Subgraph Query Generation with Fairness and Diversity Constraints

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***Abstract*—This paper studies the problem of subgraph query generation with guarantees on both diversity and group fairness. Given a query template (with parameterized search predicates) and a set of node groups in a graph, it is to compute a set of subgraph queries that instantiate the query template, and each query ensures diversified answers that meanwhile covers each group with desired number of nodes. Such need is evident in web and social search with fairness constraints, query optimization, and query benchmarking. We formalize a bi-criteria optimization problem that aims to find a Pareto optimal set of query instances in terms of diversity and fairness measures. We show the problem is in Δ_2^P and verify its hardness (NP-hard and fixed-parameter tractable). We provide (1) two efficient algorithms that approximate Pareto optimal set with ϵ-dominance relation that yield a bounded number of representative query instances, and (2) an online algorithm that progressively generates and maintains ϵ-Pareto set with fastshort? delay time. We experimentally verify that our algorithms can efficiently generate queries with desired diversity and coverage properties for targeted groups.**

I. Introduction

Subgraph queries have been routinely used in applications such as social search [25] and knowledge search [34]. Given a graph G and a subgraph query $Q(u_o)$ with a designated output node u_o, it computes a set of nodes (matches) of u_o in G. A number of algorithms [7] have been developed to process subgraph queries in terms of subgraph isomorphism or its approximate variants [34].

The emerging need for data systems that requires both result diversity and fairness [6], [2], [27], [13] poses new challenges to graph querying. In such scenarios, queries are expected to return diversified matches that meanwhile ensure a required coverage of designated groups (node sets) of interests from G. Such groups may refer to the population of vulnerable social groups which are characterized in terms of sensitive attributes (*e.g.,* gender, race, professions) [16], relevant articles yet with diversified labels for Web exploration [1] and recommendation engines [17], or designated columns for query benchmark [4].

Consider the following real-world search scenarios.

Example 1: Talent search. A talent search over a professional network G [17] finds strong candidates with desired skills. Each node in G denotes a user with attributes such as *title*, *skill*, *profession*, or an organization with attributes such as number of *employees*. Each edge indicates affiliation (worksAt) of a user or recommendation (recommend) between users. A recruiter issues a graph search query q_1 (illustrated in Fig. 1) to find directors u_o who have expertise in managing IT business, and moreover, recommended by at least two users from large companies. In our test (Section VI), this query returns a set of qualified candidates $q(G)$, yet with a skewed distribution of 375 male users and 173 female users.

The recruiter pursues a desired gender distribution and diversity of the candidates, and wonders how to revise the search such that (1) the new answer can properly cover $q(G)$ with a equal number of male and female candidates (*e.g.,* both with 200 candidates; a case of "Equal opportunity" [17]); and (2) the candidates are also more diversified in their majors. A more desirable query q_2 can be suggested, which finds 202 male and 198 female candidates that spans 10 majors. The difference between q_1 and q_2 suggest that a relaxed condition on recommendation (removing the edge from u_1 to u_3) and a relaxation to recommends from smaller business (reducing '1000' employees to '500' employees) help to achieve the desired answer with proper coverage of the gender groups. □

The above example highlights the need to suggest subgraph queries with diversified answers that can meanwhile cover each of a set of designated groups with desired cardinalities. Moreover, there may be multiple choices, and representative queries should be suggested.

Example 2: Continuing the search, q_3 and q_4 can also be suggested, which specifies those recommenders from large business, but meanwhile for q_3 the recommanders relax the years of experiences of u_1 from more than 10 years to more than 6 years and refine the the years of experiences of u_2 from more than 10 years to nore than 12 years. For q_4, recommanders just relax u_2 from more than 10 years to 5. These query returns more skewed results over gender group than q_2, yet finds candidates from more than 30 majors. As both q_2, q_3 and q_4 achieve more desired answers than q_1, and each one finds a best answer in terms of diversity or group

fairness, respectively, it is desirable to suggest all these queries for different user preference. □

This calls for efficient generation of representative queries with desirable guarantees on diversity and coverage over specified groups of interests. The need is evident in social search and recommendation with group fairness [17], workload generation for query benchmark [4], and query optimization [22].

We study a new problem called *subgraph query generation with diversity and fairness constraints* (FairSQG), which has a general form below:

- **Input:** graph G, an initial query (template) $Q(u_o)$, and a set of groups $\mathcal{P}$, where each group $P_i \in \mathcal{P}$ is associated with a coverage constraint c_i ($c_i \leq |P_i|$);
- **Output:** a set of subgraph queries $\mathcal{Q}$ obtained by revising $Q(u_o)$; each query can retrieve a set of diversified matches ("*Diversity*") from G that also cover each group P_i with desired cardinality c_i ("*Group fairness*").

Several methods have been developed to generate graph queries that lead to desired answers. Notable examples include query suggestion with diversified answers [26], [19], coverage of similar counterparts of "examples" [32], [28], or cardinality constraints on output sizes [23], [3]. These approaches are designed to revise queries towards specific properties rather than ensuring both group fairness and answer diversity, thus cannot be directly applied to our problem.

Contributions & Organization. This paper formally analyzes subgraph query generation problem with group fairness constraints. We propose both feasible approximation scheme as well as practical exact algorithms for large graphs. We refer to subgraph query simply as "query" in the rest of the paper.

A Bi-objective formalization. We provide a practical formalization of the FairSQG problem (Section III). (1) We introduce a class of *query templates* (denoted as $Q(u_o)$). A template carries variables defined on search predicates and edges yet to be instantiated at processing time. Query generation yields value binding to the variables to produce a set of query instances. (2) We introduce diversity and fairness measures to measure the quality of an instance. Despite the need of optimizing both, they may come in conflict: a single optimal instance may not exist. On the other hand, a complete Pareto set is of substantial size for users to inspect. Instead, we introduce a bi-objective optimization problem to computes an ϵ-Pareto set of instances based on a query dominance relation. ϵ-Pareto set is a desirable subset approximation of the Pareto optimal set that strikes a balance between dominance relation and the number of instances to be returned.

We show FairSQG is solvable in Δ_2^P (a class of P^{NP} problems with an NPoracle) for templates with fixed variables, and show its hardness varies from PTIMEto NP-hard if Q has fixed size and variable sizes, and for templates without range variables. These results verify useful upper and lower bound results for query generation scenarios in practice.

Query generation with quality guarantees (Section IV). Generating an excessive number of instances in the Pareto set may not be desirable and practical. We first introduce an algorithm to approximate the exact Pareto set $\mathcal{Q}$ with quality guarantee controlled by an error bound ϵ. The algorithm ensures to find a subset of $\mathcal{Q}$, denoted as $\mathcal{Q}_\epsilon$, such that for each possible instance of Q_{u_o}, there is an instance in $\mathcal{Q}_\epsilon$ that ϵ-dominant it on both diversity and coverage with a factor no worse than ϵ. Better still, the algorithm ensures to return a bounded size of representative instances. We also introduce optimization strategies to reduce the cost of query generation.

Online generation (Section V). Our analysis show that one often needs to sacrifice query instance quality (ϵ) in trade for a smaller, representative set to inspect. We follow up with an online algorithm, which progressively constructs and incrementally maintains a ϵ-Pareto set with k instances and an ϵ as small as possible. The online algorithm uses a sliding window strategy to dynamically swap or replace queries, incrementally updates ϵ-Pareto set only when necessary, and incurs a small delay time.

Real-world evaluation and case analysis (Section VI). Using real-life graphs, we verify the effectiveness and efficiency of our algorithms (Section VI). We show that our algorithms can generate subgraph queries with both desired diversity and small errors in covering designated groups. These algorithms are also feasible. For example, it takes up to 570 seconds to produce instances with desired coverage in real-life graphs with $4.9M$ nodes and $45.6M$ edges. We illustrate the applications of our algorithms for *e.g.,* talent recommendation.

Related Work. We categorize the related work as follows.

Graph query suggestion. Several methods have been studied to suggest subgraph queries towards answers with various desired properties. Graph query by example [19] induces subgraph queries from subgraphs that contain similar nodes to specified examples. Diversified query suggestion [26] expands an initial query with new edges that can lead to relevant and diversified matches. Why-questions [32], [28] suggest queries with both relaxation and refinement operators towards exact or similar matches. These methods cope with a single objective or weighted combination of diversity, similarity or output size constraints on the entire match set. Bi-objective query suggestion in terms of both diversity and group fairness is not addressed by prior methods.

Set selection with group fairness. Subset selection with diversity and fairness constraints has been studied [33], [27]. Given a universal set and a set of groups (subsets), it computes a diverse subset that can cover each group with individual cardinality constraints. Approximation algorithms have been studied to generate subsets for max-sum and max-min diversification [27]. These methods study set coverage properties and cannot be directly used to suggest graph queries. Our formal analysis establishes the hardness of subgraph query suggestion with group fairness, which is a more general counterpart.

