

# **Time-Decayed Caching: A Novel Rule Benchmarked Against Established Policies**

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**Abstract:**

AMS Mathematical Subject Classification: 68M20, 68P05

ACM Computing Reviews Categories and Subject Descriptors: Theory of computation, Design and analysis of algorithms, Online algorithms, Caching and paging algorithms

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# 1. Introduction

We propose a time-decayed policy based on the nuclear physics concept of half-life. We introduce a one-parameter eviction rule that maintains a per-key decayed score using a half-life ( $H$ ) constant. The policy blends recency and frequency and evicts the item with the lowest current decayed score.

## 1.1. Motivation

1. Summarize classical policies such as LRU, LFU, SLRU
2. Explain the need for a new policy that combines both recency and frequency

## 1.2. Contributions

1. Go into more detail for half-life policy. In short,  $H$  = the horizon where an old hit is worth half a new hit. If  $H = 100$  requests, then a hit that happened 100 requests ago counts  $\frac{1}{2}$ ; 200 requests ago counts  $\frac{1}{4}$ , etc.
2. Explain automatic adjustment of  $H$ -constant.  $H = c * R$ , where  $c$  is a predetermined constant and  $R$  = exponential weighted moving average ( $R[i + 1] = (1 - \eta) * R[i] + \eta * \text{gap}$ , where  $\text{gap} = \text{now} - \text{last\_hit[key]}$  in requests)

## 1.3. Research Questions

1. On different workloads and fixed capacity, does HL-Cache improve hit rate and tail latency over LRU/SLRU/LFU?
2. Does the new policy offer CPU/memory savings?
3. How sensitive is the policy to half-life mis-specification, and does online auto-tuning (formula in section above) keep performance better?

# 2. Background and Related Work

## 2.1. Terminology and Metrics

1. Explain cache-related terminology (hit etc.)
2. Explain what we would count as success for the proposed policy (possible metric is hit-rate gain compared to other classical policies etc.)

## 2.2. Existing Policies

1. Go into detail on LRU, LFU etc.

## 2.3. Other Related Work

1. Go over some of the more recent policies and optimizations (from the bibliography)
2. Explain general use of caching policies (in OS, databases etc.)

## 3. The Proposed Policy: Time-Decayed Caching

### 3.1. Formulation

We formalize the time-decayed caching policy in terms of a per-key decayed popularity score and an associated eviction rule. The key idea is that each access to an object contributes a unit of “popularity mass” that decays exponentially over subsequent requests, parameterized by a half-life  $H$ .

#### Problem setting and notation

We work in discrete time, indexed by request number. Let:

- $\mathcal{U}$  be the universe of objects.
- $r_1, r_2, \dots, r_t, \dots$  be the request sequence, where each  $r_t \in \mathcal{U}$ .
- The cache have capacity  $C$  (considered in number of objects).
- $\mathcal{C}(t) \subseteq \mathcal{U}$  denote the set of objects in the cache immediately before serving request  $r_t$ .

For each object  $i \in \mathcal{U}$ , let

$$t_1^i < t_2^i < \dots < t_{k_i(t)}^i \leq t$$

be the times (request indices) at which  $i$  has been requested up to and including time  $t$ , and let  $k_i(t)$  be the number of such requests.

#### Time-decayed popularity score

The policy associates to each object  $i$  a real-valued popularity score  $S_i(t)$  at time  $t$ . This score is defined as an exponentially decayed sum of past accesses:

$$S_i(t) = \sum_{j=1}^{k_i(t)} e^{-\lambda(t-t_j^i)}, \lambda > 0.$$

Equivalently, if we parameterize the decay in terms of a **half-life**  $H > 0$  measured in number of requests, we set

$$\lambda = \frac{\ln 2}{H}$$

and obtain the equivalent form

$$S_i(t) = \sum_{j=1}^{k_i(t)} 2^{-(t-t_j^i)/H}.$$

In this formulation:

- Each request to object  $i$  at time  $t_j^i$  contributes a term that is initially 1 and then decays over subsequent requests.
- After exactly  $H$  requests have passed since that access, its contribution is halved:

$$2^{-H/H} = \frac{1}{2}$$

- After  $2H$  requests, the contribution is  $2^{-2} = \frac{1}{4}$ , after  $3H$  it is  $2^{-3} = \frac{1}{8}$ , and so on.

