AI – Theo Assignment 2

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Question 1

Begin with the initial plan containing the dummy steps:
$\underline{START}$:

Preconditional: $Null$
effects: $Clear(C), On(C, Tab), Clear(D), On(D, A), On(A, B), On(B, Tab)$

$\underline{END}$:

precondition: $On(A, B), On(B, C)$
effects: $null$

Constraints order: $[START \leftarrow END]$

This state is not a solution, so we pick the unsatisfied precondition $On(B, C)$ in END.
This precondition cannot be satisfied by existing step, so add:

$\underline{STEP\ 1}$: $PutOn(B, C)$

Preconditional: $Clear(B), On(B, x), Clear(C)$
effects: $\neg Clear(C), On(B, C), \neg On(B, x)$

Constraints order: $[START \leftarrow STEP\ 1 \leftarrow END]$

Now, precondition $Clear(C)$ of $STEP\ 1$ can be satisfied by effect $Clear(C)$ of $START$ so this link is added.
Now, precondition $On(B, x)$ of $STEP\ 1$ can be satisfied by effect $On(B, Tab)$ of $START$ so this link is added.
Now, precondition $Clear(B)$ cannot be satisfied by existing step, so add:

$\underline{STEP\ 2}$: $PutOn(A, Tab)$

Preconditional: $Clear(A), On(A, x)$
effects: $On(A, Tab), Clear(x), \neg On(A, x)$

Constraints order: $[START \leftarrow STEP\ 2 \leftarrow STEP\ 1 \leftarrow END]$

Add the link from effect $Clear(x)$ with $x = B$ of $STEP\ 2$ to the same precondition of $STEP\ 1$.
Now, precondition $On(A, x)$ of $STEP\ 2$ can be satisfied by effect $On(A, B)$ of $START$ so this link is added.
Now, precondition $Clear(A)$ cannot be satisfied by existing step, so add:

$\underline{STEP\ 3}$: $PutOn(D, Tab)$

Preconditional: $Clear(D), On(D, x)$
effects: $On(D, Tab), Clear(x), \neg On(D, x)$

Constraints order: $[START \leftarrow STEP\ 3 \leftarrow STEP\ 2 \leftarrow STEP\ 1 \leftarrow END]$

Add the link from effect $Clear(x)$ with $x = A$ of $STEP$ 3 to the same precondition of $STEP$ 2.
Now, precondition $On(D, x)$ of $STEP$ 3 can be satisfied by effect $On(D, A)$ of $START$ so this link is added.
Now, precondition $Clear(D)$ of $STEP$ 3 can be satisfied by effect $Clear(D)$ of $START$ so this link is added.

However, $STEP$ 3 clobbers the precondition $On(A, B)$ of END, so add:

$\underline{STEP\ 4}$: $PutOn(A, B)$

Preconditional: $Clear(A), On(A, x), Clear(B)$
effects: $\neg Clear(B), On(A, B), \neg On(A, x)$

Constraints order: $[START \leftarrow STEP\ 3 \leftarrow STEP\ 2 \leftarrow STEP\ 1 \leftarrow STEP\ 4 \leftarrow END]$

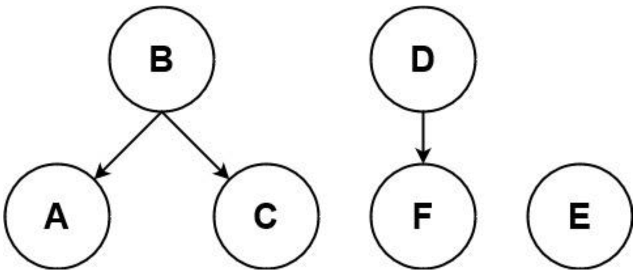
Now, precondition $Clear(B)$ of $STEP$ 4 can be satisfied by effect $Clear(B)$ of $STEP$ 2 so this link is added.
Now, precondition $On(A, x)$ of $STEP$ 4 can be satisfied by effect $On(A, Tab)$ of $STEP$ 2 so this link is added.
Now, precondition $Clear(A)$ of $STEP$ 4 can be satisfied by effect $On(A, Tab)$ of $STEP$ 3 so this link is added.

No more clobbering remaining, and all preconditions are met.
We can declare on the final plan with above steps and constraints:

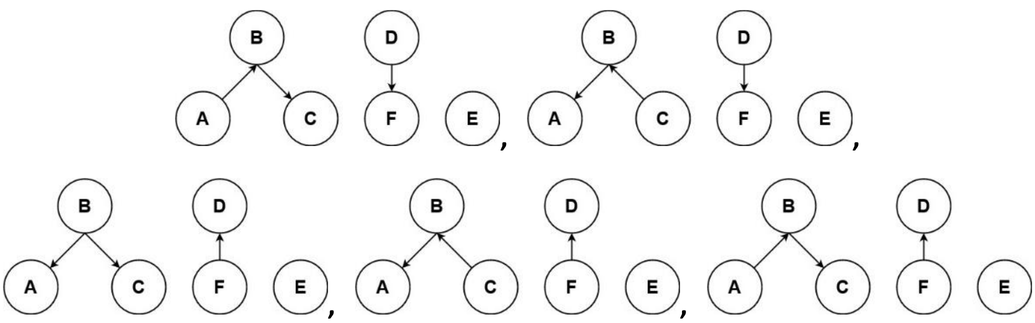
$$[\boldsymbol{START} \leftarrow \boldsymbol{STEP\ 3} \leftarrow \boldsymbol{STEP\ 2} \leftarrow \boldsymbol{STEP\ 1} \leftarrow \boldsymbol{STEP\ 4} \leftarrow \boldsymbol{END}]$$

