Reinforcement Learning and Automated Planning

Class 1: Introduction

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Course Goals

Understand models and algorithms developed, mostly in AI, to support autonomous systems, and to automate and support decision making processes.

Systems We Want to Build

Autonomous Systems

- Systems that can figure out how to perform a task without being explicitly programmed to do so
- Systems that can make decisions on their own without being explicitly programmed to do so
- Examples: Autonomous robots and droned, Autonomous Cars, Smart houses (IoT)

Decision Support Systems

- Systems that help us make decisions
- Examples: medical expert systems, logistics planning systems

Planning and RL

We will consider two cases:

- Automated Planning: We have a model of the system and we want to use it to make good decisions.
- Reinforcement Learning: We don't have a model of the system, and we want to learn to make good decisions through trial and error.

Planning for NASA's Mars Exploration Rovers

Autonomous Vehicle for Border Patrol

- Continuously monitor longer borders (fences)
- Able to recognize bombs planted and safely remove them
- Either fully autonomously, or operated remotely
- Saves lives!
- Joint project with IAI

The AOS – Tic Tac Toe

https://www.youtube.com/watch?v=ZV5-6NZ7H70

The AOS – Armadilo Simulation

https://www.youtube.com/watch?v=10sTQ8a_N6c

Some Well-Known RL Success Stories

- https://www.youtube.com/watch?v=0JL04JJjocc
- https://www.youtube.com/watch?v=dltN4MxV1RI
- https://www.youtube.com/watch?v=pkGa8ICQJS8
- https://www.youtube.com/watch?v=l9U8X6I1vow

General Information

- Instructor: Prof. Ronen Brafman
  - Office: 37/209
  - Office Hours: Sundays 14-16 on zoom
    - `https://us02web.zoom.us/j/5842390812`
  - Email: brafman@cs.bgu.ac.il

Course Outline

1. Introduction
2. Markov Processes; Markov decision processes (MDPs);
3. Classical MDP solution algorithms;
4. Specifying MDPs: factored models, dynamic Bayesian networks (DBN), deterministic models, and the AOS language;
5. Search-based solution algorithms for deterministic and stochastic MDPs: A*,AO*, RTDP, MCTS;
   - Assignment: Specify MDP using AOS language. Solve using AO*.
6. Model-Based Reinforcement Learning and Rmax;
   - Assignment: Use model-based RL to learn toy problem model in OpenAI Gym. Solve using value or policy iteration.

Course Outline - Continued

7. Model-Free Prediction;
8. Model-Free Control;
   - Assignment: Solve a problem using Q-learning and SARSA.
9. Value-Function Approximation;
10. Neural nets;
11. Deep RL: DQN and its descendants;
12. Policy Gradient methods;
    - Implement a PG algorithm.
13. POMDPs: Basic properties, specification with the AOS;
    - Assignment: Specify POMDP model using AOS language.
14. Point-based algorithms, Particle Filters, POMCP;
15. Dec-POMDPs;
    - Written or programming assignment on POMDPs.

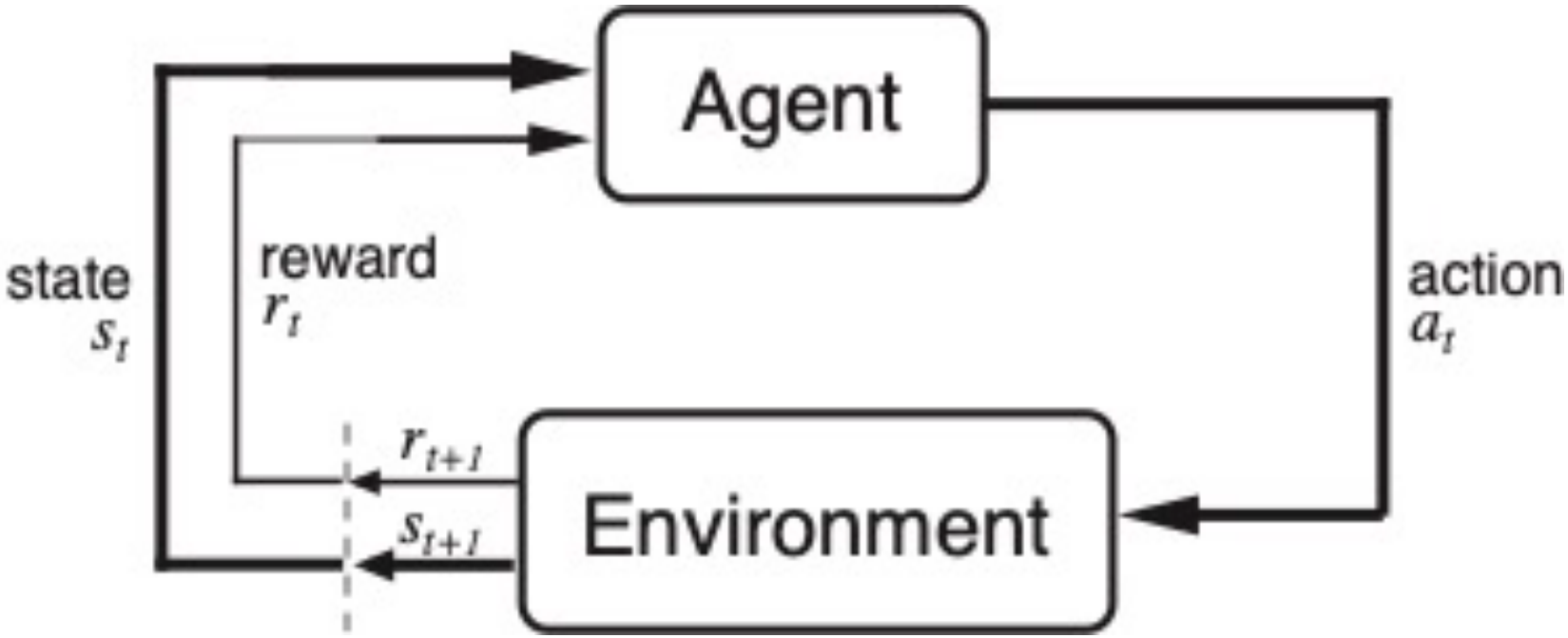
Workload and Grade

- Homework Assignments: 55%
- Exam 40% – Must pass to pass course.
- Physical Attendance 5% (0.715 pts per lecture – up to 7 lectures)
  - Reserve duty: 1 week = 2 lectures. Starting 17/12.

Resources

- David Silver's RL Course (on youtube)
- Sergey Levine's RL Course (on youtube)
- Book: Reinforcement Learning: An Introduction (2^{nd} edition). Sutton and Barto.

Sequential Decision Making: The Basic View



Three Key Concepts

Key Concepts

1. Models – what we mean (semantics)
2. Languages – how we specify it
3. Algorithms – how we solve it

Example: Finite State Machine

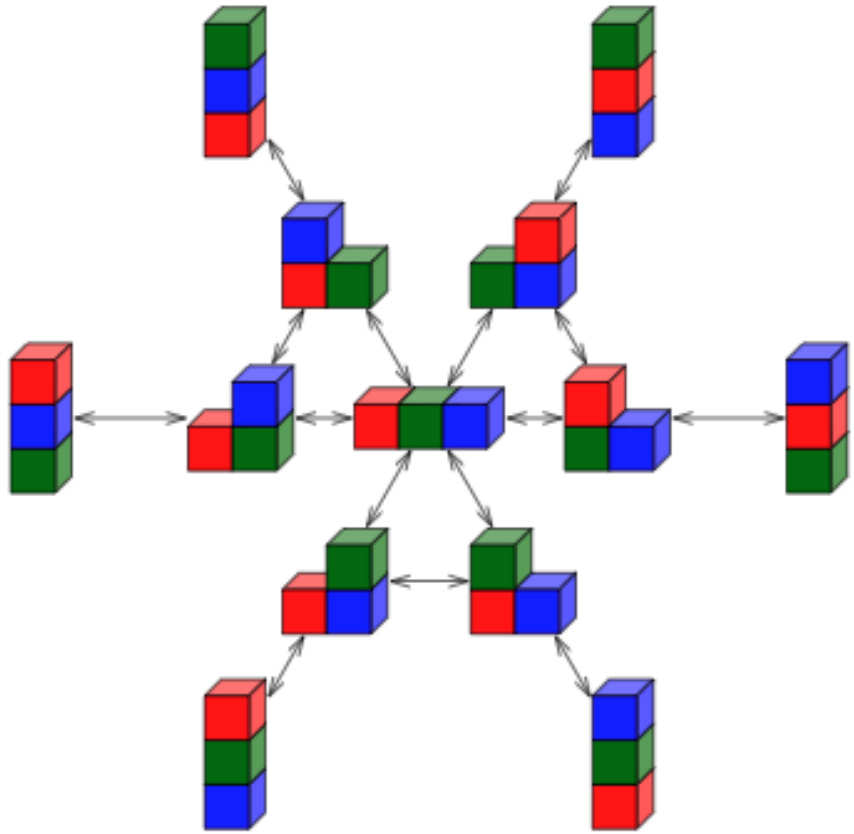
1. Model = Finite State Machine
2. Language = Finite State Machine $\langle S, \Sigma, Tr, s_0, A \rangle$
3. Algorithms = Find an accepted word

FSM is a Decision model

- A basic decision model
- Replace $\langle S, \Sigma, Tr, s_0, A \rangle$ by $\langle S, A, Tr, s_0, G \rangle$ where
  - A is a set of actions or possible decisions
  - G is a set of goal states
- Challenge: What actions should we take to reach a goal state?

Example: The Block's World

- Model $\rightarrow$
- Language = Model (= FSM)
- Algorithm = Dijkstra's



So What's the Language For?

If Language = Model why do we need a language?

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blocks	states
1	1
2	3
3	13
4	73
5	501
6	4051
7	37633
8	394353
9	4596553
10	58941091

So What's the Language For?

If Language = Model, why do we need a language?

blocks	states
1	1
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6	4051
7	37633
8	394353
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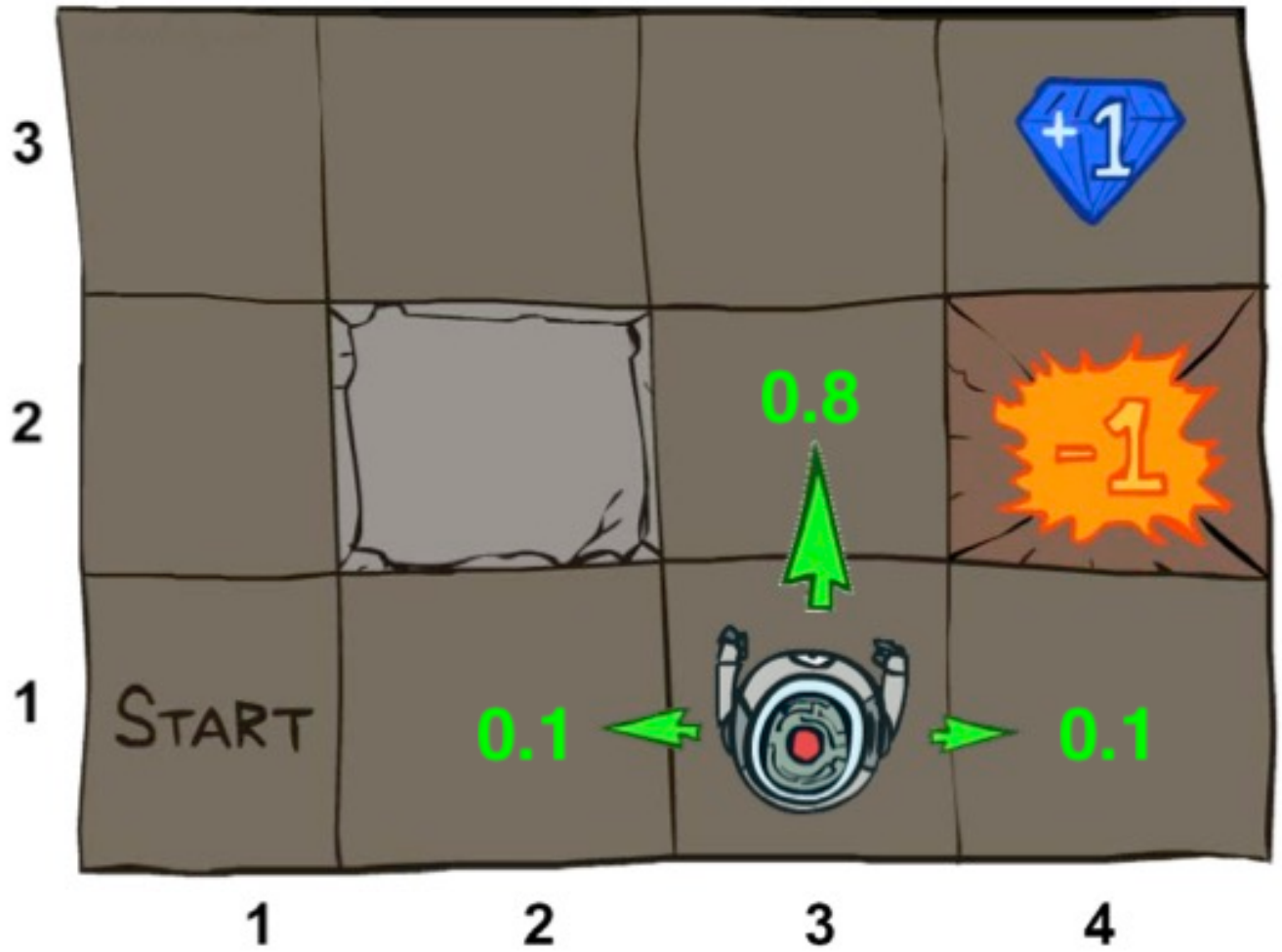
10	58941091

