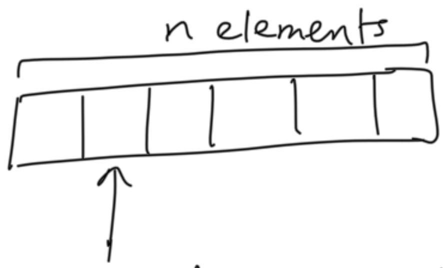
Average case

$Pr(\text{found}) = p$ $\qquad\qquad$ $Pr(!\text{found}) = 1-p$



$$Pr(\text{found at position } i, \text{ given key is in array}) = \frac{1}{n}$$

$$\therefore Pr(\text{found at position } i) = Pr(\text{found}) \times Pr(\text{found at position } i, \text{ given key in array})$$

$$= p \times \frac{1}{n} = p/n$$

Average number of comparisons

$$C_{avg}(n) = \underbrace{\sum}_{\text{sum over all possible outcomes}} (\text{number of comparisons for outcome } i) \times (\text{probability that outcome } i \text{ occurs})$$

$$= \underbrace{\sum_{i=1}^{n} \overbrace{C(i)}^{\#\text{of comparisons for finding at position } i} Pr(\text{found at } i)}_{\text{key in array outcome}} + \underbrace{C(n)\, Pr(!\text{found})}_{\text{key not in array outcome}}$$

$$\therefore C_{avg}(n) = C(1)\, Pr(\text{found at } 1)$$
$$+ C(2)\, Pr(\text{found at } 2)$$
$$+ \ldots$$
$$+ C(n)\, Pr(\text{found at } n)$$
$$+ C(n)\, Pr(!\text{found})$$

From sequential search algorithm, we know the # of comparisons when key is found at position i is

$$C(i) = i$$

$$\therefore C_{avg}(n) = 1 \times P/n + 2 \times P/n + \dots + n \times P/n + n(1-P)$$

$$= \frac{P}{n} \sum_{i=1}^{n} i + n(1-P)$$

$$= \frac{P}{n} \frac{(n+1)n}{2} + n(1-P)$$

[Applying R2 shown in lecture slide]

$$= \frac{P(n+1)}{2} + n(1-P)$$