<u>Question 2</u>

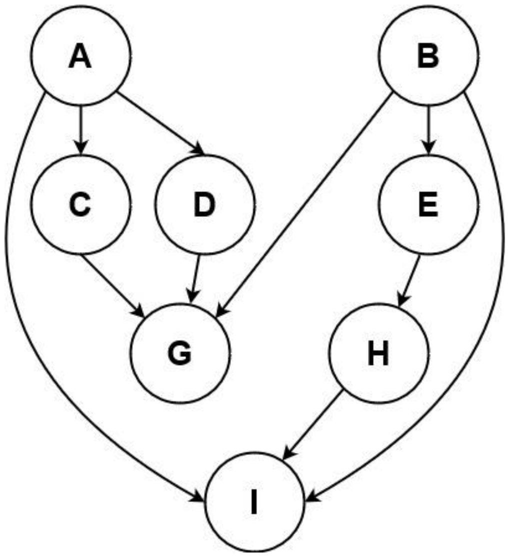
a) An example for a possible Bayes network:



b) No, it is not unique. You can also create the following networks:



Question 3



a) No, it is not a poly-tree. The underlying undirected graph contains a cycle path:

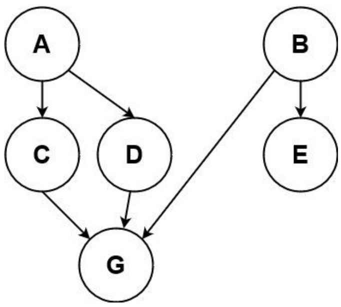
$$[A, C, G, D, A]$$

b) No, it is not singly connected. We can go to I from B in two paths: $[B, I]$ and $[B, E, H, I]$

c)

1) $\boldsymbol{I}(\{\boldsymbol{D}\}, \{\boldsymbol{E}\} \mid \{\})$ → Yes, every path from D to E is blocked by G or I.
2) $\boldsymbol{I}(\{\boldsymbol{D}\}, \{\boldsymbol{E}\} \mid \{\boldsymbol{A}\})$ → Yes, every path from D to E is blocked by G or I.
3) $\boldsymbol{I}(\{\boldsymbol{D}\}, \{\boldsymbol{E}\} \mid \{\boldsymbol{A}, \boldsymbol{H}\})$ → Yes, every path from D to E is blocked by G or I.
4) $\boldsymbol{I}(\{\boldsymbol{B}\}, \{\boldsymbol{E}\} \mid \{\boldsymbol{D}\})$ → No, B and E are connected.
5) $\boldsymbol{I}(\{\boldsymbol{B}\}, \{\boldsymbol{C}\} \mid \{\})$ → Yes, every path from B to C is blocked by G or I.
6) $\boldsymbol{I}(\{\boldsymbol{B}\}, \{\boldsymbol{C}\} \mid \{\boldsymbol{G}\})$ → No, path $[D, G, B, E]$ isn't blocked now.

d) For computing $P(E = true|B = true, G = true)$ we can remove unnecessary nodes and get the following graph:



Now, we can see that E is independent from A, C, D, G because B blocks all paths from E to them. Therefore: $P(E = true|B = true, G = true) = P(E = true|B = true)$.
And we know that $P(E = true|B = true) = 0.8$.
So, $P(E = true|B = true, G = true) = 0.8$ as well.

Question 4

a)

1. For undiscounted rewards:
   Without filing a request is guaranteed to gain 1000 MEMU.
   So, $U(\boldsymbol{A}) = \mathbf{1000}$

   By filing a request with no facilitator, we lose 200 MEMU, and now can calculate the utility value:
   $$U(\boldsymbol{B}) = -200 + [0.02 \cdot 2500 + 0.9 \cdot 1000] = \mathbf{830}$$

   By filing a request with facilitator, we lose 600 MEMU (200 for the request and 400 for the officials to "see reason") and gain 2500 MEMU for sure, and now can calculate the utility value:
   $$U(\boldsymbol{C}) = -600 + 2500 + [0.8 \cdot [0.5 \cdot -10000 + 0.5 \cdot 0] + 0.2 \cdot 0] = -\mathbf{2100}$$

   Therefore, the preferred option is to go with **option A.**

2. For discounted rewards:
   The only changed value will be of option C:
   $$U(\boldsymbol{C}) = -600 + 2500 + 0.2 \cdot [0.8 \cdot [0.5 \cdot -10000 + 0.5 \cdot 0] + 0.2 \cdot 0] = \mathbf{1100}$$
   Now, the preferred option is to go with **option C.**

b)

1. For undiscounted rewards & taking legal advice:
   Without filing a request is guaranteed to gain 1000 MEMU and lose 100 MEMU for the attorney.
   So, $U(\boldsymbol{D}) = \mathbf{900}$