We need a compact way to specify the model

FSM Are Not Enough: *Get the Diamond*



We need a more general model!

Markov Decision Processes (MDPs)

- In a nutshell – an MDP is a state machine with probabilistic transitions
- Markov decision processes formally describe an environment for most reinforcement learning problems
- They are a good model for planning when your actions have stochastic effects, but you can observe their outcome
- There are three key assumptions:
  - The transition function is stochastic
  - The state of the environment is fully observable
  - The current state completely characterizes the process

Markov Property

"The future is independent of the past given the present"

Definition

A state S_t is *Markov* if and only if

$$\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, ..., S_t]$$

- The state captures all relevant information from the history
- Once the state is known, the history may be thrown away
- i.e. The state is a sufficient statistic of the future

State Transition Matrix

For a Markov state s and successor state s', the *state transition probability* is defined by

$$\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$$

State transition matrix $\mathcal{P}$ defines transition probabilities from all states s to all successor states s',

$$\mathcal{P} = \text{from} \begin{matrix} \\ \\ \\ \end{matrix} \begin{matrix} \text{to} \\ \\ \\ \end{matrix} \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{n1} & \dots & \mathcal{P}_{nn} \end{bmatrix}$$

where each row of the matrix sums to 1.

Markov Process

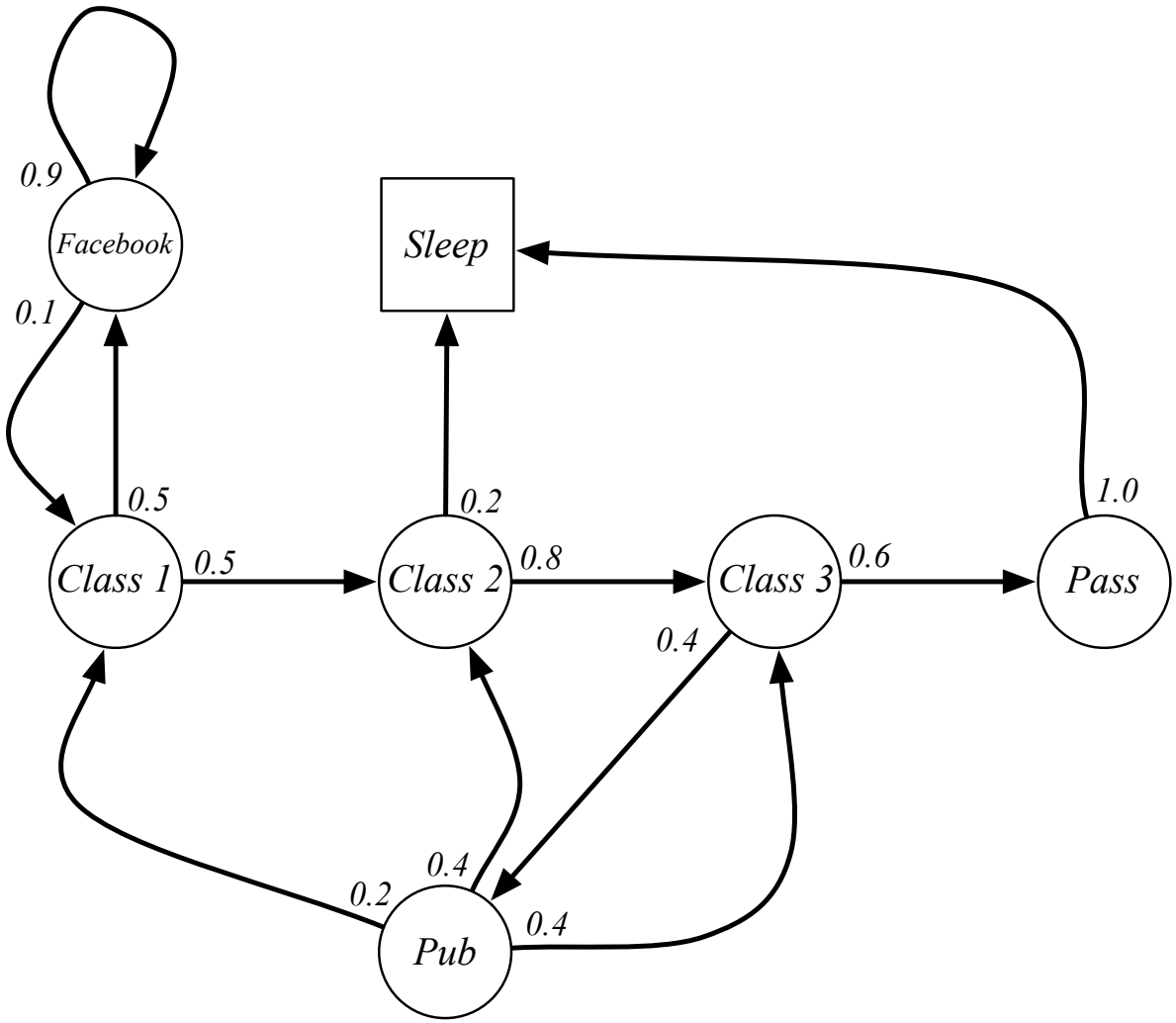
A Markov process is a memoryless random process, i.e. a sequence of random states $S_1, S_2, \ldots$ with the Markov property.

Definition

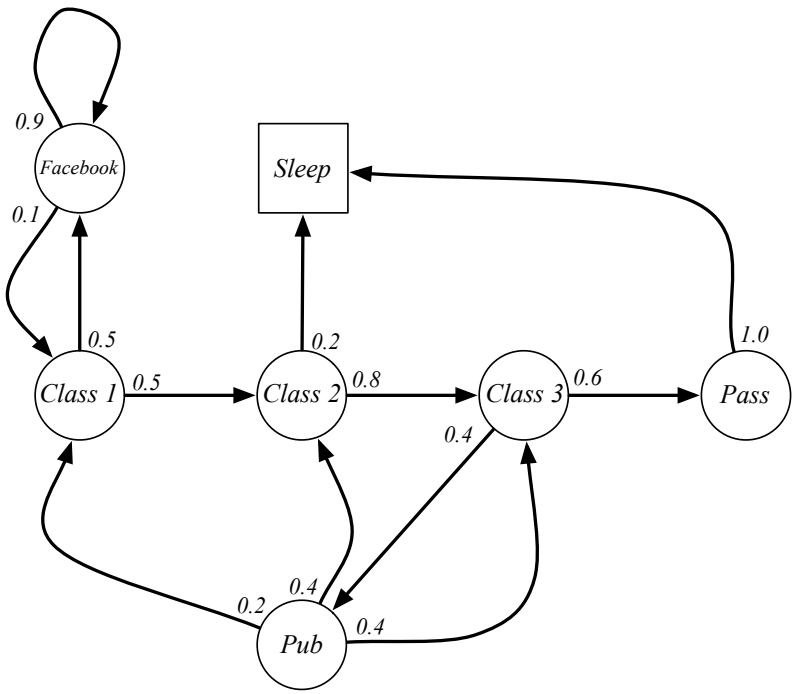
A *Markov Process* (or *Markov Chain*) is a tuple $\langle \mathcal{S}, \mathcal{P} \rangle$

- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$

Example: Student Markov Chain



Example: Student Markov Chain Episodes

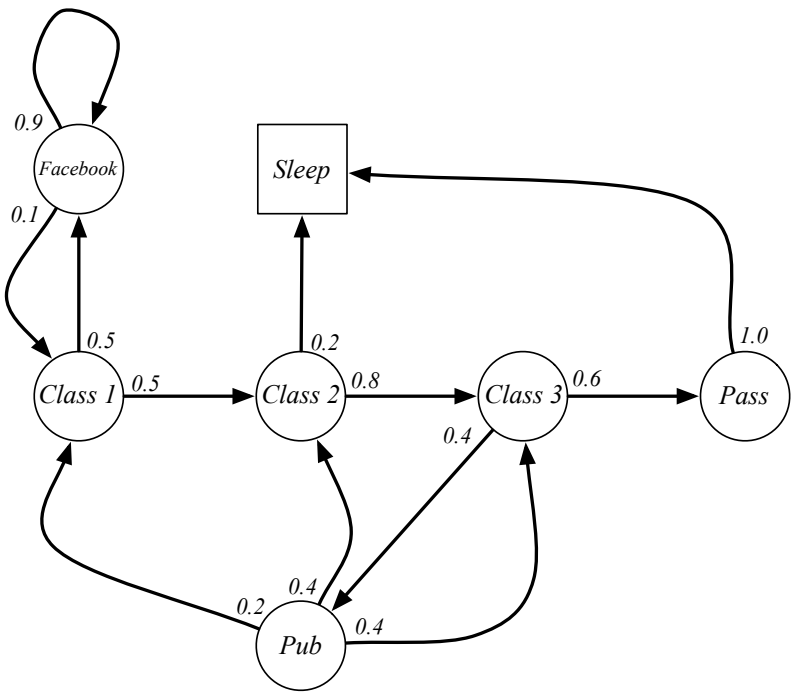


Sample episodes for Student Markov Chain starting from $S_1 = \text{C1}$

$$S_1, S_2, ..., S_T$$

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

Example: Student Markov Chain Transition Matrix



$$\mathcal{P} = \begin{array}{c} \\ C1 \\ C2 \\ C3 \\ Pass \\ Pub \\ FB \\ Sleep \end{array} \begin{array}{c} \begin{array}{ccccccc} C1 & C2 & C3 & Pass & Pub & FB & Sleep \end{array} \\ \begin{bmatrix} & 0.5 & & & & 0.5 & \\ & & 0.8 & & & & 0.2 \\ & & & 0.6 & 0.4 & & \\ & & & & & & 1.0 \\ 0.2 & 0.4 & 0.4 & & & & \\ 0.1 & & & & & 0.9 & \\ & & & & & & 1 \end{bmatrix} \end{array}$$