Skyline search. Multiobjective search such as skyline queries [9], [10] has been extensively studied. Our problem can be considered as computing a representative bi-objective

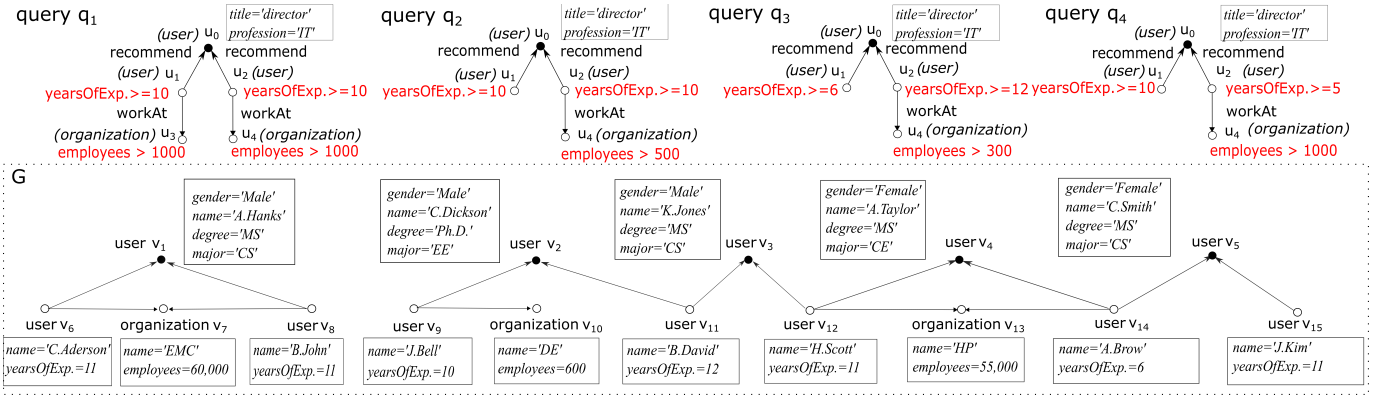


Fig. 1. Subgraph queries with Diversity and Group Fairness: Talent Search

skyline front in subgraph query space with diversity and coverage preferences. Existing skyline algorithms are optimized to compute Pareto optimal sets and their variants over a set of tuples. Our problem is more involved due to the expensive query processing cost in terms of subgraph isomorphism (an NP-hard problem). It is also not desirable to return a large Pareto set [9] for practical query generation scenarios. ϵ-Pareto set [29], [21] has been studied as a desirable approximation for Pareto optimal set. We advocate feasible query generation with such approximation, and introduce practical algorithms that can (1) efficiently generate and maintain ϵ-Pareto instance sets for large graphs, and (2) strike a balance between the solution quality and size.

Query workload generation. Query generation with output size constraints and distribution properties have been investigated for graph benchmarking and query optimization. [3] generates regular path queries from given schema that can output answers with required cardinalities when projected to pre-defined attributes. [22] generates SPARQL queries that can cover the answer of given queries with cheaper plans for multi-query optimization. In contrast to these work, (1) We study query generation under fairness constraints for arbitrary groups with guaranteed coverage requirement; and (2) We consider queries defined in subgraph isomorphism with search predicates, a more involved query class compared with regular path queries. Our algorithms can be readily applied to generate queries for benchmarking graph databases.

II. Graph, Query Templates and Instances

Graphs. We consider directed graphs $G = (V, E, L, T)$, where (1) V is a finite set of nodes, (2) $E \subseteq V \times V$ is a set of edges, (3) each node $v \in V$ (resp. edge $e \in E$) carries a label $L(v)$ (resp. $L(e)$); and (4) each node v carries a tuple $T(v) = <(A_1, a_1),\ldots,(A_n, a_n)>$, where each A_i $(i \in [1, n])$ is a distinct node attribute with a value a_i.

We denote the finite set of all the node attributes in G as $\mathcal{A}$. The *active domain* $\mathsf{adom}(A)$ of an attribute $A \in \mathcal{A}$ refers to the set of values of $v.A$ as the node v ranges over V.

Query Template. A *query template* (or simply "template") $Q(u_o)$ is a connected graph (V_Q, E_Q, L_Q, T_Q), where V_Q (resp. $E_Q \subseteq V_Q \times V_Q$) is a set of query nodes (resp. query edges). Each query node $u \in V_Q$ (resp. query edge $e \in V_E$) has a label $L_Q(u)$ (resp. $L_Q(e)$). Specifically, there is a designated *output node* $u_o \in V_Q$.

Variables. A template allows "placeholders" in search predicates that can be bound to specific values when executed. It extends parameterized queries [8] for graph query generation. We consider two types of variables in a template $Q(u_o)$. (a) For each node $u \in V_Q$, $T_Q(u)$ is a set of literals. A literal l is in the form of $u.A$ op x_l, where op is from $\{>, >=, =, <=, <\}$, and x_l is a *range variable* that can be assigned to a constant. (b) For each edge $e \in E_Q$, $T_Q(e)$ is a Boolean *edge variable* x_e (either '0' or '1'). The set of all the variables in $Q(u_o)$ is denoted as $\bar{X} = \bar{X}_L \cup \bar{X}_E$.

Query Instances. Given a template $Q(u_o)$, an *instantiation* of $Q(u_o)$ is a function I, such that for each variable $x \in \bar{X}$, $I(x)$ is either a constant or a wildcard '_'. A *query instance* (or simply "instance") $q(u_o)$ of $Q(u_o)$ induced by an instantiation I is a conntected graph (V_Q, E_q, L_Q, T_q) with the same V_Q, output node u_o and L_Q, and moreover,

- for each literal $l \in T_Q(u)$ in $Q(u_o)$, if $I(x_l)$ is a constant, then there is a literal $l = u.A$ op $I(x_l)$ in $T_q(u)$; and
- there is an edge $e \in E_Q$ if and only if (a) $I(x_e)$ = '1', and (b) e is in the same connected component of u_o.

An instance q of $Q(u_o)$ contains no variables but only literals and the edges in the connected component where u_o resides, induced by the constant binding from I. We denote the set of all the possible instances of $Q(u_o)$ as $\mathcal{I}(Q)$.

Matches. Given an instance $q(u_o)$ and a graph G, a matching from $q(u_o)$ to G is a function $h \subseteq V_q \times V$, where (1) for each node $u \in V_Q$, $L_Q(u) = L(h(u))$, and for each literal $u.A$ op c in L_q, $h(u).A$ op c; and (2) for each edge $e = (u, u')$ in $q(u_o)$, $h(e) = (h(u), h(u'))$ is an edge in G, and $L_Q(e) = L(h(e))$.

The *matches* of a query node u of $q(u_o)$ in G, denoted as $q(u, G)$, refers to the set of all the nodes in G that can match node u via a matching $h(u)$ from q to G. The *result* of q in G, denoted as $q(G)$, refers to the match set $q(u_o, G)$.

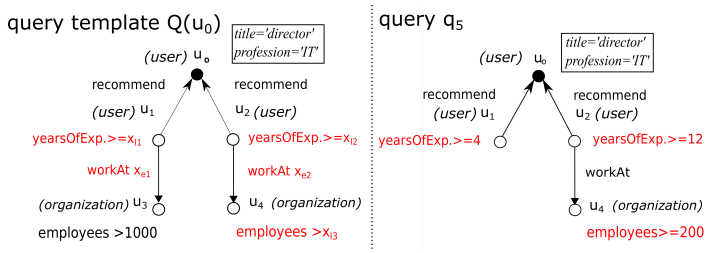


Fig. 2. Query Template and Query Instance

Example 3: A template $Q(u_o)$ that searches for directors in a graph G [17] is illustrated in Fig. 1. (1) $Q(u_o)$ has five variables $\bar{X} = \{x_{l1}, x_{l2}, x_{l3}, x_{e1}, x_{e2}\}$, with three range variables in literals u_1.yearsOfExp. $\geq x_{l1}$, u_2.yearsOfExp. $\geq x_{l2}$, and u_4.employees $\geq x_{l3}$ respectively, and edge variables x_{e1}, x_{e2}. (2) A corresponding instantiation $\{10, 10, 1000, '1', '1'\}$ (resp. $\{10, 10, 500, '0', '1'\}$, $\{10, 5, 1000, '0', '1'\}$, $\{6, 12, 300, '0', '1'\}$) of $\bar{X}$ induces an instance q_1 (resp. q_2 and q_3 and q_4) of $Q(u_o)$. (3) Given G, $q_1(G) = \{v_1\}$, $q_3(G) = \{v_1, v_2\}$, $q_2(G) = \{v_1, v_2, v_3\}$ and $q_4(G) = \{v_3, v_4, v_5\}$. □

Remarks. The instances are well-defined for a "partial" instantiation in which some variables are assigned a wildcard '_' ("don't care"). For such case, q is induced by removing corresponding parameterized predicates or edges to ensure valid $q(G)$. A user-defined "initial" query (*e.g.,* q_1 in Example 1) can be captured by a template with a partial instantiation.

III. Query Generation Problem

Given a template $Q(u_o)$, graph G and m disjoint node groups $\mathcal{P}$ in G, where each group $P_i \in \mathcal{P}$ has a cardinality constraint $c_i \in [0, |P_i|]$, the query generation problem aims to compute a set of instances $\mathcal{Q} \subseteq \mathcal{I}(Q)$ of Q with maximized diversity and required coverage properties.

A. Quality Measures

We consider two functions to quantify the "goodness" of instances in terms of diversity and fairness. For simplicity, (1) we define a constant $C = \sum_1^{|\mathcal{P}|} c_i$, and (2) we use $\bigcup(S)$ to denote $\bigcup_{s \in S} s$ for a set S.

Diversification. We consider Max-sum diversity as a natural objective for result diversification [18]. Given an instance q and G, the diversity of q is defined as:

$$\delta(q, G) = (1-\lambda) \sum_{v \in q(G)} r(u_o, v) + \frac{2\lambda}{|V_{u_o}| - 1} \sum_{v, v' \in q(G)} d(v, v')$$

where (1) $\lambda \in [0, 1]$ is a constant to balance relevance and diversity; (2) the function $r(u_o, v) \in [0, 1]$ (resp. $d(v, v') \in [0, 1]$) computes a relevance score between u_o and a match v (resp. difference between two matches v and v'). In practice, $d(v, v')$ can be the edit distance between tuples $T(v)$ and $T(v')$ [20], and $r(u_o, v)$ can be an entity linkage score or impact of v in social networks [14].

Here V_{u_o} is the set $\{v | L(v) = L(u_o), v \in V\}$, *i.e.,* the nodes in G with the same label of u_o. Given G, the pairwise dissimilarity is normalized with a constant $\frac{|V_{u_o}|-1}{2}$, as there are at most $\frac{|V_{u_o}|(|V_{u_o}|-1)}{2}$ pairs but $|V_{u_o}|$ relevance numbers. That is, $\delta(q, G) \in [0, |V_{u_o}|]$.