   By filing a request with no facilitator, we lose 200 MEMU, lose 100 MEMU for the attorney. Now, can calculate the utility value:
   $$U(\boldsymbol{E}) = -300 + [0.02 \cdot 2500 + 0.9 \cdot 1000] = \mathbf{730}$$

   By filing a request with facilitator, we lose 100 MEMU for the attorney. There are now two options based on his answer: F1 and F2
   F1: Filing a request with facilitator, gain 2500 MEMU and if charges will be brought, they won't stick.
   $$U(\boldsymbol{F1}) = -100 - 600 + 2500 + [0.8 \cdot 0 + 0.2 \cdot 0] = \mathbf{1800}$$
   F2: Filing a request with facilitator, gain 2500 MEMU and if charges will be brought, they will stick.
   $$U(\boldsymbol{F2}) = -100 - 600 + 2500 + [0.8 \cdot -10000 + 0.2 \cdot 0] = -\mathbf{6200}$$

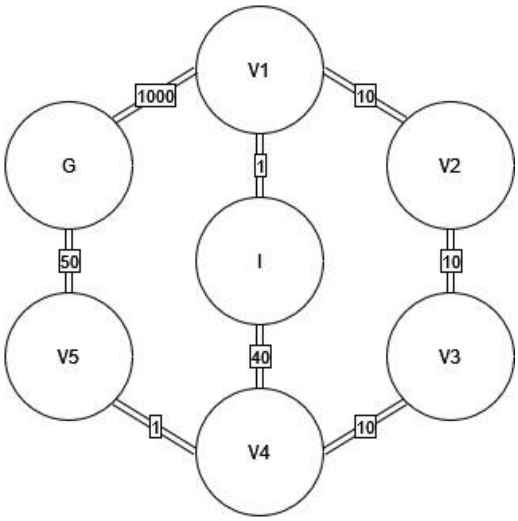
   Now, being convicted based on charges can happen in 50% chances. Meaning that the information from receiving the layer advice will also be split accordingly. We can calculate the utility value for the taking legal advice G:
   $$U(\boldsymbol{G}) = [0.5 \cdot 1800 + 0.5 \cdot 900] = \mathbf{1350}$$
   Therefore, the preferred option is to go with legal advice**.**

2. <u>For discounted rewards:</u>
   The only changed value will be of option F1 and F2:
   F1: Filing a request with facilitator, gain 2500 MEMU and if charges will be brought, they won't stick.
   $$U(\boldsymbol{F1}) = -100 - 600 + 2500 + 0.2 \cdot [0.8 \cdot 0 + 0.2 \cdot 0] = \mathbf{1800}$$
   F2: Filing a request with facilitator, gain 2500 MEMU and if charges will be brought, they will stick.
   $$U(\boldsymbol{F2}) = -100 - 600 + 2500 + 0.2 \cdot [0.8 \cdot -10000 + 0.2 \cdot 0] = \mathbf{200}$$

   We can calculate the utility value for the taking legal advice G:
   $$U(\boldsymbol{G}) = [0.5 \cdot 1800 + 0.5 \cdot 900] = \mathbf{1350}$$
   So, the preferred option is still to go with the legal advice.

<u>Question 5</u>

Our graph:



b1)

- Belief state $S = \{L, B2, B3, B5, Cost\}$ is composed of the following state variables:

  $L \in \{I, V1, V2, V3, V4, V5, G\}$ // The current location in state

  $Bi \in \{T, F, U\}$ for every $i \in \{2,3,5\}$ // T if blocked, F if not, U if unknown. And $I, V1, V4, G$ are never blocked.

  $Cost \in \mathbb{N}^{+}$ // The accumulating cost until now

- Transition probabilities are stochastic when the resulting location is adjacent to one or more edges with state U, in which case its state becomes T or F, depending on the blockage probability of its vertex.

- Our reward will be given only when arriving to G.
  So, $Reward(S) = 0$ for all states, except when $S = S_{goal} = \{G, -, -, -, Cost\}$.
  Which for it: $Reward(G, -, -, -, Cost) = -Cost$. Where "$-$" can receive any of the following values $\{T, F, U\}$.

b2) Let's starts with the quickest path $[I, V1, V2, V3, V4, V5, G]$.
For state $S_0 = \{I, U, U, U, 0\}$ we have two traversals to vertex $V1$:

For S_1: $P\{S_1 = (V1, F, U, U, 1) | S_0, I \to V1\} = 0.5$
For S_2: $P\{S_2 = (V1, T, U, U, 1) | S_0, I \to V1\} = 0.5$

From S_1 we have 2 traversals to vertex $V2$:

For S_3: $P\{S_3 = (V2, F, F, U, 11) | S_1, V1 \to V2\} = 0.5$
For S_4: $P\{S_4 = (V2, F, T, U, 11) | S_1, V1 \to V2\} = 0.5$

From S_3 we have 1 traversal to vertex $V3$:

For S_5: $P\{S_5 = (V3, F, F, U, 21) | S_3, V2 \to V3\} = 1$

From S_5 we have 2 traversals to vertex $V4$:

For S_6: $P\{S_6 = (V4, F, F, F, 31) | S_5, V3 \to V4\} = 0.5$
For S_7: $P\{S_7 = (V4, F, F, T, 31) | S_5, V3 \to V4\} = 0.5$

In state S_6 we know the values of all variables in this state. So, we can easily see that the optimal path will be through $[V5, G]$. And therefore, we receive that $\boldsymbol{U(S_6) = -82}$. The cost along the route $[I, V1, V2, V3, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_5 to S_6 and we go to S_7 instead. In state S_7 we know the values of all variables in this state. We can easily see that the optimal path will be returning from where we came from. And therefore, we receive that $\boldsymbol{U(S_7) = -1061}$. The cost along the route $[I, V1, V2, V3, V4, V5, G]$.