Markov Reward Process

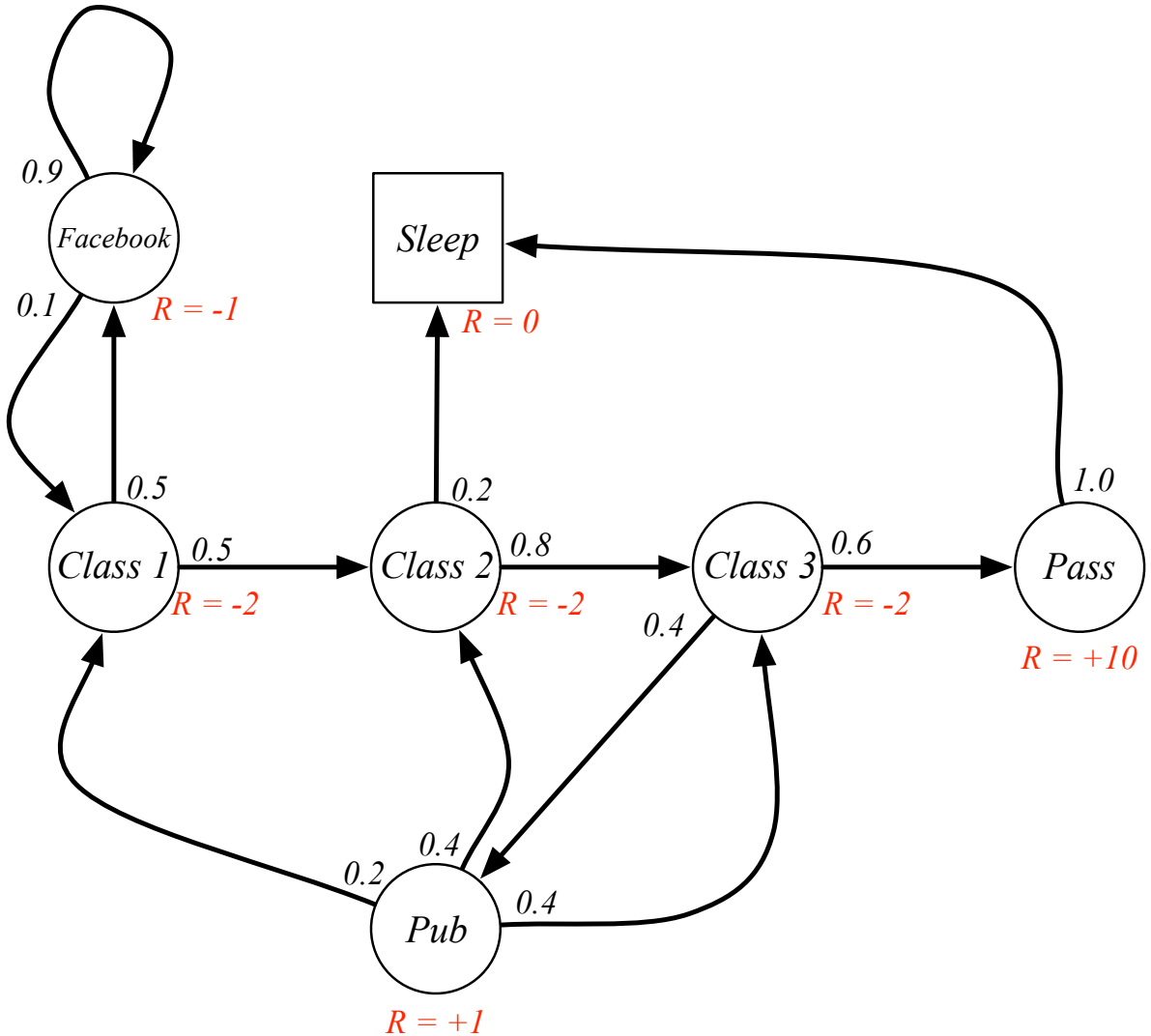
A Markov reward process is a Markov chain with values.

Definition

A *Markov Reward Process* is a tuple $\langle \mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{P}$ is a state transition probability matrix, $\mathcal{P}_{ss'} = \mathbb{P}[S_{t+1} = s' \mid S_t = s]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}_s = \mathbb{E}[R_{t+1} \mid S_t = s]$
- γ is a discount factor, $\gamma \in [0, 1]$

Example: Student MRP



Return

Definition

The *return* G_t is the total discounted reward from time-step t.

$$G_t = R_{t+1} + \gamma R_{t+2} + ... = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The *discount* $\gamma \in [0, 1]$ is the present value of future rewards
- The value of receiving reward R after $k + 1$ time-steps is $\gamma^k R$.
- This values immediate reward above delayed reward.
  - γ close to 0 leads to "myopic" evaluation
  - γ close to 1 leads to "far-sighted" evaluation

Why discount?

Most Markov reward and decision processes are discounted. Why?

- Mathematically convenient to discount rewards
- Avoids infinite returns in cyclic Markov processes
- Uncertainty about the future may not be fully represented
- If the reward is financial, immediate rewards may earn more interest than delayed rewards
- Animal/human behaviour shows preference for immediate reward
- It is sometimes possible to use *undiscounted* Markov reward processes (i.e. $\gamma = 1$), e.g. if all sequences terminate.

Value Function

The value function $v(s)$ gives the long-term value of state s

Definition

The *state value function* $v(s)$ of an MRP is the expected return starting from state s

$$v(s) = \mathbb{E}[G_t \mid S_t = s]$$

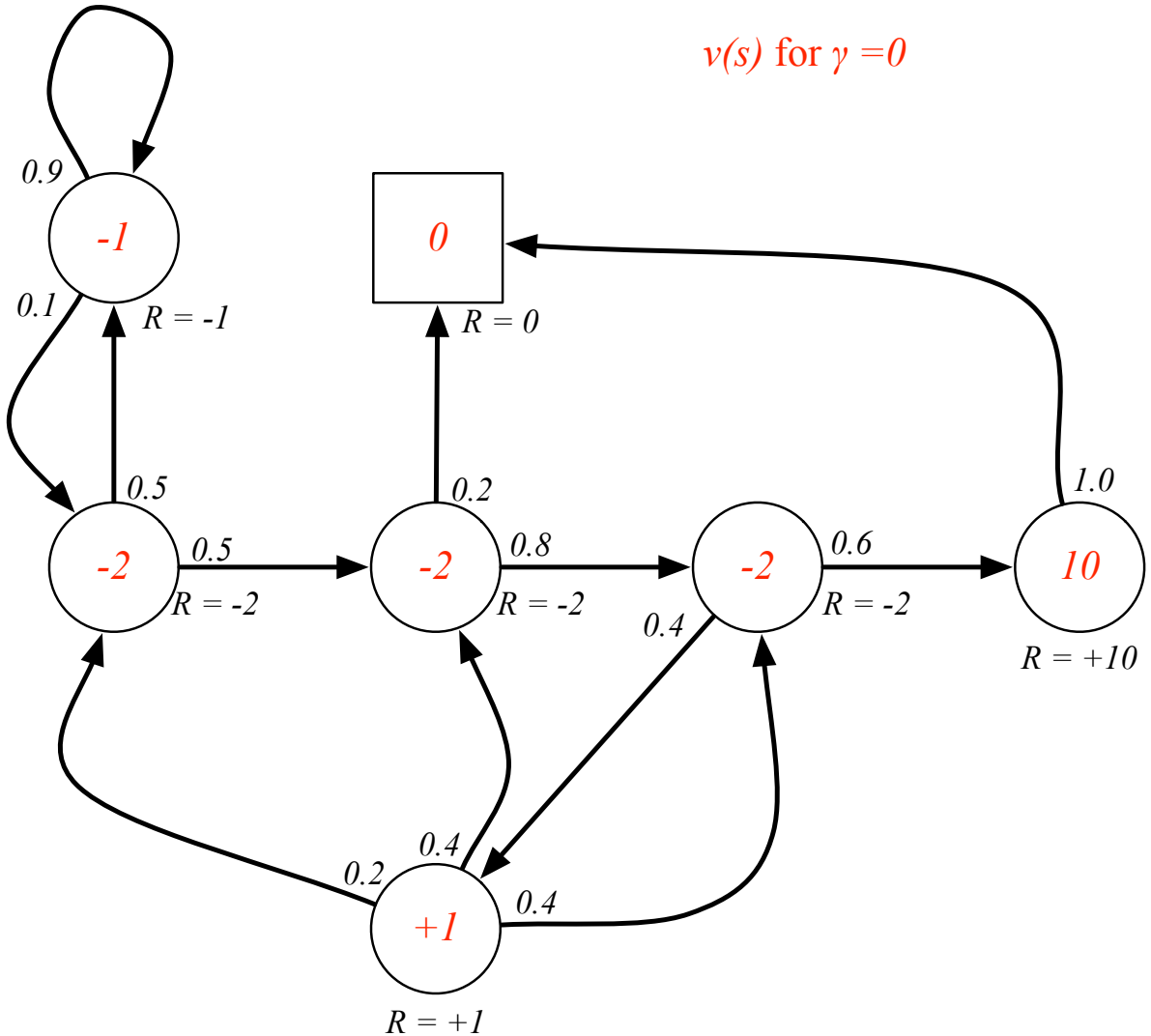
Example: Student MRP Returns

Sample returns for Student MRP:
Starting from $S_1 = \text{C1}$ with $\gamma = \frac{1}{2}$

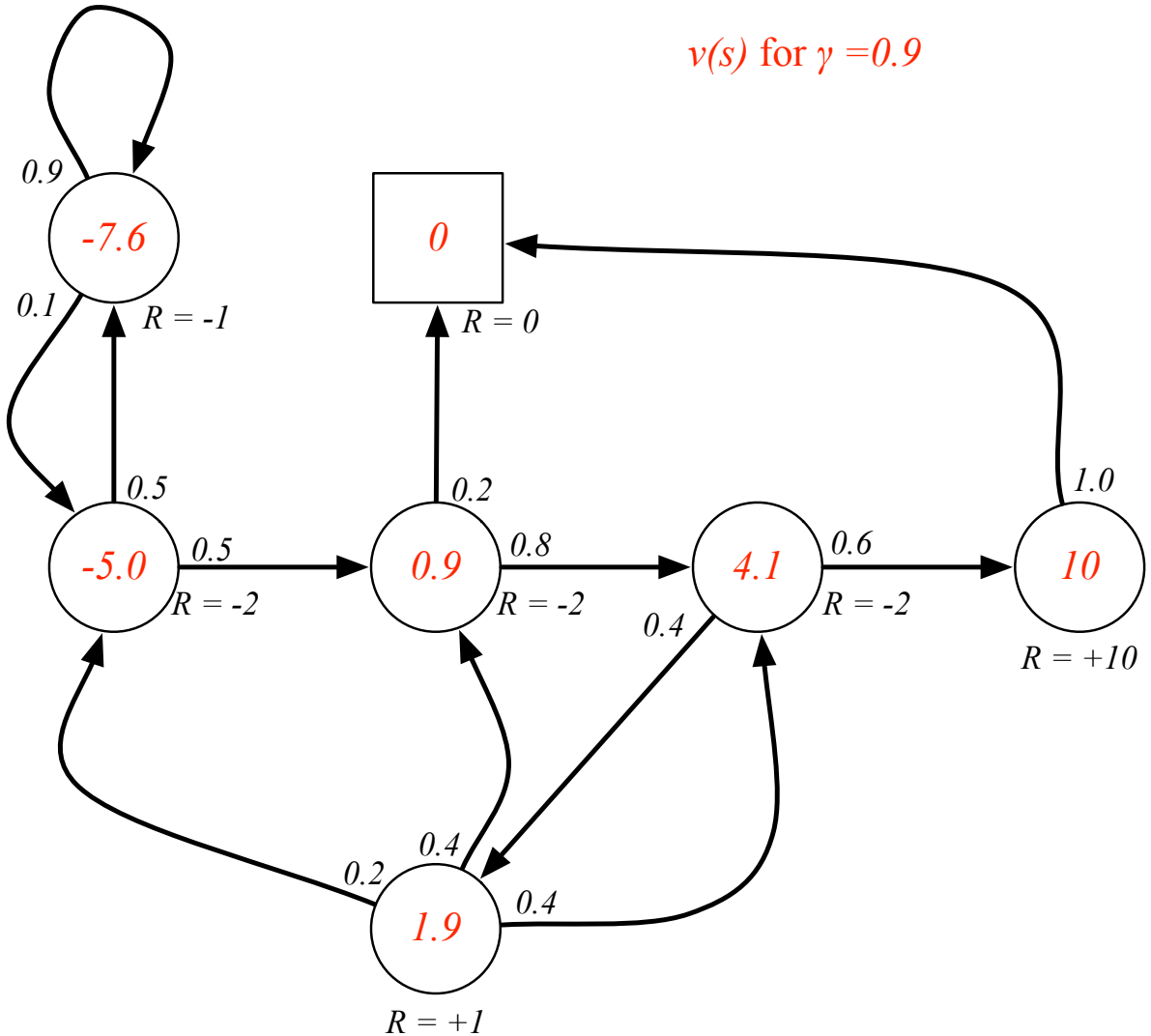
$$G_1 = R_2 + \gamma R_3 + ... + \gamma^{T-2} R_T$$

