.

Variable Length Codes – Prefix codes

Prefix Codes: Variable length codewords where no codeword is a prefix of any other codeword. Prefix codes are uniquely decodeable.

Symbol	Codeword	ℓ_i
a	0	1
b	100	3
c	110	3
d	111	3
e	1010	4

Huffman's Code – Sketch

Huffman Algorithm generates prefix codes that are optimal in the average number of bits per symbol.

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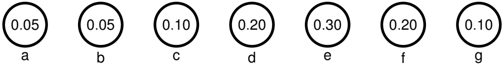
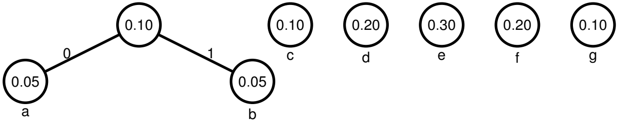
5. When only one candidate node remains, a tree has been formed, and codewords can be read from the edge labels of a tree.



Construct a Huffman Tree from the Alphabet
$\{(a, 0.05), (b, 0.05), (c, 0.1), (d, 0.2), (e, 0.3), (f, 0.2), (g, 0.1)\}$.



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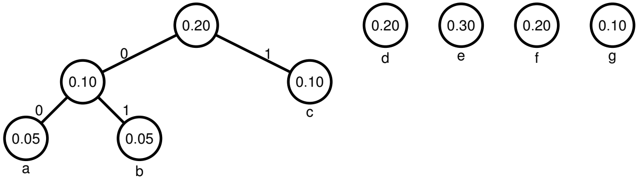

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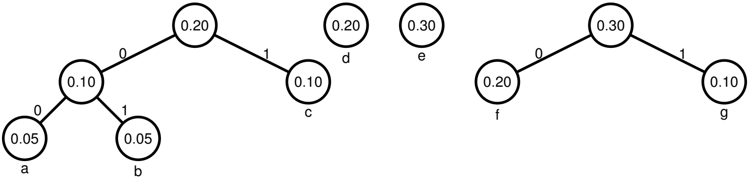
Huffman Trees – Example



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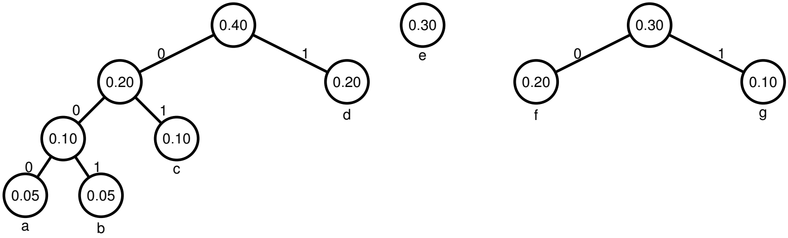

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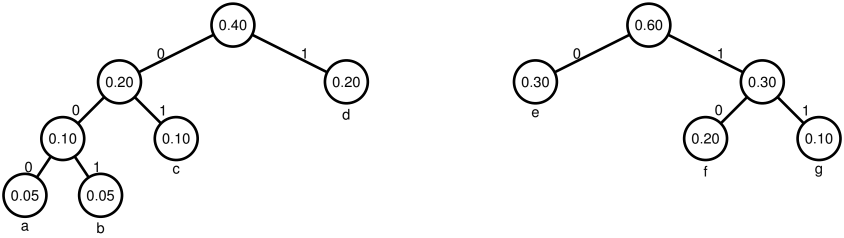

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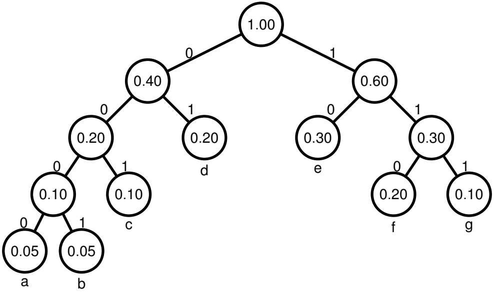

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Huffman Codes

Symbol	Codeword	ℓ_i
a	`0000`	4
b	`0001`	4
c	`001`	3
d	`01`	2
e	`10`	2
f	`110`	3
g	`111`	3

The Huffman Codes and the corresponding codeword lengths.

Huffman Codes

- This approach requires $\Theta(n \log n)$ time if a min heap (priority queue) is used to manage the set of candidates and their weights.
- If the input list is already sorted by their probabilities, then the codes can be constructed in $\Theta(n)$ time.

Summary

- Understand and be able to apply the greedy approach to solving problems.
- Examples:
  - spanning tree – Prim's algorithm
  - spanning tree – Kruskal's algorithm
  - single source shortest-path – Dijkstra's algoirthm
  - data compression