That means that $\boldsymbol{U(S_5)} = \boldsymbol{U(S_3)} = [0.5 \cdot -82 + 0.5 \cdot -1061] = \boldsymbol{-571.5}$

Now, if V3 is blocked, we can't go from S_1 to S_3, and we go to S_4 instead. We can easily see that the optimal path will be returning from where we came from. For that let's continue building possible states on the route $[I, V1, V2, V1, I, V4, V5, G]$

From S_4 we have 1 traversal to vertex $V1$:

For S_8: $P\{S_8 = (V1, F, T, U, 21) | S_4, V2 \to V1\} = 1$

From S_8 we have 1 traversal to vertex I:

For S_9: $P\{S_9 = (I, F, T, U, 22) | S_8, V1 \to I\} = 1$

From S_9 we have 2 traversals to vertex $V4$:

For S_{10}: $P\{S_{10} = (V4, F, T, F, 62) | S_8, I \to V4\} = 0.5$
For S_{11}: $P\{S_{11} = (V4, F, T, T, 62) | S_8, I \to V4\} = 0.5$

In state S_{10} we know the values of all variables in this state. So, we can easily see that the optimal path will be through $[V5, G]$. And therefore, we receive that $\boldsymbol{U(S_6) = -113}$. The cost along the route $[I, V1, V2, V1, I, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_9 to S_{10} and we go to S_{11} instead. In state S_{11} we know the values of all variables in this state. We can easily see that the optimal path will be returning from where we came from. And therefore, we receive that $\boldsymbol{U(S_{11}) = -1103}$. The cost along the route $[I, V1, V2, V1, I, V4, I, V1, G]$.

That means that $\boldsymbol{U(S_9)} = \boldsymbol{U(S_4)} = [0.5 \cdot -113 + 0.5 \cdot -1103] = \boldsymbol{-608}$

And that means that $\boldsymbol{U(S_1)} = [0.5 \cdot -571.5 + 0.5 \cdot -608] = \boldsymbol{-589.75}$

Now, if V2 is blocked, we can't go from S_0 to S_1, and we go to S_2 instead. We can easily see that the optimal path will be returning from where we came from. For that let's continue building possible states on the route $[I, V1, I, V4, V5, G]$

From S_2 we have 1 traversal to vertex I:

For S_{12}: $P\{S_{12} = (I, T, U, U, 2) | S_1, V1 \to I\} = 1$

From S_{12} we have 4 traversals to vertex $V4$:

For S_{13}: $P\{S_{13} = (V4, T, F, F, 42) | S_{12}, I \to V4\} = 0.25$
For S_{14}: $P\{S_{14} = (V4, T, F, T, 42) | S_{12}, I \to V4\} = 0.25$
For S_{15}: $P\{S_{15} = (V4, T, T, F, 42) | S_{12}, I \to V4\} = 0.25$
For S_{16}: $P\{S_{16} = (V4, T, T, T, 42) | S_{12}, I \to V4\} = 0.25$

In state S_{13} and S_{15} we know the values of all variables in this state.
So, we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{13})} = \boldsymbol{U(S_{15})} = \boldsymbol{-93}$.
The cost along the route $[I, V1, I, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_{12} to S_{13} or S_{15} and we go to S_{14} or S_{16} instead.
In state S_{14} and S_{16} we know the values of all variables in this state.
We can easily see that the optimal path in both cases will be returning from where we came from. And therefore, we receive that $\boldsymbol{U(S_{14})} = \boldsymbol{U(S_{16})} - \boldsymbol{1083}$.
The cost along the route $[I, V1, I, V4, I, V1, G]$.

That means that $\boldsymbol{U(S_{12})} = \boldsymbol{U(S_2)} = [0.25 \cdot -93 + 0.25 \cdot -1083 + 0.25 \cdot -93 + 0.25 \cdot -1083] = \boldsymbol{-588}$

Now, we can start working on a different path from I: $[I, V4, V5, G]$
For state $S_0 = \{I, U, U, U, 0\}$ we have 4 traversals to vertex $V4$:

For S_{17}: $P\{S_{17} = (V4, U, F, F, 40) | S_0, I \to V4\} = 0.25$
For S_{18}: $P\{S_{18} = (V4, U, F, T, 40) | S_0, I \to V4\} = 0.25$
For S_{19}: $P\{S_{19} = (V4, U, T, F, 40) | S_0, I \to V4\} = 0.25$
For S_{20}: $P\{S_{20} = (V4, U, T, T, 40) | S_0, I \to V4\} = 0.25$

In state S_{17} and S_{19} we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{17})} = \boldsymbol{U(S_{19})} = \boldsymbol{-91}$.
The cost along the route $[I, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_0 to S_{17} or S_{19} and we can go to S_{18} instead.
Let's try to go for the optimal new path: $[I, V4, V3, V2, V1, G]$.