C1 C2 C3 Pass Sleep	$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 10 * \frac{1}{8}$	$=$	-2.25
C1 FB FB C1 C2 Sleep	$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16}$	$=$	-3.125
C1 C2 C3 Pub C2 C3 Pass Sleep	$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 1 * \frac{1}{8} - 2 * \frac{1}{16} \dots$	$=$	-3.41
C1 FB FB C1 C2 C3 Pub C1 ... FB FB FB C1 C2 C3 Pub C2 Sleep	$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16} \dots$	$=$	-3.20

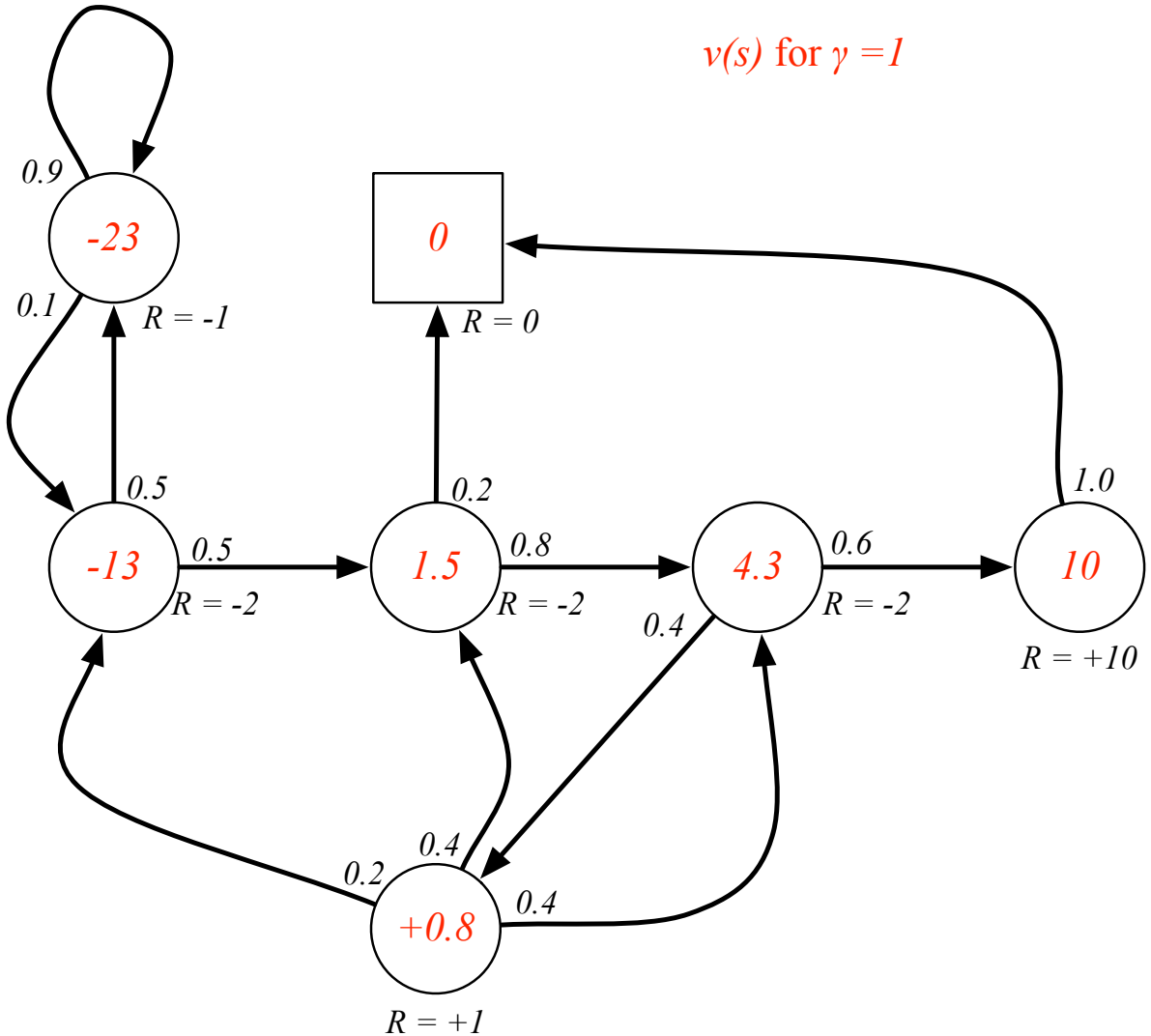
Example: State-Value Function for Student MRP (1)



Example: State-Value Function for Student MRP (2)



Example: State-Value Function for Student MRP (3)



Bellman Equation for MRPs

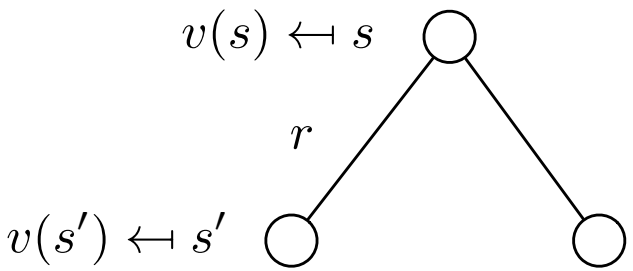
The value function can be decomposed into two parts:

- immediate reward R_{t+1}
- discounted value of successor state $\gamma v(S_{t+1})$

$$
\begin{aligned}
v(s) &= \mathbb{E}\left[G_t \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma\left(R_{t+2} + \gamma R_{t+3} + \ldots\right) \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s\right]
\end{aligned}
$$

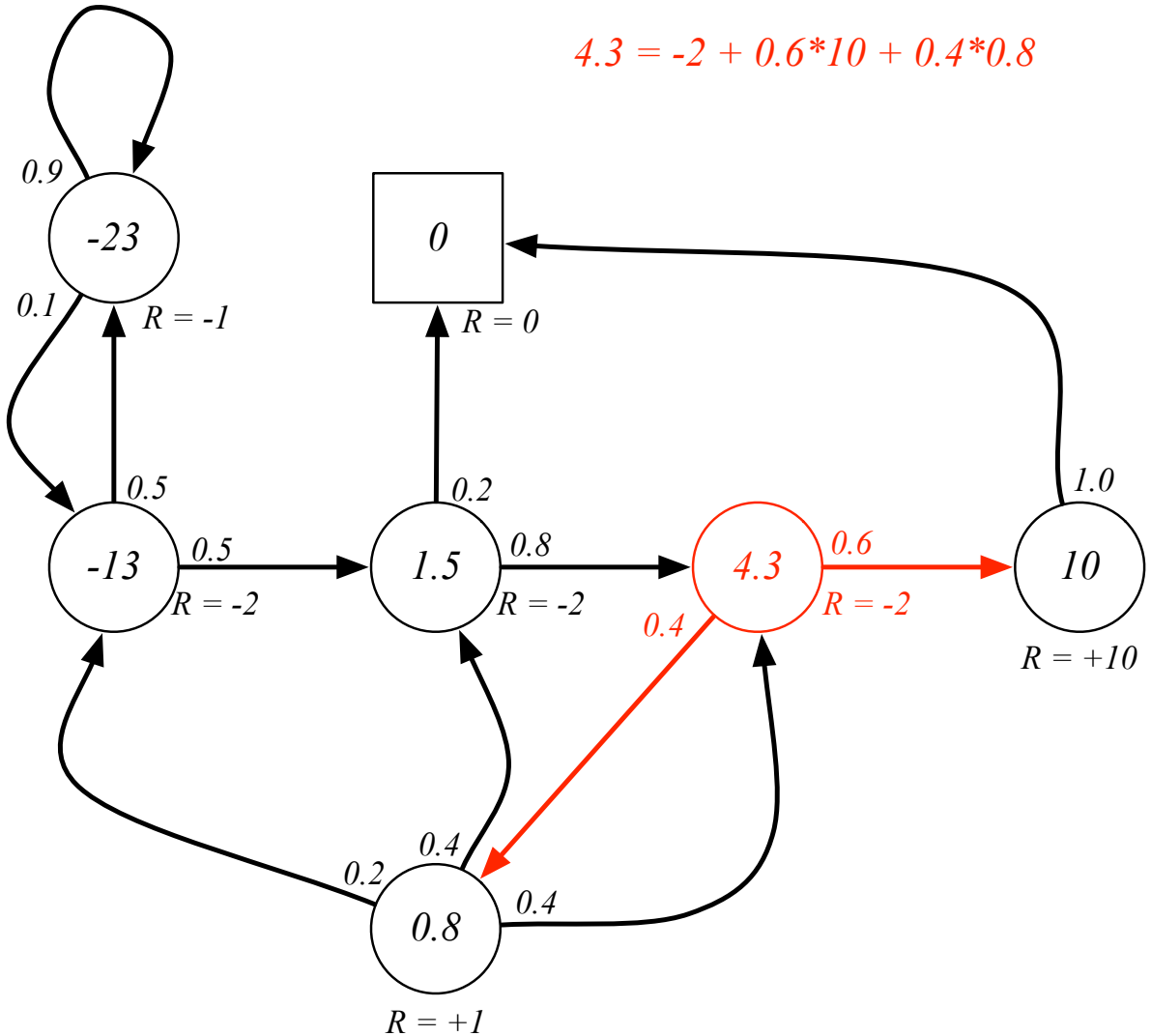
Bellman Equation for MRPs (2)

$$v(s) = \mathbb{E}\left[R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s\right]$$



$$v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')$$

Example: Bellman Equation for Student MRP



Bellman Equation in Matrix Form

The Bellman equation can be expressed concisely using matrices,

$$v = \mathcal{R} + \gamma \mathcal{P} v$$

where v is a column vector with one entry per state

$$\begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_1 \\ \vdots \\ \mathcal{R}_n \end{bmatrix} + \gamma \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{11} & \dots & \mathcal{P}_{nn} \end{bmatrix} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix}$$

Solving the Bellman Equation

- The Bellman equation is a linear equation
- It can be solved directly:

$$
\begin{aligned}
v &= \mathcal{R} + \gamma \mathcal{P} v \\
(I - \gamma \mathcal{P})\, v &= \mathcal{R} \\
v &= (I - \gamma \mathcal{P})^{-1} \mathcal{R}
\end{aligned}
$$

- Computational complexity is $O(n^3)$ for n states
- Direct solution only possible for small MRPs
- There are many iterative methods for large MRPs, e.g.
  - Dynamic programming
  - Monte-Carlo evaluation
  - Temporal-Difference learning