Coverage. Ideally, an instance should satisfy the coverage requirement c_i posed on each group $P_i \in \mathcal{P}$, and cover exactly c_i nodes in each $P_i \in \mathcal{P}$. Given $\mathcal{P}$, G and template Q, an instance is *feasible*, if for each $P_i \in \mathcal{P}$, $|q(G) \cap P_i| \geq c_i$.

We next introduce a function to quantify the quality in terms of desired coverage as:

$$f(q, \mathcal{P}) = C - \sum_{P_i \in \mathcal{P}} (|q(G) \cap P_i| - c_i)$$

The function penalizes the total accumulated errors between desired coverage and actual counterpart by $q(G)$ over each group in $\mathcal{P}$. The larger $f(q, \mathcal{P})$ is, the better ($f(q, \mathcal{P}) \in [0, C]$).

Example 4: Continue with the talent search queries in G as illustrated in Fig. 1. In a simple scenario where we want to cover exactly 2 male and 2 female over the qualified candidates, one may verify the following: $\delta(q_1, G) = 0$ and $f(q_1, \mathcal{P}) = 1$; $\delta(q_2, G) = 1$ and $f(q_2, \mathcal{P}) = 1$; and $\delta(q_3, G) = 0.75$ and $f(q_3, \mathcal{P}) = 2$, and $\delta(q_4, G) = 0.5$ and $f(q_3, \mathcal{P}) = 3$. □

In the rest of the paper, we only consider feasible instances. We shall denote $\delta(q, G)$ and $f(q, \mathcal{P})$ simply as $\delta(q)$ and $f(q)$, respectively, when G and $\mathcal{P}$ are specified in the context.

B. Query Generation Problem

Pareto Optimality. Given G, $\mathcal{P}$ and a template Q, an "optimal" instance q^* should maximize both diversity and relative coverage, (*i.e.,* a Pareto optimal instance):

$$q^* = \arg\max_{q \in \mathcal{I}(Q(u_o))} \delta(q); \qquad q^* = \arg\max_{q \in \mathcal{I}(Q(u_o))} f(q)$$

While desirable, such a solution may not always exist, as diversity (which favors instances with diversified matches) and group fairness (which requires desired coverage) may be in conflict. A proper option is to compute a Pareto set. Given two instances q and q' in $\mathcal{I}(Q)$, we say q *dominants* q', if either (1) $\delta(q) \geq \delta(q')$ and $f(q) > f(q')$, or (2) $\delta(q) > \delta(q')$ and $f(q) \geq f(q')$. A set $\mathcal{Q}^* \subseteq \mathcal{I}(Q)$ is a *Pareto instance set* if (1) there is no pair of instances (q, q') from $\mathcal{Q}^*$, such that q dominants q', and (2) for any instance $q'' \in \mathcal{I}(Q)$, there exists an instance $q \in \mathcal{Q}^*$ that dominants q''.

A Pareto instance set $\mathcal{Q}^*$ provides solutions for users with different preferences on diversity and fairness in query generation. We show it is well defined for query generation.

Lemma 1: *Given a template $Q(u_o)$, graph G and groups $\mathcal{P}$, there exists a unique, finite and maximum Pareto set $\mathcal{Q}^*$.* □

Nevertheless, the exact $\mathcal{Q}^*$ is often of substantial size in practice. For a small template of 3 edges and 3 variables, a complete Pareto set in real-world graphs may already contain 100 instances (see Section VI), a daunting task for users to inspect. We next introduce a notion of *ϵ-Pareto instance*

set, which approximates $\mathcal{Q}^*$ with quality guarantee, and is of polynomially bounded size.

ϵ-Pareto instance set. Given Q, G, and $\mathcal{P}$, we say an instance q *ϵ-dominants* q' for some $\epsilon > 0$, denoted as $q \succ_\epsilon q'$, if and only if $(1+\epsilon)\delta(q) \geq \delta(q')$, and $(1+\epsilon)f(q) \geq f(q')$.

We show the following properties of ϵ-dominance relation.

Lemma 2: *Given template Q and G, the ϵ-dominance defined on $\mathcal{I}(Q)$ is a preorder.* □

Lemma 2 can be verified by observing that ϵ-dominance is reflexive and transitive on query instances.

A set of instances $\mathcal{Q}_\epsilon^* \subseteq \mathcal{I}(Q(u_o))$ is an *ϵ-Pareto instance set*, if (a) $\mathcal{Q}_\epsilon^* \subseteq \mathcal{Q}^*$, and (b) for any instance $q' \in \mathcal{I}(Q)$, there exists an instance $q \in \mathcal{Q}_\epsilon^*$, such that $q \succeq_\epsilon q'$.

An ϵ-Pareto instance set is desirable: it not only ϵ-dominants all the instances in $\mathcal{I}(Q)$, with a configurable quality controlled by ϵ, but also only contains instances from the Pareto instance set Q^*. That is, it approximates Q^* with a subset of representative but less number of instances. On the other hand, there exist multiple such ϵ-Pareto instance sets.

We next state the FairSQG problem.

Problem statement. A *configuration* of query generation is a tuple $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, which contains a graph G, template $Q(u_o)$, disjoint groups $\mathcal{P}$ with coverage constraints, and a constant $\epsilon > 0$. Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, the FairSQG problem is to compute an ϵ-Pareto instance set $\mathcal{Q}_\epsilon$.

Example 5: Given the q_1, q_2, q_3 and q_4, the Pareto set of the query instances from q_1 to q_5 is $\{q_2, q3, q4\}$ since q_1, q_2 ans q_3 all dominant $q1$. Given the δ and values of these query instances (See **??**). We can easily compute the □

Although desirable, FairSQG remains nontrivial. We provide the following upper and lower bound analysis.

Theorem 3: *Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, the* FairSQG *problem (1) is in Δ_2^P when Q has fixed number of variables $|\hat{X}|$, (2) remains* NP*-hard when Q has no range variables, and (3) is fixed-parameter tractable, for Q with fixed size (number of edges) and fixed $|\hat{X}|$.* □

Proof sketch: The decision problem of FairSQG is to determine whether there is a non-empty ϵ-Pareto instance set $\mathcal{Q}_\epsilon$. (1) FairSQG is solvable in Δ_2^P for fixed template Q. Here Δ_2^P is the class of problems in P^{NP}. A Δ_2^P algorithm first enumerates $\mathcal{I}(Q)$. For each instance, it consults an NPoracle to verify $f(q)$ and $\delta(q)$, and computes an ϵ-Pareto set with a pairwise comparison (*e.g.,* nested loop). As $|\mathcal{I}(Q)| \leq 2^{|X_e|}|\mathsf{adom}_m|^{|X_L|}$ ($|X_e|$ and $|X_L|$ are constants), the verification is in PTIME. Here adom_m refers to the largest active domain in G. (2) The NP-hardness can be verified from the hardness of deciding subgraph isomorphism, even when X_e is fixed (thus in total $2^|X_e|$ instances).

To see (3), we observe that the Δ_2^P algorithm in (1) takes polynomial time when both $|\hat{X}|$ and $|Q|$ are fixed, given that it takes PTIMEto verify the coverage and diversity for $\mathcal{I}(Q)$ with polynomially bounded size. □

The above analysis provides a naive algorithm (denoted as EnumQGen): enumerates up to $2^{|X_e|}|\mathsf{adom}_m|^{|X_L|}$ instances, verifies each instance to find feasible ones, and invokes a nested loop comparison to generate ϵ-Pareto instance set. This is infeasible when G is large. We next show that an ϵ-Pareto instance set $\mathcal{Q}_\epsilon^*$ with a *bounded size* can be efficiently computed (Section IV) and dynamically maintained (Section V), where the size bound is only determined by ϵ and the range of diversity and coverage. This enables flexible query generation that strikes a balance between quality and instance sizes.

IV. Approximating Pareto Instance Sets

We present a generic query generation algorithm, denoted as QGen, for FairSQG without enumeration. We start with the auxiliary structures it maintains.

Auxiliary structures. To characterize the search space, we start with a notion of *refinement* relation defined on $\mathcal{I}(Q)$.

Instance Refinement. Given a template Q and instantiations I and I' of Q, I' *refines* I at a variable x (denoted as $I' \succeq_I$) if it binds a constant to x that makes the predicate parameterized by x no less selective than the counterpart from I. Specifically, (1) for a literal l in the form of $u.A > x_l$ or $u.A \geq x_l$ (resp. $u.A < x_l$ or $u.A \leq x_l$), $I'(x_l)$ refines $I(x_l)$ if $I'(x_l) \geq I(x_l)$ (resp. $I'(x_l) \leq I(x_l)$) ("refines a selection condition"); (2) for edge variable x_e, $I'(x_e)$ refines $I(x_e)$ if $I'(x_e)$ = 1 and $I(x_e)$ = 0 ("adds a query edge"); (3) $I' \succeq_x I$ if $I(x) = '\_'$.

We say I' refines I (denoted as $I' \succeq I$) if for every variable x in Q, $I' \succeq_x I'$. Given two instances q' and q induced by I' and I respectively, q' refines q (denoted as $q' \succeq_\mathcal{I} q$) if $I' \succeq I$.

We observe the following result.