For state S_{18} we have 2 traversals to vertex $V3$:

For S_{21}: $P\{S_{21} = (V3, F, F, T, 40) | S_{18}, V4 \to V3\} = 0.5$
For S_{22}: $P\{S_{22} = (V3, T, F, T, 40) | S_{18}, V4 \to V3\} = 0.5$

In state S_{21} we know the values of all variables in this state.
So, we can easily see that the optimal path will be through $[V3, V2, V1, G]$.

And therefore, we receive that $U(\boldsymbol{S_{21}}) = -\mathbf{1070}$.
The cost along the route $[I, V4, V3, V2, V1, G]$.

Now, if $V2$ is blocked, we can't go from S_{18} to S_{21} and we can go to S_{22} instead.
Let's try to go for the optimal new path: $[I, V4, V3, V4, I, V1, G]$.

In state S_{22} we know the values of all variables in this state.
So, we can easily see that the optimal path will be through $[V3, V2, V1, G]$.
And therefore, we receive that $U(\boldsymbol{S_{22}}) = -\mathbf{1101}$.
The cost along the route $[I, V4, V3, V4, I, V1, G]$.

And that means that $\boldsymbol{U(S_{18})_{temp}} = [0.5 \cdot -1070 + 0.5 \cdot -1101] = -\mathbf{1085.5}$, however, just getting back to I from $V4$ will get us a cost of -1081. And therefore $\boldsymbol{U(S_{18})} = -\mathbf{1081}$

Now, if $V3$ is blocked, we can't go from S_0 to S_{18} and we can go to S_{20} instead.
In this state we can easily see that the optimal path will be through $[I, V1, G]$.
And therefore, we receive that $U(\boldsymbol{S_{20}}) = -\mathbf{1081}$.
The cost along the route $[I, V4, I, V1, G]$.

Now, when we finished calculating the optimal policy for the given start vertex, we know that by going to $V1$ (S_1 or S_2) we will get that the utility is:

$$[0.5 \cdot -589.75 + 0.5 \cdot -588] = -\mathbf{588.875}$$

By going to $V4$ (S_{17} or S_{18} or S_{19} or S_{20}) we will get that the utility is:

$$[0.25 \cdot -91 + 0.25 \cdot -1081 + 0.25 \cdot -91 + 0.25 \cdot -1081] = -\mathbf{586}$$

So, we can see that our <u>optimal policy</u> will be to go to $\boldsymbol{V4}$ from the initial state.
Then if $\boldsymbol{V5}$ is not blocked, going through it will be the quickest. If it is blocked, it will be best to just go back as well to $\boldsymbol{I, V1, G}$ and don't try our luck with $\boldsymbol{V2}$.

We've reached our destination.

b3) The new information SENSING can give us is only about the state of $V2$. Therefore, the decision to do the sensing or not to do it can happen as the first thing.
For state $S_0 = \{I, U, U, U, 0\}$ we have two traversals to make a SENSING action:

For S_{23}: $P\{S_{23} = (I, F, U, U, 1) | S_0, SENSING\} = 0.5$
For S_{24}: $P\{S_{24} = (I, T, U, U, 1) | S_0, SENSING\} = 0.5$

Let's starts with the quickest path $[I, SENSING, V1, V2, V3, V4, V5, G]$.

For state S_{23} we have 1 traversal to vertex $V1$:

For S_{25}: $P\{S_{25} = (V1, F, U, U, 2) | S_{23}, I \to V1\} = 1$

For state S_{25} we have 2 traversals to vertex $V2$:

For S_{26}: $P\{S_{26} = (V2, F, F, U, 12) | S_{25}, V1 \to V2\} = 0.5$
For S_{27}: $P\{S_{27} = (V2, F, T, U, 12) | S_{25}, V1 \to V2\} = 0.5$

For state S_{26} we have 1 traversal to vertex $V3$:

For S_{28}: $P\{S_{28} = (V3, F, F, U, 22) | S_{26}, V2 \to V3\} = 0.5$

For state S_{28} we have 2 traversals to vertex $V4$:

For S_{29}: $P\{S_{29} = (V4, F, F, F, 32) | S_{28}, V3 \to V4\} = 0.5$
For S_{30}: $P\{S_{30} = (V4, F, F, T, 32) | S_{28}, V3 \to V4\} = 0.5$

In state S_{29} we know the values of all variables in this state.
So, we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{29}) = -83}$.
The cost along the route $[I, SENSING, V1, V2, V3, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_{28} to S_{29} and we go to S_{30} instead.
In state S_{30} we know the values of all variables in this state.
We can easily see that the optimal path will be returning from where we came from.
And therefore, we receive that $\boldsymbol{U(S_{30}) = -1062}$.
The cost along the route $[I, SENSING, V1, V2, V3, V4, V3, V2, V1, G]$.

