Algorithmic Operation Research

"Do dogs know Calculus?"

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Introduction

Definition of the problem

Finding the optimal path, from point A to point B.

"Optimal":

- Minimizing the time of travel
- Optimizing travel conditions
- Available paths must transverse two different mediums,involving different rates of speed.

Application of such problem

Paper purpose

The writer is curious to find out if his dog is aware of techniques to solve such problems, by playing "fetch".

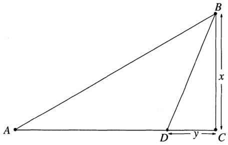
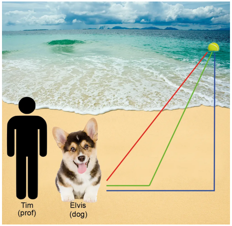


Figure 1. Paths to the ball



- **A**: Source, water's edge
- **B**: Destination of the ball
- **C**: Dry point nearest to the ball
- **D**: Point in the lake

Approaches for solving the problem

With the goal being to reach point B, we have some options for the dog's strategy:

- **Try to minimize the time by minimizing the distance traveled.** Thus he could immediately jump into the surf and swim the entire distance.
- **Minimize the swimming distance.** Thus, he could sprint down the beach to the point on shore closest to the ball, C, and then turn a right angle and swim to it.
- **Combine the solutions.** Thus, he has the option of running a portion of the way, and then plunging into the lake at D and swimming diagonally to the ball.

Choosing the right approach

Optimal solution

Depending on the relative running and swimming speeds, this last option usually turns out to minimize the time.

Declarations

- **r**: running speed
- **s**: swimming speed
- $T(y)$: Time to get to the ball, given that the dog jumps into the water at D.
- **y**: Distance form D to C.
- **z**: Total distance from A to C.

Mathematical formula

Since $time = \frac{distance}{speed}$, we have:

$$T(y) = \frac{z - y}{r} + \frac{\sqrt{x^2 + y^2}}{s}$$

Minimization of T(y)

Happens when $T'(y) = 0$ and when T has a minimum at y.

By solving $T'(y) = 0$, we get

Least distance from C to D to get to the ball quickly

$$y = \frac{x}{\sqrt{\frac{r}{s+1}}\sqrt{\frac{r}{s-1}}}$$

Dog's behavior

Observations

- When playing fetch, he uses the third strategy of jumping into the lake at D.
- His y values were roughly proportional to the x values.

Thus, we can assume that the dog indeed chooses the optimal path to get to the ball as quickly as possible.

Next stage: experiment

Calculate the dog's values of r and s and then checking how closely his ratio of y to x coincided with the exact value provided by the mathematical model.

Relations between variables

After several testings, we can suppose that

Measurements' results

- $r = 6.40 \frac{meters}{second}$.
- $s = 0.90 \frac{meters}{second}$.
- $y = 0.114x$

To find out if the dog really follows the optimal solution to the problem, we must have had optimal environmental conditions. Thus this is not possible, we(at least) can ensure that:

Experiment conditions

- Measurements were made on a calm day when the waves were small.
- The times when the dog chose on of the first 2 approaches were not taken into consideration

Results

Table 2. Throw and fetch trials

x	y	x	y	x	y	x	y	x	y
10.5	2.0	17.0	2.1	4.7	0.9	10.9	2.2	15.3	2.3
7.2	1.0	15.6	3.9	11.6	2.2	11.2	1.3	11.8	2.2
10.3	1.8	6.6	1.0	11.5	1.8	15.0	3.8	7.5	1.4
11.7	1.5	14.0	2.6	9.2	1.7	14.5	1.9	11.5	2.1
12.2	2.3	13.4	1.5	13.5	1.8	6.0	0.9	12.7	2.3
19.2	4.2	6.5	1.0	14.2	1.9	14.5	2.0	6.6	0.8
11.4	1.3	11.8	2.4	14.2	2.5	12.5	1.5	15.3	3.3

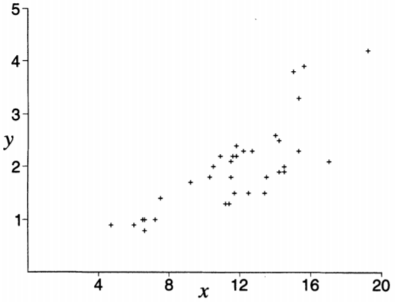


Figure 2. Scatter plot of Elvis's choices

Comparison between method and results

To find out if dogs actually know calculus, we must compare the results from figure to with our optimal relation between x and y ($y = 0.144x$).

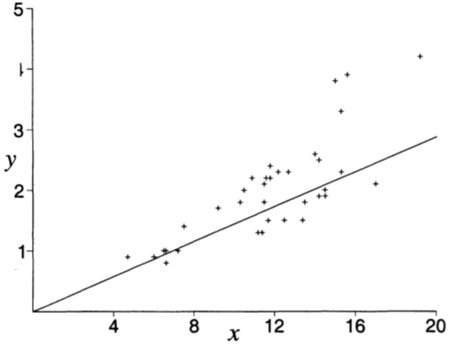


Figure 3. Scatter plot with optimal line

Conclusion

The result looks good. It seems clear that in most cases the dog chose a path that agreed remarkably closely with the optimal path.

Simplifying assumptions

- There was a definite line between shore and lake.
- When he entered the water, he started swimming. Actually, he ran a short distance in the water.
- The ball was stationary in the water. Actually, the waves, winds, and currents moved it a slight distance.
- The values of r and s are constant, independent of the distance run or swum.

Conclusion (cont'd)

- Given these complicating factors as well as the error in measurements, it is possible that the dog chose paths that were actually **better** than the calculated ideal path.
- Although he made good choices, **he does not know calculus**.
- Although he does not do the calculations, the dog's behavior is an example of the **uncanny way in which nature often finds optimal solutions.**

Conclusion (cont'd)

- This could be a consequence of **natural selection**, which gives a slight but consequential advantage to those animals that exhibit better judgment.