Lemma 4: *Given a configuration $\mathcal{C} = (G, Q, \mathcal{P}, \epsilon)$, (1) the refinement relation is a preorder; and (2) for any instances q and q' in $\mathcal{I}(Q)$, if $q' \succeq_\mathcal{I} q$, then (a) $\delta(q) \geq \delta(q')$; and (b) $f(q) \leq f(q')$ when both q and q' are feasible.* □

Proof sketch: The above results can be verified by observing that (a) refinement relation is reflexive and transitive, and (b) $q'(G) \subseteq q(G)$ if $q' \succeq_\mathcal{I} q$, *i.e.,* any match of u_o in q' remains to be a match of u_o in q if q' refines q at some variables. □

These results justify the following. Lemma 4 (1) provides a convenient lattice encoding of the search space induced by the refinement preorder. Lemma 4 (2) verifies useful monotonicity properties on diversity and coverage measures that shall be exploited for effective pruning.

Instance Lattice. Following Lemma 4, the algorithmQGen maintains a lattice encoding of the instance space Ł = $(\mathcal{I}(Q), \prec_\mathcal{I})$ induced by refinement. (1) It initializes $(\mathcal{I}(Q), \prec_\mathcal{I})$ with a single root q_r (upper bound) induced by the "most relaxed" instantiation, and a single lower bound q_b induced by the "most refined" instantiation. (2) Each node q in $(\mathcal{I}(Q), \prec_\mathcal{I})$ is an instance. For each node q, QGen maintains

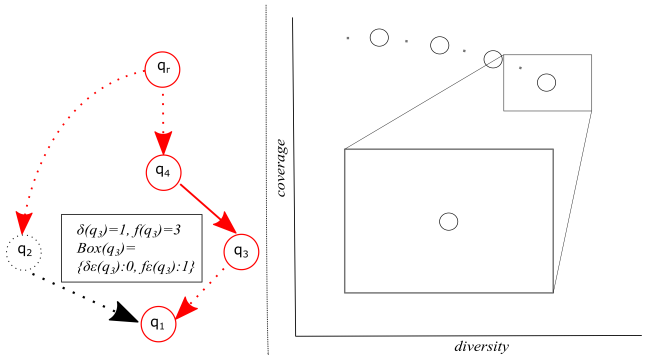


Fig. 3. Lattice illustration

- (a) two Boolean flags: 'verified' to record if q is verified, and 'feasible' to indicate if q is a feasible instance;
- (b) $q.(G)$, the (estimated) query answer; and
- (c) a coordinate $(\delta(q), f(q))$, and a "boxing" coordinate Box(q) = $(\delta_\epsilon(q)), f_\epsilon(q)$. The coordinate value $\delta_\epsilon(q)$ (resp. $f_\epsilon(q)$) is defined as $1 + \frac{\log \delta(q)}{\log(1+\epsilon)}$ (resp. $1 + \frac{\log f(q)}{\log(1+\epsilon)}$). Box(q) specifies a box region in the bi-objective (2-dimensional) space of instance q to decide the ϵ-dominance relation (see "Updater" below).

(3) There is an edge (q, q') with a label x if (a) $q' \succeq q$, and (b) q' and q differs in the value of only one variable x, and q' refines q by modifying the value of x to its closest counterpart in the corresponding active domain (e.g., changing 0 to 1 if x is an edge variable). Intuitively, an edge indicates a stepwise refinement action of q by adjusting the value of x only.

Example 6: A fraction of the lattice structure Ł that contains $\{q_r, q_1, \ldots q_4\}$ is shown on the left hand side of Fig. 3. Algorithm QGen maintains the auxiliary information of *e.g.,* q_3 once it is verified, including the coordinates $\delta(q_3)$, $f(q_3)$, and the boxing coordinates $Box(q_3)$. As q_3 refines q_4 at variable $x_l 1$ at the node u_1, (q_3, q_4) is a direct edge in Ł. □

Generic Algorithm. Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, algorithm QGen maintains an instance set $\mathcal{Q}$ and iteratively refines $\mathcal{Q}$ towards an ϵ-Pareto instance set of $\mathcal{I}(Q)$. At each iteration i, it refines the solution $\mathcal{Q}_i$ from the last iteration by interacting two procedures.

(1) *Spawner*. A spawner (Spawn) constructs new instances to be verified that may contribute to the current instance set $\mathcal{Q}_i$ with new ϵ-dominance relation in diversity and coverage. In each iteration, Spawn(a) refines the current configuration given the quality of $\mathcal{Q}_i$ to reduce unnecessary generation, (b) constructs a *front set* of instances $\mathcal{Q}_F$ (thus a fraction of lattice $(\mathcal{I}(Q), \prec_{\mathcal{I}})$) on-the-fly, and (c) prunes unpromising instances that are already ϵ-dominanted by $\mathcal{Q}_i$ whenever possible. The spawner performs no actual query processing and verification.

(2) *Updater*. An updater (Update) refines $\mathcal{Q}_i$ with the front set $\mathcal{Q}_F$ from Spawntowards a better solution $\mathcal{Q}_{i+1}$. Our idea extends [21] to maintain "boxes" of instances that discretize the bi-objective (2-dimensional) space of answer diversity and coverage of groups. Each box is represented by a single instance q and specified by its boxing coordinates $(\delta_\epsilon(q), f_\epsilon(q))$. To verify ϵ-dominance, it then suffices to verify the dominance of boxing coordinates at both box level and instance level.

Algorithm RfQGen
Input: configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$;
Output: an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$.
1. set $\mathcal{Q}^*_\epsilon := \emptyset$; initializes Ł := $\{q_r\}$;
2. BFExplore $(\mathcal{C}, q_r, Ł, \mathcal{Q}^*_\epsilon)$;
3. **return** $\mathcal{Q}^*_\epsilon$;

Procedure BFExplore$(\mathcal{C}, q_r, Ł, \mathcal{Q}^*_\epsilon)$
1. **if** q.verified **then return** ;
2. incVerify $(q, Ł, G)$; q.verified:= true;
3. **if** !q.feasible **then return** ;
4. Update $(q, \mathcal{Q}^*_\epsilon)$;
5. set $\mathcal{Q}_F$:= Spawn$(q, \mathcal{C})$;
6. **for each** $q' \in \mathcal{Q}_F$ **do**
7. BFExplore $(\mathcal{C}, q', Ł, \mathcal{Q}^*_\epsilon)$;

Fig. 4. Algorithm RfQGen

We present our main result below.

Theorem 5: *Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, there exists an algorithm that (1) correctly maintains an ϵ-Pareto instance set $\mathcal{Q}^i_\epsilon$ over all generated instances upon any time i the updater is invoked; (2) ensures a size-bounded $\mathcal{Q}_i$, where $|\mathcal{Q}^i_\epsilon| \leq \frac{\log |V|}{\log(1+\epsilon)}$, and (3) take $O(|\mathsf{adom}_\mathsf{m}|^{|\hat{X}|}(\frac{\log(|V|}{\log(1+\epsilon)} + T_q))$ time to compute $\mathcal{Q}^*_\epsilon$, where adom_m refers to the largest active domain, and T_q is the cost of verifying a single instance.* □

We next present two efficient algorithms as constructive proof of Theorem 5. Each implements QGen with different exploration strategies of the instance lattice Ł, and convergences to instances with high diversity, or a more balanced distribution of diversity and coverage, for different user preference. Both have provable guarantees in Theorem 5.

A. Query Generation by Refinement

Our first algorithm, denoted as RfQGen, uses a "refine as always" strategy to compute $\mathcal{Q}^*_\epsilon$. Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, it starts from the root q_r of the instance lattice Ł (which carries search predicates with most "relaxed" conditions), and performs a depth-first exploration of Ł. The algorithm uses Lemma 4 (2) to achieve early pruning of infeasible instances, and reduce unnecessary update.

Algorithm. Algorithm RfQGen is shown in Fig. 4. It initializes set $\mathcal{Q}^*_\epsilon$, and the lattice Ł with a single root q_r. It then invokes a recursive procedure RfExplore to perform depth first exploration, which interacts spawn and update process and generates a front set $\mathcal{Q}_F$ to be explored at each level of Ł. RfQGen early terminates if no new instance can be spawned (as RfExplore backtracks), and returns set $\mathcal{Q}^*_\epsilon$.

Procedure BFExplore. The recursive procedure RfExplore starts by verifying an unvisited instance q from the current front set $\mathcal{Q}_F$. (1) It invokes a procedure incVerify (line 2; not shown) to incrementally update the match set $q(G)$ [15],

along with the coordinates $(\delta(q), f(q))$ and boxing coordinates $(\delta_\epsilon(q), f_\epsilon(q))$. Following Lemma 4, incVerify only determines which matches should be excluded from the counterparts of the verified "parent" of q in Ł. (2) It then invokes a procedure Update (line 4) to maintain $\mathcal{Q}_f^*$ given a feasible instance q. (3) For a feasible instance q, it invokes a procedure Spawn(line 5) to generate the frontier set $\mathcal{Q}_F$ of refined instances (thus spawns a set of children of q in Ł), by modifying one variable at a time using the next closest active domain value. It then starts a next-level exploration for each instance in $\mathcal{Q}_F$. BFExplore backtracks whenever q is not feasible (line 3), as no refined counterparts is feasible (Lemma 4).

Procedure Update. Given a feasible instance q and current ϵ-Pareto instance set $\mathcal{Q}_\epsilon^*$, procedure Update maintains ϵ-dominance by verifying the dominance of Boxing coordinates $Box(q)$, with a case analysis below.

(Case 1) *Replacing boxes* (lines 1-5). This case verifies a box-level dominance relation. Using the boxing coordinates Box(q) = $(\delta_\epsilon(q), f_\epsilon(q))$ of q, it verifies if q introduces a box that also already dominants a set of boxes in the bi-objective space of diversity and coverage. If so, it removes all the representative instances of those boxes from $\mathcal{Q}_f^*$, and adds q.

(Case 2) *Replacing instances* (lines 6-7). If q falls into a box which is represented by another instance q', Update simply keeps the one that can dominate the other.

(Case 3) *Adding a non-dominated box* (lines 8-9). If no box can dominate Box(q), Update simply adds q to $\mathcal{Q}_f^*$ (which represents a new box). Here we use $Box(q') \succeq Box(q)$ to denote that $Box(q') \succ Box(q)$ or $Box(q') = Box(q)$.