That means that $\boldsymbol{U(S_{28})} = \boldsymbol{U(S_{26})} = [0.5 \cdot -83 + 0.5 \cdot -1062] = \boldsymbol{-572.5}$

Now, if V3 is blocked, we can't go from S_{25} to S_{26}, and we go to S_{27} instead. We can easily see that the optimal path will be returning from where we came from. For that let's continue building possible states on the route $[I, SENSING, V1, V2, V1, I, V4, V5, G]$

From S_{27} we have 1 traversal to vertex $V1$:

For S_{31}: $P\{S_{31} = (V1, F, T, U, 21) | S_{27}, V2 \to V1\} = 1$

From S_{31} we have 1 traversal to vertex I:

For S_{32}: $P\{S_{32} = (I, F, T, U, 22) | S_{31}, V1 \to I\} = 1$

From S_{32} we have 2 traversals to vertex $V4$:

For S_{33}: $P\{S_{33} = (V4, F, T, F, 62) | S_{32}, I \to V4\} = 0.5$
For S_{34}: $P\{S_{34} = (V4, F, T, T, 62) | S_{32}, I \to V4\} = 0.5$

In state S_{33} we know the values of all variables in this state.
So, we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{33}) = -114}$.
The cost along the route $[I, SENSING, V1, V2, V1, I, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_{32} to S_{33} and we go to S_{34} instead.
In state S_{34} we know the values of all variables in this state.
We can easily see that the optimal path will be returning from where we came from.
And therefore, we receive that $\boldsymbol{U(S_{34}) = -1104}$.
The cost along the route $[I, SENSING, V1, V2, V1, I, V4, I, V1, G]$.

That means that $\boldsymbol{U(S_{32})} = \boldsymbol{U(S_{27})} = [0.5 \cdot -114 + 0.5 \cdot -1104] = \boldsymbol{-609}$

And that means that $\boldsymbol{U(S_{25})} = \boldsymbol{U(S_{23})_{temp}} = [0.5 \cdot -572.5 + 0.5 \cdot -609] = \boldsymbol{-590.75}$

Another option is to go to $[I, SENSING, V4, V5, G]$ from state S_{23}.
From S_{23} we have 4 traversals to vertex $V4$:

For S_{35}: $P\{S_{35} = (V4, F, F, F, 40) | S_{23}, I \to V4\} = 0.25$
For S_{36}: $P\{S_{36} = (V4, F, F, T, 40) | S_{23}, I \to V4\} = 0.25$
For S_{37}: $P\{S_{37} = (V4, F, T, F, 40) | S_{23}, I \to V4\} = 0.25$
For S_{38}: $P\{S_{38} = (V4, F, T, T, 40) | S_{23}, I \to V4\} = 0.25$

In state S_{35} and S_{37} we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{35})} = \boldsymbol{U(S_{37})} = \mathbf{-92}$.
The cost along the route $[I, SENSING, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_{23} to S_{35} or S_{37}. We can go to S_{36} instead.
In state S_{36} we know the values of all variables in this state.
We can easily see that the optimal path will be returning from $[V3, V2, V1, G]$.
And therefore, we receive that $\boldsymbol{U(S_{36})} = \mathbf{-1071}$.
The cost along the route $[I, SENSING, V1, V2, V1, I, V4, I, V1, G]$.

Now, if $V3$ is blocked, we can't go from S_{23} to S_{36} either. We can go to S_{37} instead.
In state S_{36} we know the values of all variables in this state.
We can easily see that the optimal path will be back through $[I, V1, G]$.
And therefore, we receive that $\boldsymbol{U(S_{36})} = \boldsymbol{U(S_{38})} = \mathbf{-1082}$.
The cost along the route $[I, SENSING, V4, I, V1, G]$.

That means that we can now update $U(S_{23})$:

$$\boldsymbol{U(S_{23})} = [0.25 \cdot -92 + 0.25 \cdot -1071 + 0.25 \cdot -92 + 0.25 \cdot -1082] = \mathbf{-584.25}$$

Now, if V2 is blocked, we know from the SENSING action that we better try going to V4.
Let's starts with the path $[I, SENSING, V4, V5, G]$. From S_0 we will go to S_{24} instead.

From S_{24} we have 4 traversals to vertex $V4$:

For S_{39}: $P\{S_{39} = (V4, T, F, F, 40) | S_{24}, I \to V4\} = 0.25$
For S_{40}: $P\{S_{40} = (V4, T, F, T, 40) | S_{24}, I \to V4\} = 0.25$
For S_{41}: $P\{S_{41} = (V4, T, T, F, 40) | S_{24}, I \to V4\} = 0.25$
For S_{42}: $P\{S_{42} = (V4, T, T, T, 40) | S_{24}, I \to V4\} = 0.25$

In state S_{39} and S_{41} we can easily see that the optimal path will be through $[V5, G]$.
And therefore, we receive that $\boldsymbol{U(S_{39})} = \boldsymbol{U(S_{41})} = \mathbf{-92}$.
The cost along the route $[SENSING, I, V4, V5, G]$.

