

Data Structures and Patterns Revision 2





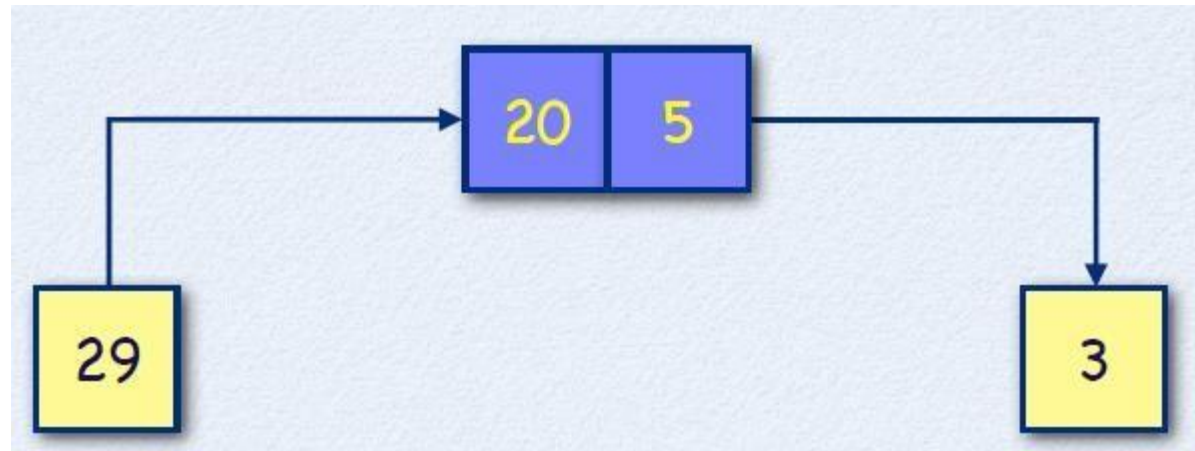
Lecture 7





Queue

- Queue is another important basic data structure.
- A queue is a special version of a linear sequence where access to element is **only** possible at its front and end.





Queue Behaviour

- Queues manage elements in first-in, first-out (FIFO) manner, just like a real queue.
- A queue underflow happens when one tries to dequeue on an empty queue.
- A queue overflow happens when one tries to enqueue on a full queue.



Requirements for a Priority Queue

- The underlying data structure for a priority queue must be sorted.
- Elements are queued using an integer to specify priority. We can use a pair<Key, T> to store elements with their associated priority.
- We need to provide a matching operator $<$ on key values to sort elements in the priority queue.



Static variables

- C++ allows for two forms of global variables:
 - Static non-class variables,
 - Static class variables.
- Static variables are mapped to the global memory. Access to them depends on the visibility specified.
- Generally, local variables and function arguments are stored on the **stack**, while global and static variables are stored on the **heap**.
 - Static variables get created only once no matter how many times the function is called or how many class instances are created.



The Keyword **static**

- The keyword static can be used to
 - mark the linkage of a variable or function internal,
 - retain the value of a local variable between function calls,
 - declare a class instance variable,
 - define a class method.



Program Memory: Stack

- All value-based objects are stored in the program's stack.
- The program stack is automatically allocated and freed.
- References to stack locations are only valid when passed to a callee (a function called by another).
- References to stack locations cannot be returned from a function.

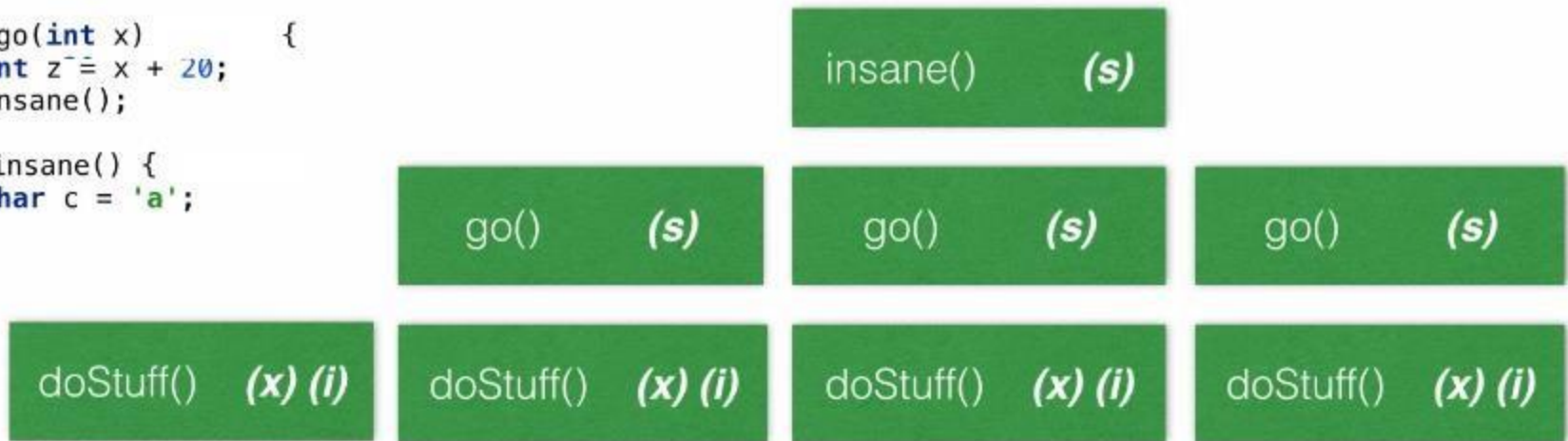
Example of Stack



```
void doStuff() {  
    boolean b = true;  
    go(3);  
}
```

```
void go(int x) {  
    int z = x + 20;  
    insane();  
}
```

```
void insane() {  
    char c = 'a';  
}
```



More readings on stack:

<http://cryptroix.com/2016/10/16/journey-to-the-stack/>



Program Memory: Heap

- Every program maintains a heap for dynamically allocated objects.
- Each heap object is accessed through a pointer.
- Heap objects are not automatically freed when pointer variables become inaccessible (i.e., go out of scope).
- Memory management becomes essential in C++ to reclaim memory and to prevent the occurrences of so-called **memory leaks**.
 - a memory leak is a type of resource leak that occurs when a computer program incorrectly manages memory allocations in such a way **that memory which is no longer needed is not released**



The Copy Constructor

- The **copy constructor** is a **constructor** which creates an object by initializing it with an object of the same class, which has been created previously. The **copy constructor** is used to: Initialize one object from another of the same type.
- Whenever one defines a new type, one needs to specify implicitly or explicitly what has to happen when objects of that that type are copied, assigned, and destroyed.
 - The copy constructor is a special member, taking just a **single parameter** that is a **const reference** to an object of the class itself.



Rule Of Thumb

- Copy control in C++ requires three elements:
 - a copy constructor
 - an assignment operator (=)
 - a destructor
- Whenever one defines a copy constructor, one must also define an assignment operator and a destructor.



What if I want to use “=”

■ Overload the operator “=”

```
SimpleString.cpp | SimpleString.h* X
SimpleString      | operator+(const char aChar)
class SimpleString{
    char* myChar;

public:
    SimpleString();
    ~SimpleString();
    SimpleString(const SimpleString& aString);
    SimpleString& operator=(const SimpleString& aString);
    SimpleString& operator+(const char aChar);
    const char* operator*() const;
};
```

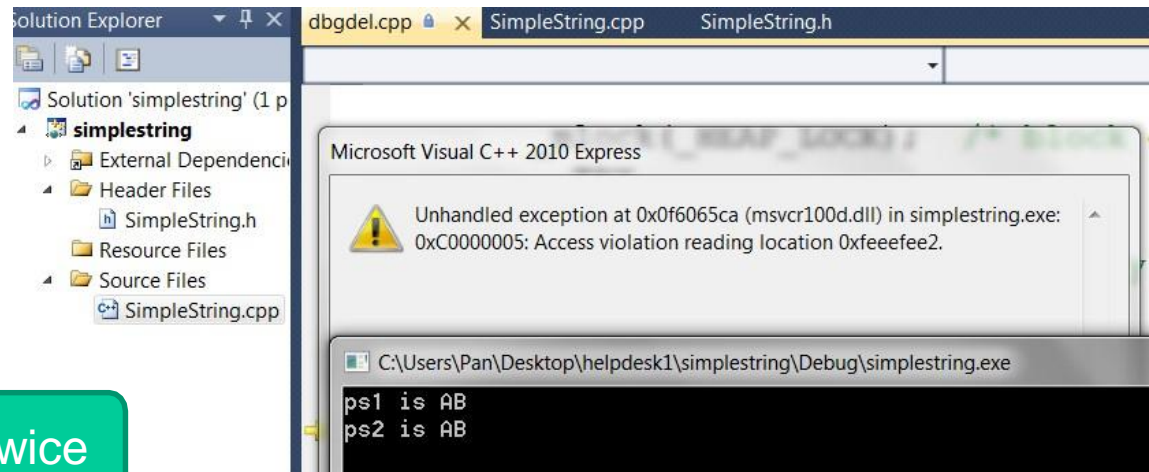
Copying Pointers



```
int main() {  
  
    SimpleString* ps1= new SimpleString();  
    *ps1+'A';  
  
    SimpleString* ps2=ps1;  
    *ps2+'B';  
  
    cout<<"ps1 is "<<*ps1<<endl;  
    cout<<"ps2 is "<<*ps2<<endl;  
    |  
    delete ps1;  
    delete ps2;  
}
```

Shallow
Copy

Free twice



Solution: A clone() function



It is best to define the destructor of a class as virtual in order to avoid problems later.

```
SimpleString.cpp  SimpleString.h x
SimpleString
class SimpleString{
    char* myChar;

public:
    SimpleString();
    virtual ~SimpleString();
    SimpleString(const SimpleString& aString);

    virtual SimpleString* clone();

    //SimpleString& operator=(const SimpleString& aStr
    SimpleString& operator+(const char aChar);

    const char* operator*() const;
};
```

The Use of clone()



```
SimpleString.cpp x SimpleString.h
(Global Scope)

SimpleString* SimpleString::clone() {
    return new SimpleString(*this);
}

int main() {

    SimpleString* ps1= new SimpleString();
    *ps1+'A';

    SimpleString* ps2=ps1->clone();
    *ps2+'B';
}
```

```
C:\Users\Pan\Desktop\helpdesk1\simplestring\Debug\simplestring.exe
ps1 is A
ps2 is AB
Press any key to continue . . .
```




Note with clone()

- if a class has *any* virtual function, it should have a virtual destructor,
- The member function clone() must be defined virtual to allow for proper redefinition in subtypes.
- The member function clone() can only return one type. When a subtype redefines clone(), only the super type can be returned



Reference Counting

- A simple technique to record the number of active uses of an object is **reference counting**.
- Each time a heap-based object is assigned to a variable, the object's reference count is incremented and the reference count of what the variable previously pointed to is decremented.
- Some compilers emit the necessary code, but in case of C++ reference counting must be defined (semi-)manually.



