Algorithmic Operation Research

Homework 4
Instructor: Anna Karasoulou
Fall 2019

Exercise 1. Consider the problem

$$\begin{aligned} \min \ & 2x_1 + 3|x_2 - 10| \\ s.t. \ & |x_1 + 2| + |x_2| \leq 5 \end{aligned}$$

and reformulate it a linear programming problem.

Exercise 2. (Road lighting) Consider a road divided in n segments that is illuminated by m lamps. Let p_j be the power of the jth lamp. The illumination I_i of the ith segment is assumed to be $\sum_{j=1}^{m} a_{ij} p_j$ where a_{ij} are known coefficients. Let I_i^* be the desired illuminationof road i. We are interested in choosing the lamp powers p_j so that the illuminations I_i are close to the desired I_i^*. Provide a reasonable linear programming formulation of this problem.

Exercise 3. Consider a school district with I neighborhoods, J schools and G grades at each school. Each school j has a capacity C_{jg} for grade g. In each neighborhood i, the student population of grade i is S_{ig}. Finally the distance of school j from neighborhood i is d_{ij}. Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. (You may ignore the fact that numbers of students must be integer).

Exercise 4. Consider a set P described by linear inequality constraints

$$P = \{x \in \mathbb{R}^n : a_i' x \leq b_i, i = 1, \ldots, m\}$$

A ball with center y and radius r is defined as the set of all points within distance r from y. We are interested in finding a ball with the largest possible radius, which is entirely contained within the set P. Provide a linear programming formulation of this problem.