Now, if $V5$ is blocked, we can't go from S_{24} to S_{39} or S_{41}. Because we know that $V2$ is blocked, we shouldn't go to $V3$, and the quickest path from here is through I. So, for both S_{40}, S_{42} we can easily see that the optimal path will be back through $[I, V1, G]$.
And therefore, we receive that $\boldsymbol{U(S_{40})} = \boldsymbol{U(S_{42})} = \mathbf{-1082}$.
The cost along the route $[I, SENSING, V4, I, V1, G]$.

That means that

$$\boldsymbol{U(S_{24})} = [0.25 \cdot -92 + 0.25 \cdot -1082 + 0.25 \cdot -92 + 0.25 \cdot -1082] = \mathbf{-587}$$

Now:

$$\boldsymbol{U(S_0)} = [0.5 \cdot -584.25 + 0.5 \cdot -587] = \mathbf{-585.625}$$

So, we can see that doing the SENSING action did help us and got our reward a bit higher. Our <u>optimal policy</u> now will be doing the SENSING action and then go to $V4$ instead of risking our ways with the passage in more blocking vertices (such as $V3$). If $V5$ is not blocked we will go through there, but if it is, since we know $V2$ blockage, we can choose to go through $V3$ if it is not blocked or go through I in the case that it is.

Question 6

a)

A	B	C	D	decision
2	F	L	0	N
2	F	H	1	Y
2	T	L	0	N
3	F	H	0	N
3	T	H	2	Y
3	T	L	0	Y
3	T	H	1	Y

Need to check all 4 attributes as candidate for root. We get:

With A as root:

A=2: 2N, 1Y $\quad bits(A=2) = 3 \cdot \left(-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right) = 2.7549$

A=3: 1N, 3Y $\quad bits(A=3) = 4 \cdot \left(-\frac{1}{4}\log\left(\frac{1}{4}\right) - \frac{3}{4}\log\left(\frac{3}{4}\right)\right) = 3.2451$

With B as root:

B=F: 2N, 1Y $\quad bits(B=F) = 3 \cdot \left(-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right) = 2.7549$

B=T: 1N, 3Y $\quad bits(B=T) = 4 \cdot \left(-\frac{1}{4}\log\left(\frac{1}{4}\right) - \frac{3}{4}\log\left(\frac{3}{4}\right)\right) = 3.2451$

With C as root:

C=L: 2N, 1Y $\quad bits(C=L) = 3 \cdot \left(-\frac{2}{3}\log\left(\frac{2}{3}\right) - \frac{1}{3}\log\left(\frac{1}{3}\right)\right) = 2.7549$

C=H: 1N, 3Y $\quad bits(C=H) = 4 \cdot \left(-\frac{1}{4}\log\left(\frac{1}{4}\right) - \frac{3}{4}\log\left(\frac{3}{4}\right)\right) = 3.2451$

With D as root:

D=0: 3N, 1Y $\quad bits(D=0) = 4 \cdot \left(-\frac{3}{4}\log\left(\frac{3}{4}\right) - \frac{1}{4}\log\left(\frac{1}{4}\right)\right) = 3.2451$

D=1: 0N, 2Y $\quad bits(D=1) = 2 \cdot \left(-\frac{0}{2}\log\left(\frac{0}{2}\right) - \frac{2}{2}\log\left(\frac{2}{2}\right)\right) = 0$

D=2: 0N, 1Y $\quad bits(D=2) = 1 \cdot \left(-\frac{0}{1}\log\left(\frac{0}{1}\right) - \frac{1}{1}\log\left(\frac{1}{1}\right)\right) = 0$

We easily see that D has the best separation in features. For $D = 1$ or $D = 2$ we can make a final decision and wouldn't need to make another branching. Let's continue for $D = 0$.

A	B	C	D	decision
2	F	L	0	N
2	T	L	0	N
3	F	H	0	N
3	T	L	0	Y

With A as root:

A=2: 2N, 0Y $\quad bits(A=2) = 2 \cdot \left(-\frac{2}{2}\log_2\left(\frac{2}{2}\right) - \frac{0}{2}\log_2\left(\frac{0}{2}\right)\right) = 0$

A=3: 1N, 1Y $\quad bits(A=3) = 2 \cdot \left(-\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right)\right) = 2$

With B as root:

B=F: 2N, 0Y $\quad bits(B=F) = 2 \cdot \left(-\frac{2}{2}\log_2\left(\frac{2}{2}\right) - \frac{0}{2}\log_2\left(\frac{0}{2}\right)\right) = 0$

B=T: 1N, 1Y $\quad bits(B=T) = 2 \cdot \left(-\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right)\right) = 2$

With C as root:

C=L: 2N, 1Y $\quad bits(C=L) = 3 \cdot \left(-\frac{2}{3}\log\left(\frac{2}{3}\right) - \frac{1}{3}\log\left(\frac{1}{3}\right)\right) = 2.7549$

C=H: 1N, 0Y $\quad bits(C=H) = 1 \cdot \left(-\frac{1}{1}\log\left(\frac{1}{1}\right) - \frac{0}{1}\log\left(\frac{0}{1}\right)\right) = 0$

We can choose either A or B as they have the same entropy. For $A=2$ we can make a final decision and wouldn't need to make another branching. Let's continue for $A=3$.