Thus,  $H$  defines an effective horizon over which past hits remain significantly influential: a hit that occurred  $nH$  requests ago counts as  $2^{-n}$  of a new hit. Small values of  $H$  emphasize very recent activity, while large values integrate information over a longer history.

Conceptually, this popularity score has the following properties:

- It increases whenever the object is requested (each new hit contributes an additional term of size 1).
- It decreases smoothly as time passes without accesses, since every term in the sum is multiplied by a factor smaller than 1 at each step.
- It captures both frequency (more hits add more terms) and recency (more recent hits are less decayed and thus have larger weight).

For later implementation and analysis, it is helpful to note that this definition admits a recursive characterization. Let  $\Delta_i(t)$  denote the number of requests that have occurred since the previous request for object  $i$  (i.e., the gap in request indices). Then, immediately before processing a new request for  $i$  at time  $t$ , we can write

$$S_i(t) = S_i(t^-) \cdot 2^{-\Delta_i(t)/H} + 1,$$

where  $S_i(t^-)$  denotes the score right before applying the current hit. This shows that the full history of accesses to  $i$  can be compressed into a single scalar  $S_i$ , updated by exponential decay plus a unit increment on each hit.

## Eviction rule

The time-decayed policy uses the scores  $S_i(t)$  to make eviction decisions. At a cache miss at time  $t$ , when the cache is already full (i.e.,  $|\mathcal{C}(t)| = C$ ), the policy proceeds conceptually as follows:

1. For each object  $j \in \mathcal{C}(t)$ , consider its current time-decayed popularity score  $S_j(t)$ .
2. Identify an object with minimal score:

$$j = \arg \min_{j \in \mathcal{C}(t)} S_j(t).$$

3. Evict  $j$  from the cache.

4. Insert the newly requested object  $r_t$  into the cache and initialize its score with 1.

The half-life parameter  $H$  controls how rapidly the cache “forgets” past popularity:

- For small  $H$ , scores concentrate on recent accesses, and the policy behaves more like a recency-driven strategy.
- For large  $H$ , scores retain the influence of past accesses over longer spans, and the policy behaves more like a smoothed frequency-based strategy.

In subsequent sections we describe how to maintain the popularity scores efficiently in an online setting and how to adapt the half-life  $H$  to the observed workload.

## 3.2. Complexity

A naïve implementation would maintain, for each object  $i$  in the cache, its explicit decayed score

$$S_i(t) = \sum_{j=1}^{k_i(t)} e^{-\lambda(t-t_j^i)}$$

and, at a miss, recompute all  $S_j(t)$  for  $j \in \mathcal{C}(t)$  in order to select the victim with minimal score. This leads to  $\mathcal{O}(|\mathcal{C}(t)|)$  work per miss, which is unacceptable for large caches. Instead, we exploit the algebraic structure of  $S_i(t)$  to maintain a time-normalized key  $K_i$  for each cached object and store these keys in a priority queue (min-heap), so that extracting the victim remains efficient.