Smart Pointers in C++

- There are several smart pointers available:
 - `std::unique_ptr`
 - `std::shared_ptr`
 - `std::weak_ptr`
- Smart pointers are used to guarantee automatic deletion of heap-allocated class instances.
- With the unique and shared smart pointers, you do not have to call `delete` yourself for an object, as it will be done automatically.



Smart Pointers in C++

■ `std::unique_ptr`

- The unique pointer is a scoped pointer. When this pointer goes out of scope, it will get destroyed and call delete on the object. It cannot be copied.

■ `std::shared_ptr`

- The shared pointer keeps track of how many references you have to a pointer. Reference count will increase each time the pointer is shared. The object will get deleted when the reference count gets to zero.

■ `std::weak_ptr`

- The weak pointer will not increase the reference count. It is used only for storing a reference to an object to check if it is still valid or has expired.



Lecture 8





Basics

- A tree T is a finite, non-empty set of nodes, $T = \{r\} \cup T_1 \cup T_2 \cup \dots \cup T_n$,
- with the following properties:
 - A designated node of the singleton, r , is called the root of the tree.
 - The remaining nodes are partitioned into $n \geq 0$ subsets T_1, T_2, \dots, T_n , each of which is a tree.



Definition

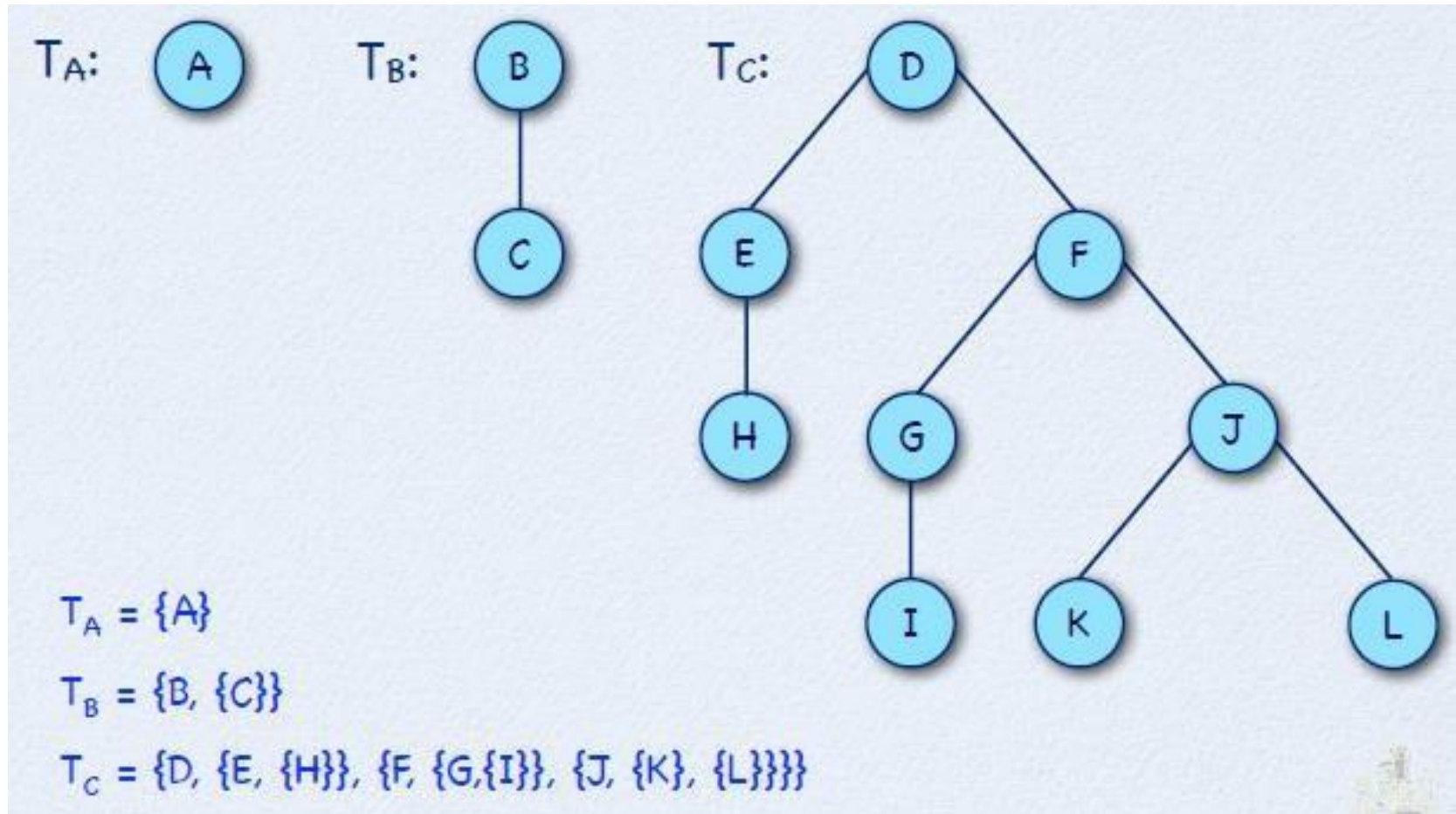
■ Tree

- ☐ undirected graph
- ☐ any two nodes are connected by exactly one edge
- ☐ no loop

Why are trees in computer science generally drawn upside down?

<https://www.quora.com/Why-are-trees-in-computer-science-generally-drawn-upside-down-from-how-trees-are-in-real-life>

Tree Examples



Structure of a Tree

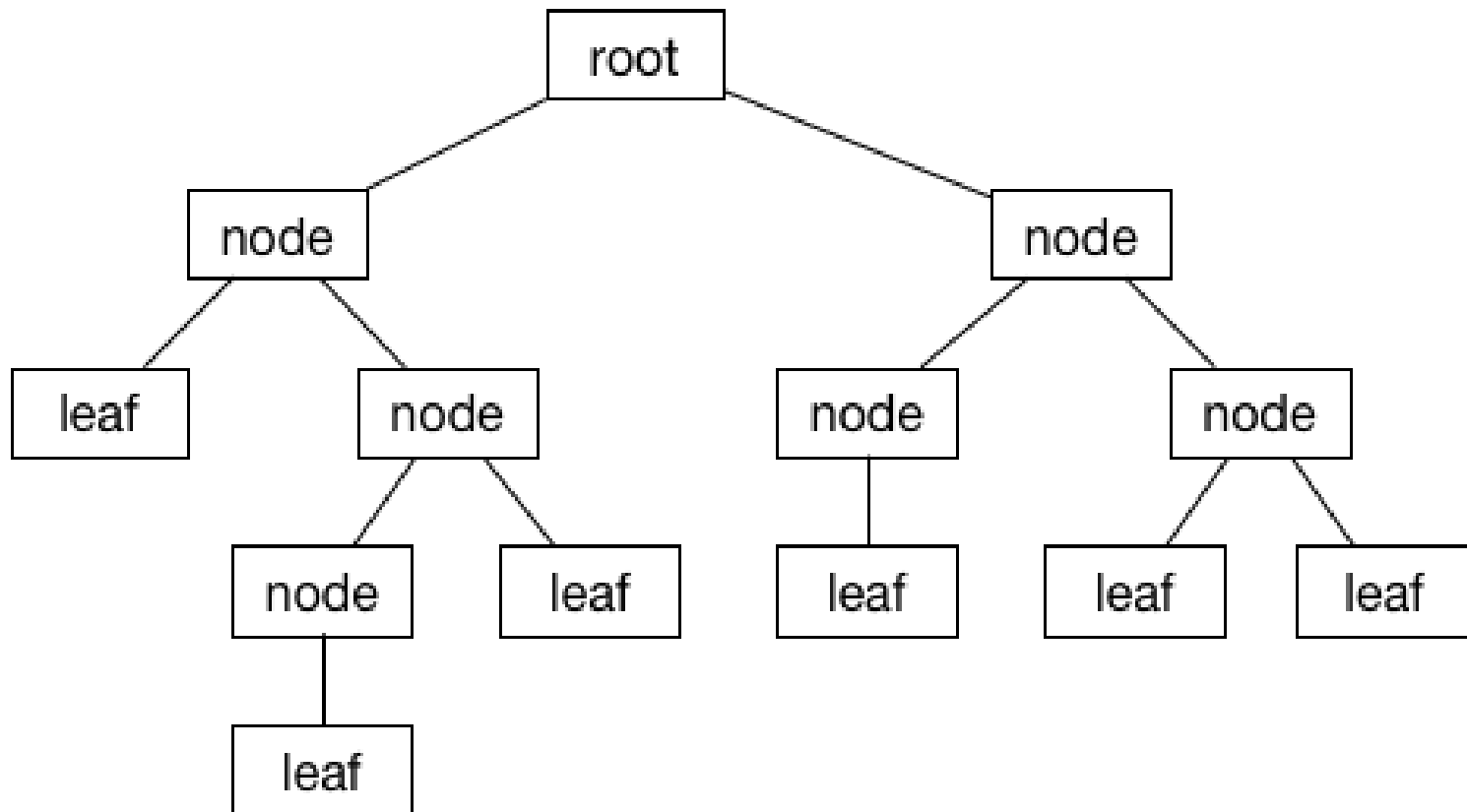


Figure from: <https://www.raywenderlich.com/990-swift-algorithm-club-swift-binary-search-tree-data-structure>



