Algorithmic Operation Research

Homework 4

Nikolaos Galanis - sdi1700019
Pantelis Papageorgiou - sdi1700115
Maria-Despoina Siampou - sdi1600151

November 9, 2019

Problem 1

The problem

Consider the problem:

$$\min\ 2x_1 + 3|x_2 - 10| \quad \text{s.t. } |x_1 + 2| + |x_2| \leq 5$$

We want to reformulate it as a linear programming problem.

Problem 1

Solution

- Idea: Reformulate the cost function!

$$2x_1 + 3|x_2 - 10| = 2x_1 + 3 \cdot max\{x_2 - 10, -x_2 + 10\}$$

- Replace the max function with the variable z_1 :

$$z_1 \geq x_2 - 10 \ \& \ z_1 \geq -x_2 + 10$$

- The minimum possible value of z_1 is $|x_2 - 10|$.

Problem 1 cont'd

Solution

- Add variables z_2 and z_3 to the restrictions:

$$z_2 \geq x_1 + 2 \ \& \ z_2 \geq -x_1 - 2, \quad z_3 \geq x_2 \ \& \ z_3 \geq -x_2$$

- The original restriction becomes:

$$z_2 + z_3 \leq 5$$

Problem 2

The problem

We want to minimize the differences between the illumination I_i of each segment and the desired illumination of that segment, $I^*{}_i$.

Problem 2

Solution

Linear Program

$$\sum_{i=1}^{n} |I_i - I_i^*|,$$

where I_i is given by the summation in the problem statement, and our variables are then the p_j , subject to the condition that each $p_j \geq 0$.

Problem 3

Solution

- Let x_{ijg} be the amount of students from neighborhood i of grade g travelling to $school_j$. Then for an assignment of students to schools, the total distance travelled by all students is given as:

$$\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} d_{ij} x_{ijg}$$

- For a feasible assignment, every student of every neighborhood and grade must be assigned to a school, this gives the constraint:

$$\sum_{j \in J} x_{ijg} = S_{ig} \quad \forall i \in I, g \in G$$

Problem 3

Solution

- The number of students each school can take of the respective grades is bounded by C_{jg}, thus must hold:

$$\sum_{i \in I} x_{ijg} \leq C_{jg} \quad \forall j \in J, g \in G$$

- Finally there can be no negative numbers of assignments:

$$x \geq 0$$

Problem 3

Solution

Linear Program

This gives the following linear program:

- min
  - $\sum_{i \in I} \sum_{j \in J} \sum_{g \in G} d_{ij} x_{ijg}$
- s.t.
  - $\sum_{j \in J} x_{ijg} = S_{ig} \quad \forall i \in I, g \in G$
  - $\sum_{i \in I} x_{ijg} \leq C_{jg} \quad \forall j \in J, g \in G$
  - $x \geq 0$

Problem 4

Solution

We are trying to pick a point y that maximizes the shortest distance from y to the boundary of the set P.

The dot product of our potential point y with the normal vector to each hyperplane defining the boundary will help us calculate this distance.

Linear Problem

Our linear program is thus:

- minimize $max_i|a_i'y - b_i|$
- subject to $a_i'y \leq b_i \forall i$