For an object  $i$ , let  $t_i$  denote the time of its most recent update (hit) and  $S_i(t_i)$  the score immediately after that update. By definition of exponential decay,

$$S_i(t) = S_i(t_i) e^{-\lambda(t-t_i)}$$

for any later time  $t \geq t_i$ . We can rewrite this as

$$S_i(t) = e^{-\lambda t} (S_i(t_i) e^{\lambda t_i}).$$

This suggests defining the time-normalized key

$$K_i := S_i(t_i) e^{\lambda t_i}.$$

Then, for any current time  $t$ ,

$$S_i(t) = e^{-\lambda t} K_i.$$

The global factor  $e^{-\lambda t}$  is the same for all objects, so for any fixed  $t$  the ordering of objects by true score  $S_i(t)$  is exactly the same as the ordering by key  $K_i$ . In particular, an object with minimal  $K_i$  also has minimal current score  $S_i(t)$ . This allows us to store only  $K_i$  for each object and to use  $K_i$  as the heap key, without explicitly recomputing all  $S_i(t)$  at eviction time.

Moreover, the key representation yields a simple update rule on hits that does not require storing  $t_i$  or  $S_i(t_i)$  separately. Suppose object  $i$  is present in the cache at time  $t$ , with current key  $K_i$ . Its true score just before processing the new hit is

$$S_i^{\text{current}} = S_i(t) = K_i e^{-\lambda t}.$$

After the hit, the new score is

$$S_i^{\text{new}} = S_i^{\text{current}} + 1 = K_i e^{-\lambda t} + 1.$$

The corresponding new key is

$$K_i^{\text{new}} = S_i^{\text{new}} e^{\lambda t} = (K_i e^{-\lambda t} + 1) e^{\lambda t} = K_i + e^{\lambda t}.$$

Thus, on a hit at time  $t$ , we can update the key using the simple recurrence

$$K_i \leftarrow K_i + e^{\lambda t},$$

without explicitly tracking  $t_i$  or  $S_i(t_i)$ . On a miss for a never-seen-before object  $i$ , its first request occurs at time  $t$ , and by the definition of  $S_i(t)$  we have  $S_i(t) = 1$ , so we initialize

$$K_i = e^{\lambda t}.$$

To support efficient eviction, we maintain a hash table from object identifiers to heap positions, and a binary min-heap keyed by  $K_i$  over the objects currently in the cache. For each request, the operations are:

- Hit: we look up the object in the hash table (expected  $\mathcal{O}(1)$ ), compute  $e^{\lambda t}$  once for the current time step, update  $K_i \leftarrow K_i + e^{\lambda t}$ , and perform a key-increase operation in the heap. In a binary heap this takes  $\mathcal{O}(\log C)$  time, where  $C$  is the cache capacity.
- Miss with free space: we insert a new object with key  $K_i = e^{\lambda t}$  into the hash table and heap, which costs  $\mathcal{O}(\log C)$ .
- Miss with full cache: we extract the minimum key from the heap to obtain the victim

$$j = \arg \min_{j \in \mathcal{C}(t)} K_j$$

in  $\mathcal{O}(\log C)$ , remove  $j$  from the hash table, and then insert the new object as above, for a total  $\mathcal{O}(\log C)$  time.

Therefore, each request is processed in  $\mathcal{O}(\log C)$  worst-case time, dominated by heap operations, compared to the  $\mathcal{O}(1)$  updates of pointer-based LRU or counter-based LFU. In practice, cache capacities are bounded and  $\log C$  is small (e.g.,  $\log_2 10^4 \approx 14$ ), so the additional overhead is modest, while the policy gains the ability to track a smooth time-decayed popularity measure without scanning the full cache. The space complexity is  $\mathcal{O}(C)$ : for each cached object we store its key  $K_i$ , the cached value (or a pointer to it), and the metadata required by the hash table and heap.

## 4. Implementation

This chapter aims to describe how the time-decayed caching policy is implemented in practice. We first outline the data structures and state maintained by the cache, and then present the core operations in pseudocode. A full C++ implementation is available as open-source code on GitHub [1].