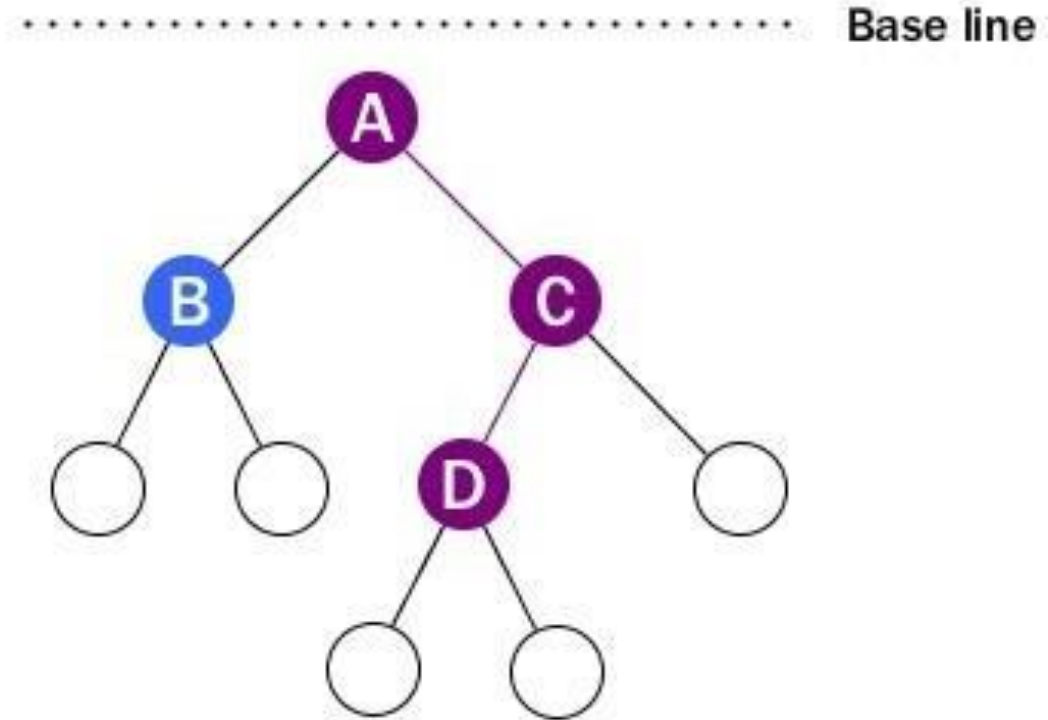
Depth, Height and Level

- The **depth** of a node is the number of edges from the node to the **tree's** root node.
 - A root node will have a **depth** of 0.
- The **height of a node** is the number of edges on **the longest path** from the node to a leaf.
 - *Height of tree* – The height of a tree is the number of edges on the longest downward path between the root and a leaf.
 - A leaf node will have a **height** of 0.
- The **level of a node** is defined by 1 + the number of connections between the node and the root.

$$\text{i.e. level} = \text{depth} + 1$$

Example

- The height of the tree is 3
- The height of Node C is 2 (count from bottom)
- The depth of Node D is 2 (count from top)



Nodes With the Same Degree



- The general case allows each node in a tree to have a different degree (The number of subtrees of a node is called the **degree** of the node). We now consider a variant of trees in which each node has the same degree.
- Unfortunately, it is not possible to construct a tree that has a finite number of nodes, which all have the same degree N , except the trivial case $N = 0$.
- We need a special notion, called empty tree, to realize trees in which all nodes have the same degree.

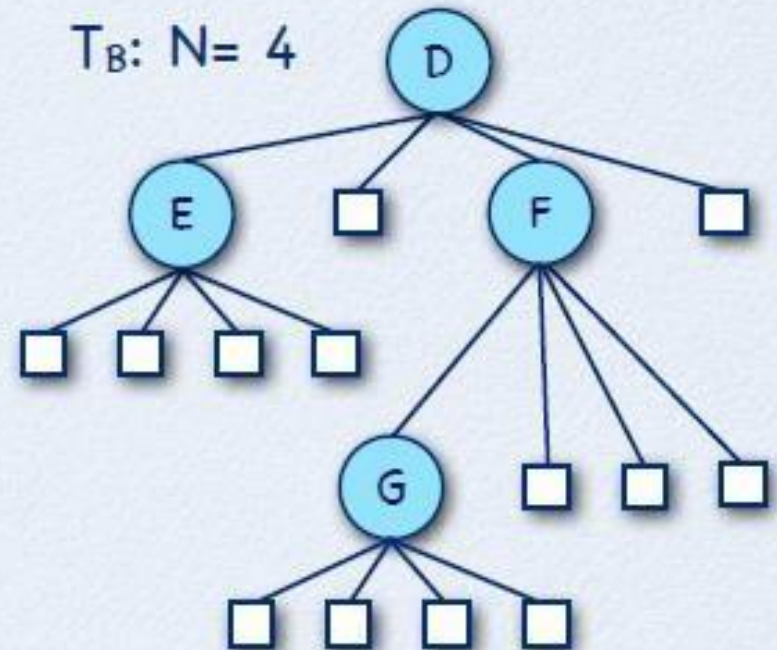


N-ary tree

- An N-ary tree T , $N \geq 1$, is a finite set of nodes with one of the following properties:
 - Either the set is empty, $T = \emptyset$, or
 - The set consists of a root, R , and exactly N distinct
- N-ary trees, That is, the remaining nodes are partitioned into $N \geq 1$ subsets, T_1, T_2, \dots, T_N , each of which is an N-ary tree such that

$$T = \{R, T_1, T_2, \dots, T_N\}.$$

N-ary Tree Examples



Ntree Class



```
NTree.h
6  template<class T, int N>
7  class NTree
8  {
9  private:
10     const T* fKey;           // 0 for empty NTree
11     NTree<T,N>* fNodes[N];
12
13     NTree();                 // empty NTree
14
15 public:
16     static NTree<T,N> NIL;    // sentinel
17
18     NTree( const T& aKey );    // root with N subtrees
19     ~NTree();
20
21     bool isEmpty() const;
22     const T& key() const;
23     NTree& operator[] ( int aIndex ) const;
24     void attachNTree( int aIndex, NTree<T,N>* aNTree );
25     NTree* detachNTree( int aIndex );
26 };
27
```

Line: 4 Column: 16 C++ Tab Size: 4 NTREE.H

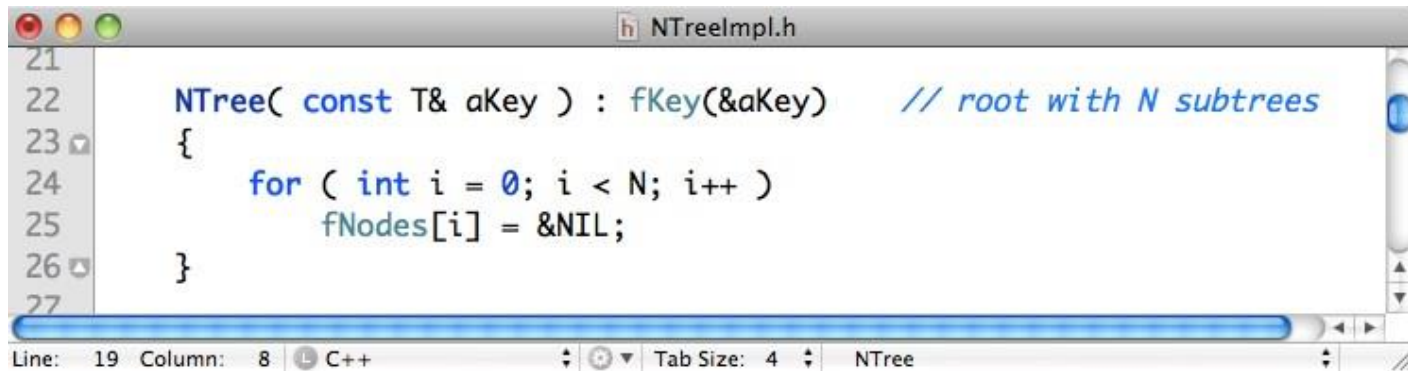
The Private Empty NTree Constructor



```
NTreeImpl.h
13
14     NTree() : fKey((T*)0)           // empty NTree
15     {
16         for ( int i = 0; i < N; i++ )
17             fNodes[i] = &NIL;
18     }
19
Line: 21 Column: 1 C++ Tab Size: 4 NTree
```

- We use $(T^*)0$, the null pointer, to initialize the fKey with a suitable value of type pointer to T.
- Each subtree-node is set to the location of NIL, the sentinel node for NTree.
- This constructor is only used to set up the sentinel for NTree.

The Public NTree Constructor



```
21
22 NTree( const T& aKey ) : fKey(&aKey)    // root with N subtrees
23 {
24     for ( int i = 0; i < N; i++ )
25         fNodes[i] = &NIL;
26 }
27
```

Line: 19 Column: 8 C++ Tab Size: 4 NTree

- We store the address of the reference aKey in fKey.
- Each child node in a non-empty NTree leaf node is set to the location of NIL, the sentinel node for NTree.

In computer programming, a **sentinel node** is a specifically designated **node** used with linked lists and **trees** as a traversal path terminator. This type of **node** does not hold or reference any data managed by the data structure.



The NTree Destructor

```
NTreempl.h
27
28 ~NTree()
29 {
30     for ( int i = 0; i < N; i++ )
31         if ( fNodes[i] != &NIL )
32             delete fNodes[i];
33 }
34
```

Line: 20 Column: 5 C++ Tab Size: 4 NTree

- In the destructor of NTree only non-sentinel nodes are destroyed.



The NTree Sentinel

```
93  
94  template<class T, int N>  
95  NTree<T,N> NTree<T,N>::NIL;  
96
```

Line: 20 Column: 5 C++ Tab Size: 4 NTree

- Static instance variables, like the NTree sentinel NIL, need to be initialized **outside** the class definition.
- Here, NIL is initialized using the private default constructor.
- The scope of NIL is Ntree. It means that all members of Ntree are available, including the private constructor to initialize NIL.

The NTree Auxiliaries



```
h NTreeImpl.h
35 bool isEmpty() const
36 {
37     return this == &NIL;
38 }
39
40 const T& key() const
41 {
42     if ( isEmpty() )
43         throw std::domain_error( "Empty NTree!" );
44
45     return *fKey;
46 }
47
```

Line: 32 Column: 1 C++ Tab Size: 4 ~NTree

Attaching a New Subtree



```
NTreeImpl.h
60
61 void attachNTree( int aIndex, NTree<T,N>* aNTree )
62 {
63     if ( isEmpty() )
64         throw std::domain_error( "Empty NTree!" );
65
66     if ( (aIndex >= 0) && (aIndex < N) )
67     {
68         if ( fNodes[aIndex] != &NIL )
69             throw std::domain_error( "Non-empty subtree present!" );
70
71         fNodes[aIndex] = aNTree;
72     }
73     else
74         throw std::out_of_range( "Illegal subtree index!" );
75 }
76
```

Line: 20 Column: 5 C++ Tab Size: 4 NTree

Accessing a Subtree (Operator [])



```
NTreeImpl.h
47
48 NTree& operator[]( int aIndex ) const
49 {
50     if ( isEmpty() )
51         throw std::domain_error( "Empty NTree!" );
52
53     if ( (aIndex >= 0) && (aIndex < N) )
54     {
55         return *fNodes[aIndex];    // return reference to subtree
56     }
57     else
58         throw std::out_of_range( "Illegal subtree index!" );
59 }
60
```

Line: 42 Column: 25 C++ Tab Size: 4 key

- We return a reference to the subtree rather than a pointer. This way, we prevent accidental manipulations outside the tree structure.