Example 7: refer to 2D in Fig 3. □

Procedure Spawn. To further reduce generation and verification cost, procedure Spawnuses the following strategy to actively refines the values each variables can take, and "simplifies" template $Q(u_o)$ when possible. The refined templates are restored when BFExplore backtracks to ensure the correctness.

Template refinement. Given a verified instance q, Spawn dynamically tracks the subgraph induced by d-hop neighbors of $q(G)$ (d is the diameter of $Q_(u_o)$), denoted as G_q^d.

(1) For each literal $u.A$ op x of $Q(u_o)$, it refines the values x can take to $\{T(v.A)\} \subseteq \mathsf{adom}(A)$, where v ranges over the nodes in G_q^d and $L(v) = L(u)$.

(2) For each edge variable x_e on edge $e = (u, u')$ in $Q(u_o)$, it "fixes" x_e to be 0 if there is no path from any match of u_o in G_q^d with an edge (v, v') such that $L_Q(e) = L((v, v'))$. Moreover, if e is a bridge of $Q(u_o)$, *i.e.,* removing e leads to two connected components in $Q(u_o)$, Spawnremoves e and the entire connected component that does not contain u_o.

Example 8: Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$ where $\mathcal{P}$ is defined on gender group of users in G, and $\epsilon = 3$. Algorithm RfQGen starts with the root q_r in the lattice Ł, as illustrated in Fig. 3. (1) Following a depth first strategy,

Procedure Update $(q, \mathcal{Q}_\epsilon^*)$
1. set $\mathcal{Q}_B := \emptyset$;
   /* *verify "box-level" dominance* */
2. **for each** $q' \in \mathcal{Q}_\epsilon^*$ **do**
3. **if** $Box(q') \prec Box(q)$ **then** $\mathcal{Q}_B := \mathcal{Q}_B \cup \{q'\}$;
4. **if** $\mathcal{Q}_B \neq \emptyset$ **then**
5. $\mathcal{Q}_\epsilon^* := (\mathcal{Q}_\epsilon^* \setminus \mathcal{Q}_B) \cup \{q\}$;
   /* *verify "instance-level" dominance* */
6. **else if** there is an instance $q' \in \mathcal{Q}_\epsilon^*$ and $Box(q') = Box(q)$ **then**
7. **if** $q' \prec q$ **then** $\mathcal{Q}_\epsilon^* := (\mathcal{Q}_\epsilon^* \setminus \{q'\} \cup \{q\}$;
   /* *adding a new instance and a non-dominated box* */
8. **else if** there is no instance $q' \in \mathcal{Q}_\epsilon^*$ such that $Box(q') \succeq Box(q)$ **then**
9. $\mathcal{Q}_\epsilon^* := \mathcal{Q}_\epsilon^* \cup \{q\}$;
10. **return** $\mathcal{Q}_\epsilon^*$;

Fig. 5. Algorithm Update

BFExplore reaches q_4 and invokes Spawnto refine q_4 to q_3. In particular, Spawnselects variable X_l1 at node u_1. While the active domain of "yearsOfExp" suggests three values $\{10, 11, 12, 20\}$, Spawnrecognizes that it suffices to explore only $\{10, 11, 12\}$ with the next available value value, given that no neighbors of the current match has "yeasOfExp" more than 20. It then generates q_3 and adds it to the front set for further exploration. (2) As the exploration reaches q_2, it finds a ϵ-Pareto set $\{q_2, q_3\}$ and returns the set. □

Correctness. Algorithm RfQGen correctly maintains an ϵ-Pareto instance set $\mathcal{Q}_\epsilon^i$ over the generated instances $\mathcal{I}^i(Q)$ upon Update is invoked at time i. To see this, assume $\mathcal{Q}_\epsilon^i$ is not an ϵ-Pareto instance set. Then either (a) there exists an instance $q \in \mathcal{I}^i(Q) \setminus \mathcal{Q}_\epsilon^i$ that is not ϵ-dominated by any instance in $\mathcal{Q}_\epsilon^i$, or (b) $q \in \mathcal{Q}_\epsilon^i$ but not in the Pareto set of $\mathcal{I}^i(Q)$. For case (a), Update only removes q if there is another verified instance $q' \in \mathcal{I}^i(Q)$ that either dominants q (line 7), or ϵ-dominants q (line 3). In either cases, it contradicts the assumption. Similarly, case (b) indicates that there exists at least an instance $q' \in \mathcal{I}^i(Q)$ that $q' \succeq q$. Thus q' is verified at some time and either remains in $\mathcal{Q}_\epsilon^i$ or leaves a box $Box(q'')$, where $q'' \succeq q'$. In either case, q should be excluded by Update from $\mathcal{Q}_\epsilon^i$ (at line 7 or line 3), which contradicts that $q \in \mathcal{Q}_\epsilon^i$.

Size bound. We next show that at any time i, $|\mathcal{Q}_\epsilon^i| \leq \frac{\log |V|}{\log(1+\epsilon)}$. To see this, observe that (a) Update ensures that each box is represented by a single instance; (b) there are in total $\frac{\log(|V|}{\log(1+\epsilon)} \frac{C}{\log(1+\epsilon)})$ boxes in the bi-objective 2D space, thus at most $\frac{\log(|V|}{\log(1+\epsilon)}$ instances having same coverage that ϵ-dominant the rest (as $\delta(q) \in [0, |V|]$), or $\frac{\log C}{\log(1+\epsilon)}$ instances that ϵ-dominate the rest having same diversity (as $f(q) \in [0, C]$). As $C \leq |V|$ for disjoint groups, the size bound follows.

Time Cost. Following Theorem 3, Spawngenerates up to $2^{|X_e|}|\mathsf{adom}_m|^{X_L}$ instances. Thus it takes in $O(|\mathsf{adom}_\mathsf{m}|^{|\hat{X}|})$ runs of BFExplore. For each instance q, it takes Update $O(\frac{\log(|V|}{\log(1+\epsilon)} + T_q)$ time to verify q (in time T_q) and update $\mathcal{Q}_\epsilon^i$. Thus the total time cost is in $O(|\mathsf{adom}_\mathsf{m}|^{|\hat{X}|}(\frac{\log(|V|}{\log(1+\epsilon)} + T_q))$

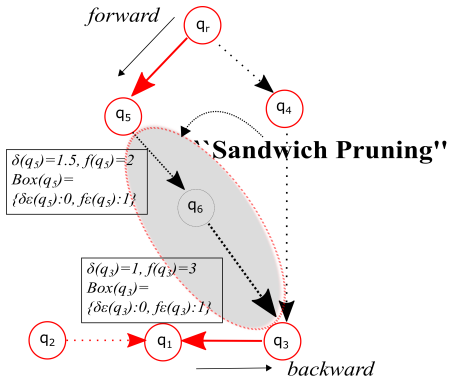


Fig. 6. Bi-directional Query Generation

time. We found that $\mathsf{adom_m}$ and $|\hat{X}|$ are usually small in real-world graphs. Moreover, the early pruning of infeasible instances significantly reduce on average 40% of generated ones, compared with the naive algorithm EnumQGen (Section VI).

The above analysis completes the proof of Theorem 5.

B. Bi-directional Query Generation

Algorithm RfQGen achieves early convergences to an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$, where a majority of instances may have high answer diversity (Lemma 2 (2a)).

We next present a second algorithm, denoted as BiQGen. It adopts a bi-directional strategy that explores Ł from both ends. The forward exploration inspects instances with non-increasing diversity, and the backward exploration keeps "relaxing" instances towards early convergence to instances with high coverage. Following Lemma 2 (2), the computation has more chance to generate $\mathcal{Q}^*_\epsilon$ with a more "balanced" distribution on instances with high diversity and those with desired coverage, as also verified by our experiments (Section VI).

Algorithm. The algorithm BiQGen (shown in Fig. 7) uses the same procedure Update as in RfQGen to maintain $\mathcal{Q}^*_\epsilon$ at any time, but specifies two separate spawners: SpawnF, same as Spawnin RfQGen, and SpawnB, a reversed "relaxation" counterpart that yields new instances by relaxing the search predicates. The procedure SpawnFand SpawnBare invoked to generate a front set $\mathcal{Q}^f_F$ (line 9) and $\mathcal{Q}^b_F$ (line 15) in a "forward" refinement-based exploration from q_r, and a "backward" relaxation-based exploration from q_b, respectively. It uses two queues S_f and S_b to control the iterative forward and backward exploration (line 2). It iteratively performs forward (lines 3-9) and backward exploration (lines 10-15), and terminates if no new instances can be generated (line 3).

"Sandwidch" pruning. The bidirectional strategy enables an effective pruning strategy that exploits the monotonicity properties of diversity and coverage in Lemma 4(2).

Lemma 6: *For any feasible instances $q \in \mathcal{Q}^f_F$ and $q' \in \mathcal{Q}^b_F$, if $q' \succeq_\mathcal{I} q$, and (a) $Box(q).\delta = Box(q').\delta$ or (b) $Box(q).f = Box(q').f$, then for any instance $q'' \in \mathcal{I}(Q)$ where $q \prec_\mathcal{I} q'' \prec_\mathcal{I} q'$, $q'' \notin Q^*_\epsilon$.* □

Algorithm BiQGen

Input: configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$;
Output: an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$.