### 4.1. Data Structures and Maintained State

The implementation follows the formulation from section 3.2. Thus, we store only a single scalar key  $K_i$  per cached object, rather than explicitly maintaining its decayed score  $S_i(t)$ . We recall that for an object  $i$  updated last at time  $t_i$  with score  $S_i(t_i)$ , we define

$$K_i := S_i(t_i) e^{\lambda t_i},$$

so that for any current time  $t$ ,

$$S_i(t) = e^{-\lambda t} K_i.$$

Because the factor  $e^{-\lambda t}$  is the same for all objects, the ordering of objects by  $K_i$  coincides with the ordering by their true decayed scores  $S_i(t)$  at time  $t$ .

The cache maintains the following global state:

- A global time index  $t \in \mathbb{N}$ , interpreted as the number of read requests processed so far. Each access operation increments  $t$  by one.
- The cache capacity  $C$ , measured in number of objects.
- The decay parameter  $\lambda > 0$ , computed based on half-life  $H$ :  $\lambda = \ln 2/H$ .

For the objects currently stored in the cache, we maintain two core data structures:

- A hash table `hashTable` that maps object identifiers to internal cache entries. This provides expected  $\mathcal{O}(1)$  lookup by key.
- A binary min-heap `minHeap` containing all cached entries, ordered by their time-normalized key  $K_i$ . The heap supports extracting the entry with minimal  $K_i$  and updating an existing entry in  $\mathcal{O}(\log C)$  time.

Each cached object  $i$  is represented by a cache entry that stores:

- `object`: the identifier of the cached object,
- `K`: the current time-normalized key  $K_i$  associated with that object.

The explicit score  $S_i(t)$  and the last update time  $t_i$  are not stored. Conceptually, they can be recovered from  $K_i$ , but the reconstruction is not required by the core operations of the policy.

## 4.2. Core Operations

We model cache usage through a single access operation on objects. Each request in a trace corresponds to one call to `ACCESS(object)`, which either hits or misses in the cache. The cache tracks only which objects are present and their time-decayed popularity scores.

On a hit, the cached entry's time-normalized key  $K_i$  is updated. On a miss, if the object has never been seen before, its first request at time  $t$  should give a decayed score  $S_i(t) = 1$ . By the definition, this corresponds to initializing

$$K_i = e^{\lambda t}.$$

If the cache is full, the least popular entry is evicted before inserting the new object.

In the pseudocode below,  $\exp(\cdot)$  denotes the natural exponential  $e^{(\cdot)}$ . `HEAP_UPDATE`, `HEAP_INSERT`, and `HEAP_POP_MIN` represent classic heap operations on `minHeap`. Similarly, `HASH_TABLE_SEARCH`, `HASH_TABLE_INSERT`, and `HASH_TABLE_REMOVE` represent hash table operations on `hashTable`.

```

procedure ACCESS(object) :
    t ← t + 1 // advance time counter
    entry ← HASH_TABLE_SEARCH(hashTable, object)

    if entry ≠ NONE then // cache hit
        UPDATE_POPULARITY(entry)
        return HIT

    else // cache miss
        if SIZE(minHeap) = C then
            EVICT()

        entry ← new cache entry
        entry.object ← object
        entry.K ← exp(λ · t)

        HEAP_INSERT(minHeap, entry)
        HASH_TABLE_INSERT(hashTable, object, entry)

        return MISS
    
```

On a hit, we update the time-normalized key  $K_i$  for the corresponding entry. If the current time is  $t$  and the entry has key  $K_i$ , then after accounting for the decayed past and adding one new access, the updated key is

$$K_i \leftarrow K_i + e^{\lambda t}.$$

```

procedure UPDATE_POPULARITY(entry) :
    delta ← exp(λ · t)
    entry.K ← entry.K + delta
    HEAP_UPDATE(minHeap, entry)

```

Here `HEAP_UPDATE(minHeap, entry)` performs the appropriate sift-up or sift-down operation to restore the min-heap invariant after `entry.K` changes; in a binary heap this runs in  $O(\log C)$  time.

```

procedure EVICT() :
    victim ← HEAP_POP_MIN(minHeap)
    HASH_TABLE_REMOVE(hashTable, victim.object)

```

During eviction, the cache entry with the smallest time-normalized key  $K_i$ , i.e. the least popular object according to the time-decayed score, is removed from `minHeap`. Its mapping from `hashTable` is removed as well.