Removing a Subtree



```
NTreeImpl.h
77 NTree* detachNTree( int aIndex )
78 {
79     if ( isEmpty() )
80         throw std::domain_error( "Empty NTree!" );
81
82     if ( (aIndex >= 0) && (aIndex < N) )
83     {
84         NTree<T,N>* Result = fNodes[aIndex];
85         fNodes[aIndex] = &NIL;
86         return Result;           // return subtree
87     }
88     else
89         throw std::out_of_range( "Illegal subtree index!" );
90 }
91
```

Line: 44 Column: 1 C++ Tab Size: 4 key

A NTree Example



```
NTreeTest.cpp

8  int main()
9  {
10     string s1( "Hello World!" );
11     string s2( "A" );
12     string s3( "B" );
13     string s4( "C" );
14
15     NTree<string,3> A3Tree( s1 );
16
17     NTree<string,3> STree1( s2 );
18     NTree<string,3> STree2( s3 );
19     NTree<string,3> STree3( s4 );
20
21     A3Tree.attachNTree( 0, &STree1 );
22     A3Tree.attachNTree( 1, &STree2 );
23     A3Tree.attachNTree( 2, &STree3 );
24
25     cout << "Key: " << A3Tree.key() << endl;
26     cout << "Key: " << A3Tree[1].key() << endl;
27
28     A3Tree.detachNTree( 0 );
29     A3Tree.detachNTree( 1 );
30     A3Tree.detachNTree( 2 );
31
32     return 0;
33 }
```

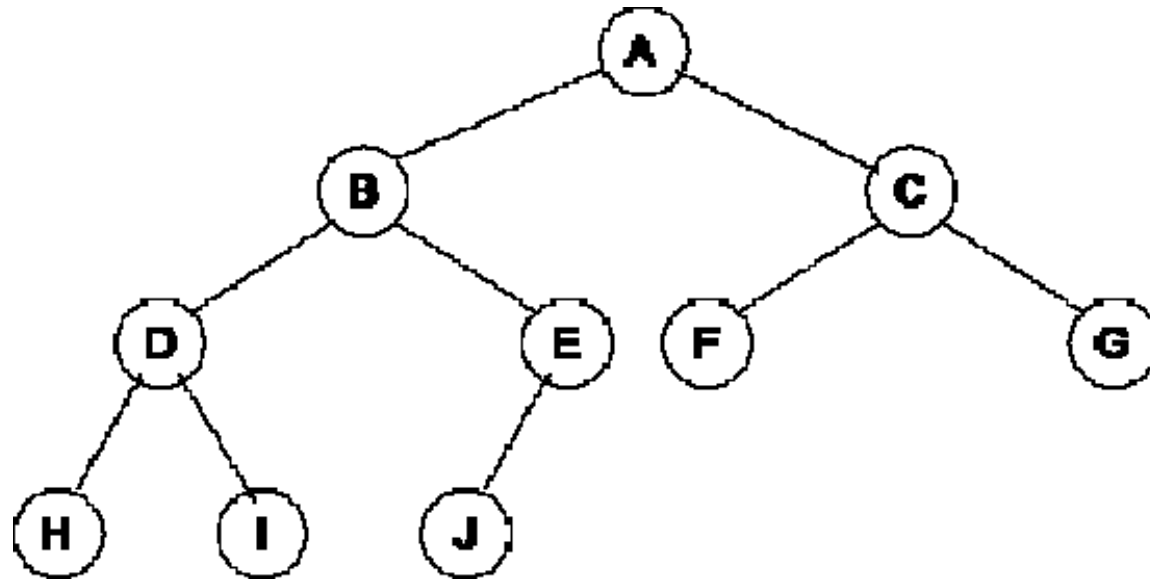
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Binary Trees (2-ary Trees)

- Definition: A Binary Tree is a Tree in which each node has a degree of **at most** 2.
- A binary tree T is a finite set of nodes with one of the following properties:
 - Either the set is empty, $T = \emptyset$, or
 - The set consists of a root, R , and exactly 2 distinct binary trees TL and TR , $T = \{R, TL, TR\}$.
- The tree TL is called the **left** subtree of T and the tree TR is called the **right** subtree of T .

Example : Binary Tree



BTree Class



```
h BTree.h
6  template<class T>
7  class BTree
8  {
9  private:
10     const T* fKey;           // 0 for empty BTree
11     BTree<T>* fLeft;
12     BTree<T>* fRight;
13
14     BTree();                 // empty BTree
15
16 public:
17     static BTree<T> NIL;     // sentinel
18
19     BTree( const T& aKey );   // root with 2 subtrees
20     ~BTree();
21
22     bool isEmpty() const;
23     const T& key() const;
24     BTree& left() const;
25     BTree& right() const;
26     void attachLeft( BTree<T>* aBTree );
27     void attachRight( BTree<T>* aBTree );
28     BTree* detachLeft();
29     BTree* detachRight();
30 };
31
```

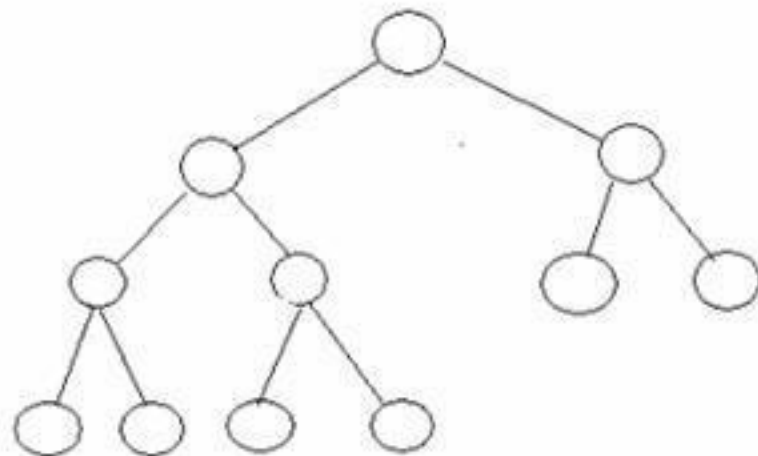
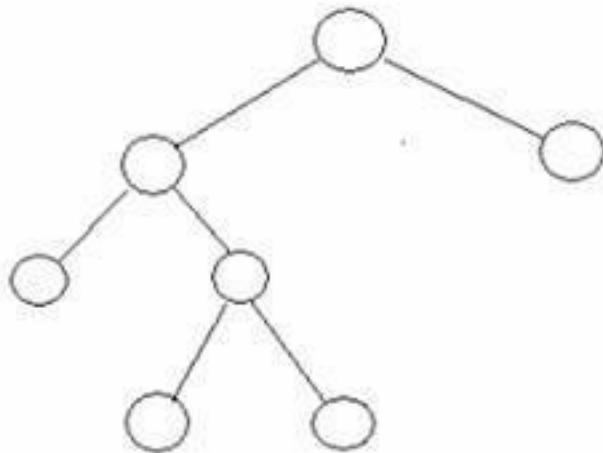
Line: 4 Column: 21 C++ Tab Size: 4 BTREE.H



More on binary tree

■ Full Binary tree:

- Every node should have exactly 2 nodes except the leaves. It is also called as **Strict Binary Tree** or **Proper Binary Tree**.
- Every node must have either 0 or 2 children.

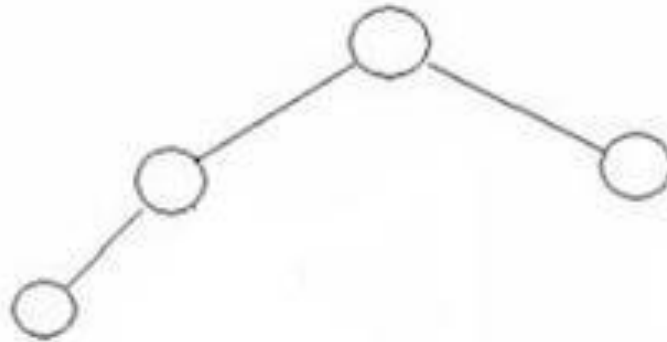
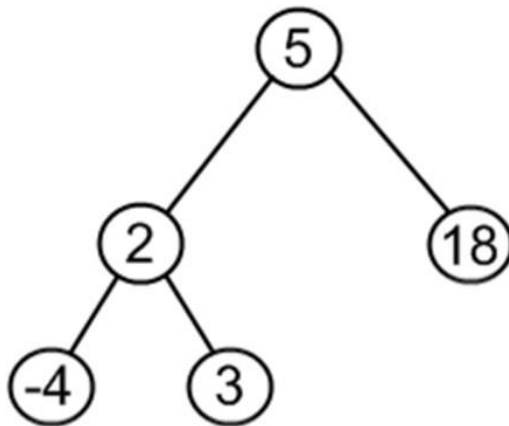




More on binary tree

■ Complete Binary Tree:

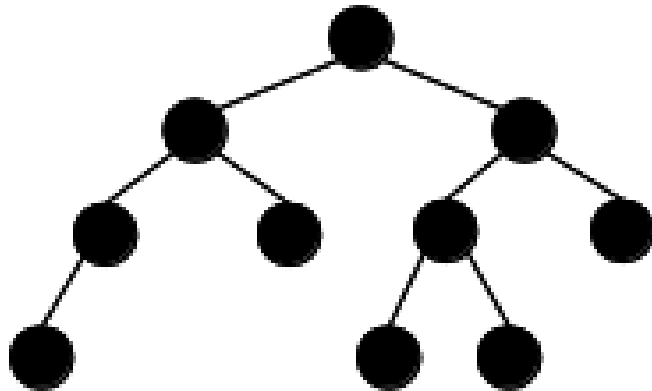
- It is a binary tree in which every level (except possibly the last) is completely filled, and all nodes are **as far left as possible**.