1. queue $S_f := \{q_r\}$; queue $S_b := \{q_b\}$; set $\mathcal{Q}^*_\epsilon$:=$\emptyset$;
2. set $\mathcal{Q}_f := \emptyset$; set $\mathcal{Q}_b := \emptyset$; set SBounds:= $\emptyset$;
3. **while** $S_f \neq \emptyset$ or $S_b \neq \emptyset$ **do**
   /* *forward exploration* */
4. **if** $S_f \neq \emptyset$ **then** instance q:= $\mathcal{Q}_f$.dequeue();
5. **if** q.verified or !q.feasible **then** continue;
6. **if** SPrune(q,SBounds) **then** continue;
7. incVerify (q); q.verified:=true;
8. **if** q.feasible **then** Update $(q, \mathcal{Q}^*_\epsilon)$;
9. $\mathcal{Q}^f_F$:= SpawnF$(q, \mathcal{C})$; S_f.enqueue($\mathcal{Q}^f_F$);
   /* *backward exploration* */
10. **if** $S_b \neq \emptyset$ **then** instance q':= $\mathcal{Q}_b$.dequeue();
11. **if** q.verified or !q.feasible **then** continue;
12. **if** SPrune(q, SBounds) **then** continue;
13. incVerify (q'); q.verified:=true;
14. **if** q'.feasible **then** Update $(q', \mathcal{Q}^*_\epsilon)$;
15. $\mathcal{Q}^b_F$:= SpawnB$(q, \mathcal{C})$; S_b.enqueue($\mathcal{Q}^b_F$);
    /**update "Sandwidch" bounds with feasible pair (q, q')*/
16. **if** $q' \succ_\mathcal{I} q$ **and** $(Box(q').\delta = Box(q').\delta$
    or $Box(q').f = Box(q').f)$ **then**
17. update SBounds with (q, q');
18. **return** $\mathcal{Q}^*_f$;

Fig. 7. Algorithm BiQGen

Proof sketch: Consider an instance $q'' \in \mathcal{I}(Q)$ where $q \prec_\mathcal{I} q'' \prec_\mathcal{I} q'$. Following Lemma 4, $\delta(q') \leq \delta(q'') \leq \delta(q)$, and $f(q') \geq f(q'') \geq f(q)$. For Case (a), if $Box(q).\delta = Box(q').\delta$, then $Box(q'').\delta = Box(q').\delta$ and $f(q') \geq f(q'')$. Thus $q' \succ q''$ or $q' \succ_\epsilon q''$. Similarly, for Case (b), if $Box(q).f = Box(q').f$, then $Box(q'').f = Box(q).f$. Thus $q \succ q''$ or $q \succ_\epsilon q''$. In either case, Update rejects q'' (line 5 or line 7). □

During the bi-directional exploration, BiQGen keep tracks of the occurrences of "sandwidch" pairs (q, q') that satisfy the condition in Lemma 6 in a set SBounds. Upon a new pair (q, q') is identified (line 16), it updates SBounds by (a) replacing any pair (q_1, q_2) with (q, q_2) (resp. (q_1, q')) if $q \prec_\mathcal{I} q_1$ (resp. $q_2 \prec_\mathcal{I} q'$), or (q, q') if $q \prec_\mathcal{I} q_1$ and $q_2 \prec_\mathcal{I} q'$; or (b) adding (q, q') (line 17). This in turn allows more instances to be pruned (by a procedure SPrune; lines 6 and 12), for both forward and backward exploration.

These pruning strategies are fast and effective. Checking the refinement preorder $\succ_\mathcal{I}$ takes $O(|\hat{X}|)$ time per instance. We found on average 60% of the generated instances from EnumQGen are pruned by BiQGen (see Section VI).

Example 9: Given $\mathcal{I}(Q)$ that contains $\{q_r, q_1, q_2, q_3 q_4\}$, BiQGen starts a forward search from q_r as in RfQGen, and a backward search from q_1. (1) In the first round of bidirectional search, SpawnFrefines q_r to q_5, and SpawnBrelaxes q_1 to q_3. Upon the verification of q_3 and q_5, BiQGen finds that $Box(q_5).f = Box(q_3).f = 1$. It then creates a pair (q_5, q_3) and adds it to SBounds. (2) In the next round, the backward search reaches q_2. Meanwhile, as $q_5 \prec_\mathcal{I} q_6 \prec_\mathcal{I} q_3$, q_6 is skipped in the forward search without further exploration. □

Analysis. The correctness of BiQGen follows from the cor-

rectness analysis of Update, SpawnFand SpawnB(following the analysis of Spawn) and the invariant that forward and backward exploration verifies and safely prunes $\mathcal{I}(Q(u_o))$ (Lemma 4 and Lemma 6). The size bound and time cost follows from similar analysis as in RfQGen.

V. Online Query Generation

Another need of query generation is to produce workloads with arbitrary size k with diversity and coverage requirements over interested groups for query benchmarking [30], [3], [4], [5]. We consider the following problem. Given a configuration $\mathcal{C}$=$(G, Q(u_o), \mathcal{P}, k)$, maintain an ϵ-Pareto instance set $\mathcal{Q}^t_{(\epsilon,k)}$ over a large set of instances (due to *e.g.,* active domains and G), such that at any time t, (a) $|\mathcal{Q}^t_{(\epsilon,k)}| = k$, and (b) ϵ is as small as possible. This is nontrivial: as more instances arrives, one needs to compromise with a larger ϵ (thus worse approximation) for smaller k (Theorem 5).

We extend QGen to an online algorithm, denoted as OnlineQGen, to maintains an ϵ-Pareto instance set with a fixed size k and a small ϵ at any time. To cope with large instance space, it treats $\mathcal{I}(Q)$ as a stream of instances from a query generator. Unlike RfQGen and BiQGen, it does not assume an ordered processing (*e.g.,* "refinement"). Instead, (1) it uses a sliding window W_Q with a bounded size w to cache a certain number of instances that can help reduce ϵ, while keeping k fixed; and (2) it *incrementalizes* the maintenance of $Q^t_{(\epsilon,k)}$ upon the arrival of a new instance, and only perform necessary maintenance when Update causes size growth (in particular, Case (3) in Update).

Online Algorithm. The algorithm, as shown in Fig. 8, takes as input a stream of instances from an arbitrary generator that instantiates $Q(u_o)$, and a small initial constant ϵ_m¿0. It starts by populating $Q^t_{(\epsilon,k)}$ with Update upon newly arrived instances (in arbitrary order), until $|Q^t_{(\epsilon,k)}| = k$ (lines 7-10). If an instance q is rejected by Update, OnlineQGen includes it to W_Q (line 9) for future consideration (up to at most w timestamps before it "expires"; lines 5-6). This is to "temporally" keep the instances that may be accepted by Update again, thus reduce $|Q^t_{(\epsilon,k)}|$ (hence ϵ remains unchanged).

When $|Q^t_{(\epsilon,k)}| = k$ and a new instance q can be added, OnlineQGen incrementalizes Update by individually checking (a) if adding q increases $|Q^t_{(\epsilon,k)}|$ (Case (3)) or not (Case (1) and (2); see Update in Section IV). if the latter, it simply adds q (lines 12-13); otherwise, it first finds the nearest neighbor of q in $Q^t_{(\epsilon,k)}$ to be replaced by q. To this end, it enlarges ϵ as the Euclidean distance of the coordinates of q and q', in order to include q and q' in a larger box (line 16). It also verifies if a cached instance can be added without increasing $|Q^t_{(\epsilon,k)}|$ (lines 18-20). It returns a size-k ϵ-Pareto set $Q^t_{(\epsilon,k)}$ upon request (line 21) or no new instance is generated (line 22).

Analysis. OnlineQGen correctly maintains an ϵ-Pareto instance set with a fixed size k for the "seen" fraction of $\mathcal{I}(Q)$ at time t. We first observe the following property.

Lemma 7: *If $q \prec_\epsilon q'$, then $q \prec_{\epsilon'} q'$ for any $\epsilon' > \epsilon$.* □

Algorithm OnlineQGen

Input: a configuration $(G, Q(u_o), \mathcal{P}, k)$, ϵ_m;
a stream of instances $\mathcal{I}(Q)$, a cache size w;
Output: a size-k ϵ-Pareto instance set $\mathcal{Q}^t_{(\epsilon,k)}$ at any time t.

1. set $\mathcal{Q}_{(\epsilon,k)}$:=$\emptyset$; set W_Q:= $\emptyset$; integer t:= 0; ϵ:=ϵ_m;
2. **while** $\mathcal{I}(Q)$.hasNext() **do**
3. instance q:=$\mathcal{I}(Q).getNext()$;
4. verify q; q.ts:=i; i:=i+1;
   /* *remove "expired" query instances* */
5. **for each** $q' \in W_Q$ **do**
6. **if** q'.ts< $i - w + 1$ **then** W_Q:=$W_Q \setminus \{q'\}$;
7. **if** $|\mathcal{Q}_{(\epsilon,k)}| < k$ **then**
8. Update $(q, \mathcal{Q}_{(\epsilon,k)})$;
   /* *cache an ϵ-dominated instance for future update* */
9. **if** $q \notin \mathcal{Q}_{(\epsilon,k)}$ **then** W_Q :=$W_Q \cup \{q\}$;
10. **else continue** ;
11. **if** $|\mathcal{Q}_{(\epsilon,k)}|$ = k **then**
12. **if** Update accepts q with Case (1) or (2) **then**
13. $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q\}$; Continue;
14. **if** Update accepts q with Case (3) **then**
    /* *replace an instance with q* */
15. q' := NearestNeighbor$(q, \mathcal{Q}_{(\epsilon,k)})$;
16. ϵ := dist$((q.\delta, q.f), (q'.\delta, q'.f))$;
17. $\mathcal{Q}_{(\epsilon,k)}$:= $\mathcal{Q}_{(\epsilon,k)} \setminus \{q'\}$;
    /* *check if a cached instance can be added without impact* */
18. **if** there is a $q_b \in W_Q$ such that Update accepts q_b in Case (1) or Case (2) **then**
19. $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q_b\}$;
20. $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q\}$;
21. **return** $\mathcal{Q}_{(\epsilon,k)}$ upon request;
22. **return** $\mathcal{Q}_{(\epsilon,k)}$;

Fig. 8. Algorithm OnlineQGen

The correctness follows from the following invariant: at any time t, (1) either Update correctly rejects an instance that is already ϵ-dominated by an instance in $|Q^t_{(\epsilon,k)}|$, including those "expired" in W_Q; or (2) ϵ is adjusted to a larger counterpart to reduce the size of $|Q^t_{(\epsilon,k)}|$ to k (Theorem 5), and preserves any previous ϵ-dominance relation (Lemma 7).