# 5. Evaluation

## 5.1. Methodology

We aim to describe how we evaluate the proposed time-decayed cache replacement policy (“Proposed”) against three baselines: LRU, LFU and ARC. We first define the validation metrics, then describe the experimental design, and finally present the workloads and the choice of decay parameters.

### Validation metrics and baselines

We compare four algorithms:

$$\mathcal{A} = \{\text{LRU}, \text{LFU}, \text{ARC}, \text{Proposed}\}.$$

A request trace is a finite sequence

$$R = (r_1, r_2, \dots, r_T),$$

where  $r_t$  is the identifier of the object requested at time  $t$ , and  $T$  is the trace length.

For each algorithm  $A \in \mathcal{A}$ , cache capacity  $C$ , and trace  $R$ , we simulate the trace and record, for every request  $t \in \{1, \dots, T\}$ , a hit indicator

$$H_t(A, C, R) \in \{0, 1\},$$

which is 1 if the request  $r_t$  is a cache hit under algorithm  $A$  with capacity  $C$ , and 0 otherwise.

The hit ratio and miss ratio are defined as

$$\begin{aligned} \text{HR}(A, C, R) &= \frac{1}{T} \sum_{t=1}^T H_t(A, C, R), \\ \text{MR}(A, C, R) &= 1 - \text{HR}(A, C, R). \end{aligned}$$

The miss ratio MR is the main effectiveness metric.

To normalize performance across heterogeneous traces and cache capacities, we use a relative gap-to-best measure. Let traces be indexed by  $j$ , and capacities by an index  $c$  corresponding to a capacity level (defined precisely further in this section). For a fixed trace  $R_j$  and capacity level  $c$ , we define the best miss ratio among all algorithms as

$$\text{MR}_{\min}(j, c) = \min_{A' \in \mathcal{A}} \text{MR}(A', C_{j,c}, R_j),$$

where  $C_{j,c}$  is the absolute cache capacity (number of objects) corresponding to level  $c$  on trace  $j$ .

For the proposed algorithm we define, on each trace  $j$  and capacity level  $c$ , the relative gap

$$\Delta(j, c, \text{Proposed}) = \frac{\text{MR}(\text{Proposed}, C_{j,c}, R_j) - \text{MR}_{\min}(j, c)}{\text{MR}_{\min}(j, c)} \cdot 100\%.$$

A value  $\Delta(j, c, \text{Proposed}) = 0\%$  means that the proposed policy is best or tied-best for that  $(j, c)$ ; small positive values indicate that it is close to the best algorithm among LRU, LFU, ARC and itself.

We then analyze the set

$$\{\Delta(j, c, \text{Proposed}): j \text{ ranges over traces, } c \text{ over capacities}\}$$

and report, only for the proposed algorithm:

- the fraction of configurations  $(j, c)$  where  $\Delta(j, c, \text{Proposed}) = 0\%$  (best or tied-best), and
- the fraction of configurations where  $\Delta(j, c, \text{Proposed}) \leq 10\%$  (“within 10% of best”).

These two statistics summarize how often and by how much the proposed policy matches or exceeds the best of LRU, LFU and ARC across all traces and capacities.

To study adaptivity on non-stationary traces (i.e., traces with periodically changing popular items), we additionally use a sliding-window hit ratio. We fix a window length  $W$ . For  $t \geq W$ , the windowed hit ratio is

$$\text{HR}_W(A, C, R, t) = \frac{1}{W} \sum_{\tau=t-W+1}^t H_\tau(A, C, R).$$

On non-stationary traces with known phase boundaries (i.e., known times at which the hot set of popular items changes), we plot  $\text{HR}_W(A, C, R, t)$  over time for all algorithms  $A \in \mathcal{A}$  and visually compare:

- the drop in hit ratio after each phase change, and
- the speed at which each algorithm’s  $\text{HR}_W$  recovers.