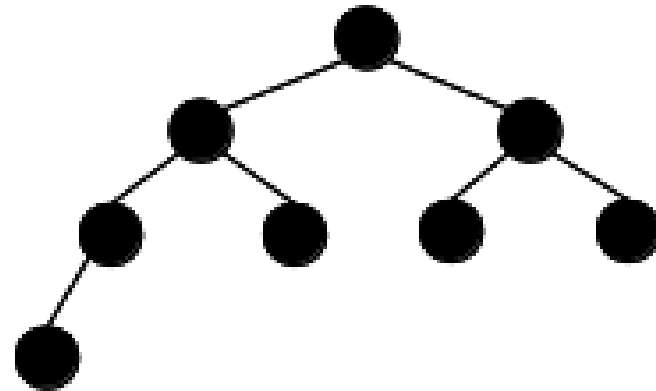
More on binary tree



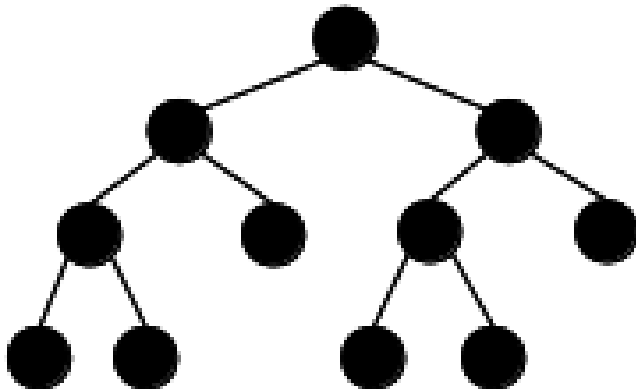
Neither complete nor full



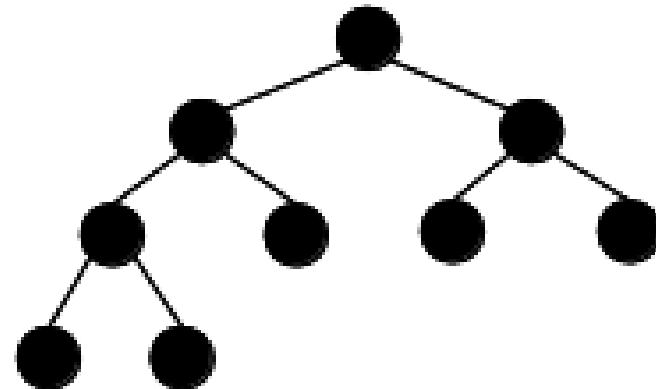
Complete but not full



Full but not complete



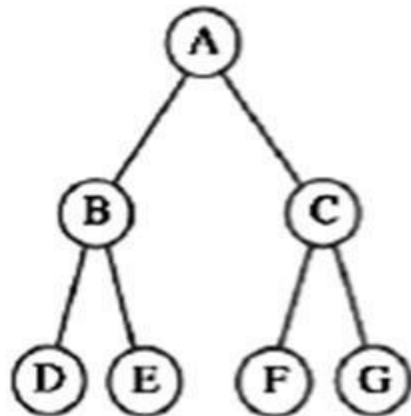
Complete and full



More on binary tree

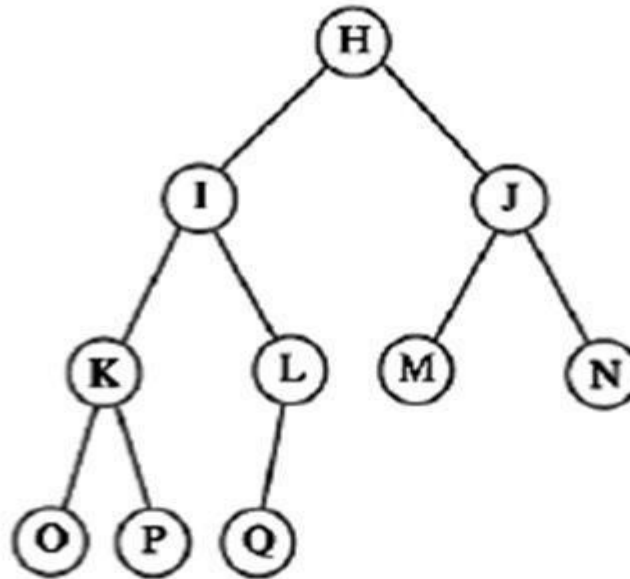


(a) Full tree

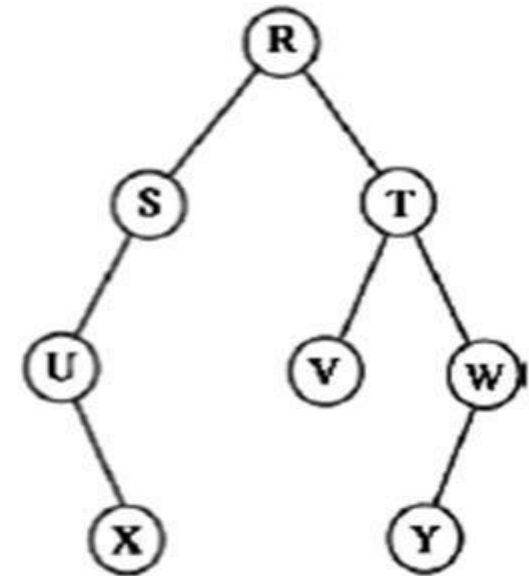


Left children: B, D, F
Right children: C, E, G

(b) Complete tree



(c) Tree that is not full and not complete





Tree Traversal

- Many different algorithms for manipulating trees exist, but these algorithms have in common that they systematically visit all the nodes in the tree.
- There are essentially two methods for visiting all nodes in a tree:
 - Depth-first traversal (Depth First Search, DFS),
 - Breadth-first traversal (Breadth First Search, BFS)

Depth-first Traversal



■ Pre-order traversal:

- Visit the **root first**; and then
- Do a preorder traversal of each of the subtrees of the root one-by-one in the order given (from **left to right**).

■ Post-order traversal:

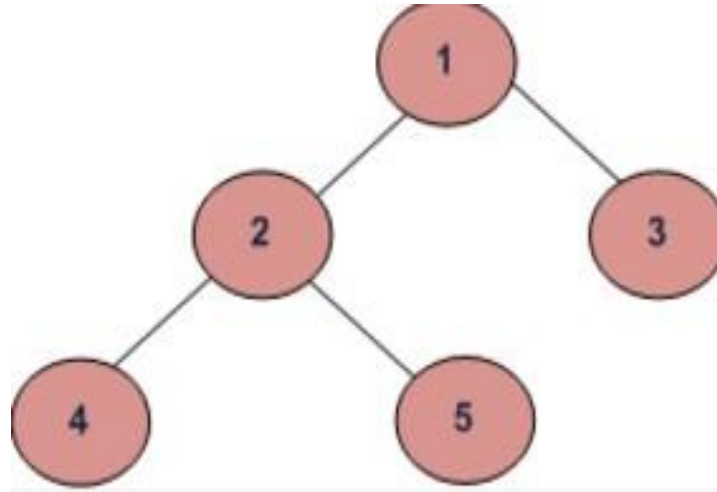
- Do a postorder traversal of each of the subtrees of the root one-by-one in the order given (from left to right); and
- **then** visit the root.

■ In-order traversal:

- Traverse the left subtree; and then
- Visit the root; and then
- Traverse the right subtree.



Example of Depth-first Traversal

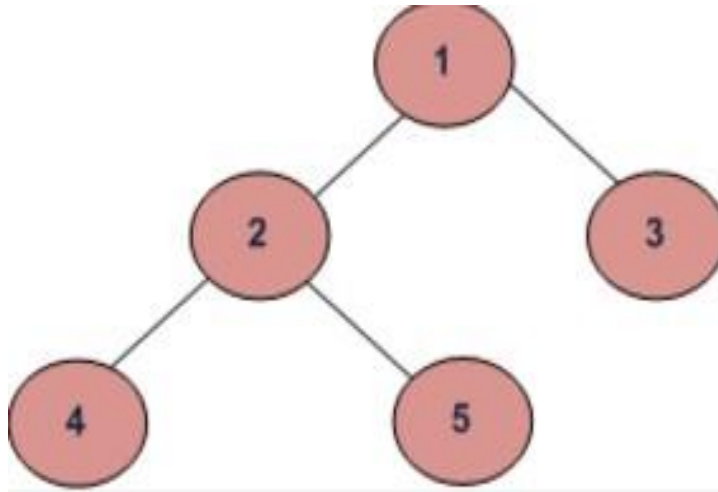


- (a) Preorder (Root, Left, Right) : 1 2 4 5 3
- (b) Postorder (Left, Right, Root) : 4 5 2 3 1
- (c) Inorder (Left, Root, Right) : 4 2 5 1 3



Breadth-first Traversal

- Breadth-first traversal visits the nodes of a tree in the order of their depth (from left to right).

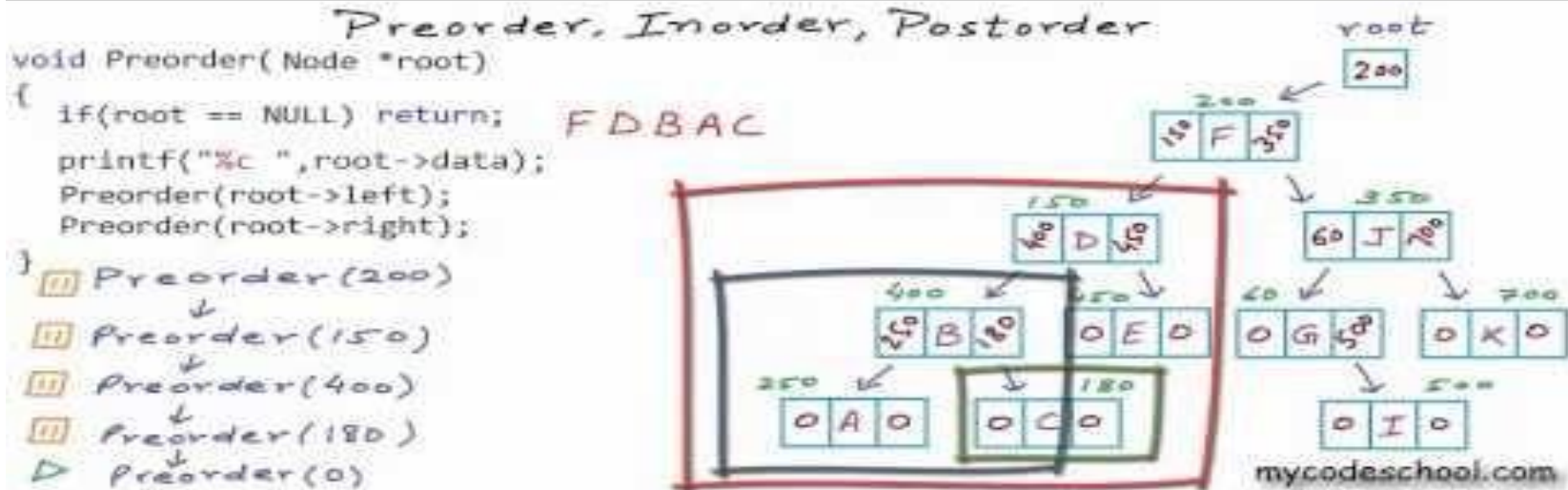


- BFS: 1 2 3 4 5

DFS: Preorder, Inorder and Postorder



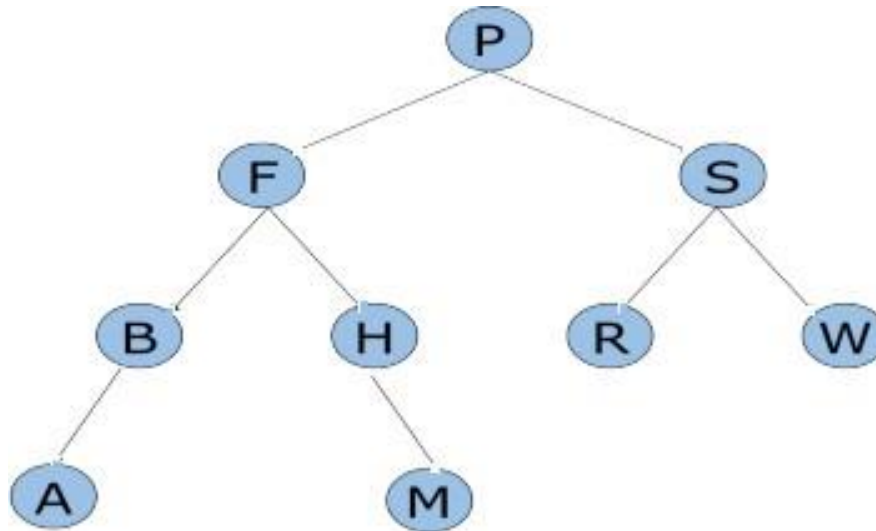
Note: Slight mistake for Inorder example, there is no H in the actual answer.