Delay time. OnlineQGen efficiently maintains ϵ-Pareto instance set with a delay time in $O(T_q + w + k)$ time, where T_q is the cost of verifying a single instance. This verifies practical application of OnlineQGen in query generation and selection with desired diversity and coverage for large workload.

VI. Experiments

Based on real-world graphs, we experimentally verify the effectiveness and efficiency of our algorithms for FairSQG.

Experiment Setting. We used the following setting.

Datasets and Groups. We use three real-life data graphs.
(1) DBP is a movie knowledge graph induced from DBpedia [24] with $1M$ entities (*e.g.,* movies, directors, actors) and $3.18M$ relations (*e.g.,* directed, collaboration). Each node has attributes such as title, genre, and years, and on average **TBF** attributes. The diversity is measured by **TBF**. We induce movie groups based on their genres or countries, and create up to **TBF** groups.
(2) LKI [35] contains $3M$ nodes (*e.g.,* users, organizations)

with labels (professions, skills) and 26M edges (*e.g.,* co-review). Each node has attributes such as **TBF** with on average **TBF** attributes. We induce gender groups $\mathcal{P}$ with synthetic genders generated by gender inference tools [12].

(3) Cite [31] contains 4.9M nodes (*e.g.,* papers, authors) with labels (*e.g.,* topics) and 46M edges (*e.g.,* citations, authorship). Each node has attributes such as **TBF**. The nodes have on average **TBF** attributes. We induce groups $\mathcal{P}$ of papers having different topics.

Queries and Templates. We developed a generator to produce query templates with practical search conditions, controlled by number of variables $|\hat{X}|$ (specifically, the number of edge variables and range variables), query size $|Q(u_o)|$ (in terms of the number of edges) and topologies.

For each dataset, we generated a set of $Q(u_o)$ and $\mathcal{P}$ to ensure the existence of a set of feasible query instances. The largest set of instances $\mathcal{I}(Q)$ for DBP, LKI, and Cite are **TBF**, **TBF** and **TBF**, respectively.

Algorithms. We implemented the following algorithms in Java: (1) EnumQGen, which enumerates and verifies the instances in $\mathcal{I}(Q)$, and performs a simple nested loop to compute the ϵ-Pareto optimal instance set. (2) RfQGen, with "refine as always" strategy; (3) BiQGen, the algorithm that adopts bi-directional search with "Sandwidch" pruning; and (4) the online query generation algorithm OnlineQGen, which maintains an ϵ-Pareto instance set with a fixed size k and small ϵ. (5) To verify the quality of query generation, we also implemented Kungs, an algorithm that enumerates and verifies the instances in $\mathcal{I}(Q)$, and invokes Kung's algorithm [11] to compute the Pareto optimal non-dominated set.

Experimental results. We next present our findings.

Exp-1: Effectiveness. We first evaluated the effectiveness of our algorithms. As the diversity and coverage of generated queries vary over different graphs, we quantify the effectiveness with two established relative indicators for Pareto set approximations: R-indicator, and ϵ-indicator[36], for a fair comparison for FairSQG. Specifically, we choose a theoretically optimal reference point q^*

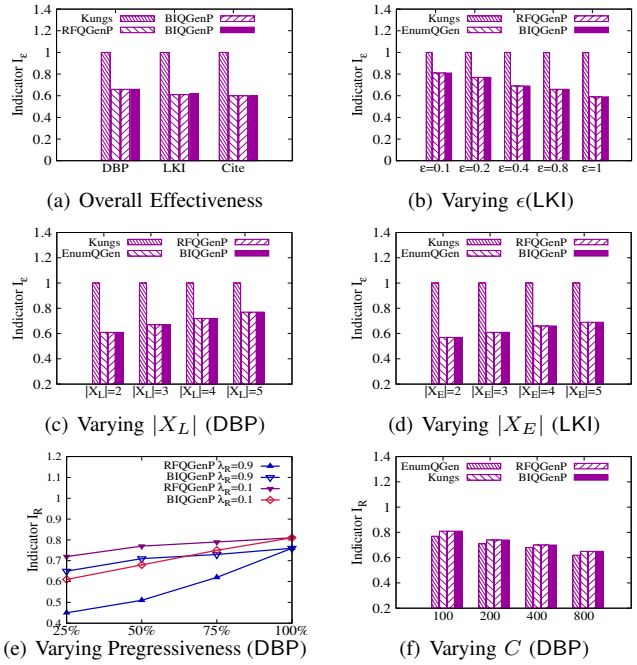
ϵ-Indicator (I_ϵ). Given a set of data points $\mathcal{Q}$, the ϵ-indicator [36] finds the minimum ϵ, denoted by ϵ_{min}, for which $\mathcal{Q}$ is an ϵ_{min}-Pareto set. Given ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$ that conform to a given constant ϵ, we define a *normalized ϵ-Indicator* (denoted as I_ϵ), which is computed as $I_\epsilon(\mathcal{Q}^*_\epsilon)$ = 1 -

$$\frac{\epsilon_{min}}{\epsilon}$$

, where

$$\epsilon_{min}$$

refers to the smallest constant such that for any instance $q \in \mathcal{I}(Q)$, there still exist an instance q' and $q' \succeq_{\epsilon\ min} q'$. The larger $I_\epsilon(\mathcal{Q}^*_\epsilon)$ is, the better. For the complete Pareto optimal set $\mathcal{Q}^*$, $I(\mathcal{Q}^*)$ = 1.



(a) Overall Effectiveness (b) Varying ϵ(LKI)
(c) Varying $|X_L|$ (DBP) (d) Varying $|X_E|$ (LKI)
(e) Varying Pregressiveness (DBP) (f) Varying C (DBP)

Fig. 9. Effectiveness for Query Generation

R-indicator (I_R). Given a configuration $\mathcal{C}$, let $\mathcal{Q}^*$ be the complete Pareto optimal set over $\mathcal{I}(Q)$ and $\mathcal{Q}^*_\epsilon$ the computed ϵ-Pareto instance set, the R-indicator aggregates the total differences of diversity and coverage between $\mathcal{Q}^*_\epsilon$ and $\mathcal{Q}^*$, weighted by a preference. Specifically, to compute R-indicator I_R[36], we normalized $f(q)$ and $\delta(q)$ in $[0, 1]$ - are we still doing this?. Given an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$ and a weight factor λ_R, we defined R-indicator as R-indicator evaluates the weighted average distance between $\mathcal{Q}^*_\epsilon$ to the reference set - what's this? Is this one referring to the Pareto set $\mathcal{Q}^*$? The "reference set" - whatever it is - is not used in your definition any way - seems incorrect. In our experiment, we set the reference set as a set that contains a single query instance q with the maximal normalized f and δ scores ($f(q) = 1$ and $\delta(q) = 1$). this is not correct - you are using each $\mathcal{Q}_\epsilon$ as its own reference set - not a fair comparison. Fix this!

Overall Effectiveness (ϵ-indicator). We compare the performance of Kungs, EnumQGen, RfQGen andBiQGen over the three real life datasets (Fig. 9(a)). We set $|Q|$ = 3 with 3 variables (1 edge variables, and 2 range variables), $|\mathcal{P}|$ = 2, and C=200. We use an "Equal opportunity" scenario and set c = 100 for both groups. (1) Kungs always can achieve scores as 1 over all the datasets as it computes the Pareto-optimal sets. (2) EnumGen,RfQGen and BiQGen achieved comparable quality. Specially, we found that BiQGen achieves I_ϵ = 0.6 on average and inspects 40% less instances compared with EnumGen.

Varying ϵ (ϵ-indicator). Fixing $|Q|$ = 4, $|\bar{X}|$ = 3 (with 1 range variables and 2 edge variables), and C= 200, we varied ϵ from 0 to 1. As shown in Fig. 9(b), both EnumQGen,RfQGen

and BiQGen achieve comparable performance with Kungs. EnumQGen,RfQGen and BiQGen reports less score when ϵ is larger. This is because the larger ϵ is, the smaller ϵ-Pareto instance sets are, given the trade off between the quality and the size of the ϵ-Pareto set. Thus, it is more difficult to use less amount of representative instances to approximate the Pareto set. On the other hand, all methods report an ϵ-indicator constantly above 0.6. These suggest our methods can compute good approximation of the Pareto set over various ϵ.

Varying $|X_l|$ (ϵ-indicator). Using the same setting as in Fig. 10(c) over DBP, EnumQGen,RfQGen and BiQGen approximates the Pareto set better when $|X_L|$ becomes larger. Interestingly, on one hand, the larger $|X_L|$ is, the more ϵ-dominating query instances we may generate for testing to approach Pareto set; on the other hand, the increased query complexity lead to less number of matches, which in turn reduces feasible instances and the sizes of Pareto instance sets, thus make it "easier" to approximate Pareto set with fewer representative instances.