Markov Decision Process

08/01/24

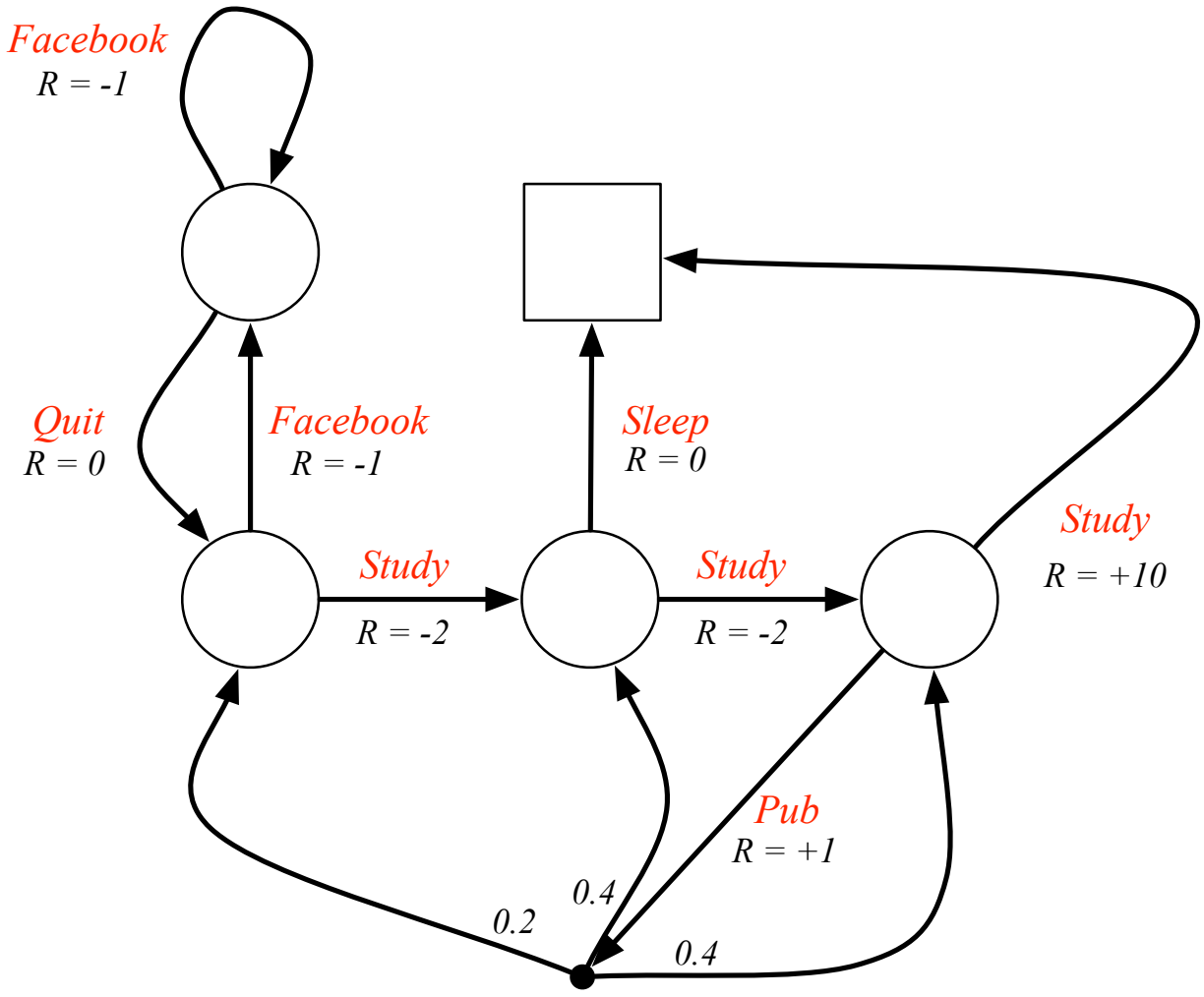
A Markov decision process (MDP) is a Markov reward process with decisions. It is an *environment* in which all states are Markov.

Definition

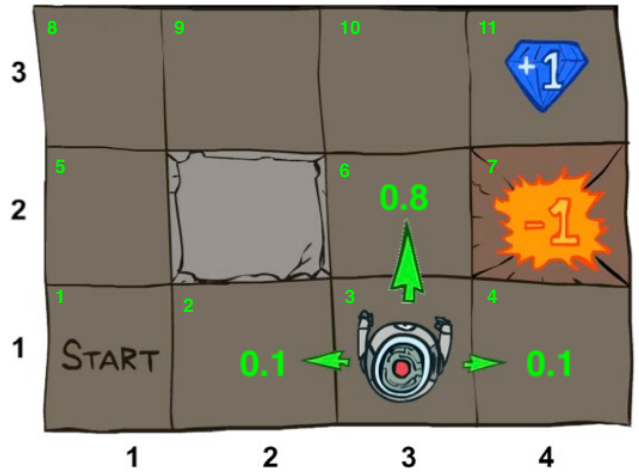
A *Markov Decision Process* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix, $\mathcal{P}^{a}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}^{a}_{s} = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right]$
- γ is a discount factor $\gamma \in [0, 1]$.

Example: Student MDP



Formalizing *Get the Diamond*



- $S = \{s_1, s_2, \ldots, s_{11}\}$
- $A = \{U, D, L, R, S\}$
- Tr : 5 transition matrices
  - Example entires: $Tr(s_1, U, s_5) = 0.8$, $Tr(s_1, D, s_1,) = 0.9$
- $R(s, a) = -0.04 \quad \forall s \in S \setminus \{s_7, s_{11}\}$
- $R(s_7, *) = -1, R(s_1 1, *) = 1$

Policies (1)

Definition

A *policy* π is a distribution over actions given states,

$$\pi(a|s) = \mathbb{P}\left[A_t = a \mid S_t = s\right]$$

- A policy fully defines the behaviour of an agent
- MDP policies depend on the current state (not the history)
- i.e. Policies are *stationary* (time-independent),
  $A_t \sim \pi(\cdot|S_t), \forall t > 0$

Policies (2)

- Given an MDP $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ and a policy π
- The state sequence $S_1, S_2, ...$ is a Markov process $\langle \mathcal{S}, \mathcal{P}^\pi \rangle$
- The state and reward sequence $S_1, R_2, S_2, ...$ is a Markov reward process $\langle \mathcal{S}, \mathcal{P}^\pi, \mathcal{R}^\pi, \gamma \rangle$
- where

$$\mathcal{P}^\pi_{s,s'} = \sum_{a \in \mathcal{A}} \pi(a|s) \mathcal{P}^a_{ss'}$$

$$\mathcal{R}^\pi_s = \sum_{a \in \mathcal{A}} \pi(a|s) \mathcal{R}^a_s$$

Value Function

Definition

The *state-value function* $v_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π

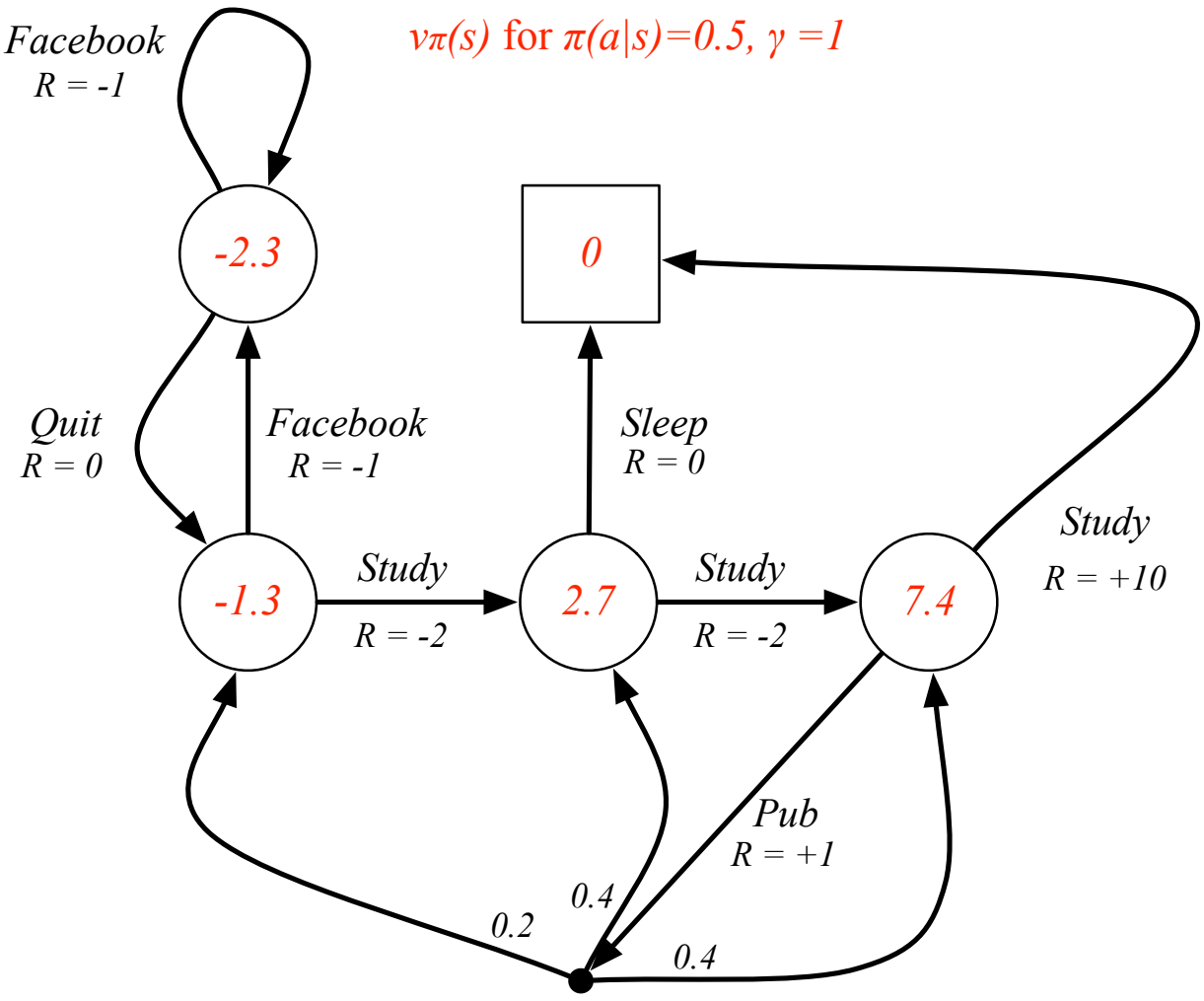
$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

Definition

The *action-value function* $q_\pi(s, a)$ is the expected return starting from state s, taking action a, and then following policy π

$$q_\pi(s, a) = \mathbb{E}_\pi \left[G_t \mid S_t = s, A_t = a \right]$$

Example: State-Value Function for Student MDP



Bellman Expectation Equation

The state-value function can again be decomposed into immediate reward plus discounted value of successor state,

$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

The action-value function can similarly be decomposed,

$$q_\pi(s, a) = \mathbb{E}_\pi \left[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a \right]$$