Exercise

- Write the pre-order, in-order, post-order traversal of the following binary tree:



- Is the binary tree complete or full?



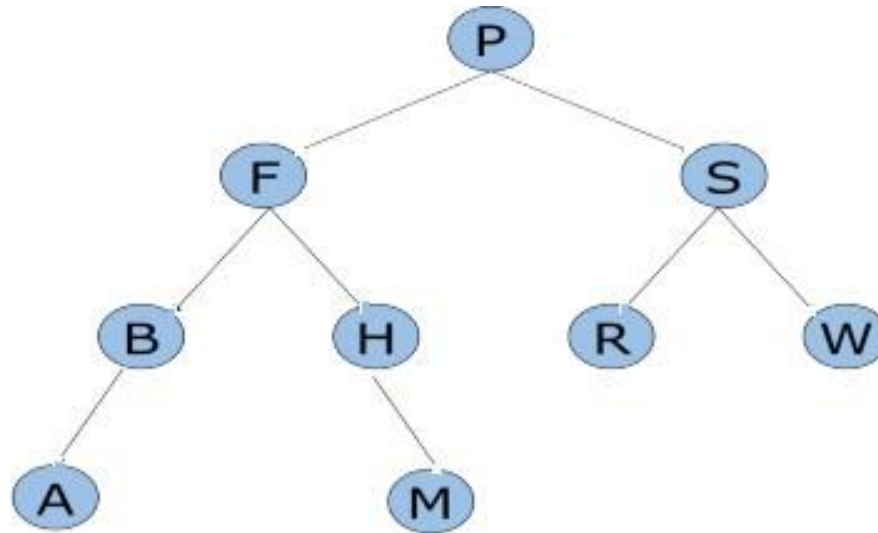
Tricks for Preorder, Inorder and Postorder

Note: The video is in Hindi but the tricks for solving are great.



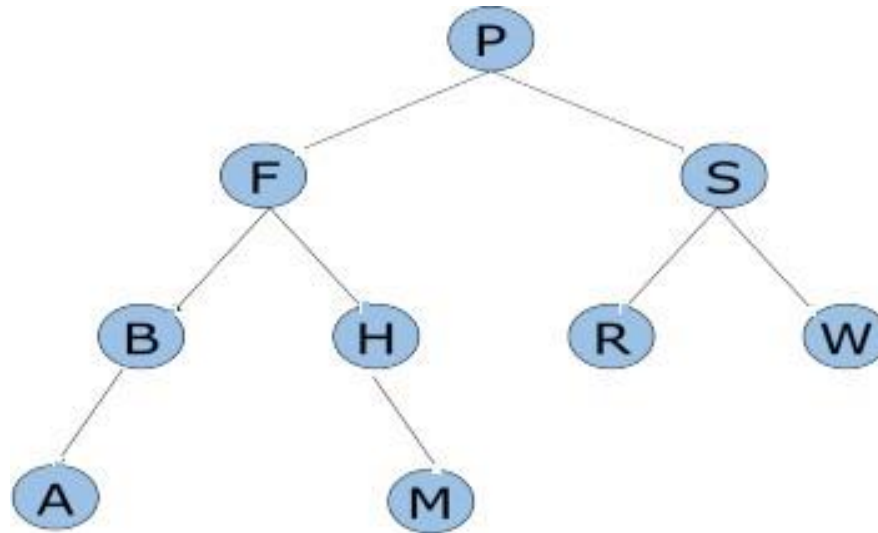
**TRICK for Preorder,
Inorder, Postorder with
Example (Imp)**

Exercise (Answers)



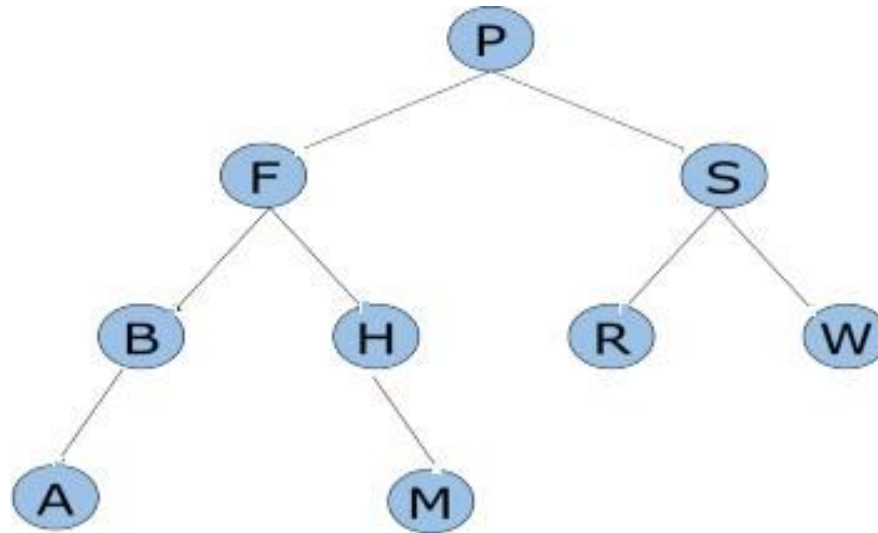
- Pre-order Traversal (Root, Left, Right):
 - P, F, B, A, H, M, S, R, W

Exercise (Answers)



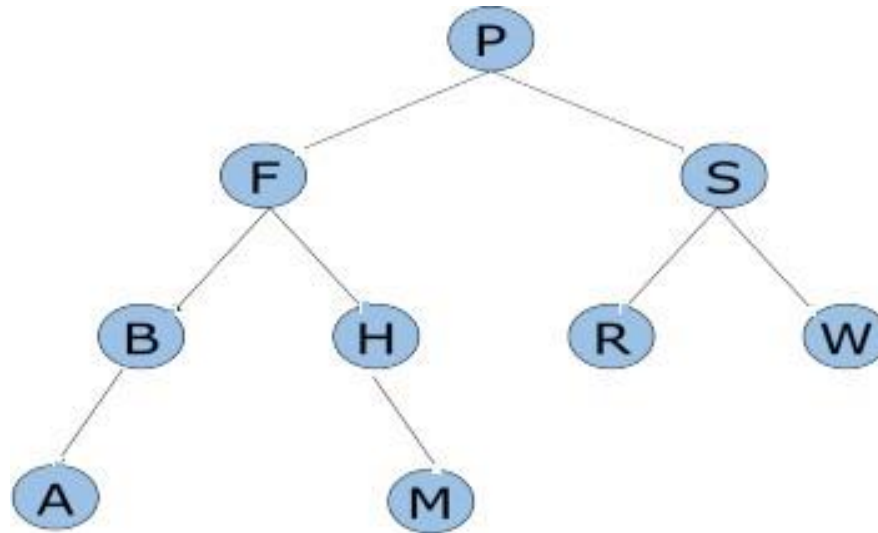
- In-order Traversal (Left, Root, Right):
 - A, B, F, H, M, P, R, S, W

Exercise (Answers)



- Post-order Traversal (Left, Right, Root):
 - A, B, M, H, F, R, W, S, P

Exercise (Answers)



- The binary tree is not complete and not full.
- It is not complete as some of the nodes have 1 leaf. In a complete binary tree, each nodes much have 0 or 2 children.
 - It is not full as not all nodes are as far left as possible.



Lecture 9



The Visitor Pattern



- The **Visitor Pattern** falls under the category of behavioral design patterns
- **Intent**
 - Represent an operation to be performed on the elements of an object structure. Visitor lets you **define a new operation without changing the classes of the elements** on which it operates.
 - The classic technique for recovering lost type information.
 - Do the right thing based on the type of **two objects**.
 - **Double Dispatch** is used to invoke an overloaded method where the parameters vary among an inheritance hierarchy



The Visitor Pattern

- Collaborations:
- A client that uses the Visitor pattern must create a ConcreteVisitor object and then traverse the object structure, visiting each element with the visitor.
- When an element is visited, it calls the Visitor operation that corresponds to its class. The element supplies itself as an argument to this operation to let the visitor access its state, if necessary.

A Tree Visitor



```
TreeVisitor.h
4  #include <iostream>
5
6  template<class T>
7  class TreeVisitor
8  {
9  public:
10     virtual ~TreeVisitor() {}    // virtual default destructor
11
12     // default behavior
13     virtual void preVisit( const T& aKey ) const {}
14     virtual void postVisit( const T& aKey ) const {}
15     virtual void inVisit( const T& aKey ) const {}
16
17     virtual void visit( const T& aKey ) const
18     {
19         std::cout << aKey << " ";
20     }
21 };
22
```

Pre-Order,
Post-Order
In-Order
from DFS

PreOrderVisitor



```
22
23 template<class T>
24 class PreOrderVisitor : public TreeVisitor<T>
25 {
26 public:
27
28     // override pre-order behavior
29     virtual void preVisit( const T& aKey ) const
30     {
31         visit( aKey ); // invoke default behavior
32     }
33 };
34
```

Line: 17 Column: 13 C++ Tab Size: 4 visit

PostOrderVisitor



```
TreeVisitor.h
34
35 template<class T>
36 class PostOrderVisitor : public TreeVisitor<T>
37 {
38 public:
39
40     // override post-order behavior
41     virtual void postVisit( const T& aKey ) const
42     {
43         visit( aKey ); // invoke default behavior
44     }
45 };
46
```

Line: 17 Column: 13 C++ Tab Size: 4 visit

InOrderVisitor



```
46
47 template<class T>
48 class InOrderVisitor : public TreeVisitor<T>
49 {
50 public:
51
52     // override in-order behavior
53     virtual void inVisit( const T& aKey ) const
54     {
55         visit( aKey ); // invoke default behavior
56     }
57 };
58
```

