RMHD-CHM Equation

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1 Expanding the RMHD-CHM Equations

We begin with the 3 RMHD-CHM Equations

$$d_t U + \nabla_{\parallel} J = 0 \tag{1}$$

$$\partial_\tau \psi + \nabla_{\parallel} \phi = \eta J + \alpha \nabla_{\parallel} \chi \tag{2}$$

$$d_t \chi + \nabla_{\parallel} J = 0 \tag{3}$$

To simplify the notation, I will use $\partial_{x_i} f = f_{x_i}$ So we have:

$$U = \nabla_{\perp}^2 \phi = \phi_{xx} + \phi_{yy}, J = \psi_{xx} + \psi_{yy} \tag{4}$$

$$d_t f = f_t + [\phi, f] = f_t + \phi_x f_y - \phi_y f_x, \nabla_{\parallel} f = f_z - [\psi, f] = f_z - \psi_x f_y + \psi_y f_x \tag{5}$$

$$U_t + \phi_x U_y - \phi_y U_x + J_z - \psi_x J_y + \psi_y J_x = 0 \tag{6}$$

$$\psi_\tau + \phi_z - \psi_x \phi_y + \psi_y \phi_x = \eta J + \alpha(\chi_z - \psi_x \chi_y + \psi_y \chi_x) \tag{7}$$

$$\chi_t + \phi_x \chi_y - \phi_y \chi_x + J_z - \psi_x J_y + \psi_y J_x = 0 \tag{8}$$

Now evaluate for equation 4

$$\phi_{xxt} + \phi_{yyt} + \phi_x(\phi_{xxy} + \phi_{yyy}) - \phi_y(\phi_{xxx} + \phi_{yyx}) + \psi_{xxz} + \psi_{yyz} - \psi_x(\psi_{xxy} + \psi_{yyy}) + \psi_y(\psi_{xxx} + \psi_{yyx}) = 0 \tag{9}$$

$$\psi_\tau + \phi_z - \psi_x \phi_y + \psi_y \phi_x = \eta(\psi_{xx} + \psi_{yy}) + \alpha(\chi_z - \psi_x \chi_y + \psi_y \chi_x) \tag{10}$$

$$\chi_t + \phi_x \chi_y - \phi_y \chi_x + \psi_{xxz} + \psi_{yyz} - \psi_x(\psi_{xxy} + \psi_{yyy}) + \psi_y(\psi_{xxx} + \psi_{yyx}) = 0 \tag{11}$$

We now have 3 equations with 3 dependent variables (ϕ, ψ, χ) and 5(?) independent variables (x, y, z, t, τ), and 2(?) constants (α, η).

These equations should be able to be inputted into Cantwell's Mathematica code to get the determining equations without too much difficulty.