A_{t+1} is chosen by policy π and therefor $\rightarrow$ $V_\pi(S_{t+1})$

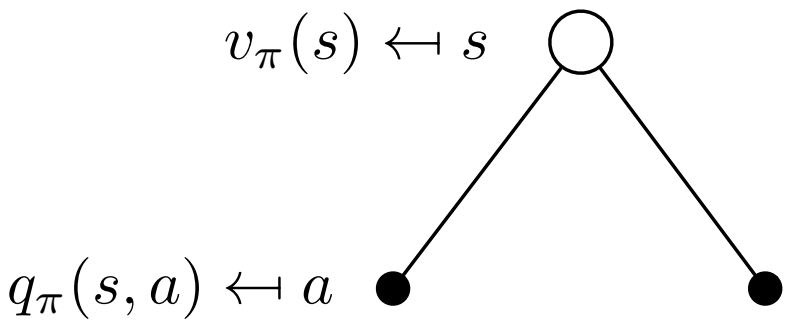
If: $\pi(s) = a'$

Then: $V_\pi(s) = q_\pi(s, a')$

But nothing guarantee that: $V_\pi(s) = q_\pi(s, a^*)$

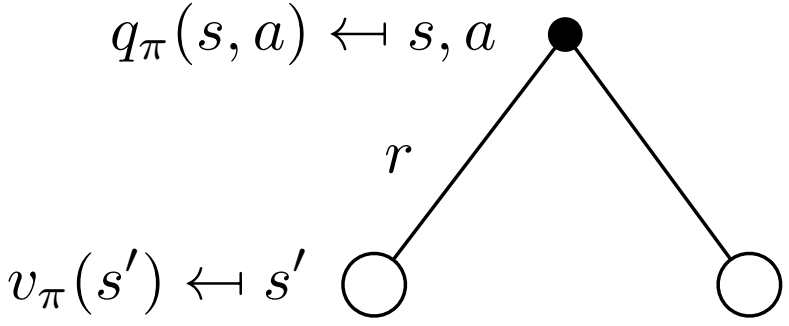
The relation between V_π and q_π is: $V_\pi(s) = q(s, \pi(s))$

Bellman Expectation Equation for V^{π}



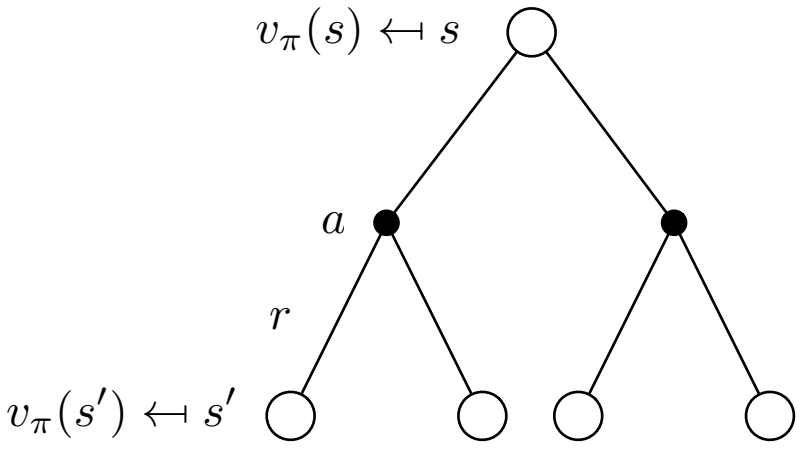
$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s,a)$$

Bellman Expectation Equation for Q^π



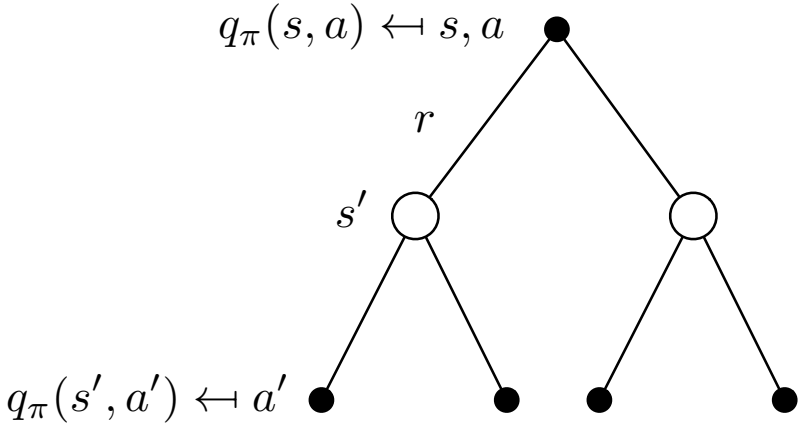
$$q_\pi(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s')$$

Bellman Expectation Equation for v_π (2)



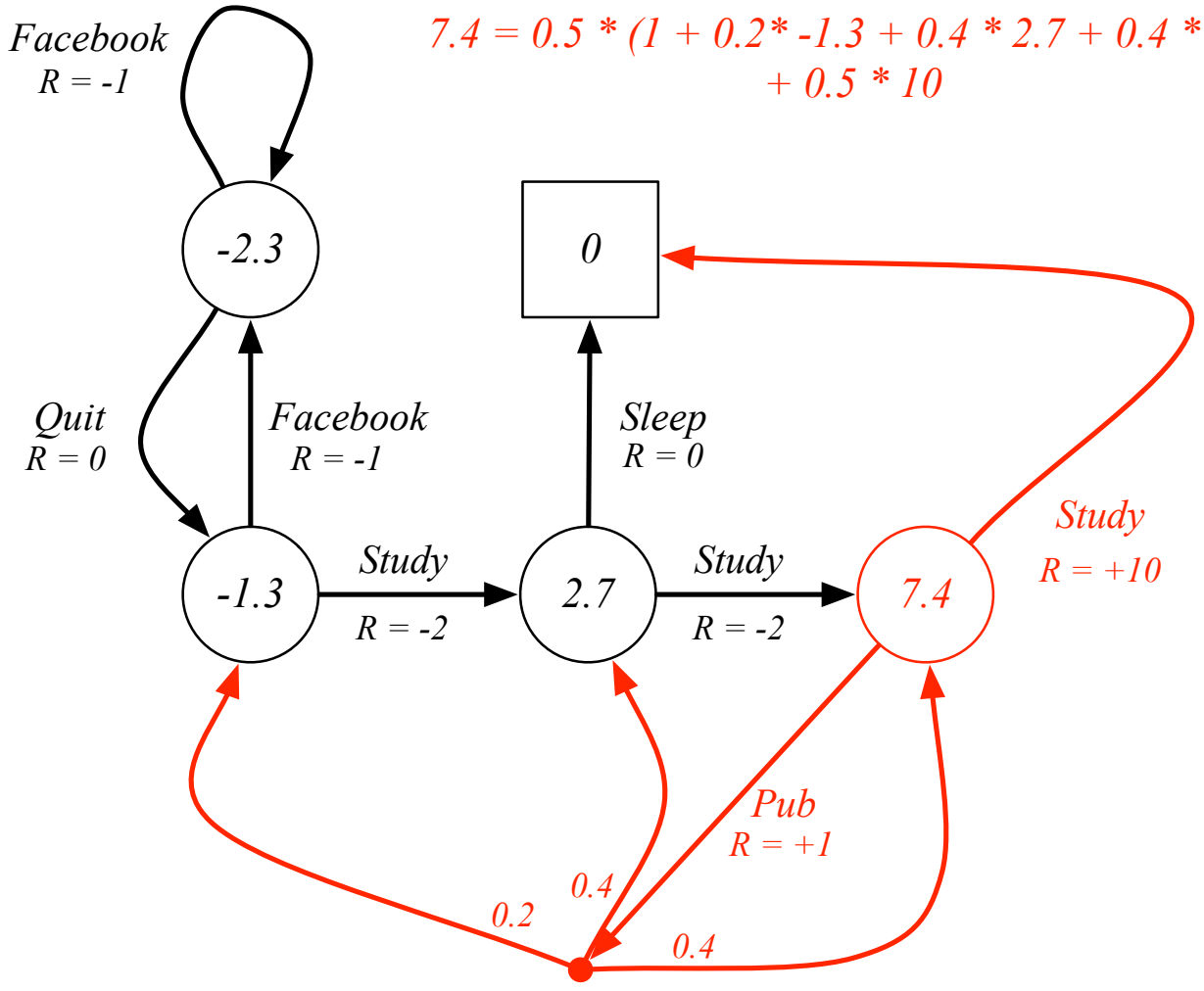
$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

Bellman Expectation Equation for q_π (2)



$$q_\pi(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \sum_{a' \in \mathcal{A}} \pi(a'|s') q_\pi(s', a')$$

Example: Bellman Expectation Equation in Student MDP



Bellman Expectation Equation (Matrix Form)

The Bellman expectation equation can be expressed concisely using the induced MRP,

$$v_\pi = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi v_\pi$$

with direct solution

$$v_\pi = (I - \gamma \mathcal{P}^\pi)^{-1} \mathcal{R}^\pi$$

Optimal Value Function

Definition

The *optimal state-value function* $v_*(s)$ is the maximum value function over all policies

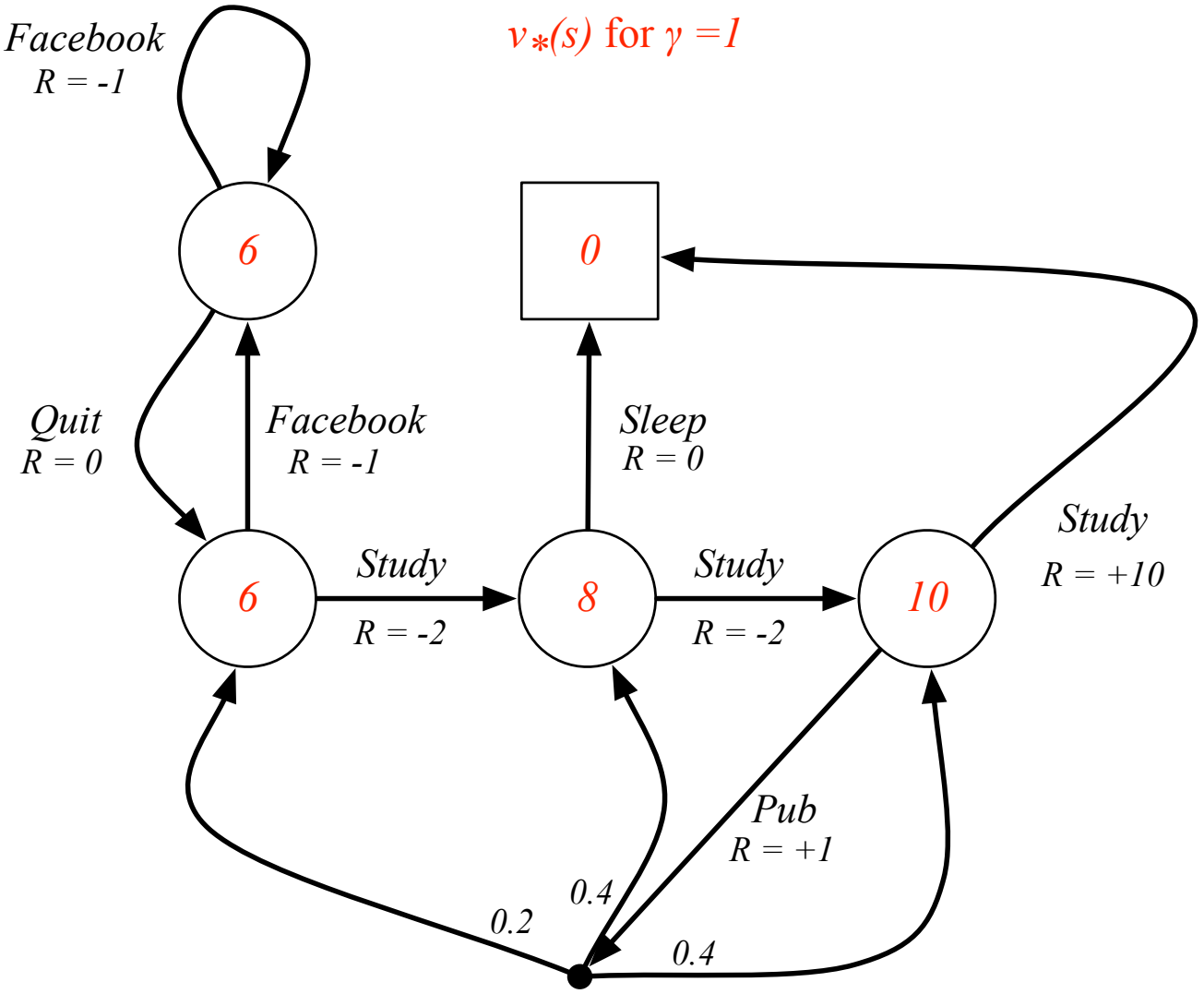
$$v_*(s) = \max_\pi v_\pi(s)$$

The *optimal action-value function* $q_*(s, a)$ is the maximum action-value function over all policies