Line: 17 Column: 13 C++ Tab Size: 4 visit

Depth-first Traversal for BTree



```
BTreImpl.h
103
104 void traverseDepthFirst( const TreeVisitor<T>& aVisitor ) const
105 {
106     if ( !isEmpty() )
107     {
108         aVisitor.preVisit( key() );           // Show if PreOrder
109         left().traverseDepthFirst( aVisitor );
110         aVisitor.inVisit( key() );           // Show if InOrder
111         right().traverseDepthFirst( aVisitor );
112         aVisitor.postVisit( key() );         // Show if PostOrder
113     }
114 }
115
```

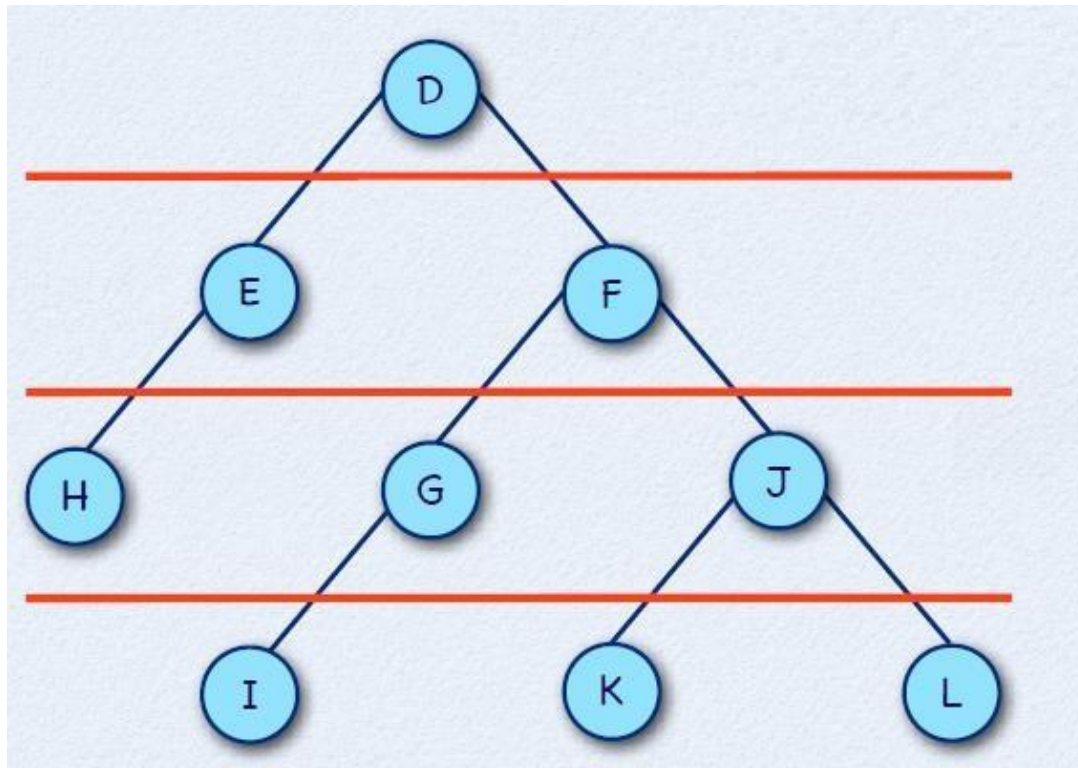


```
BTreeTest.cpp

8  int main()
9  {
10     string s1( "Hello World!" );
11     string s2( "A" );
12     string s3( "B" );
13     string s4( "C" );
14
15     BTree<string> A2Tree( s1 );
16
17     BTree<string> STree1( s2 );
18     BTree<string> STree2( s3 );
19     BTree<string> STree3( s4 );
20
21     A2Tree.attachLeft( &STree1 );
22     A2Tree.attachRight( &STree2 );
23     A2Tree.left().attachLeft( &STree3 );
24
25     cout << "Key: " << A2Tree.key() << endl;
26     cout << "Key: " << A2Tree.left().left().key() << endl;
27
28     A2Tree.traverseDepthFirst( PreOrderVisitor<string>() );
29     cout << endl;
30
31     A2Tree.left().detachLeft();
32     A2Tree.detachLeft();
33     A2Tree.detachRight();
34
35     return 0;
36 }
```

```
Terminal
Sela:HIT3303 Markus$ ./BTreeTest
Key: Hello World!
Key: C
Hello World! A C B
Sela:HIT3303 Markus$ _
```

Breadth-first Traversal Implementation



■ Traversal : D-E-F-H-G-J-I-K-L

Breadth-first Traversal for BTree



```
BTreImpl.h
116
117 void traverseBreadthFirst( const TreeVisitor<T>& aVisitor ) const
118 {
119     Queue< BTree<T> > lQueue;
120
121     if ( !isEmpty() ) lQueue.enqueue( *this );           // start with root node
122
123     while ( !lQueue.isEmpty() )
124     {
125         const BTree<T>& head = lQueue.dequeue();
126
127         if ( !head.isEmpty() ) aVisitor.visit( head.key() );           // output
128         if ( !head.left().isEmpty() ) lQueue.enqueue( head.left() );   // enqueue left
129         if ( !head.right().isEmpty() ) lQueue.enqueue( head.right() ); // enqueue right
130     }
131 }
132
```




```
BTreeTest.cpp
8 int main()
9 {
10     string s1( "Hello World!" );
11     string s2( "A" );
12     string s3( "B" );
13     string s4( "C" );
14
15     BTree<string> A2Tree( s1 );
16
17     BTree<string> STree1( s2 );
18     BTree<string> STree2( s3 );
19     BTree<string> STree3( s4 );
20
21     A2Tree.attachLeft( &STree1 );
22     A2Tree.attachRight( &STree2 );
23     A2Tree.left().attachLeft( &STree3 );
24
25     cout << "Key: " << A2Tree.key() << endl;
26     cout << "Key: " << A2Tree.left().left().key() << endl;
27
28     A2Tree.traverseBreadthFirst( PreOrderVisitor<string>() );
29     cout << endl;
30
31     A2Tree.left().detachLeft();
32     A2Tree.detachLeft();
33     A2Tree.detachRight();
34
35     return 0;
36 }
```

```
Terminal
Sela:HIT3303 Markus$ ./BTreeTest
Key: Hello World!
Key: C
Hello World! A B C
Sela:HIT3303 Markus$
```

AVL Tree



■ AVL trees are **height-balanced binary** search trees

- A tree where no leaf is much farther away from the root than any other leaf.
- 1) Left subtree of Tree is balanced
2) Right subtree of Tree is balanced
3) The **difference between heights of left subtree and right subtree** is **not more than 1**.

■ **Balance factor** of a node

- **$\text{height}(\text{left subtree}) - \text{height}(\text{right subtree})$**

AVL Tree

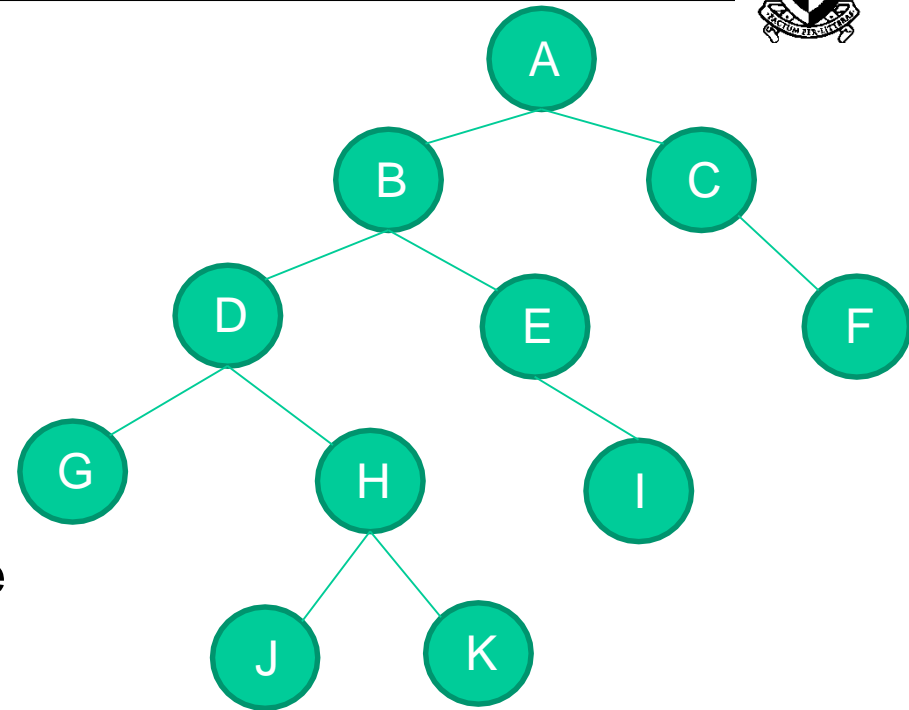


- An AVL tree has balance factor calculated at every node
- For every node, heights of left and right subtree can differ by no more than 1
- Store current heights in each node



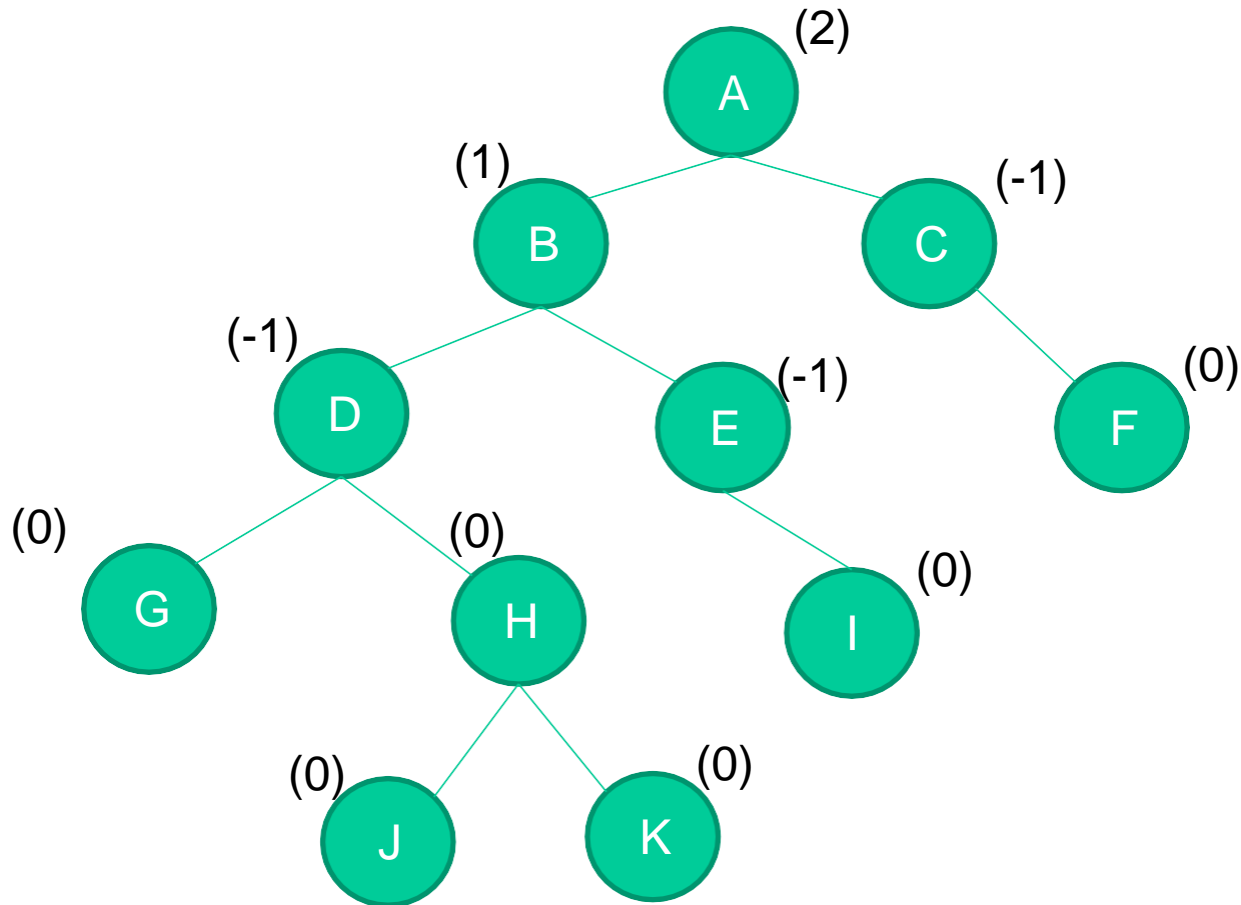
Is this a AVL tree?