This highlights how quickly the proposed policy adapts relative to LRU, LFU and ARC.

## Experimental design

We assume a finite object universe

$$\mathcal{O} = \{o_1, o_2, \dots, o_N\},$$

where each  $o_i$  is a distinct object identifier. Each trace  $R = (r_1, \dots, r_T)$  is a sequence of requests with  $r_t \in \mathcal{O}$ .

For a given trace  $R$  and length  $T$ , we define the footprint

$$U(R, T) = |\{r_t : 1 \leq t \leq T\}|,$$

the number of distinct objects requested in the first  $T$  requests. In our experiments,  $T$  is the full trace length; we write  $U_j = U(R_j, T_j)$  for trace  $j$  with length  $T_j$ .

Cache capacities are expressed as percentages of the footprint. We fix a set of capacity fractions

$$\alpha_c \in \{0.01, 0.02, 0.04, 0.08, 0.16\},$$

corresponding to 1%, 2%, 4%, 8% and 16% of the footprint. For trace  $j$  and capacity level  $c$ , the absolute cache capacity (in number of objects) is

$$C_{j,c} = \lfloor \alpha_c \cdot U_j \rfloor.$$

All algorithms are evaluated on the same pairs  $(R_j, C_{j,c})$ .

We consider two main experiments:

### 1. Overall performance experiment:

For each trace  $R_j$  and each capacity level  $\alpha_c$ :

- a. Compute  $U_j$  and the corresponding capacity

$$C_{j,c} = \lfloor \alpha_c \cdot U_j \rfloor.$$

- b. For each algorithm  $A \in \mathcal{A}$ , simulate the full trace  $R_j$  with capacity  $C_{j,c}$ , record the hit indicators  $H_t(A, C_{j,c}, R_j)$  and compute the miss ratio  $MR(A, C_{j,c}, R_j)$ .
- c. From the resulting set of miss ratios, compute  $MR_{\min}(j, c)$  and the relative gap  $\Delta(j, c, \text{Proposed})$  for the proposed algorithm.
- d. The statistics defined in the previous subsection are then computed over all values  $\Delta(j, c, \text{Proposed})$  obtained in this experiment.

## 2. Adaptivity experiment:

To isolate adaptivity, we consider non-stationary traces with a known number of phases  $P$ , each of equal length  $L$ , so that the total length is

$$T = P \cdot L.$$

Each phase  $p \in \{1, \dots, P\}$  has its own hot set and request distribution; phase changes occur at times

$$t = L, 2L, \dots, (P - 1)L.$$

We fix a capacity  $C$  and window length  $W$ . Then for each algorithm  $A \in \mathcal{A}$ :

- a. Simulate the full non-stationary trace  $R$  with capacity  $C$ , recording the hit indicators  $H_t(A, C, R)$ .
- b. For each  $t \geq W$ , compute the sliding-window hit ratio

$$\text{HR}_W(A, C, R, t) = \frac{1}{W} \sum_{\tau=t-W+1}^t H_\tau(A, C, R).$$

- c. We then plot  $\text{HR}_W(A, C, R, t)$  over time for all algorithms, marking the phase boundaries at  $t = L, 2L, \dots$ . Adaptivity is judged qualitatively by the magnitude of the drop in  $\text{HR}_W$  immediately after phase changes, and the recovery speed (i.e., how quickly  $\text{HR}_W$  returns to its steady level in each new phase).

## Workloads and decay parameter selection

We use four synthetic workloads and one real-world trace. Synthetic workloads let us control the popularity dynamics; the real trace demonstrates behavior under realistic access patterns. For all experiments, the baselines use standard parameter settings, and the proposed policy uses a decay parameter  $\lambda$  chosen according to the rules below.