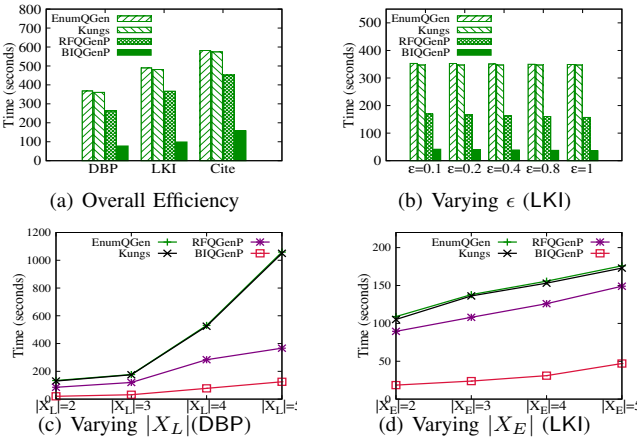
Varying $|X_e|$ (ϵ-indicator). Using the same setting as in Fig. 10(d) over LKI, we observe consistent trend for EnumQGen,RfQGen and BiQGen over larger $|X_E|$. Similar with 10(d), more edge variables will enable us to spawn more query instances and help us identify more dominating query instances, approximating Pareto set with fewer feasible instances.

The above results verify that our methods suggest better approximation for higher query complexity, due to the reduction of size of feasible answers and Pareto optimal set, and a larger query instance space to explore.

Progressiveness (R-indicator). Fixing $|Q|$ = 4, $|\mathcal{P}|$ = 200, $|\bar{X}| = 3$, we varies the portion of all the query instances we can generate and evaluate the I_R for both RfQGen and BiQGen. Here, Λ is the weight vector represents the user preference (diversity($\Lambda = 0.1$) or coverage($\Lambda = 0.9$)). We can observe that RfQGen converges faster then BiQGen for $\Lambda = 0.1$ since RfQGen always retrieve the best diversity earlier comparing with BiQGen. While BiQGen converges earlier when $\Lambda = 0.9$ since it can achieve better coverage earlier due to the bi-directional search strategy.

Varying C (R-indicator). Fixing $|Q|$ = 4, $|\mathcal{P}|$ = 200, $|\bar{X}| = 3$ and $\Lambda = 0.5$ which represents equal preference for diversity and coverage. We evaluate the the I_R score for both EnumQGen,RfQGen, BiQGen andKungs. We can observe that Increasing C decrease I_R as the C is larger the fewer feasible dominanting instances we may generate based on current query template. Thus, it is harder to approximate Pareto set with high quality. On the other hand, both our algorithms slightly performs Kungs due to the optimal pareto set (from Kungs) contains more Pareto optimal instances outside of the ϵ-Pareto instance set. Thus, it may have higher chance to decrease I_R.

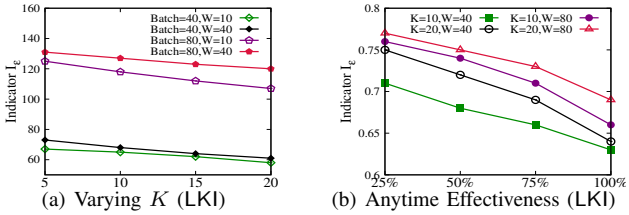
Varying k. Fig. 11(a) reports the delay time performance of OnlineQGen algorithm over a batch (with size 40 or 80) of



(a) Overall Efficiency (b) Varying ϵ (LKI)

(c) Varying $|X_L|$(DBP) (d) Varying $|X_E|$ (LKI)

Fig. 10. Efficiency of Query Generation



(a) Varying K (LKI) (b) Anytime Effectiveness (LKI)

Fig. 11. Performance of OnlineQGen

query instances from the stream. Here, we varies k from 5 to 20 and set the w = 10 or 40. On average OnlineQGen takes 63 seconds for the batch with size 40 and 121 seconds for the batch with size 80 on average. We can observe that the K is larger OnlineQGen take less time to main $\mathcal{Q}_{(\epsilon,k)}$ for the current batch as it can avoid updating more query instances comparing with smaller k. On the other hand, the larger w is the more unexpired query instances in the cache that need to be evaluated. Thus, OnlineQGen takes more time to maintain $\mathcal{Q}_{(\epsilon,k)}$ when the w is larger.

Exp-2: Efficiency. We next evaluate the efficiency of the algorithms Kungs, EnumQGen, RfQGen andBiQGen.

Efficiency over real-life graphs. Fig. 10(a) reports the performance of Kungs, EnumQGen, RfQGen and BiQGen, over the real datasets. For all the datasets, BiQGen achieves the best performance. On average, it outperforms EnumQGen by 4.4 times due to the pruning. The late pruning strategy of BiQGen improves the efficiency of RfQGen by 2.5 times. In general, RfQGen and BiQGen are feasible. For example, while EnumQGen takes 490s over LKI, it takes BiQGen (resp. RfQGen) 78s (resp. 367s) with $3M$ nodes and $26M$ edges to generate queries with comparable quality.

Varying ϵ. Fig. 10(b) reports the performance of Kungs, EnumQGen, RfQGen and BiQGen over LKI. We verify the performance of all the algorithms in by varying ϵ from 0.1 to 1. (1) BiQGen achieves the best performance among all the algorithms and is less sensitive compared with others. BiQGen

(resp. RfQGen) outperforms EnumGen (resp. QGenEq$_n$) by 6 times (resp. 2.2 times) on average. On the other hand, by increasing ϵ. BiQGen and RfQGen take less time due to the fewer number of query query instances that need to invoke $updater$.

Varying $|X_L|$. Fixing $|Q| = 4$, $|\mathcal{P}| = 200$, $\epsilon = 0.01$, we varied the number of range variables from 2 to 5 and evaluated the impact to the performance of Kungs, EnumQGen, RfQGen andBiQGen. As shown in Fig. 10(c) BiQGen achieves the best performance among all the algorithms, and is less sensitive compared with others with others BiQGen (resp. RfQGen) outperforms EnumQGen (resp. QGenEq$_n$) by 7.5 times (resp. 5.6 times) on average over DBP.

Varying $|X_E|$. Fixing $|Q| = 5$, $|\mathcal{P}| = 200$, $\epsilon = 0.01$, we varied the number of range variables from 2 to 5 and evaluated the impact to the performance of Kungs, EnumQGen, RfQGen andBiQGen. Fig. 10(d) that (1)BiQGen achieves the best performance among all the algorithms. BiQGen (resp. RfQGen) outperforms EnumQGen by 3 times (resp. 2.1 times) on average due to the pruning over LKI.(2) All the algorithm are less sensitive to $|X_E|$ comparing with $|X_L|$ due to edge variables spawn less query instances.

Anytime Effectiveness (R-indicator). Using the same setting as in Fig. 11(a) over LKI, we evaluate the anytime effectiveness of OnlineQGen by setting $k = 10, 20$ and $w = 40, 80$. We can observe that (1)the I_R decreases as the OnlineQGen evaluate more query instances from the stream. The reason is to accept the coming query instance that can not be ϵ-donminanted by any of existing instances in $\mathcal{Q}^*_\epsilon$ (when $|\mathcal{Q}^*_\epsilon| = k$), OnlineQGen enlarges ϵ to update $\mathcal{Q}^*_\epsilon$ to guarentee the property of ϵ-Pareto instance. However, the larger ϵ may harm the quality of $\mathcal{Q}_{(\epsilon,k)}$. (2) The larger k and w are the I_R scores are higher. The reason is that OnlineQGen can maintain more query instances without enlarging current ϵ comparing with smaller k. On the other hand, OnlineQGen can cache more query instances with larger w. In this case, OnlineQGen may have more opportunities to update $\mathcal{Q}_{(\epsilon,k)}$ without sacrificing the quality.

The above results verify that our methods are practical and feasible for generating queries over complex query templates and large graphs.

Exp-3: Case Study. We manually verified 2=2 instances found by BiQGen for talent search with fairness constraints, and report a case in Fig. 12. The query template Q (not shown) contains a range variable on skills and an edge variable between two users (u_3, u_1). An initial query with an instantiation $\{\emptyset, `1'\}$ returns a set of 62 male and 12 female candidates. Posing a gender equality requirement, q_4 and q_5 are suggested, resulting a balanced set of 30 female and 30 male. The queries suggest to relax long "co-review" chains while specify two different skillsets.

VII. Conclusions

We have introduced and studied the subgraph query generation with both diversity and group fairness constraints.

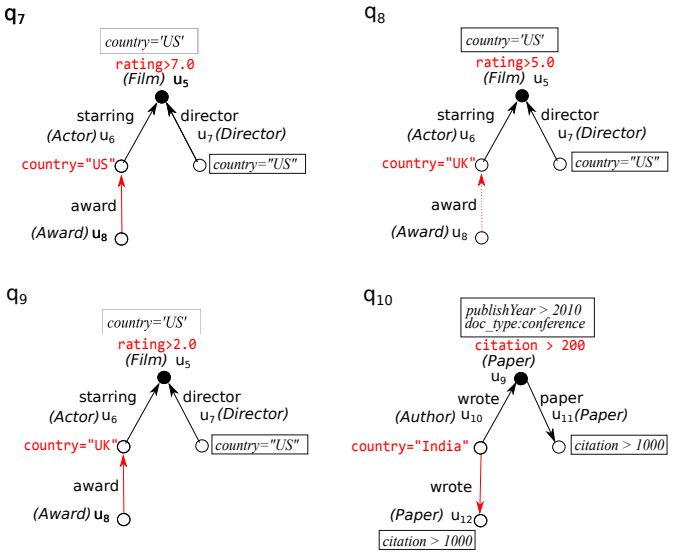


Fig. 12. Case study: Query Generation

We have introduced a formalization that aims to compute bi-objective Pareto optimal set in terms of answer diversity and coverage of groups of interests. We have provided the upper and lower bound analysis of the problem. We have provided two feasible algorithms that approximates the Pareto-optimal query instance set with ϵ-Pareto instance set, with effective pruning strategies, and and an online algorithm that maintains the ϵ-Pareto instance set with a fixed size and high quality (small ϵ). As verified analytically and experimentally, our methods are feasible for large graphs, and can achieve desirable diversity and coverage properties over targeted groups. A future topic is to study parallel query generation over large-scale graphs with diversity and group fairness. Another topic is to incorporate more fairness measures such as top-k fairness.

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