- The height of the tree is 4, meaning the length of the longest path from the root to a leaf node.
- The height of the left subtree of the root is 3, meaning that the length of the longest path from the node B to one of the leaf nodes (G, J, K or I).
- For finding the balancing factor of the **root** we subtract the height of the right subtree and the left subtree : $3 - 1 = 2$.
- The balancing factor of the node with the key I is very easy to determine. We notice that **the node has no children so the balancing factor is 0**



- For finding the balancing factor of the node with key D we subtract the height of the right subtree from the height of the left subtree: $0 - 1 = -1$.

Balance factor



The binary tree is **balanced** when all the balancing factors of all the nodes are -1,0,+1.

This binary tree is not balanced at the Root.



Insert and Rotation in AVL Trees

- Insert operation may cause balance factor to become 2 or -2 for some node
- only nodes on the path from insertion point to root node have possibly changed in height
- So after the Insert, go back up to the root node by node, updating heights
- If a new balance factor (the difference $h_{\text{left}} - h_{\text{right}}$) is 2 or -2 , adjust tree by *rotation* around the node



Insertions in AVL Trees

- Let the node that needs rebalancing be α .
- There are 4 cases:
 - **Outside Cases** (require single rotation) :
 1. Insertion into **left** subtree **of left** child of α .
 2. Insertion into **right** subtree **of right** child of α .
 - **Inside Cases** (require double rotation) :
 3. Insertion into **right** subtree of **left** child of α .
 4. Insertion into **left** subtree of **right** child of α .
- The rebalancing is performed through four separate rotation algorithms, **left-left rotation**, **right-right rotation**, **right-left rotation** and **left-right rotation**.



Lecture 10





Big-O notation

- Wiki : *Big O notation* is a mathematical *notation* that describes **the limiting behaviour** of a function when the argument tends towards a particular value or infinity
- It is used to describe the **worse case**, or ceiling of growth for an algorithm or a function.
- It is a simplified analysis of an algorithm's efficiency/ complexity.
- **Asymptotic** analysis
- It focuses on basic computing steps



Big-O notation : General rules

- Ignores the **constant** coefficient
 - $n, 2n, 5n, \dots O(n)$
- Functions/algorithms with “higher growth rate” dominate others
 - $O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$

Big-O notation : General rules



Big-O Complexity Chart

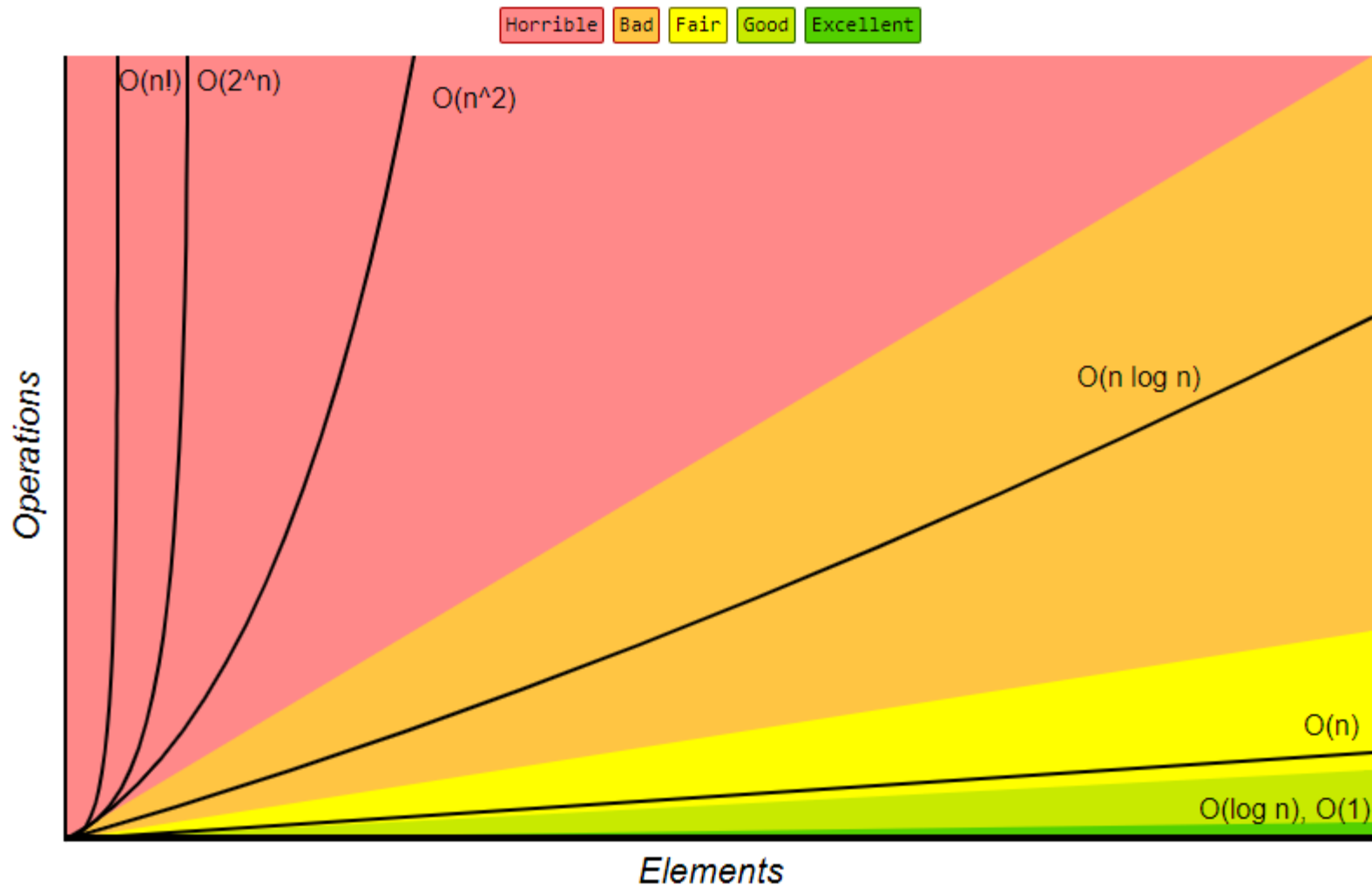


Figure from bigocheatsheet.com



Big-O : Constant time, $O(1)$

- Notation: $O(1)$
- $x = 3 * (5 + 12)$
- x is independent from any input or additional variables
- We call such complexity, constant time

$x = 3 * (5 + 12);$

$y = 15 - 2;$

`print x - y;`

- What is big-O for this code?

Total = $O(1) + O(1) + O(1) = 3 * O(1) \Rightarrow O(1)$

Since constants are ignored.



Big-O : Linear time, $O(n)$

- Notation: $O(n)$

```
for(int i =0; i<n; ++i)  
    cout<<i<<endl;
```

- $O(1)$ statement repeat n times. n is not a constant coefficient but it is possibly a variable.
- So it is $n * O(1) = O(n)$

Big-O: Quadratic time, $O(n^2)$



■ Two for loops

```
for(i =0; i<n; ++i)
    for(j =0; j<n; ++j)
        cout<<i*j<<endl;
```



Big-O notation: exercise 1

- What is the Big-O for this?

```
x = 5 + (15 * 20);  
for x in range (0, n):  
    print x;  
for x in range (0, n):  
    for y in range (0, n):  
        print x * y;
```

$O(1)$
 $O(n)$
 $O(n^2)$

The largest run time will be the Big-O notation: $O(n^2)$



Big-O notation: exercise 2

```
a=5
b=6
c=10
for i in range(n):
    for j in range(n):
        x = i * i
        y = j * j
        z = i * j
    for k in range(n):
        w = a*k + 45
        v = b*b
d = 33
```

$O(n^2)$

$O(n)$

The largest run time will be the Big-O notation: $O(n^2)$



What is computable?

- Computation is usually modeled as a mapping from inputs to outputs, carried out by a “formal machine”, or program, which processes its input in a sequence of steps.
- An “effectively computable” function is one that can be computed in a finite amount of time using finite resources.



Halting Problem

- A problem that cannot be solved by any machine in **finite time** (or any equivalent formalism) is called uncomputable.
- An uncomputable problem cannot be solved by any real computer.
- **The Halting Problem:**
 - Given a program and its input, determine whether the program will complete or run forever.
 - The Halting Problem is provably uncomputable – which means that it cannot be solved in practice.

```
green = ON
red = amber = OFF
while(true){
    amber = ON; green =
    OFF;
    wait 10 seconds;
    red = ON; amber = OFF;
    wait 40 seconds;
    green = ON; red = OFF;
}
```



Algorithmic Patterns

- Direct solution strategies:
 - Brute force and greedy algorithms
- Backtracking strategies:
 - Simple backtracking and branch-and-bound algorithms
- Top-down solution strategies:
 - Divide-and-conquer algorithms
- Bottom-up solution strategies:
 - Dynamic programming
- Randomized strategies:
 - Monte Carlo algorithms

End of Revision 2

All the best for your finals! :D