### Stationary synthetic workloads

In the stationary workloads, popularity does not change over time. In total, we will be using two workloads generated from a Zipf distribution over  $\mathcal{O}$ . We index objects as  $o_1, \dots, o_N$ , and define

$$\mathbb{P}(r_t = o_i) \propto i^{-\alpha},$$

for indices  $i \in \{1, \dots, N\}$ , where the rank  $i$  corresponds to object  $o_i$ . More specifically, the object with rank 1 will be hottest, rank 2 is colder, and so on. This gives us a heavy-tailed popularity: a few objects get a lot of requests, and there is a long tail of rarely used ones. In order for the probabilities to sum up to 1 and maintain the proportionality described above, we use the following probability formula

$$\mathbb{P}(r_t = o_i) = \frac{i^{-\alpha}}{\sum_{j=1}^N j^{-\alpha}}.$$

We will use a different choice of  $\alpha$  for each of the two workloads (e.g., moderately and highly skewed values).

### Non-stationary synthetic workloads

In the non-stationary workloads, popularity changes over time in a controlled way. We will be using two workloads that consist of  $P$  phases of equal length  $L$ , so that  $T = P \cdot L$ . For each phase  $p \in \{1, \dots, P\}$ , we choose a hot set  $H_p \subset \mathcal{O}$  of size  $k$ , with limited overlap between consecutive hot sets  $H_p$  and  $H_{p+1}$ . Within phase  $p$ , requests are generated as

$$r_t = \begin{cases} \text{a key chosen uniformly from } H_p \text{ with probability } p_{\text{hot}} \\ \text{a key chosen uniformly from } \mathcal{O} \setminus H_p \text{ with probability } 1 - p_{\text{hot}} \end{cases}.$$

The hot set thus “jumps” at times  $t = L, 2L, \dots$ , creating clear phase changes.

### Real-world trace

In addition to the synthetic workloads, we evaluate the algorithms on a real-world trace drawn from a web proxy cache. This serves as a realistic counterpart to the previous non-stationary workloads, since web-access traces are known to exhibit strong temporal locality and evolving hot sets as user interests shift over time.

### Decay parameter selection

The proposed policy uses exponential decay with rate  $\lambda > 0$ , equivalently described by a half-life  $H$ :

$$H = \frac{\ln 2}{\lambda}.$$

We choose  $\lambda$  in a simple, trace-independent way based on a desired memory horizon  $A$  (in units of requests), and then keep  $\lambda$  fixed across all traces within a given regime:

#### 1. Stationary workloads

Let  $T$  be the number of simulated requests per stationary trace. We choose a memory horizon

$$A = \beta \cdot T,$$

with a small  $\beta$  (for example  $\beta = 0.01$ , corresponding to a horizon of approximately 1% of the trace length). We then set

$$H = \frac{A}{3}, \lambda = \frac{\ln 2}{H}.$$

With this choice, hits that occurred about  $A$  requests ago have their weight reduced by a factor of

$$2^{A/H} = 2^3 = 8,$$

i.e. to roughly  $1/8 \approx 12.5\%$  of their original contribution. In our setting, we consider contributions at this level (around ten percent of the original weight) to be effectively negligible, so  $A$  can be interpreted as the point where past hits become largely irrelevant. A single  $\lambda_s$  chosen in this way is used for all stationary synthetic traces and all capacity levels.

## 2. Non-stationary workloads

Let  $L$  denote the characteristic length of a stable hot phase. We choose a memory horizon  $A$  on the order of  $L$  (for example  $A = L$ ) and again set

$$H = \frac{A}{3}, \lambda = \frac{\ln 2}{H}.$$

This aligns the effective memory of the policy with the time scale over which each hot set remains stable. A single  $\lambda_{ns}$  chosen in this way is used for all non-stationary synthetic traces and capacities.

## 5.2. Results

1. Summarize results and provide information visually (graphs etc.)

## 6. Future Work

## 7. Conclusion

## 8. Bibliography

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