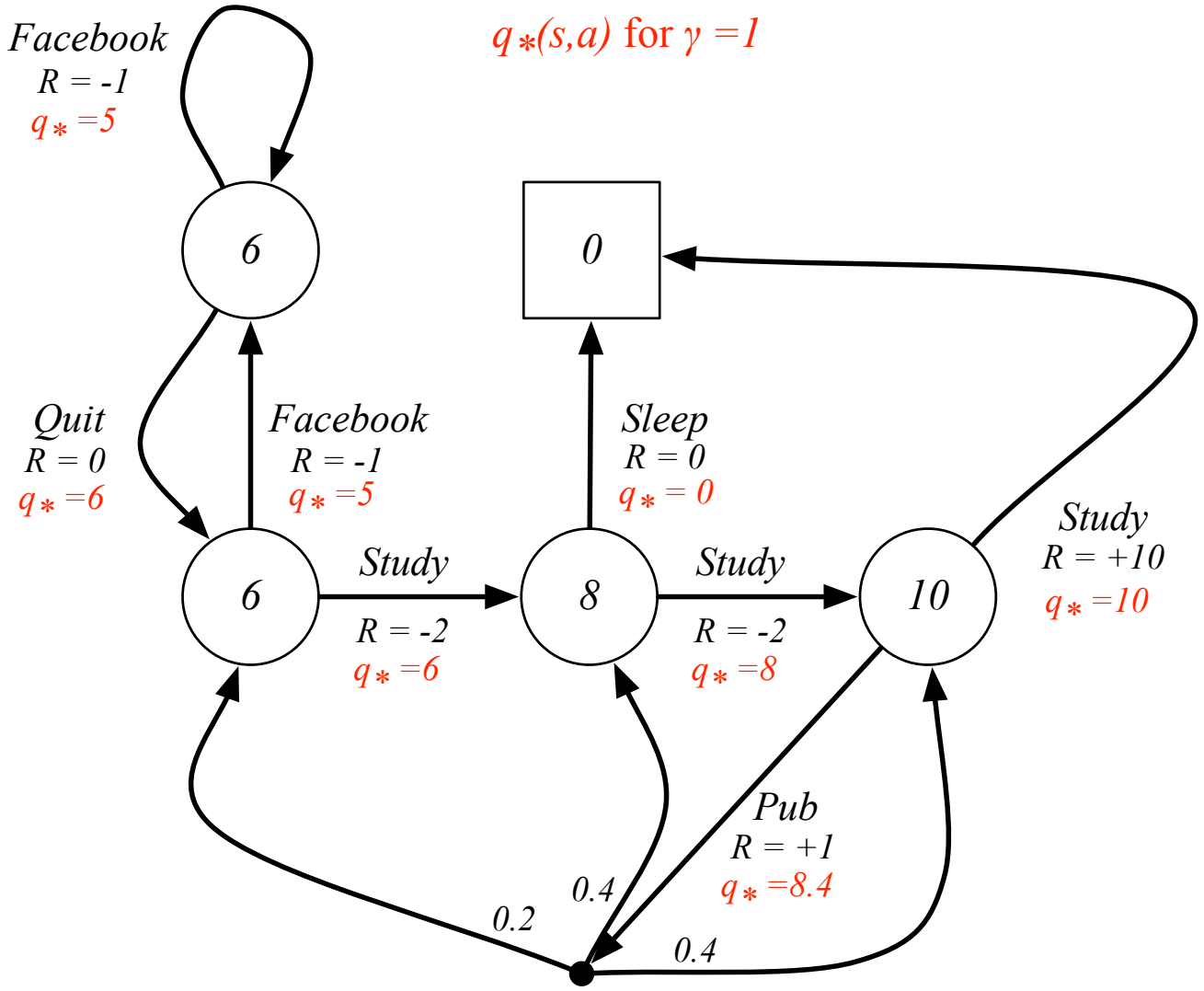
$$q_*(s, a) = \max_\pi q_\pi(s, a)$$

- The optimal value function specifies the best possible performance in the MDP.
- An MDP is "solved" when we know the optimal value fn.

Example: Optimal Value Function for Student MDP



Example: Optimal Action-Value Function for Student MDP



Optimal Policy

Define a partial ordering over policies

$$\pi \geq \pi' \text{ if } v_\pi(s) \geq v_{\pi'}(s), \forall s$$

Theorem

For any Markov Decision Process

- *There exists an optimal policy π_* that is better than or equal to all other policies, $\pi_* \geq \pi, \forall \pi$*
- *All optimal policies achieve the optimal value function, $v_{\pi_*}(s) = v_*(s)$*
- *All optimal policies achieve the optimal action-value function, $q_{\pi_*}(s, a) = q_*(s, a)$*

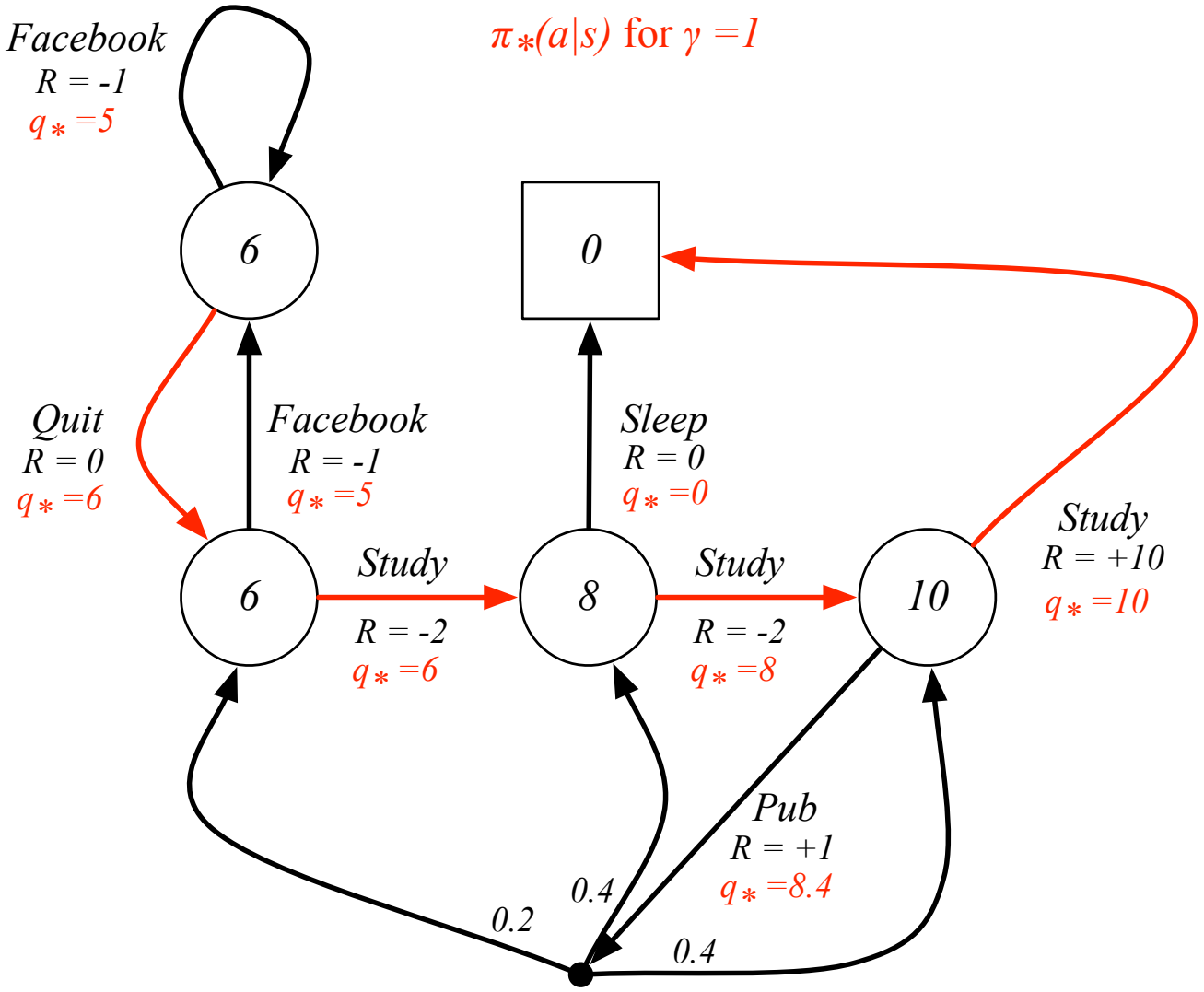
Finding an Optimal Policy

An optimal policy can be found by maximising over $q_*(s, a)$,

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\text{argmax}}\ q_*(s, a) \\ 0 & \textit{otherwise} \end{cases}$$

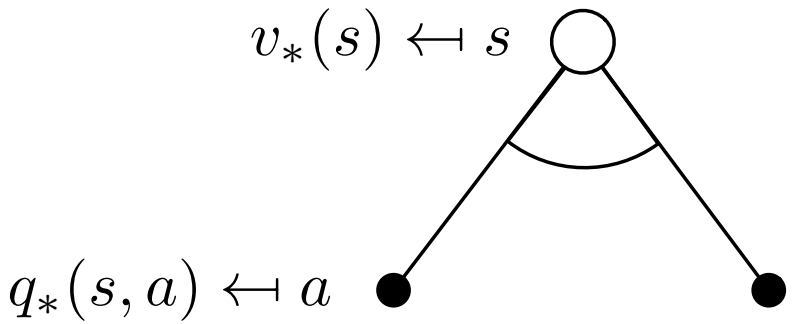
- There is always a deterministic optimal policy for any MDP
- If we know $q_*(s, a)$, we immediately have the optimal policy

Example: Optimal Policy for Student MDP



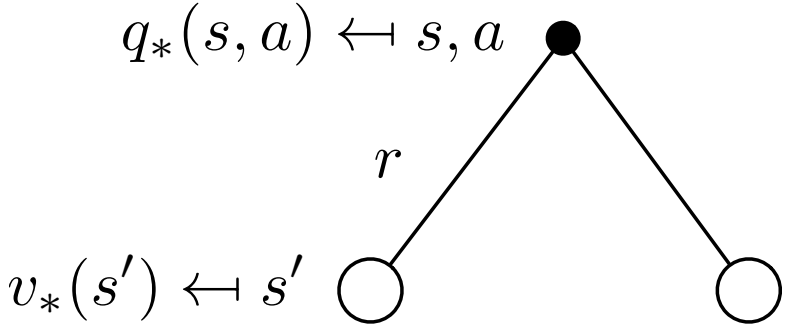
Bellman Optimality Equation for v_*

The optimal value functions are recursively related by the Bellman optimality equations:



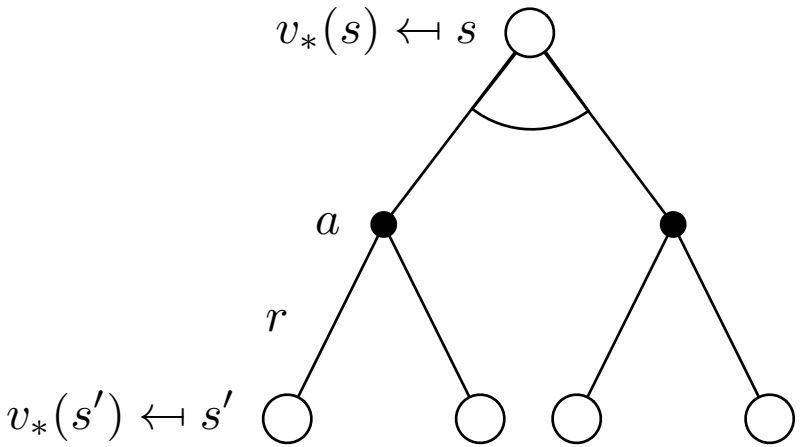
$$v_*(s) = \max_a q_*(s,a)$$

Bellman Optimality Equation for Q^*



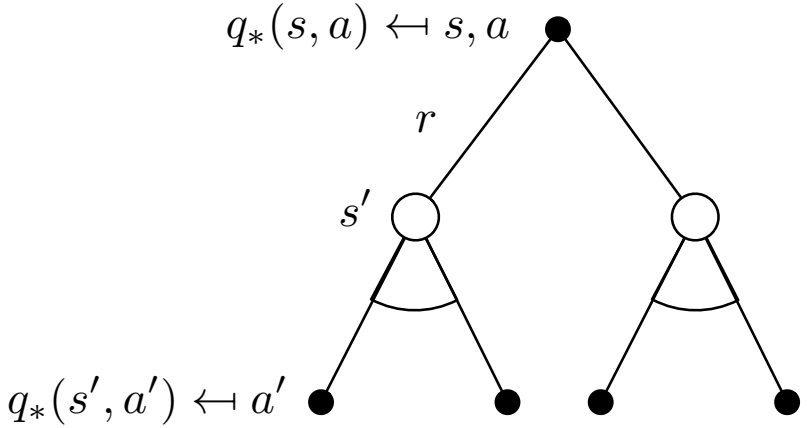
$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

Bellman Optimality Equation for V^* (2)



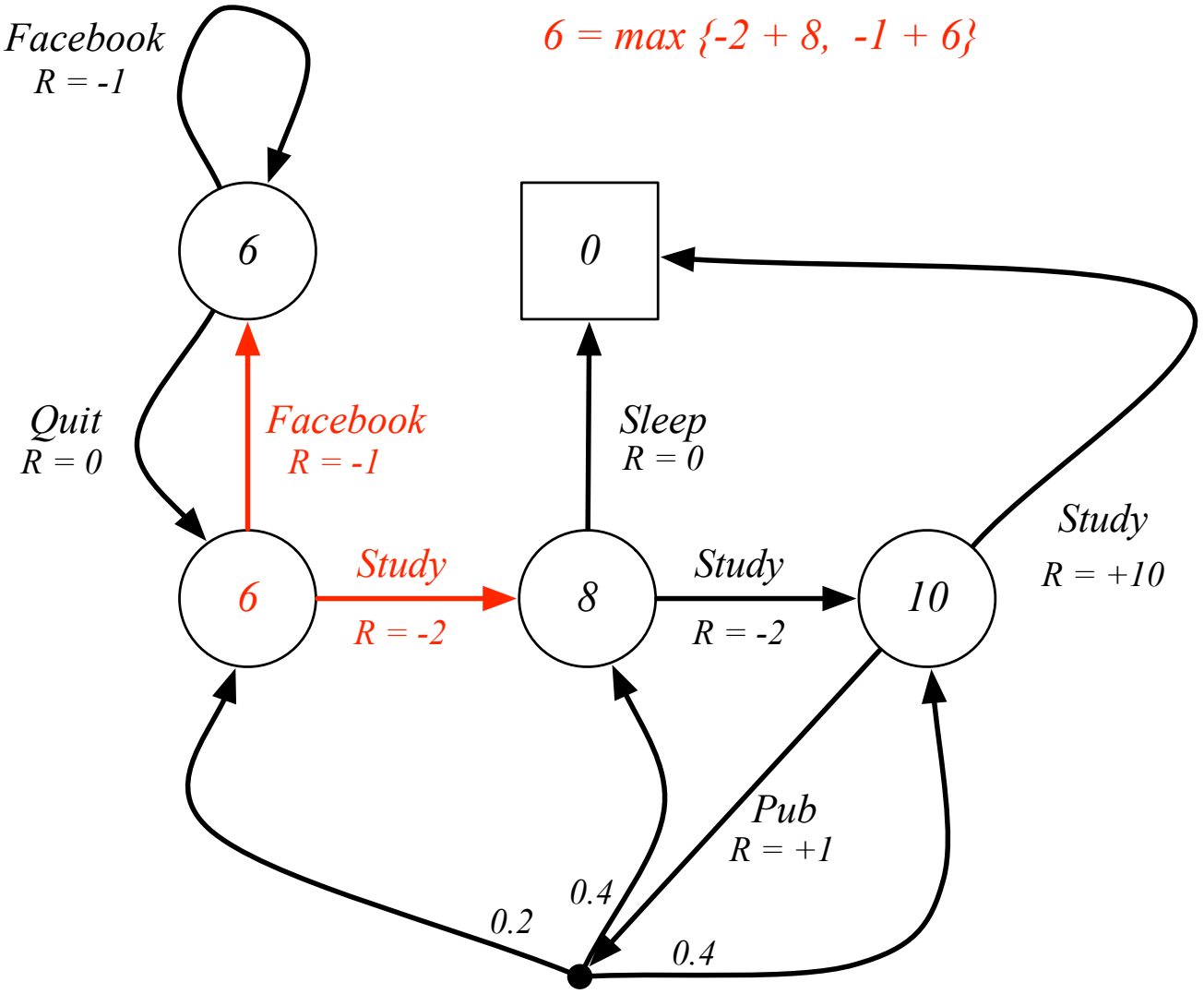
$$v_*(s) = \max_a \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

Bellman Optimality Equation for Q^* (2)



$$q_*(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \max_{a'} q_*(s', a')$$

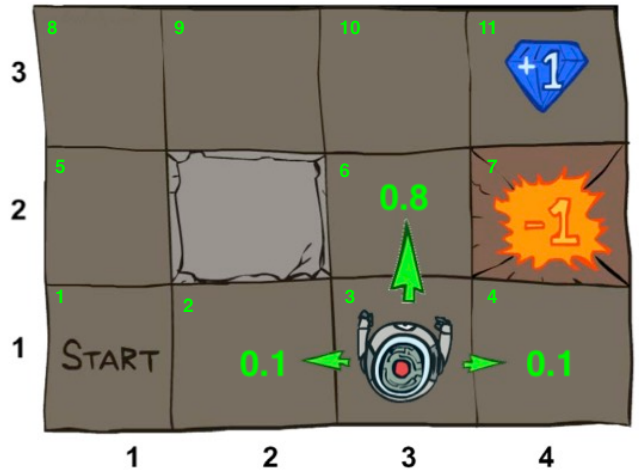
Example: Bellman Optimality Equation in Student MDP



Solving the Bellman Optimality Equation

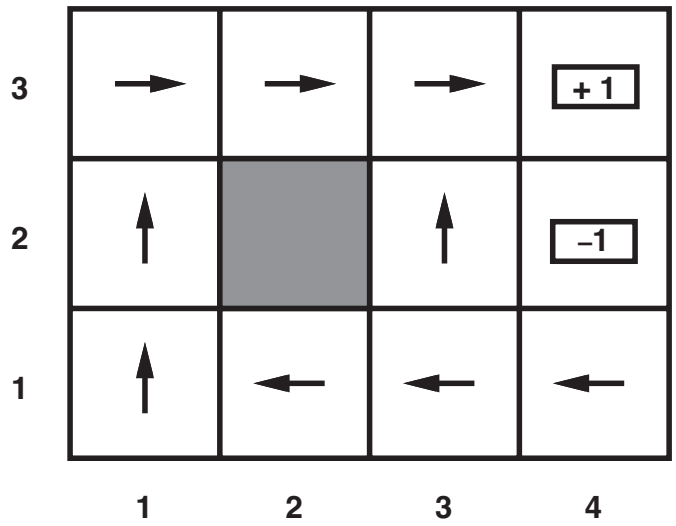
- Bellman Optimality Equation is non-linear
- No closed form solution (in general)
- Many iterative solution methods
  - Value Iteration
  - Policy Iteration
  - Q-learning
  - Sarsa

Formalizing *Get the Diamond*

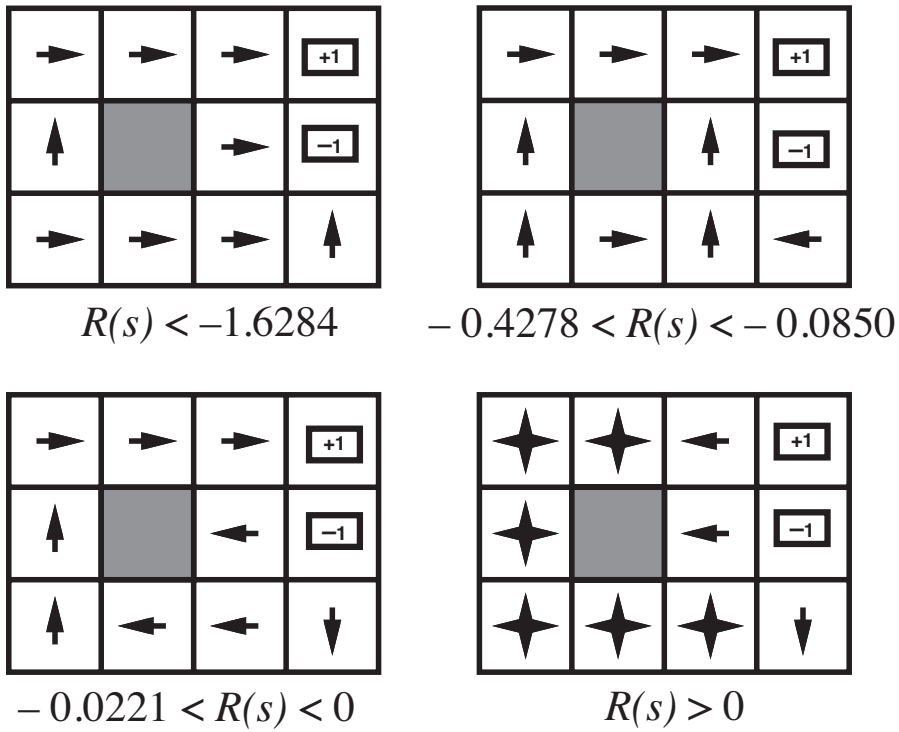


- $S = \{s_1, s_2, \ldots, s_{11}\}$
- $A = \{U, D, L, R, S\}$
- Tr : 5 transition matrices
  - Example entires: $Tr(s_1, U, s_5) = 0.8$, $Tr(s_1, D, s_1,) = 0.9$
- $R(s, a) = -0.04 \quad \forall s \in S \setminus \{s_7, s_{11}\}$
- $R(s_7, *) = -1, R(s_1 1, *) = 1$

Optimal Policies: *Get the Diamond*



(a)



(b)

R(s) = -0.04

Other values

Ergodic Markov Process (no exam)

An ergodic Markov process is

- *Recurrent*: each state is visited an infinite number of times
- *Aperiodic*: each state is visited without any systematic period

Theorem

An ergodic Markov process has a limiting stationary distribution $d^{\pi}(s)$ *with the property*

$$d^{\pi}(s) = \sum_{s' \in \mathcal{S}} d^{\pi}(s') \mathcal{P}_{s's}$$

Ergodic MDP (no exam)

Definition

An MDP is ergodic if the Markov chain induced by any policy is ergodic.

For any policy π, an ergodic MDP has an *average reward per time-step* ρ^π that is independent of start state.

$$\rho^\pi = \lim_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\sum_{t=1}^{T} R_t\right]$$

POMDPs (no exam)

A Partially Observable Markov Decision Process is an MDP with hidden states. It is a hidden Markov model with actions.

Definition

A *POMDP* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, \mathcal{P}, \mathcal{R}, \mathcal{Z}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{O}$ is a finite set of observations
- $\mathcal{P}$ is a state transition probability matrix, $\mathcal{P}^a_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}^a_s = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right]$
- $\mathcal{Z}$ is an observation function, $\mathcal{Z}^a_{s'o} = \mathbb{P}\left[O_{t+1} = o \mid S_{t+1} = s', A_t = a\right]$
- γ is a discount factor $\gamma \in [0, 1]$.