A	B	C	D	decision
3	F	H	0	N
3	T	L	0	Y

With B as root:

B=F: 1N, 0Y $\quad bits(B=F) = 1 \cdot \left(-\frac{1}{1}\log\left(\frac{1}{1}\right) - \frac{0}{1}\log\left(\frac{0}{1}\right)\right) = 0$

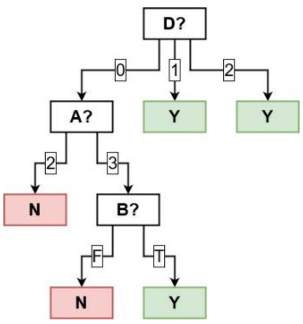
B=T: 0N, 1Y $\quad bits(B=T) = 1 \cdot \left(-\frac{0}{1}\log\left(\frac{0}{1}\right) - \frac{1}{1}\log\left(\frac{1}{1}\right)\right) = 0$

With C as root:

C=L: 0N, 1Y $\quad bits(C=L) = 1 \cdot \left(-\frac{0}{1}\log\left(\frac{0}{1}\right) - \frac{1}{1}\log\left(\frac{1}{1}\right)\right) = 0$

C=H: 1N, 0Y $\quad bits(C=H) = 1 \cdot \left(-\frac{1}{1}\log\left(\frac{1}{1}\right) - \frac{0}{1}\log\left(\frac{0}{1}\right)\right) = 0$

We can choose either B or C as they have the same entropy. Let's choose B and get the following decision tree:

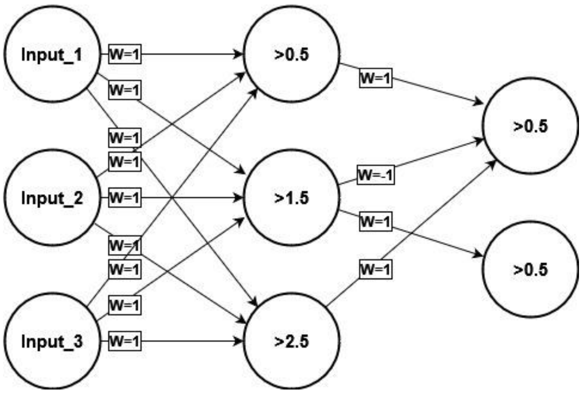


b) Less than 3 nodes are not possible. If we choose a different node than D as our root, then a concise decision still cannot be made (since the entropy wasn't 0 in for the other cases), and we would have gotten 3 internals for sure. If we choose D as our root but a different node for the case where $D=0$, then we would still have another branch with incomplete classification. Therefore, a more compact decision tree is not possible for the above table.

Question 7

a) No, in case where we will have a 0 in some input bit of the input layer, we will encounter a XOR scenario that we could not solve. The network needs to determine if the other input bits are 0 and 1 or 1 and 0 to decide if the number of the lightened bits is odd.

b) Let's look at the following neural network:



Let's create a hidden unit to understand the number of overall lighted bits.
The fully connected first layer with weights 1 for each edge will tell us:

If passed the > 0.5 threshold: at least 1 bit is lightened.
If passed the > 1.5 threshold: at least 2 bit are lightened.
If passed the > 2.5 threshold: at least 3 bit are lightened.

Now, if > 0.5, > 1.5 and > 2.5 thresholds does not passed in the hidden layer, we can deduce that all bits are not lightened up and the number of inputs with value 1 is even. And there aren't at least 2 inputs with value 1.
So overall, for output 1 we get that:

$$0 \cdot 1 + 0 \cdot -1 + 0 \cdot 1 = 0 < 0.5$$

and for output 2 we get that:

$$0 \cdot 1 = 0 < 0.5$$

So, the output will be $[0,0]$ as needed.

Now, if > 0.5 threshold pass but > 1.5 and > 2.5 does not passed in the hidden layer, we can deduce that only 1 bit is lightened up and the number of inputs with value 1 is odd. But there aren't at least 2 inputs with value 1.
So overall, for output 1 we get that:

$$1 \cdot 1 + 0 \cdot -1 + 0 \cdot 1 = 1 > 0.5$$

and for output 2 we get that:

$$0 \cdot 1 = 0 < 0.5$$

So, the output will be $[1,0]$ as needed.

Now, if > 0.5 and > 1.5 thresholds pass but > 2.5 does not passed in the hidden layer, we can deduce that only 2 bits are lightened up and the number of inputs with value 1 is even. And there are at least 2 inputs with value 1.
So overall, for output 1 we get that:

$$1 \cdot 1 + 1 \cdot -1 + 0 \cdot 1 = 0 < 0.5$$

and for output 2 we get that:

$$1 \cdot 1 = 1 > 0.5$$

So, the output will be $[0,1]$ as needed.

Now, if > 0.5, > 1.5 and > 2.5 thresholds pass in the hidden layer, we can deduce that all 3 bits are lightened up and the number of inputs with value 1 is odd.
And there are at least 2 inputs with value 1.
So overall, for output 1 we get that:

$$1 \cdot 1 + 1 \cdot -1 + 1 \cdot 1 = 1 > 0.5$$

and for output 2 we get that:

$$1 \cdot 1 = 1 > 0.5$$

So, the output will be $[1,1]$ as needed.